# How to determine best diversity ordering method for a community data set? 

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## ECOLOGY


#### Abstract

Background: We conducted a study about how to determine best diversity ordering method for a community data set. Using 12 hypothetical and one ecological datasets, we tested the performances of 20 diversity ordering (divo) methods based on four criteria. Number of intersections (ints) amongst the diversity curves was taken the most important criterion into account. We defined the other criteria considering whether parametric values of a divo method contains SHD (species richness, Shannon entropy and Simpson index), potentially $u S H D$ (unbiased values of $S H D$ ), and potentially true species diversity, $t S H D$ (bias corrected values as effective number of $S H D$ ). All the criteria were collected into an equation called the relative selection value, $r$ Vd $d_{i}$


Results: According to the $r V d_{i}$ values of hypothetical community data sets, the best performances in seven community data sets were provided by $N_{a^{\text {t }}}$. This was followed by intrinsic diversity related methods with five community data sets. For ecological data set, the best results were obtained from the methods, $\left(i, M_{i}\right),\left(\log i, k_{i}\right)$ and $N_{d}$ with the $r V d_{i}$ values of 6.883, 6.881 and 6.859 , respectively.

Conclusion: Findings suggest that the characteristics of community data sets play important role in defining the best diversity ordering method. This tells us that diversity is certainly a multifaceted phenomenon for a single community but perhaps it is a single phenomenon for a group of communities.

Keywords: complexity, diversity measures, entropy, diversity profile, numbers equivalent, species-distribution abundance, community

## HIGHLIGHTS

The problem of diversity index choice is well-known in the literature.
The solution is to use of diversity ordering methods or one-parametric diversity index families.
Numerous diversity ordering methods have been used for drawing diversity profiles of ecological assemblages. Choosing the best diversity ordering method is another well-known problem.
Various features, especially number of intersections amongst the diversity curves, play essential role in the selection of the best diversity ordering method for a community data set.

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## INTRODUCTION

Biodiversity plays an essential role for balance, health, dynamism, stability, productivity, and sustainability of the ecosystems. It is therefore the central topic in many scientific disciplines such as conservation biology, environmental ecology, and biogeography (Pärtel et al., 2011; Özkan and Berger 2014). The importance of biodiversity was better understood especially after the Rio Declaration in 1992 and the Lisbon Conference in 1998 (Neumann and Starlinger 2001) and, the number of studies dealing with biodiversity has been steadily increasing since then.

As reported by Peet (1974), biodiversity, in essence, has always been defined by using the measures. An enormous number of diversity measures have been proposed in the literature. All these measures are broadly divided into three groups which are known as species richness indices, heterogeneity indices and species abundance models (Özkan, 2016). From those diversity measures, species richness (Peet, 1974), Shannon's entropy (Shannon, 1948), and Simpson's diversity index (Simpson, 1949) are the most popular metrics. Species richness solely presents the number of species whereas the letter two combine measures of richness together with abundance or incidence data (Abrams et al., 2021; Liu et al., 2007).

As stated by Patil (2014), while diversity increases according to one index, it may decrease according to the other index. This situation demonstrates the difficulties in quantifying biodiversity in a single number (Daly et al. 2018; Abrams et al., 2021). In other words, the question of which diversity index is best remains unanswered. To overcome this shortcoming, some researchers have preferred to define the representative diversity measure using principal component analysis (PCA) (Wilsey et al., 2005; Morris et al., 2014) or produce a component diversity equation derived from different diversity indices (Negiz and Özkan, 2019). Such approaches are reasonable but more or less information loss is inevitable in diversity estimations. To avoid from this information loss, the several other researchers have proposed to use parametric families of diversity indices.

Parametric families of diversity indices can also be called diversity profile techniques or diversity ordering methods. Those methods are divided into 4 groups as reported by Liu et al., (2007). They are intrinsic diversitybased methods, rank type index based method, expected number of species based methods and information based methods. The group of intrinsic diversity-based methods is basically composed of four method which are right tail-sum method (Patil and Taillie, 1979; 1982), logarithmic dominance plot (Tóthmérész, 1995), majorization method (Solomon, 1979) and $k$ - dominance plot (Shaw et al., 1983). Rank type index-based method includes only one method that called rank type index (Patil and Taillie, 1982). Hurlbert's family of diversity index (Hurlbert, 1971) and Hurlbert-Smith-Grassle index of order $w$ (Patil and Taillie, 1979) are found in the group of the expected number of species-based methods. The widest diversity profile group is composed of informationbased methods which includes Rényi's generalized entropy (Tóthmérész, 1995), Tsallis generalized entropy (Tsallis, 2002), Hill's diversity number of a (Hill, 1973), Daróczy's
entropy of type a (Daróczy, 1970), Numbers equivalent (Patil and Taillie, 1979, 1982) and diversity index of degree (Patil and Taillie, 1979, 1982). We can also join Landsberg Vedral entropy (Beck, 2009), information function (Bromiley et al., 2010), Kaniadakis entropy (Kaniadakis and Scarfone 2002; Sparavigna 2015), Arimoto entropy (Arimoto, 1971), Boekee and Lubbe's entropy (Boekee and Lubbe, 1980; Hooda and Sharma, 2008), Simpson entropy (Grabchak et al., 2017) and nonextensive Gaussion (Oikonomou and Tirnakli, 2009) in this group since they are the members of information theoretic measures.

It is obvious that more than one dozen methods have been used for drawing diversity profiles of ecological assemblages. In this case, a new question arises about which diversity ordering method is best. To response to this question, the studies conducted by Tóthmérész (1995) and Liu et al. (2007) are of great importance. According to their studies, the evaluation criteria in the selection of the best diversity ordering method is essentially based on number of intersections that occurrences throughout the diversity profiles in a community data set.

Number of intersections probably plays the most important role for selection of best method. However, it is not a unique evaluation criterion. For selection of best method, there are other characteristics to be questioned or considered. In this context, the questions given in the following items should be responded for a diversity ordering method to be selected or used.

1. Does the diversity ordering method to be used contain SHD (species richness ( $S$ ), Shannon entropy ( $H$ ) and/or Simpson index (D)) values?
2. Does the diversity ordering method to be used have unbiased forms of $S, H$ and/or $D(U S H D)$ ?
3. Does effective number of species (true diversity) corresponding to $S, H$ and $D_{1}(t S H D)$ can be obtained from the diversity ordering method considered to be used?

For a community data set, if number of intersections of a diversity ordering method has a greater value than those of the other diversity ordering methods, and for that method, all the answers to the questions of the items 1, 2 and 3 are "yes", then it is certainly the most suitable diversity ordering method for that community data set.

As can be understood in the light of the information given, for selection of best method, our approach is based on not only the number of intersections but also the other characteristics of diversity profiles. In the present study, for a community data set, we discuss how to select the best one among the diversity ordering methods considering all the characteristics mentioned above.

## MATERIAL AND METHODS

We use the terms "community", "assemblage" and "sampling plot" interchangeably in this article. In the present study, evaluation material consists of 12 hypothetical community data sets $\left(\mathrm{HS}_{1}-\mathrm{HS}_{12}\right)$ and one real ecological data set. Each of hypothetical data sets contains 5 communities and varied number of species (see supplementary files Table A1). Real ecological data set includes vegetation data taken from Yazili Canyon Nature Park, a small-scale district in the Mediterranean region of Turkey.

Table 1. Diversity ordering methods.

| Group1 | Intrinsic diversity-based methods | References |
| :---: | :---: | :---: |
| Right tail-sum method | $\left(i, T_{i}\right)=\left(i, \sum_{j=i+1}^{S} p_{[j]}\right), i=1,2, \ldots, S-1$ | Patil and Taillie (1979; 1982) |
| Logarithmic dominance plot | $\left(\log i, L_{i}\right)=\left(\log i, \sum_{j=i+1}^{S} p_{[j]}\right), i=1,2, \ldots, S-1$ | Tóthmérész (1995) |
| Majorization method | $\left(i, M_{i}\right)=\left(i, \sum_{j=1}^{i} p_{[j]}\right), i=1,2, \ldots, S$ | Solomon (1979) |


| Group 2 | Rank type index-based method |  |
| :---: | :---: | :---: |
| Rank type index | $\Delta_{\rho}^{(\text {rank })}=\sum_{i=1}^{S}\left(1-\sum_{j=1}^{i} p_{[j]}\right) \rho^{i-1}, \rho \geq 0$ | Patil and Taillie (1982) |
| Group 3 | Expected number of species-based methods |  |
| Hurlbert's family of diversity index | $s(m)=\sum_{i=1}^{S}\left[1-\left(1-p_{i}\right)^{m}\right], m \geq 1$ | Hurlbert (1971) |
| Hurlbert-Smith-Grassle index of order w | $\Delta_{\omega}^{\mathrm{HSG}}=\sum_{i=1}^{S}\left(1-p_{i}\left[1-\left(1-p_{i}\right)^{\omega}\right], \omega \geq 0\right.$ | Patil and Taillie (1979) |
| Group 4 | Information based methods |  |
| Rényi's generalized entropy | $H_{\alpha}=\left[\ln \left(\sum_{i=1}^{S} p_{i}^{\alpha}\right)\right] /(1-\alpha), \quad \alpha \geq 0, \alpha \neq 1$ | Tóthmérész (1995) |
| Tsallis generalized entropy | $H_{q}^{T}=\left(1-\sum_{i=1}^{S} p_{i}^{q}\right) /(q-1), q \geq 0, q \neq 1$ | Tsallis (2002) |
| Hill's diversity number of $a$ | $N_{\alpha}=\left(\sum_{i=1}^{S} p_{i}^{\alpha}\right)^{1 /(1-\alpha)}, \alpha \geq 0, \alpha \neq 1$ | Hill (1973) |
| Daróczy's entropy of type a | $H^{(\alpha)}=\left(\sum_{i=1}^{S} p_{i}^{\alpha}-1\right) /\left(2^{1-\alpha}-1\right), \alpha \geq 0, \alpha \neq 1$ | Daróczy (1970) |
| Numbers equivalent | $S_{\beta}=\left(\sum_{i=1}^{S} p_{i}^{\beta+1}\right)^{-1 / \beta}, \beta \geq-1, \beta \neq 0$ | Patil and Taillie (1979, 1982) |
| Diversity index of degree $\beta$ | $\Delta_{\beta}=\left(1-\sum_{i=1}^{S} p_{i}^{\beta+1}\right) / \beta, \beta \geq-1, \beta \neq 0$ | Patil and Taillie (1979, 1982) |
| Kaniadakis entropy | $S_{\kappa}=-\sum_{i=1}^{S}\left(\left[p_{i}^{1+\kappa}-p_{i}^{1-\kappa}\right] / 2 \kappa\right),-1>\kappa>1, \kappa \neq 0$ | Kaniadakis and Scarfone (2002), Sparavigna (2015) |
| Landsberg- Vedral entropy | $S_{q}^{L}=(1 / q-1)\left[\left(1 / \sum_{i=1}^{S} p_{i}^{q}\right)-1\right], q \neq 1$ | Beck (2009) |
| Information function | $I_{q}(P)=(1 / q-1)\left(\sum_{i=1}^{S}\left(p_{i}^{q} / q-1\right)\right), q \neq 1$ | Bromiley et al. (2010) |
| Arimoto entropy | $A_{\alpha}(P)=\left(2^{\alpha-1}-1\right)^{-1}\left[\left(\sum_{i=1}^{S} p_{i}^{\frac{1}{\alpha}}\right)^{\alpha}-1\right], \alpha>0, \alpha \neq 1$ | Arimoto (1971) |
| Boekee and Lubbe entropy | $H_{R}(P)=(R / R-1)\left[1-\left(\sum_{i=1}^{S} p_{i}^{R}\right)^{\frac{1}{R}}\right], R>0, R \neq 1$ | Boekee and Van der Lubbe (1980), Hooda and Sharma (2008) |
| Generalized Simpson entropy | $\zeta_{r}=\sum_{i=1}^{S} p_{i}\left(1-p_{i}\right)^{r}, r=1,2, \ldots$ | Grabchak et al. (2017) |
| Nonextensive Gaussian | $\begin{aligned} & S_{q}^{G}=\ln _{\mathrm{q}}^{\mathrm{T}}\left(\prod_{i=1}^{S}\left(1 / p_{i}\right)^{p_{i}}\right) \\ & \text { where } \ln _{\mathrm{q}}^{\mathrm{T}}(x):=\left(x^{1-q}-1\right) /(1-q), q \neq 1 \end{aligned}$ | Oikonomou and Tirnakli, (2009) |

This data set consists of 107 sampling plots and totally 180 species (see supplementary files Table A2). The nature park located in Mediterranean region of Turkey ( $37^{\circ} 27^{\prime} 22^{\prime \prime}$ $\mathrm{N}-37^{\circ} 29^{\prime} 37^{\prime \prime} \mathrm{N}$ and $30^{\circ} 54^{\prime} 16^{\prime \prime} \mathrm{E}-30^{\circ} 58^{\prime} 26^{\prime \prime}$ E) covers an area of approximately 600 hectares and has a karstic land structure between 100-400 m altitude. Brutian pine (Pinus brutia Ten.) and Oak species (Quercus sp.) are dominant species in the study area. The nature park is rich in endemic species with twenty-four endemic plant taxa (Özkan and Süel, 2008; Mert and Özkan, 2017). During the field survey in Yazili Canyon Nature Park, plant species composition of each plot was recorded using the Braun-Blanquet scores. Those scores were then transformed to relative cover ( r : $0.01 ;+: 0.02 ; 1: 0.04 ; 2: 0.15 ; 3: 0.375 ; 4: 0.625 ; 5: 0.875)$ prior to analysis (Fontaine et al., 2007). We used those transformed values throughout the present study.

We totally employed 20 diversity ordering (divo) methods. The names, the equations and the references belonging to the methods are given in Table 1 which has been rearranged by joining the methods, $S_{K^{\prime}} S_{q^{\prime}}^{L} I_{q}(P), A_{a}$ $(P), H_{R}(P), \zeta_{r}$ and $S_{q}^{G}$ from the table called "Methods for diversity ordering" given by Liu et al., (2007). In Table 1, S is the number of species in an assemblage, $p_{i}$ is the relative abundance of $i$-th species and $p_{[j]}$ refers to the relative abundance of the $j$-th most abundant species.

The scale values of $\left(i, T_{j}\right),\left(\log i, L_{i}\right),\left(i, M_{i}\right)$ and $\left(\log i, k_{i}\right)$ are standart which range from $1,0,1$ and 0 to $S-1, \log (S-1)$, $S$ and $\log S$, respectively. Those of the other divo methods were defined in wide ranges (Table 2). For $S_{K^{\prime}}$ we considered the recommended positive parameter value range ( $0>K>1$ ) (Kaniadakis and Scarfone, 2004). We employed the same parameter values for $s(m)$ and $\Delta_{\omega}^{H S G}$ between 1 to 30 , for $H_{d^{\prime}} H_{d^{\prime}}^{\top} N_{a}$ and $H^{(a)}$ ranging from $a, q \rightarrow 0$ to 5 , and for $I_{q}(P)$,
$S_{G}^{G}$ and $S_{q}^{L}$ from $q \rightarrow 0$ to 2. The beginning parameter values of $H_{R}(P)^{q}$ and $A_{a}(P)$ were defined by setting $R=0.5$ and $a=0.01$, respectively because the value of the $R$ parameter equal to almost 0 is most likely cause to obtain very large $H_{R}(P)$ values and, $A_{\alpha \rightarrow 0}(P)$ may have a greater value than $A_{a_{i>a \rightarrow 0}}(P)$ value which wouldn't satisfy the condition corresponding to ever-increasing curve with increasing its parameter value.

The beginning scale value of $S_{\beta}$ and $\Delta_{\beta}$ can take an integer value, $\beta=-1$. However, those of $H_{\alpha^{\prime}} H_{\alpha^{\prime}}^{\top} N_{a}$ and $H^{(a)}$ do not integer ( $\alpha, q \rightarrow 0$ ). Therefore, we used $\beta \rightarrow-1$ as the beginning scale values of $S_{\beta}$ and $\Delta_{\beta}$ corresponding to the those of $H_{d} H_{q^{\prime}}^{\top} N_{a}$ and $H^{(a)}(a, q \rightarrow 0)$. For $\zeta_{r^{\prime}}$ minimum parameter value is standard, equal to 1 (Grabchak et al., 2017). Therefore, we terminated the maximum of $\zeta_{r}$ at the value of 12 .

Relative value of $i$-th divo method $\left(R d_{i}\right)$ is defined as

$$
R d_{i}=100\left(\frac{\text { ints }_{i}}{\sum \text { ints }_{i}}\right)
$$

Which is for hypothetical data sets. For ecological (Yazili Canyon Nature Park) data set, 10000 iterations were run. Each iteration result consists of number of intersections computed using the divo methods from randomly selected 16 sampling plots amongst 107 sampling plots. After calculating total number of intersections for each of the methods, their relative values were computed by the following equation.

$$
\begin{equation*}
R d_{i}=100\left(\frac{\sum_{t=1}^{T} \text { ints }_{i}}{\sum_{t=1}^{T} \sum \text { ints }_{i}}\right), t=1,2, \ldots, T \tag{1}
\end{equation*}
$$

Where $R d_{i}$ also refers to ints\% of $i$-th divo method employed in evaluating the ecological data set.

We determined the weights in favor of number of intersections since it is the most important criterion. For

Table 2. The scale parameter values of the divo methods in Group 2, 3 and 4.

| Methods$\Delta_{\rho}^{(\text {rank })}$ | The scale parameter values |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\rho=0$ | 0 | 0.01 | 0.02 | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1 | 1.2 |
| $s(m), \Delta_{\omega}^{H S G}$ | $m, \omega=1$ | 2 | 3 | 5 | 10 | 12 | 15 | 18 | 22 | 25 | 30 |
| $H_{a^{\prime}} H_{q^{\prime}}^{\top} N_{a}$ and $H^{(a)}$ | $a, q \rightarrow 0$ | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | $\rightarrow 1$ | 2 | 3 | 4 | 5 |
| $S_{\beta} \Delta_{\beta}$ | $\beta \rightarrow-1$ | -0.9 | -0.8 | -0.6 | -0.4 | -0.2 | $\rightarrow 0$ | 1 | 2 | 3 | 4 |
| $S_{k}$ | $k \rightarrow 0$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | $\rightarrow 1$ |
| $I_{q}(P), S_{q}^{G}$ and $S_{q}^{L}$ | $q \rightarrow 0$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | $\rightarrow 1$ | 1.2 | 1.4 | 1.7 | 2 |
| $A_{a}(P)$ | $a=0.01$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | $\rightarrow 1$ | 1.2 | 1.4 | 1.7 | 2 |
| $H_{R}(P)$ | $R=0.5$ | $\rightarrow 1$ | 1.5 | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 |
| $\zeta$ | $r=1$ | 2 | 3 | 4 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

selecting the best method, the weights of the characteristics (i.e., number of intersections, $S H D, u S H D$ and $t S H D$ ) are 0.8 , $0.1,0.05$ and 0.05 , respectively. The following equations are used to find the best divo method for a community data set.

$$
\begin{equation*}
V d_{i}=0.8\left(\frac{R d_{i}}{\max \cdot R d_{i}}\right)+0.1(S H D)+0.05(b c S H D)+0.05(b c t S H D) \tag{2}
\end{equation*}
$$

$V d_{i}$ is the selection value of $i$-th divo method. Relative value of $V d_{i}$ is

$$
\begin{equation*}
r V d_{i}=100 \frac{V d_{i}}{\sum V d_{i}} \tag{3}
\end{equation*}
$$

From the divo methods, the best selection is obtained by

$$
\begin{equation*}
r V d_{i}^{*}=\max \left(r V d_{1}, r V d_{2}, \ldots, r V d_{i}\right), i=1,2, \ldots, 20 \tag{4}
\end{equation*}
$$

Where $T$ is total number of iterations $(t=1,2, \ldots T)$, ints; is number of intersections of $i$-th divo method at $t$-th iteration. SHD refers to species richness ( $S$ ), Shannon entropy $(H)$ and Simpson index (D). uSHD is bias corrected values of $S, H$ and $D$ and, $t S H D$ refers to true $S, H$ and $D$ values. $S H D$ value is $0.1 \times(1 / 3)$ if a divo computes solely any one of species richness $(S)$, Shannon entropy $(H)$ and Simpson index $(D)$. SHD value is $0.1 \times(2 \beta)$ if a divo computes any two of $S, H$ and $D$, It is 0.1 if all of $S, H$ and $D$ can
be obtained from a diversity ordering method and, 0 , otherwise. If bias corrected diversity measures (uSHD) can be obtained from a divo, the numerical weight value is 0.05 and if a divo method provides true species diversity ( $t S H D$ ), its contribution value is 0.05 .

None of $S, D$ and $H$ values are found in the profile of $\Delta_{\rho}^{\text {rank }}($ Patil and Taillie, 1982). $(i, T),(\log i, L),.(i, M),(\log i, k), s(m)$, $\Delta_{\omega}^{H S G}, S_{K} S_{d}^{L} A_{a}(P), H_{R}(P), \zeta_{r}$ and $S_{q}^{G}$ contain only one of $S, D$ and H values (Patil and Taillie,1979 and 1982; Tóthmérész, 1995; Solomon, 1979; Shaw et al.,1983; Hurlbert, 1971; Kaniadakis and Scarfone, 2002; Beck, 2009; Arimoto, 1971; Boekee and Van der Lubbe, 1980; Grabchak et al. 2017; Oikonomou and Tirnakli, 2009). $I_{a}(P)$ consists of $S$ and $H$ (Bromiley et al., 2010) and, $H_{\alpha} H_{\alpha}^{\top} \stackrel{q}{N_{\alpha}} H^{(a)}, S_{\beta}$ and $\Delta_{\beta}$ profiles include all of $S, D$ and $H$ values (SHD) (Liu et al., 2007). Additionally, $H_{q}^{\top}$ and $N_{a}$ contain unbiased $S, D$ and $H$ values (uSHD) (Marcon et al., 2014; Chao and Jost, 2015). In these two methods, effective number of species (true diversity) ( $t S H D$ ) can only be provided by $N_{a}$ (Chao and Jost, 2015) (Table 3).

All the values of diversity ordering methods and their number of intersections were computed by using spreadsheet software programs which were created by the authors of the present paper.

Table 3. The weights of the divo methods.

| Measures | SHD | USHD | $t S H D$ |
| :---: | :---: | :---: | :---: |
| $\Delta_{\rho}^{\text {(rank }}$ | 0 | 0 | 0 |
| $\left(i, T_{i}, 1\left(\log i, L_{i}\right),\left(i, M_{i}\right),\left(\log i, k_{i}\right), s(m), \Delta_{\omega}^{H S G}, S_{K^{\prime}} S_{d}^{L} A_{a}(P), H_{R}(P), \zeta, S_{q}^{G}\right.$ | 0.0333 | 0 | 0 |
| ${ }_{q}(P)$ | 0.0666 | 0 | 0 |
| $H_{d} H^{(a)}, S_{\beta^{\prime}} \Delta_{\beta}$ | 0.1 | 0 | 0 |
| $H_{q}^{\top}$ | 0.1 | 0.05 | 0 |
| $N_{a}$ | 0.1 | 0.05 | 0.05 |

## RESULTS

## Hypothetical community data sets

The results of the hypothetical community data sets (HS) are given in Table 4. For the hypothetical community set $1\left(\mathrm{HS}_{1}\right)$, Figure 1 illustrates that except for the $C_{5}$ curve based on $S_{q}^{L}$ method, all diversity curves show ever decreasing or increasing trend with increasing the values of scale parameters.

It is worth mentioning that maximum number of intersections (ints) obtained from a diversity ordering method is equal to $\left(n^{2}-n\right) / 2$ where $n$ is the number of communities in a community data set. It means that all the community pairs are non-separable. The number of intersections takes the maximum value equal to 10 for a data set consisting of 5 communities. From the hypothetical data sets, the maximum of number of
intersections was provided by the Group 1 methods from $\mathrm{HS}_{2}$ and $\mathrm{HS}_{11}$ (Table 4). For $\mathrm{HS}_{1}$, the greatest number of intersections was obtained by intrinsic diversity-based methods and generalized Simpson entropy. From those methods, the calculated number of intersections is 8 and, they have the same separable community pairs ( $C_{1}$ and $C_{4}$ and, $C_{2}$ and $\left.C_{3}\right) . s(m)$ and $\Delta_{\omega}^{H S G}$ are the second ranks. Both of the methods indicate separable four pairs of communities $\left(C_{1}\right.$ and $C_{4}$ and, $C_{2}$ and $C_{3}$ and, $C_{2}$ and $C_{5}$ and, $C_{3}$ and $C_{4}$ ). Rank type index and information theoretical measures, except for $S_{K}$ and $S_{q^{\prime}}^{G}$ have the same number of intersections with the same separable community pairs ( $C_{1}$ and $C_{4}$ and, $C_{2}$ and $C_{3}$ and, $C_{2}$ and $C_{5}$ and, $C_{3}$ and $C_{4}$ and, $C_{4}$ and $\mathrm{C}_{5}$ ). The number of non-separable community pairs $\left(C_{1}\right.$ and $C_{3}$ and, $C_{1}$ and $C_{5}$ and, $C_{2}$ and $C_{4}$ and, $C_{3}$ and $C_{5}$ ) of $S_{K}$ is equal to 4 and, all the communities are separable in accordance with the method, $S_{q}^{G}$ (Figure 1).

As can be seen in Table 4, number of intersections (ints) of the divo methods is varied. $\left(i, M_{i}\right)$ and $\left(\log i, k_{i}\right)$ have greatest total ints at the value of $83 .\left(i, T_{i}\right)$ and (log $i, L_{i}$ ) are the second-ranked methods which have a total ints value of 78. These methods are followed by $H_{\alpha^{\prime}} H_{q^{\prime}}^{\top} N_{\alpha^{\prime}} H^{(a)}$, $S_{\beta}$ and $\Delta_{\beta}$ with the same value of 68. The total ints values of the others (i.e., $s(m), \Delta_{\omega}^{H S G}, \Delta_{\rho}^{\text {rank) }}, S_{\mathcal{K}} S_{q^{\prime}}^{L} I_{q}(P), A_{q}(P), H_{R}(P)$ and $\zeta_{N}$ ), except for $S_{q^{\prime}}^{G}$ range between 41 and 63 . All the communities into the hypothetical data sets are separable
according to $S_{q}^{G}$ method. Its total number of intersections is therefore equal to 0 (Table 4).

Table 5 gives the $r V d_{i}$ and $r V d_{i}^{*}$ results of hypothetical community sets. According to $r V d_{i}^{*}$ values, $N_{a}$ is the method with highest performance at six community data sets $\left(\mathrm{HS}_{3^{\prime}} \mathrm{HS}_{6^{\prime}} \mathrm{HS}_{8^{\prime}} \mathrm{HS}_{9^{\prime}} \mathrm{HS}_{10}\right.$ and $\left.\mathrm{HS}_{12}\right)$. This is followed by $(i, T),\left(\log i, L_{i}\right),\left(i, M_{i}\right)$ and $\left(\log i, k_{i}\right)$, with 5 data sets $\left(\mathrm{HS}_{1^{\prime}}, \mathrm{HS}_{2^{\prime}} \mathrm{HS}_{4^{\prime}} \mathrm{HS}_{5}\right.$ and $\left.\mathrm{HS}_{11}\right)$. $A_{a}(P)$ gets the $V d_{i}^{*}$ twice and, $H_{R}(P)$ and $\zeta_{r}$ only once.


Figure 1. The graphical results of the divo methods for $\mathrm{HS}_{1}$.

Table 4. The number of intersections (ints) for each diversity ordering (divo) method of the hypothetical community sets.

| $\mathrm{Measures}^{2}$ | $\mathrm{HS}_{1}$ | $\mathrm{HS}_{2}$ | $\mathrm{HS}_{3}$ | $\mathrm{HS}_{4}$ | $\mathrm{HS}_{5}$ | $\mathrm{HS}_{6}$ | $\mathrm{HS}_{7}$ | $\mathrm{HS}_{8}$ | $\mathrm{HS}_{9}$ | $\mathrm{HS}_{10}$ | $\mathrm{HS}_{11}$ | $\mathrm{HS}_{12}$ | $\sum_{\text {ints }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(i, T_{p}\right)$ | 8 | 10 | 6 | 5 | 2 | 9 | 5 | 6 | 5 | 6 | 10 | 6 | 78 |
| $\left(\log i_{i} L_{i}\right)$ | 8 | 10 | 6 | 5 | 2 | 9 | 5 | 6 | 5 | 6 | 10 | 6 | 78 |
| $\left(i, M_{i}\right)$ | 8 | 10 | 7 | 5 | 2 | 9 | 5 | 6 | 7 | 6 | 10 | 8 | 83 |
| $\left(\log i_{i} k_{i}\right)$ | 8 | 10 | 7 | 5 | 2 | 9 | 5 | 6 | 7 | 6 | 10 | 8 | 83 |
| $\Delta_{\rho}^{\text {rank })}$ | 5 | 7 | 6 | 5 | 2 | 7 | 5 | 4 | 2 | 6 | 9 | 4 | 62 |
| $S_{(m)}(m)$ | 6 | 5 | 5 | 0 | 0 | 9 | 2 | 2 | 1 | 5 | 4 | 2 | 41 |
| $\Delta_{\omega}^{H S G}$ | 6 | 5 | 5 | 0 | 0 | 9 | 2 | 2 | 1 | 5 | 4 | 2 | 41 |
| $H_{a}$ | 5 | 7 | 6 | 3 | 1 | 8 | 4 | 6 | 7 | 6 | 7 | 8 | 68 |
| $H_{q}^{\top}$ | 5 | 7 | 6 | 3 | 1 | 8 | 4 | 6 | 7 | 6 | 7 | 8 | 68 |
| $N_{a}$ | 5 | 7 | 6 | 3 | 1 | 8 | 4 | 6 | 7 | 6 | 7 | 8 | 68 |
| $H^{(a)}$ | 5 | 7 | 6 | 3 | 1 | 8 | 4 | 6 | 7 | 6 | 7 | 8 | 68 |
| $S_{\beta}$ | 5 | 7 | 6 | 3 | 1 | 8 | 4 | 6 | 7 | 6 | 7 | 8 | 68 |
| $\Delta_{\beta}$ | 5 | 7 | 6 | 3 | 1 | 8 | 4 | 6 | 7 | 6 | 7 | 8 | 68 |
| $S_{k}$ | 4 | 5 | 5 | 3 | 1 | 6 | 2 | 3 | 7 | 5 | 4 | 6 | 51 |
| $S_{q}^{L}$ | 5 | 7 | 6 | 3 | 1 | 8 | 3 | 5 | 7 | 6 | 5 | 7 | 63 |
| $I_{q}(P)$ | 5 | 7 | 6 | 3 | 1 | 8 | 3 | 5 | 7 | 6 | 5 | 7 | 63 |
| $A_{a}(P)$ | 5 | 4 | 4 | 3 | 2 | 8 | 4 | 3 | 0 | 5 | 10 | 3 | 51 |
| $H_{R}(P)$ | 5 | 4 | 4 | 3 | 2 | 8 | 4 | 3 | 0 | 4 | 9 | 3 | 49 |
| $\zeta_{r}$ | 8 | 8 | 5 | 1 | 1 | 9 | 2 | 2 | 1 | 5 | 5 | 2 | 49 |
| $S_{a}^{G}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 5. $r V d_{i}$ and $r V d_{i}^{*}$ values of hypothetical community sets.

| Measures | $\mathrm{HS}_{1}$ | $\mathrm{HS}_{2}$ | $\mathrm{HS}_{3}$ | $\mathrm{HS}_{4}$ | $\mathrm{HS}_{5}$ | $\mathrm{HS}_{6}$ | $\mathrm{HS}_{7}$ | $\mathrm{HS}_{8}$ | $\mathrm{HS}_{9}$ | $\mathrm{HS}_{10}$ | $\mathrm{HS}_{11}$ | $\mathrm{HS}_{12}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left(i, T_{j}\right)$ | 6.77 | 6.98 | 5.30 | 7.82 | 7.70 | 5.52 | 6.63 | 6.37 | 5.16 | 5.38 | 6.84 | 5.10 |
| $\left(\log i, L_{i}\right)$ | 6.77 | 6.98 | 5.30 | 7.82 | 7.70 | 5.52 | 6.63 | 6.37 | 5.16 | 5.38 | 6.84 | 5.10 |
| $\left(i, M_{i}\right)$ | 6.77 | 6.98 | 6.15 | 7.82 | 7.70 | 5.52 | 6.63 | 6.37 | 7.10 | 5.38 | 6.84 | 6.71 |
| $\left(\log i, k_{i}\right)$ | 6.77 | 6.98 | 6.15 | 7.82 | 7.70 | 5.52 | 6.63 | 6.37 | 7.10 | 5.38 | 6.84 | 6.71 |
| $\Delta_{\rho}^{(r a n k)}$ | 4.06 | 4.69 | 5.06 | 7.51 | 7.40 | 4.13 | 6.36 | 4.08 | 1.95 | 5.17 | 5.91 | 3.22 |
| $s(m)$ | 5.14 | 3.63 | 4.46 | 0.31 | 0.31 | 5.52 | 2.81 | 2.29 | 1.26 | 4.52 | 2.90 | 1.88 |
| $\Delta_{\omega}^{H S G}$ | 5.14 | 3.63 | 4.46 | 0.31 | 0.31 | 5.52 | 2.81 | 2.29 | 1.26 | 4.52 | 2.90 | 1.88 |
| $H_{a}$ | 4.87 | 5.53 | 5.79 | 5.44 | 4.62 | 5.38 | 5.88 | 6.88 | 7.67 | 5.81 | 5.42 | 7.25 |
| $H_{q}^{\top}$ | 5.28 | 5.95 | 6.16 | 5.91 | 5.08 | 5.71 | 6.28 | 7.26 | 8.10 | 6.14 | 5.83 | 7.65 |
| $N_{a}$ | 5.68 | 6.37 | 6.53 | 6.38 | 5.55 | 6.04 | 6.68 | 7.64 | 8.52 | 6.46 | 6.24 | 8.05 |
| $H^{(a)}$ | 4.87 | 5.53 | 5.79 | 5.44 | 4.62 | 5.38 | 5.88 | 6.88 | 7.67 | 5.81 | 5.42 | 7.25 |
| $S_{\beta}$ | 4.87 | 5.53 | 5.79 | 5.44 | 4.62 | 5.38 | 5.88 | 6.88 | 7.67 | 5.81 | 5.42 | 7.25 |
| $\Delta_{\beta}$ | 4.87 | 5.53 | 5.79 | 5.44 | 4.62 | 5.38 | 5.88 | 6.88 | 7.67 | 5.81 | 5.42 | 7.25 |
| $S_{\kappa}$ | 3.52 | 3.63 | 4.46 | 4.82 | 4.01 | 3.76 | 2.81 | 3.31 | 7.10 | 4.52 | 2.90 | 5.10 |
| $S_{q}^{L}$ | 4.33 | 4.97 | 5.30 | 4.82 | 4.01 | 4.94 | 4.08 | 5.35 | 7.10 | 5.38 | 3.56 | 5.91 |
| $I_{q}(P)$ | 4.60 | 5.25 | 5.55 | 5.13 | 4.31 | 5.16 | 4.35 | 5.61 | 7.39 | 5.60 | 3.83 | 6.17 |
| $A_{a}(P)$ | 4.33 | 2.96 | 3.62 | 4.82 | 7.70 | 4.94 | 5.35 | 3.31 | 0.28 | 4.52 | 6.84 | 2.68 |
| $H_{R}(P)$ | 4.33 | 2.96 | 3.62 | 4.82 | 7.70 | 4.94 | 5.35 | 3.31 | 0.28 | 3.66 | 6.19 | 2.68 |
| $\zeta_{r}$ | 6.77 | 5.64 | 4.46 | 1.81 | 4.01 | 5.52 | 2.81 | 2.29 | 1.26 | 4.52 | 3.56 | 1.88 |
| $S_{q}^{G}$ | 0.27 | 0.28 | 0.25 | 0.31 | 0.31 | 0.22 | 0.27 | 0.25 | 0.28 | 0.22 | 0.27 | 0.27 |
| $r V d_{i}$ | 6.77 | 6.98 | 6.53 | 7.82 | 7.70 | 6.04 | 6.68 | 7.64 | 8.52 | 6.46 | 6.84 | 8.05 |

## Ecological data set

After 10.000 iterations, calculated relative values of the total number of intersections (ints\%) and $r V d_{i}$ of each divo method is given in Figure 2. As expected, the greatest ints\% values belong to intrinsic diversity related methods (i.e., $\left(i, M_{i}\right),\left(\log i, k_{i}\right),\left(i, T_{i}\right)$ and $\left.\left(\log i, L_{i}\right)\right)$. Majority of the member of information theoretical measures, $H_{d^{d}} H_{d^{\prime}}^{\top} N_{d^{\prime}}$ $H^{(a)}, S_{\beta}$ and $\Delta_{\beta^{\prime}}$ ranks second. Those are followed by $S_{d}^{L} I_{q}(P)$,
$A_{a}(P), H_{R}(P)$ and $\Delta_{\rho}^{\text {rank } k}$. The methods, $s(m), \Delta_{\omega}^{H S G}, S_{K^{\prime}} \zeta_{r}$ and in particular $S_{q}^{G}$ indicate insufficient performance.

The methods, $\left(i, M_{i}\right)$, (log $\left.i, k\right)$ and $N_{d^{\prime}}$ have very close $r V d_{i}$ values with $6.865\left(r V d_{i}^{*}\right), 6.862$ and 6.841 , respectively. This is followed by $\left(i, T_{)}\right)$and $(\log i, L)$ with the same value of 6.672. Majority of information theoretical measures, $H_{\alpha^{\prime}} H_{q^{\prime}}^{\top}$ $H^{(a)}, S_{\beta}$ and $\Delta_{\beta}$ also indicate good performances ranging the values between 6.017 and 6.429. The $V d_{i}$ values of the other methods are less than 0.5 (Figure 2).


Figure 2. ints\% and $r V d_{i}$ values of the ecological data set ( $\sum$ ints $\left.\%=\sum r V d_{i}=100\right)$.

## DISCUSSION

We tested 20 diversity ordering methods using different ( 12 hypothetical and one ecological) community data sets. Among the methods, $S_{q}^{L}$ (Beck, 2009) and $S_{q}^{G}$ (Oikonomou and Tirnakli, 2009) are unfavorable for drawing diversity profiles of sampling plots or communities since $S_{q}^{L}$ curves do not show always monotonically increasing trend with incresing the scale parameter values (Figure 1) and, almost all the community curves obtained from $S_{q}^{G}$ are separable (Figure 1 and 2).

Apart from $S_{q}^{L}$ and $S_{q^{\prime}}^{G}$ findings indicate that the methods that have the lowest number of intersections are $s(m), \Delta_{\omega}^{H S G}, S_{K}$ and $\zeta_{r}$. Moreover, $s(m)$ and $\Delta_{\omega}^{H S G}$ contains only species richness $\left(s(\infty)=S, \Delta_{\infty}^{H S G}=S\right.$-1) (Hurlbert, 1971; Patil and Taillie, 1979), $S_{K}$ only Shannon entropy ( $S_{K \rightarrow 0}=H=-$ $\sum_{i=1}^{S} p_{i} \ln p_{\mathrm{i}}$ where $p_{\mathrm{i}}$ is the relative abundance value of $i$-th species) (Kaniadakis and Scarfone, 2002) and $\zeta_{\text {r }}$ only Simpson index, $D=\sum_{i=1} p_{i}^{2^{s}}$, using the equation, $D^{\zeta}=1 /\left(1-\zeta^{1 / r}\right)$ where $D^{\prime}=1 /(1-\zeta)=D^{\prime=1}($ Grabchak et al. 2017) and, to the best of our knowledge, none of them have estimators proposed or improved for $u S H D$ or $t S H D$ values.

According to the ints values, the performances of the methods, $\Delta_{\rho}^{(r a n k)}, I_{q}(P), A_{q}(P)$ and $H_{R}(P)$ are better than $s(m), \Delta_{\omega}^{H S G}, S_{K}$ and $\zeta_{i}, \Delta_{\rho}^{(r a n k)}$ includes none of SHD (Patil and Taillie, 1982), $A_{a}(P), H_{R}(P)$ only $H\left(A_{a \rightarrow 1}(P)=H / \ln 2\right.$ and $\left.H_{R \rightarrow 1}(P)=H\right)$ (Arimoto, 1971; Boekee and Van der Lubbe,1980), and $I_{q}(P)$ both $S$ and $H\left(I_{q \rightarrow 0}(P)=S-1\right.$ and $\left.I_{q \rightarrow 1}(P)=H\right)$ (Bromiley
et al., 2010). uSHD or $t S H D$ cannot be defined by all these methods. Therefore, except for $I_{q}(P)$, their ints\% values lower than their rVd, values.

The greatest ints or ints\% values were provided by $\left(i, M_{i}\right)$ and $\left(\log i, k_{j}\right)$. Those are followed by $\left(i, T_{j}\right)$ and $\left(\log i, L_{j}\right)$. All of those methods are found in intrinsic diversity-related methods (Group 1). As stated by Liu et al., (2007), the methods, ( $i, M_{i}$ ) and (log $i, k_{j}$ ) are almost entirely attributed to each other because the $x$-coordinates are only log-transformed and, the $y$-coordinates are positively linearly transformed in the latter. $\left(i, T_{i}\right)$ and $\left(\log i, L_{j}\right)$ are equivalent. The only difference between them is that the x-coordinates are log transformed in the latter.

The curves of Group 1 methods comprise only species richness corresponding to $S-1, \log (S-1), S$ and $\log S$ for $\left(i, T_{j}\right),\left(\log i, L_{j}\right),\left(i, M_{i}\right)$ and (log $\left.i, k_{j}\right)$, respectively. Hence, their performance values ( $r V d_{j}$ ) are lower than their ints\% values. Moreover, intrinsic diversity-related methods directly rank dominance rather than diversity (Liu et al., 2007). Although we did not consider this characteristic in defining $r V d_{i}$ values, this is a disadvantage for application of Group 1 methods in creating diversity profiles of communities.

As expected, $H_{\alpha^{\prime}} H_{q^{\prime}}^{\top} N_{a}$ and $H^{(a)}, S_{\beta}$ and $\Delta_{\beta}$ have the same ints\% values since they are monotonic transformations of each other (Chao and Jost, 2015). $\Delta_{\beta}$ is identical to $H_{q}^{\top}$ when putting $q=\beta+1$ (Ricotta 2003 and 2005) and, $S_{\beta}=N_{\alpha+1}$ ( $\alpha=\beta$ ) in Hill's notation (Patil and Taillie, 1982).

The findings of all the information theoretical measures indicate that $r V d_{i}$ values are greater than ints\% values because they include all of $S\left(H_{\alpha=0}=N_{\alpha=0}=S_{\beta=-1}=S\right.$, $\left.H_{q=0}^{T}=H^{(\alpha=0)}=\Delta_{\beta=-1}=S-1\right), \quad H\left(H_{\alpha \rightarrow 1}=H_{q \rightarrow 1}^{T}=\Delta_{\beta \rightarrow 0}=H, \quad N_{\alpha \rightarrow 1}=S_{\beta \rightarrow 0}=e^{H}\right.$, $\left.H^{(\alpha-1)}=H / \ln 2\right)$ and $D\left(H_{\alpha=2}=-\ln D, H_{q=2}^{T}=\Delta_{\beta=1}=1-D, \quad N_{\alpha=2}=S_{\beta=1}=1 / D\right.$ $H^{(\alpha-2)}=2(1-D)$ ) values into their profiles (Liu et al., 2007). Among those methods, the better performances were obtained by $H_{q}^{\top}$ and $N_{a}$ because they also have biascorrected forms and, $N_{a}$ has a greater $r V d_{i}$ value than $H_{q}^{\top}$ since it gives the richness/diversity results as effective number of species based on unbiased $S, H$ and $D$ (Marcon et al., 2014; Chao and Jost, 2015).

## CONCLUSIONS

In our opinion, selection of a diversity profile for a community data set should be based on not only number of intersections but the other factors which were explained throughout the present study.

Based on the studies conducted by Tóthmérész (1995) and Liu et al. (2007), we assume that the larger the number of intersections between diversity profiles of communities, the lower the separability between them in terms of diversity. That provides the most accurate results to decide the separable communities.

Species richness, Shannon entropy and Simpson index are the most commonly used measures to estimate species diversity (Lewis et al., 1988; Ricotta et al., 2003; Pallmann et al., 2012; Di Battista et al., 2016; Grabchak et al. 2017; Daly et al., 2018; Abrams et al., 2021). Therefore, presences of the values belonging to those indices within the numerical value range of a diversity profile at the fixed scale values are of great importance as a selection criterion.

Numerous studies have been performed to propose the bias corrected forms of S (Chao and Lee, 1992; Gotelli and Chao, 2013; Chao and Chiu, 2016), H (Zahl, 1977; Chao et al., 1993; Grassberger, 2003; Özkan, 2020) and D (Lande, 1996, Chen et al., 2018; Nielsen et al., 2003; Droissart et al., 2012). On this context, the fact that a diversity ordering method has a potential to obtain unbiased SHD values gives a significant advantage for its selection.

Complete agreement was reached in an Ecology forum that using numbers equivalents or effective number instead of the classical diversity indices (entropies) such as Shannon entropy should be used in any diversity partitioning (Ellison, 2010). Unbiased forms of effective number have also been proposed by Gotelli and Chao (2013) and Chao and Jost (2015). This means a significant contribution to the relative selection value of a divo method that makes unbiased estimations of SHD as effective number. On this context, there is only one divo method that is called Hill numbers (Hill, 1973; Jost, 2006).

We defined the weights of the criteria mentioned above in the frame of literature, our scientific information and experiences. Scientific information and experiences are different from person to person. Therefore, number of criterion and their weights may be differently evaluated by different scientists. For selecting the best method, the better way is probably application of analytical hierarchical
process, AHP (Saaty, 1987). However, for AHP application, the weights of the factors or characteristics relative to each other should be determined by the experts studying on biodiversity computing. Further studies will probably improve the evaluation criteria in selection of the most suitable divo method for community datasets. Besides, it is worth noting that there are several the other parametric family of measures not used in the present study such as Abe entropy (Beck, 2009), Sharma-Mittal entropy (Akturk and Bagci, 2007), Havrda-Charvat entropy (Kumar and Choudhary, 2012), Deng entropy (Deng, 2016) and generalized Deng entropy (Gao et al., 2019; Liu et al., 2019) and so on. Those methods can also be evaluated whether they are favorable for a diversity ordering method or not.

The results obtained from the present study indicate that the characteristics of community data sets are important to define the best diversity ordering method. This allows us to establish the following sentence that diversity is certainly a multifaceted phenomenon for a single community, but probably a single phenomenon for a community data set. Hence, employment of the selection criteria such as ints, SHD, uSHD and tSHD is important for more accurate and objective assessments of diversities in ecological community data sets.

## AUTHORS' CONTRIBUTION

Project Idea: KÖ
Funding: KÖ, SG, AM, AŞ
Database: KÖ, SG, AM, AŞ
Processing: KÖ, SG, AM, AŞ
Analysis: KÖ, SG, AM, AŞ
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## REFERENCES

ABRAMS, J. F.; SOLLMANN; R., MITCHELL, S. L.; STRUEBIG, M. J.; WILTING, A. Occupancy-based diversity profiles: capturing biodiversity complexities while accounting for imperfect detection. Ecography, v.44, n.7, p. 975-986, 2021.
AKTURK, E.; BAGCI, G.B.; SEVER, R. Is Sharma-Mittal entropy really a step beyond Tsallis and Rényi entropies? arXiv arXiv:cond-mat/0703277, p. 1-9, 2007.

ARIMOTO, S. Information-theoretical considerations on estimation problems. Information and Control, v.19, n.3, p. 181-194, 1971.

BECK, C. Generalised information and entropy measures in physics. Contemporary Physics, v.50, n.4, p. 495-510, 2009.

BOEKEE, D.E.; VAN DER LUBBE, J.C. The R-norm information measure. Information and Control, v.45, n.2, p. 136-155, 1980.

BROMILEY, P.A.; THACKER N.A.; BOUHOVA-THACKER E. Shannon entropy, Rényi's entropy, and information. Statistics and Inf. Series, Imag. Sci. Biomed. Eng., Univ. Manchester, Manchester, U.K., Tina 2004-004, 2010.
CHAO, A.; CHIU, C. H. Species richness: estimation and comparison. Wiley StatsRef: Statistics Reference Online, p. 1-26, 2016.

CHAO, A.; JOST, L. Estimating diversity and entropy profiles via discovery rates of new species. Methods in Ecology and Evolution, v.6, n.8, p. 873-882, 2015.
CHAO, A.; LEE, S.M. Estimating the number of classes via sample coverage. Journal of the American Statistical Association, v.87, n.417, p. 210-217, 1992.

CHAO, A.; MA, M.C., YANG, M.C.K. Stopping rules and estimation for recapture debugging with unequal failure rates. Biometrika, v.80, n.1, p. 193-201, 1993.

CHEN, Y.; Wu, Y.; SHEN, T.J. Evaluation of the estimate bias magnitude of the Rao's quadratic diversity index. PeerJ, v.6, n. e5211, p. 1-13, 2018.

DALY, A. J.; BAETENS, J.M.; DE BAETS, B. Ecological diversity: measuring the unmeasurable. Mathematics, v.6, n.7, p. 119, 2018.

DAROCZY, Z. Generalized information functions. Information and Control, v.16, n.1, p. 36-51, 1970.

DENG, Y. Deng entropy. Chaos, Solitons \& Fractals, v. 91, p. 549-553, 2016.
DI BATTISTA, T.; FORTUNA, F.; MATURO, F. Environmental monitoring through functional biodiversity tools. Ecological Indicators, v.60, p. 237-247, 2016.

DROISSART, V.; HARDY, O.J.; SONKÉ, B.; DAHDOUH-GUEBAS, F.; STÉVART, T. Subsampling herbarium collections to assess geographic diversity gradients: a case study with endemic Orchidaceae and Rubiaceae in Cameroon. Biotropica, v.44, n.1, p. 44-52, 2012.

ELLISON, A.M. Partitioning diversity. Ecology, v.91, n.7, p. 1962-1963, 2010.
FONTAINE, M.; AERTS, R.; OZKAN, K.; MERT, A.; GULSOY, S.; SUEL, H.; WAELKENS, M.; MUYS, B. Elevation and exposition rather than soil types determine communities and site suitability in Mediterranean mountain forests of southern Anatolia, Turkey. Forest Ecology and Management, v.247, n.1-3, p. 18-25, 2007.

GAO, X.; LIU, F.; PAN, L.; DENG, Y.; TSAI, S.B. Uncertainty measure based on Tsallis entropy in evidence theory. International Journal of Intelligent Systems, v.34, n.11, p. 3105-3120, 2019.

GOTELLI, N.J.; CHAO, A. Measuring and estimating species richness, species diversity, and biotic similarity from sampling data. In: Levin S.A. (ed.) Encyclopedia of Biodiversity, second edition, Volume 5, Waltham, MA: Academic Press, 2013, p. 195- 211.

GRABCHAK, M.; MARCON, E.; LANG, G.; ZHANG, Z. The generalized Simpson's entropy is a measure of biodiversity. PloS ONE, v.12, n.3, e0173305, 2017.

GRASSBERGER, P. Entropy estimates from insufficient samplings. arXiv arXiv:physics/0307138, p.1-5, 2003.

HILL, M.O. Diversity and evenness: a unifying notation and its consequences. Ecology, v.54, n.2, p. 427-432, 1973.
HOODA, D.S.; SHARMA, D.K. Generalized R-norm information measures. Journal of Appl. Math, Statistics \& Informatics (JAMSI), v.4, n.2, p. 153-168, 2008.

HURLBERT, S.H. The nonconcept of species diversity: a critique and alternative parameters. Ecology, v.52, n.4, p. 577-586, 1971.

JOST, L. Entropy and diversity. Oikos, v.113, n.2, p. 363-375, 2006.
KANIADAKIS, G., SCARFONE, A.M. A new one-parameter deformation of the exponential function. Physica A: Statistical Mechanics and Its Applications, v.305, n.1-2, p. 69-75, 2002.

KANIADAKIS, G.; SCARFONE, A.M. Lesche stability of k-entropy. Physica A v.340, p. 102-109, 2004.

KUMAR, S.; CHOUDHARY, A. Some coding theorem on generalized Havrda-Charvat and Tsallis's entropy. Tamkang Journal of Mathematics, v.43, n.3, p. 437-444, 2012.

LANDE, R. Statistics and partitioning of species diversity, and similarity among multiple communities. Oikos, v.76, p. 5-13, 1996.

LEWIS, C.E.; SWINDEL, B.F.; TANNER, G.W. Species diversity and diversity profiles: concept, measurement, and application to timber and range management. Rangeland Ecology \& Management/Journal of Range Management Archives, v.41, n.6, p. 466-469, 1988.

LIU, F.; GAO, X.; ZHAO, J.; DENG, Y. Generalized belief entropy and its application in identifying conflict evidence. IEEE Access, v.7, p. 126625-126633, 2019.

LIU, C.; WHITTAKER, R. J.; MA, K.; MALCOLM, J.R. Unifying and distinguishing diversity ordering methods for comparing communities. Population Ecology, v.49, n.2, p. 89-100, 2007.

MARCON, E.; SCOTTI, I.; HÉRAULT, B.; ROSSI, V.; LANG, G. Generalization of the partitioning of Shannon diversity. PloS ONE, v.9, n. 3 e90289, 2014.

MERT, A.; ÖZKAN, K. Studies on taxonomic diversity of plant communities and modeling its potential distribution in Yazili Canyon Nature Park, Turkey. Journal of Environmental Biology, v.38, n.6, p. 1267-1274, 2017.
MORRIS, E. K.; CARUSO, T.; BUSCOT, F.; FISCHER, M.; HANCOCK, C.; MAIER, T.S.; MEINERS, T.; MULLER, C.; OBERMAIER, E.; PRATI, D.; SOCHER, S.A.; SONNEMANN, I.; WASCHKE, N.; WUBET, S.; RILLIG, M.C. Choosing and using diversity indices: insights for ecological applications from the German Biodiversity Exploratories. Ecology and Evolution, v.4, n.18, p. 3514-3524, 2014.
NEGIZ, M.G.; ÖZKAN, K. Reproducing a component diversity index for regional biodiversity assessments. Fresenius Environmental Bulletin, v.28, p. 9746-9752, 2019.

NEUMANN, M.; STARLINGER, F. The significance of different indices for stand structure and diversity in forests. Forest ecology and Management, v.145, n.1-2, p.91-106, 2001.

NIELSEN, R.; TARPY, D.R.; Reeve, H. K. Estimating effective paternity number in social insects and the effective number of alleles in a population. Molecular Ecology, v.12, n.11, p. 3157-3164, 2003.

OIKONOMOU, T.; TIRNAKLI, U. Generalized entropic structures and non-generality of Jaynes' Formalism. Chaos, Solitons \& Fractals, v.42, n.5, p. 3027-3034, 2009.

ÖZKAN, K.; BERGER, U. Predicting the potential distribution of plant diversity in the Yukarıgökdere forest district of the Mediterranean region. Polish Journal of Ecology, v.62, n.3, p. 441-454, 2014.

ÖZKAN, K. Biyolojik çeşitlilik bileşenleri ( $\alpha, \beta$ ve $\gamma$ ) nasıl ölçülür. Suleyman Demirel University, Faculty of Forestry, Press No:98, 2016, 142p.
ÖZKAN, K. A New Proposed Estimator for Reducing Bias Due to Undetected Species. Gazi University Journal of Science, v.33, n.1, p. 229-236, 2020.

ÖZKAN, K.; SUEL, H. Endemic plant species in a karstic canyon [Mediterranean Region, Turkey]: relation to relief and vegetation diversity. Polish Journal of Ecology, v.4, n.56, p. 709-715, 2008.

PALLMANN, P.; SCHAARSCHMIDT, F.; HOTHORN, L. A.; FISCHER, C.; NACKE, H.; PRIESNITZ, K.U.; SCHORK, N.J. Assessing group differences in biodiversity by simultaneously testing a user-defined selection of diversity indices. Molecular Ecology Resources, v.12, n.6, p. 1068-1078, 2012.

PATIL, G.P. Diversity Profiles. In Wiley StatsRef: Statistics Reference Online (eds N. Balakrishnan, T. Colton, B. Everitt, W. Piegorsch, F. Ruggeri and J. L. Teugels), 2014.

PATIL G.P.; TAILLIE, C. An overview of diversity. In: Grassle JF, Patil GP, Smith W, Taillie C (eds) Ecological diversity in theory and practice. International Cooperative Publ House, Fairland, 1979, p. 3-27.

PATIL, G.P.; TAILLIE, C. Diversity as a concept and its measurement. Journal of the American Statistical Association, v. 77, n.379, p. 548-561, 1982.
PÄRTEL, M.; SZAVA-KOVATS, R.; ZOBEL, M. Dark diversity: shedding light on absent species. Trends In Ecology \& Evolution, v.26, n.3, p. 124-128, 2011.

PEET, R.K. The measurement of species diversity. Annual Review of Ecology and Systematics, v.5, n.1, p. 285-307, 1974.

RICOTTA, C. Additive partition of parametric information and its associated $\beta$-diversity measure. Acta Biotheoretica, v.51, n.2, p. 91-100, 2003.
RICOTTA, C. A note on functional diversity measures. Basic and Applied Ecology, v.6, n.5, p. 479-486, 2005.

RICOTTA, C.; AVENA, G.; CHIARUCCI, A. An index of divergence from species to life-form diversity based on the notion of intrinsic diversity ordering. Plant Ecology, v.165, n.2, p. 217-222, 2003.

SAATY, R.W. The analytic hierarchy process-what it is and how it is used. Mathematical Modelling, v.9, n.3-5, p. 161-176, 1987.

SPARAVIGNA, A.C. On the generalized additivity of Kaniadakis entropy. International Journal of Sciences, v.4, n.02, p. 44-48, 2015.
SHANNON, C.E. A mathematical theory of communication. The Bell System Technical Journal, v.27, n.3, p. 379-423, 1948.

SHAW, K.M.; LAMBSHEAD, P.J.D.; PLATT, H.M. Detection of pollutioninduced disturbance in marine benthic assemblages with special reference to nematodes. Marine Ecology Progress Series. Oldendorf, v.11, n.2, p. 195-202, 1983.

SIMPSON, E.H. Measurement of diversity. Nature, v.163, n.4148, p. 688-688, 1949.
SOLOMON D.L. A comparative approach to species diversity. In: Grassle JF, Patil GP, Smith W and Taillie C (eds) Ecological diversity in theory and practice. International Cooperative Publ House, Fairland, 1979, p. 29-35.
TÓTHMÉRÉSZ, B. Comparison of different methods for diversity ordering. Journal of Vegetation Science, v.6, n.2, p. 283-290, 1995.

TSALLIS, C. Entropic nonextensivity: a possible measure of complexity. Chaos, Solitons \& Fractals, v.13, n.3, p. 371-391, 2002.

WILSEY, B.J.; CHALCRAFT, D.R.; BOWLES, C.M.; WILLIG, M.R. Relationships among indices suggest that richness is an incomplete surrogate for grassland biodiversity. Ecology, v.86, n.5, p. 1178-1184, 2005.

ZAHL, S. Jackknifing an index of diversity. Ecology, v.58, n.4, p. 907-913, 1977.

## SUPPLEMENTARY FILES

Table A1. Hypothetical community data sets.


#### Abstract

$\mathbf{H S}_{1}\left[\mathbf{C}_{1}: 462,428,271,10,9,9,8,8,8,8,7,7,7,6,6,6,4,4,3,3,3,2,2,1,1,1,1, \mathbf{C}_{2}: 336,196,68,10,10,9,9,9,8,7,7,6,5,5,5,4,3,3,3,3\right.$, $3,3,2,2,2,1,1,1,1,1,1, \mathbf{C}_{3}: 453,347,49,9,8,8,8,7,6,6,6,6,6,5,5,5,4,4,3,3,3,3,3,2,2,2,1,1,1,1, \mathbf{C}_{4}: 349,335,220,10,10,9,9,7$, $\left.7,6,6,5,5,5,5,5,4,4,4,4,3,3,3,3,2,2,1,1,1,1,1,1, \mathbf{C}_{5}: 396,127,10,9,9,9,8,8,7,7,6,6,6,6,6,5,4,4,4,3,3,2,2,2,1,1,1\right], \mathbf{H S}_{2}\left[\mathbf{C}_{1}\right.$ $489,280,275,10,9,9,8,8,8,8,8,7,6,4,4,4,4,3,3,2,2,2,2,1,1,1,1,1,1, C_{2}: 325,275,71,10,10,9,8,8,7,7,6,6,6,6,5,5,4,4,4,3$, $3,3,3,2,2,1,1,1,1,1, C_{3}: 479,61,30,10,10,9,9,8,8,7,6,6,6,5,4,4,3,3,2,2,2,2,2,2,1,1,1,1,1,1,1,1,1, C_{4}: 396,127,10,9,9,9,8,8$, $\left.7,7,6,6,6,6,6,5,4,4,4,3,3,2,2,2,1,1,1, \mathbf{C}_{5}: 274,272,64,10,10,10,10,9,9,8,7,6,6,6,5,5,5,4,4,3,2,2,1,1,1,1,1,1,1,1,\right], \mathbf{H S}\left[\mathbf{C}_{1}\right.$ $424,387,374,10,10,10,10,9,9,9,9,8,8,8,8,6,6,5,4,4,4,3,3,2,1,1,1, C_{2}: 469,248,24,10,9,9,8,8,8,7,7,7,7,6,5,5,5,4,3,3,3$, $3,2,2,2,1,1,1,1,1,1, C_{3}: 468,435,419,406,376,373,367,360,356,348,339,308,277,253,246,235,225,213,195,192,189,181,166$, $136,129,120,104,93,77,54,50,19, C_{4}: 497,260,242,10,8,8,7,6,6,5,5,4,4,4,4,3,3,3,3,2,2,1,1,1,1,1,1,1,1,1,1,1, C_{5}: 496,61,30$, $9,9,8,8,8,7,7,7,7,6,6,6,5,3,3,2,2,2,2,2,1,1,1,1,1,1,1,1,], \mathbf{H S}_{4}\left[\mathbf{C}_{1}: 499,349,302,10,10,9,8,8,8,7,7,6,6,6,6,5,5,4,4,4,4,4\right.$, $3,3,2,2,1,1,1,1,1,1,1, \mathbf{C}_{2}: 424,387,374,10,10,10,10,9,9,9,9,8,8,8,8,6,6,5,4,4,4,3,3,2,1,1,1, C_{3}: 307,113,113,113,113,113,13$, $13,13,13,13,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,2,2,2, C_{4}: 473,374,288,10,10,10,9,8,8,7,6,6,5,5,5,4,4,3,3,3,1,1,1$, $\left.1, \mathbf{C}_{5}: 274,123,2,2,2,2,2,2,2,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,\right], \mathbf{H S}_{5}\left[\mathbf{C}_{1}: 151,134,22,10,8,8,8,8,7,7,6,6,6,5,5,5,4\right.$, $4,4,3,2,2,2,2,1,1,1,1,1,1,1,1,1, \mathbf{C}_{2}: 375,372,83,10,10,10,9,8,8,8,7,7,7,6,6,6,5,5,4,2,2,2,2,1,1,1,1,1,1, C_{3}: 386,367,161,10$, $10,10,10,9,8,8,7,7,7,6,6,6,6,5,5,4,3,3,3,3,3,3,1,1,1,1,1,1, C_{4}: 231,128,43,9,8,8,8,8,8,7,7,7,7,6,6,6,5,5,5,5,5,4,4,2,1$, $\left.1,1,1,1, \mathbf{C}_{5}: 122,45,14,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,6,6,6,6,6,6,6,6,1,1,\right], \operatorname{HS}_{6}\left[\mathbf{C}_{1}: 396,127,10,9,9,9,8,8,7,7\right.$, $6,6,6,6,6,5,4,4,4,3,3,2,2,2,1,1,1, C_{2}: 275,87,10,10,10,9,9,9,7,7,7,6,6,6,6,6,5,5,4,3,3,3,3,2,1,1,1,1, C_{3}: 463,388,370,10$, $10,10,9,8,8,8,6,6,5,5,5,4,3,2,2,2,2,2,1,1,1,1,1,1,1, C_{4}: 466,220,168,10,10,10,10,8,8,8,6,5,5,5,3,3,3,2,2,2,2,1,1,1,1,1, C_{5}$ $423,322,248,10,9,8,7,7,7,6,6,5,5,5,4,4,3,2,2,2,2,1,1,1,1,1,1,1,1,1,1,], \operatorname{HS}_{7}\left[\mathbf{C}_{1}: 232,165,12,10,10,9,9,9,8,8,7,7,7,7,7,6,5\right.$, $4,4,3,3,3,3,2,2,2,1,1,1,1,1,1,1,1, \mathbf{C}_{2}: 470,337,94,10,9,9,9,9,9,8,8,7,7,7,5,4,4,4,4,4,3,3,1,1,1,1,1,1,1,1,1,1, \mathbf{C}_{3}: 491,487$, $481,479,474,460,447,418,389,362,344,340,313,313,309,237,207,203,199,195,181,174,154,148,146,125,108,94,82,80,68,66$, $58,38,22,5, C_{4}: 374,315,88,10,10,10,10,9,9,8,7,7,7,6,6,6,6,6,5,5,4,4,2,2,1,1,1, C_{5}: 463,388,370,10,10,10,9,8,8,8,6,6,5,5$, $5,4,3,2,2,2,2,2,1,1,1,1,1,1,1,], \mathbf{H S}_{8}\left[\mathbf{C}_{1}: 499,455,454,436,412,412,372,295,244,212,184,145,145,68,64,55,53,28, \mathbf{C}_{2}: 491,487\right.$ $481,479,474,460,447,418,389,362,344,340,313,313,309,237,207,203,199,195,181,174,154,148,146,125,108,94,82,80,68,66$, $58,38,22,5, C_{3}: 386,367,161,10,10,10,10,9,8,8,7,7,7,6,6,6,6,5,5,4,3,3,3,3,3,3,1,1,1,1,1,1, C_{4}: 463,388,370,10,10,10,9,8,8$, $\left.8,6,6,5,5,5,4,3,2,2,2,2,2,1,1,1,1,1,1,1, \mathbf{C}_{5}: 325,275,71,10,10,9,8,8,7,7,6,6,6,6,5,5,4,4,4,3,3,3,3,2,2,1,1,1,1,1,\right], \mathbf{H S}\left[\mathbf{C}_{1}\right.$ $487,482,467,439,418,367,347,339,315,287,262,254,252,184,165,145,142,134,10, C_{2}: 499,455,454,436,412,412,372,295,244$, $212,184,145,145,68,64,55,53,28, \mathbf{C}_{3}: 285,280,191,10,10,9,9,9,8,8,7,6,6,6,4,4,3,3,3,3,2,2,1,1,1,1,1,1,1, C_{4}: 496,61,30,9,9$, $8,8,8,7,7,7,7,6,6,6,5,3,3,2,2,2,2,2,1,1,1,1,1,1,1,1, C_{5}: 17,17,17,13,13,13,13,13,13,13,13,13,13,13,13,13,12,12,12,12,12,12$, $12,12,12,12,12,12,], \operatorname{HS}_{10}\left[\mathbf{C}_{1}: 374,315,88,10,10,10,10,9,9,8,7,7,7,6,6,6,6,6,5,5,4,4,2,2,1,1,1, \mathbf{C}_{2}: 375,372,83,10,10,10,9,8,8\right.$, $8,7,7,7,6,6,6,5,5,4,2,2,2,2,1,1,1,1,1,1, C_{3}: 307,113,113,113,113,113,13,13,13,13,13,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7,7$, $2,2,2, C_{4}: 396,127,10,9,9,9,8,8,7,7,6,6,6,6,6,5,4,4,4,3,3,2,2,2,1,1,1, C_{5}: 479,61,30,10,10,9,9,8,8,7,6,6,6,5,4,4,3,3,2$, $2,2,2,2,2,1,1,1,1,1,1,1,1,1,], \mathbf{H S}_{11}\left[\mathbf{C}_{1}: 217,163,14,10,9,9,9,8,8,7,6,6,6,5,5,5,5,4,4,3,2,2,2,1,1,1,1,1,1, \mathbf{C}_{2}: 375,372,83,10,10\right.$, $10,9,8,8,8,7,7,7,6,6,6,5,5,4,2,2,2,2,1,1,1,1,1,1, C_{3}: 499,349,302,10,10,9,8,8,8,7,7,6,6,6,6,5,5,4,4,4,4,4,3,3,2,2,1,1$, $1,1,1,1,1, C_{4}: 231,128,43,9,8,8,8,8,8,7,7,7,7,6,6,6,5,5,5,5,5,4,4,2,1,1,1,1,1, C_{5}: 424,387,374,10,10,10,10,9,9,9,9,8,8,8$, $8,6,6,5,4,4,4,3,3,2,1,1,1], \operatorname{HS}_{12}\left[\mathbf{C}_{1}: 423,322,248,10,9,8,7,7,7,6,6,5,5,5,4,4,3,2,2,2,2,1,1,1,1,1,1,1,1,1,1, \mathbf{C}_{2}: 459,412,298\right.$, $10,10,9,9,8,8,8,6,6,4,4,4,4,3,2,2,2,2,2,1,1,1,1,1,1, C_{3}: 489,280,275,10,9,9,8,8,8,8,8,7,6,4,4,4,4,3,3,2,2,2,2,1,1,1,1$, $1,1, \mathbf{C}_{4}: 424,387,374,10,10,10,10,9,9,9,9,8,8,8,8,6,6,5,4,4,4,3,3,2,1,1,1, \mathbf{C}_{5}: 17,17,17,13,13,13,13,13,13,13,13,13,13,13,13$, $13,12,12,12,12,12,12,12,12,12,12,12,12$, ]


Table A2. Ecological data set.


#### Abstract

$\mathbf{C}_{1}: 0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01,0.01,0.01,0.01, C_{2}: 0.375,0.15,0.15,0.15,0.04,0.04,0.04,0.04$, $0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.01, \mathbf{C}_{3}: 0.625,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01,0.01, C_{4}: 0.625,0.15,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{5}: 0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{6}: 0.625,0.15,0.04,0.04,0.04,0.04,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.01,0.01,0.01,0.01,0.01,0.01, \boldsymbol{C}_{7}: 0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.01, C_{8}: 0.375,0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, \mathbf{C}_{9}: 0.625,0.15,0.15,0.04,0.04,0.04,0.02,0.02,0.02,0.02, \mathbf{C}_{10}: 0.625,0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, C_{11}: 0.625,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01, \mathbf{C}_{12}: 0.625,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01$, $\mathbf{C}_{13}: 0.625,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \boldsymbol{C}_{14}: 0.375$, $0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02, \mathbf{C}_{15}: 0.625,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, C_{16}: 0.375,0.15$, $0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01$, $0.01, \mathbf{C}_{17}: 0.15,0.15,0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{18}: 0.15,0.15$, $0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.01,0.01,0.01,0.01,0.01,0.01,0.01,0.01,0.01, \mathbf{C}_{19}: 0.375,0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01$, $0.01,0.01, \mathbf{C}_{20}: 0.375,0.15,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.01, \mathbf{C}_{21}: 0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02, C_{22}:$ $0.375,0.15,0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, \mathbf{C}_{23}: 0.375,0.15$, $0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02, C_{24} ; 0.625,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $\mathbf{C}_{25}: 0.375,0.15,0.15,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, \mathbf{C}_{26}: 0.375,0.15,0.15,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01, C_{27} ; 0.375,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01, C_{28}: 0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{29}: 0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04$, $0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, C_{30}: 0.625,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01,0.01,0.01,0.01, C_{31}: 0.375,0.15,0.04,0.04,0.04,0.04,0.04$, $0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01,0.01,0.01,0.01,0.01,0.01, C_{32}: 0.15,0.15,0.04,0.04,0.04$, $0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{33}: 0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{34}: 0.375,0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $C_{35}: 0.625,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{36}: 0.625,0.15,0.04,0.04,0.04,0.04,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{37} ; 0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{38}: 0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{39}: 0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02, \mathbf{C}_{40}: 0.375,0.15,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \boldsymbol{C}_{41}: 0.625$, $0.15,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{42}: 0.15,0.15,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02, C_{43} ; 0.15,0.15,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{44}: 0.15,0.15,0.04$, $0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, C_{45}: 0.375,0.15,0.15,0.04,0.04,0.04,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{46}: 0.375,0.375,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02, C_{47} ; 0.625,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{48}: 0.375,0.15,0.04,0.04,0.04,0.04$, $0.04,0.04,0.04,0.02,0.02,0.02,0.02, \mathbf{C}_{49}: 0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01, \mathbf{C}_{50}$ : $0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01, C_{51}: 0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.01,0.01, \mathbf{C}_{52}: 0.625,0.15,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.01,0.01,0.01,0.01,0.01,0.01, \mathbf{C}_{53}: 0.375,0.15,0.15,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01, C_{54}: 0.375,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02, C_{55}: 0.15,0.15,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{56}: 0.15,0.15,0.04,0.04$, $0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02, C_{57} ; 0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02$, $0.02,0.02,0.02, C_{58}: 0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{59}: 0.375,0.15,0.15,0.15,0.15,0.15,0.04$, $0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{60}: 0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02, \mathbf{C}_{61}: 0.15,0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{62}: 0.15,0.15,0.04$, $0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{63}: 0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04$, $0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{64} ; 0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02$, $0.02,0.02,0.02, \mathbf{C}_{65}: 0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{66}: 0.375,0.04,0.04,0.04,0.04,0.04,0.04$, $0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{67}, 0.375,0.375,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02, C_{68}: 0.375,0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{69}: 0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{70}: 0.375,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, 0.02,


Table A2.(continuation) Ecological data set.
$\boldsymbol{C}_{71}: 0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \boldsymbol{C}_{72}: 0.625,0.15,0.04,0.04,0.02,0.02$, $0.02,0.02,0.02, C_{73}: 0.375,0.15,0.15,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{74}: 0.625,0.15,0.04,0.04,0.04,0.04,0.04,0.04$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{75}: 0.375,0.375,0.04,0.04,0.04,0.04$ $0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02, C_{76}: 0.15,0.15,0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{77}: 0.375,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01,0.01, \mathbf{C}_{78}: 0.375,0.04,0.04,0.04,0.02,0.02,0.02$, $0.02,0.02,0.02,0.01,0.01,0.01,0.01, \mathbf{C}_{79}: 0.625,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{80}: 0.375,0.15,0.04,0.04$, $0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{81}: 0.375,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{82}$ $0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{83} ; 0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04$, $0.02,0.02, \mathbf{C}_{84}: 0.625,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02, \mathbf{C}_{85}: 0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{86}: 0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{87}: 0.375,0.15,0.15$, $0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{88}: 0.375,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.02,0.02,0.02,0.02, \mathbf{C}_{89}: 0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{90}$ $0.625,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02, \mathbf{C}_{91}: 0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02, \mathbf{C}_{92}$ $0.375,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02, C_{93}: 0.375,0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04$, $0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02, \mathbf{C}_{94}: 0.375,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02, \mathbf{C}_{95}: 0.15,0.15$, $0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, C_{96}: 0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02$ $0.02,0.02, \mathbf{C}_{97}: 0.15,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01, \mathbf{C}_{98}: 0.15,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02$, $0.01,0.01,0.01,0.01,0.01,0.01,0.01, \mathbf{C}_{99}: 0.375,0.15,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.01,0.01,0.01,0.01,0.01,0.01,0.01,0.01, \mathbf{C}_{100}: 0.15,0.04,0.04$, $0.04,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.01,0.01, \mathbf{C}_{101}: 0.625,0.15,0.04,0.04,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01, \mathbf{C}_{102}$ : $0.15,0.15,0.15,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.02,0.01,0.01, C_{103} 30.04,0.04,0.04,0.04$, $0.02,0.02,0.02,0.02,0.01, \mathbf{C}_{104}: 0.375,0.04,0.02,0.02,0.02,0.02,0.01,0.01,0.01,0.01,0.01, \mathbf{C}_{105}: 0.15,0.15,0.04,0.02,0.02,0.02,0.01,0.01,0.01,0.01,0.01,0.01$, $C_{106}: 0.15,0.04,0.04,0.04,0.04,0.02,0.02,0.02,0.01,0.01, C_{107}: 0.375,0.15,0.04,0.04,0.04,0.02,0.02,0.02,0.02,0.02,0.01$

