

# A model based on FMEA and Fuzzy TOPSIS for risk prioritization in industrial processes<sup>1</sup>

*Um modelo baseado em FMEA e Fuzzy TOPSIS para priorização de riscos em processos industriais*

Wauires Ribeiro de Magalhães<sup>1</sup> , Francisco Rodrigues Lima Junior<sup>2</sup> 

<sup>1</sup>Universidade Tecnológica Federal do Paraná – UTFPR, Programa de Pós-graduação em Administração, Grupo Tomada de Decisão em Gestão de Operações, Curitiba, PR, Brasil. E-mail: magalhães.runner@gmail.com

<sup>2</sup>Universidade Tecnológica Federal do Paraná – UTFPR, Departamento Acadêmico de Gestão e Economia, Grupo Tomada de Decisão em Gestão de Operações, Curitiba, PR, Brasil. E-mail: eng.franciscojunior@gmail.com

**How to cite:** Magalhães, W. R., & Lima Junior, F. R. (2021). A model based on FMEA and Fuzzy TOPSIS for risk prioritization in industrial processes. *Gestão & Produção*, 28(4), e5535. <https://doi.org/10.1590/1806-9649-2020v28e5535>

**Abstract:** FMEA is one of the most used methods to support risk analysis in business processes. Nonetheless, this method has some limitations, including the use of only three decision criteria, whose weights are not considered. With the objective of adding new features to FMEA, some studies combine it with multicriteria decision methods. This study proposes a model based on FMEA and Fuzzy TOPSIS to support risk prioritization in industrial production processes. A pilot application was performed to analyze and prioritize the risks of potential failures in a nodular iron melting and casting process. Based on the opinion of four company experts, potential failure modes were defined and assessed. The experts also chose the criteria and their respective weights. The pilot application results suggest that “fading time exceeded” and “chemical composition outside of the specified” should be treated with highest priority. The sensitivity analysis test results corroborate the relevance of these failures and demonstrate the effect of criteria weight variation. The proposed model is useful to support the formulation of actions plans focused on minimizing or eliminating priority failures. Other contributions from this study consist of: considering criteria weight; allowing the use of linguistic terms to express the decision makers’ judgments; considering the costs relating to the failures; and supporting group decisions.

**Keywords:** Risk assessment; FMEA; Fuzzy TOPSIS; Multicriteria decision-making.

**Resumo:** O FMEA é um dos métodos mais utilizados para apoiar a análise de riscos em processos empresariais. Apesar disso, esse método apresenta algumas limitações, incluindo o uso de apenas três critérios de decisão, cujos pesos não são considerados. Com o objetivo de incrementar novos recursos ao FMEA, alguns estudos o combinam com métodos de decisão multicritério. Este estudo propõe um modelo baseado em FMEA e *Fuzzy-TOPSIS* para apoiar a priorização de riscos em processos de produção industrial. Uma aplicação piloto foi executada a fim de analisar e priorizar os riscos de falhas potenciais em um processo de fusão e vazamento de ferro nodular. Baseando-se na opinião de quatro especialistas da empresa, os modos de falhas potenciais foram definidos e avaliados. Os especialistas também escolheram os critérios

<sup>1</sup>Data used in this article is available at Magalhães & Lima (2019).

e seus respectivos pesos. Os resultados da aplicação piloto sugerem que as falhas “tempo de *fading* excedido” e “composição química fora do especificado” sejam tratadas com maior prioridade. Os resultados dos testes de análise de sensibilidade ratificam a relevância destas falhas e evidenciam o efeito da variação dos pesos nos critérios. O modelo proposto é útil para apoiar a formulação de planos de ação focados na minimização ou eliminação das falhas prioritárias. Outras contribuições deste estudo consistem em: considerar os pesos dos critérios; permitir o uso de termos linguísticos para expressar os julgamentos dos decisores; considerar os custos referentes às falhas; e apoiar decisões em grupo.

**Palavras-chave:** Avaliação de riscos; FMEA; *Fuzzy*-TOPSIS; Tomada de Decisão multicritério.

## 1 Introduction

Aven & Renn (2009) define risk as “a situation or event where something of human value is at stake and the outcome of this situation is uncertain”. On the other hand, the NBR ISO 31000 (ABNT, 2018) standard, which establishes guidelines on risk management, conceptualizes risk as “the effect of uncertainty on objectives”. By systematizing knowledge and analyzing uncertainty in attaining a system, it becomes possible to predict potential problems, their causes, and probable consequences, creating conditions to classify and mitigate the risks identified (Aven, 2011).

In the literature, several methodologies are found to support risk analysis in different contexts, including industrial processes, product design, transport, among others (Tixier et al., 2002). Tixier et al. (2002) identified 62 methodologies for risk identification, assessment, and ranking. This set includes methodologies based on qualitative and quantitative approaches, as well as deterministic and probabilistic. According to Tixier et al. (2002), one of the most used methods is called Failure Mode and Effect Analysis (FMEA), which allows a qualitative risk analysis by also using scores represented by deterministic values. In the traditional version of FMEA, potential failures modes are assessed based on severity, occurrence, and detection criteria, using a numerical scale ranging from 1 to 10. The multiplication of scores of each failure mode in relation to these three criteria determines the risk priority number (RPN), which indicates the failure’s priority (Liu et al., 2013).

Notwithstanding the broad use of FMEA over more than 50 years, this method still has some limitations, which contributes to the development of new versions by combining it with other techniques. One of these limitations consists in using deterministic numerical values, which do not allow the quantification of uncertain or inaccurate measurements, which are inherent to the risk assessment process (Kutlu & Ekmekçioğlu, 2012). A second limitation is the fact that the levels of relative importance (weights) of the assessed criteria are not considered (Xiao et al., 2011). Another problem is related to the use of only three criteria, as important aspects, such as economic factors, are not considered (Liu et al., 2013; Zhao et al., 2017).

With the objective of adding new features to FMEA, some studies propose decision models by combining it with existing multicriteria methods (*Multicriteria Decision Making, MCDM*). For Almeida (2013, p. 2), a decision model is “[...] a formal representation and with simplification of the problem faced, with the support of a multicriteria decision support method”. One of these methods is called Fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), which was proposed by Chen (2000) from the combination between TOPSIS and fuzzy logic. One of the advantages of Fuzzy TOPSIS over traditional multicriteria techniques is allowing the use of linguistic values, given by one or more experts, to assess the score of

alternatives in relation to multiple criteria, whose weight is also assessed by way of linguistic judgments (Chen, 2000; Lima & Carpinetti, 2015).

With the objective of circumventing the limitations of FMEA outlined in this section, the present study proposes a decision model that combines Fuzzy TOPSIS with FMEA to support risk prioritization in industrial production processes. A pilot application was performed based on judgments provided by experts from a metalworking company, manufacturer of cast iron parts used in the automotive industry. The focus of the application was to prioritize the failures of a production process called melting and casting of nodular iron. The study is organized as follows: section 2 describes the research's methodological procedures; section 3 presents the theoretical framework; section 4 details the proposed model and the results of its application and sensitivity analysis. Finally, section 5 presents the conclusions and suggestions for future studies.

## 2 Methodological procedures

Following the definitions proposed by Bertrand & Fransoo (2002), the present study can be classified as normative quantitative empirical modeling research. In this approach, the model is built from data collection, with causal relationships between the input variables and the output variable. The purpose is to obtain solutions from the model developed to eliminate or mitigate the problems identified. For the development of this study, the following procedures were adopted:

**a) Bibliographical research:** the bibliographical survey of the previous models was performed by way of the following steps: (1) entering the keywords and acronyms "FMEA" AND "RPN" AND "Multicriteria" in the search fields of the *Science Direct*, *Scopus*, *Taylor & Francis*, and *Google Scholar* databases. The search was performed considering the title, abstract and keywords of the articles; (2) using the filter to select only those studies published from 2000 onwards, whose choice was made for being the year of the advent of Fuzzy TOPSIS; (3) selecting only studies published in scientific journals; (4) selecting studies published only in English and Portuguese; (5) selecting studies which address decision models based on FMEA for risk analysis in industrial production processes; (6) eliminating studies which are repeated in more than one database. Table 1 shows the results obtained after performing this procedure. Additionally, books and journal articles related to the central themes of the study were used, such as risk analysis, FMEA, MCDM and Fuzzy-TOPSIS models. This step provided the theoretical and empirical foundations for preparing and applying the decision model;

**Table 1.** Results of the bibliographical research.

Databases	Number of resulting studies in each step					
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Science Direct</i>	14	14	11	11	5	1
Scopus	5	5	5	4	0	0
Taylor & Francis	8	8	8	7	0	0
Google Scholar	270	263	205	188	13	12
Total number of selected studies						13

Source: The authors.

**b) Computational Modeling:** the computational modeling was developed using Microsoft Excel to allow model replication to be performed in an effective and simplified manner. The equations implemented were based on Chen (2000). The linguistic scales used in the modeling were defined from Wang (2011) and Chen (2017). The criteria, on the other hand, were chosen based on Liu et al. (2013) and Banduka et al. (2018). The spreadsheet containing the computational model implemented is available in the Mendeley Data database;

**c) Pilot application:** the model was applied in prioritizing risks in a nodular iron melting and casting process of a metalworking company. The assessed application's failure modes were defined by four experts from a metalworking company, active in the areas of engineering, quality, and production. The failure scores and criteria weight were provided by them. The application of the model resulted in a ranking for prioritizing potential risks, whose sorting was also assessed by the experts. In addition, some sensitivity analysis tests were performed to assess the effect of weight variation on failure classification. The results are presented in Section 4.

### 3 Theoretical framework

#### 3.1 The FMEA method

FMEA is one of the most applied methods to identify and eliminate potential or known failures in a system, project, or process. Its application allows to improve security, reliability, and support the decision-making process (Liu et al., 2013). FMEA supports risk analysis and the development of preventive and corrective actions to systematize failure analysis. It is usually applied in the automotive, aerospace, arms, electronic, medical technologies, among others industries (Carpinetti, 2016). The IATF 16949:2016 standard, which deals with requirements for the application of ISO 9001 in the automotive chain, establishes FMEA as a quality management requirement. According to Banduka et al. (2018), this method has been applied since 1993 by Ford, Chrysler, and General Motors.

Maleki & Saadat (2013) and Carpinetti (2016) suggest that FMEA be applied according to the three steps indicated in Chart 1. In step 1, the experts employ the available information and perform "brainstorming" sessions about possible failure modes in a system, project, or process under study. In this way, they identify potential and known failure modes, as well as analyze and describe the effects of these failure modes. Next, the experts discuss the probable causes and existing means for detecting the failure mode if it occurs. For each failure mode identified, a score related to the criteria must be assigned: severity (S), which quantifies the severity of the effect; occurrence (O), related to the frequency and cause; and detection (D), which assesses the existing means of control. These scores are given using integer numerical values distributed between 1 and 10. The last step of phase I consists of calculating the RPN (Risk Priority Number) by means of Equation 1.

$$RPN = S \times O \times D \tag{1}$$

**Chart 1.** Description of the FMEA application steps.

<b>Step I</b>
a) Specify the system, design, or process under study;
b) Establish the team of experts;
c) Define process requirements or the functions of the product's components;
d) Identify potential or known failure modes;
e) Analyze and describe the effects of each failure mode and assess its severity;
f) Investigate and define the probable causes of each failure mode and assess the occurrence of these causes;
g) Verify the existing controls and assess the failure mode detection capacity by these resources;
h) Calculate the RPN using Equation 1.
<b>Step II</b>
a) Sort RPN values in descending order. Failure modes with the highest RPN results are assumed as the most important and will have higher priority for taking actions.
b) Develop a prioritized corrective or preventive action plan.
<b>Step III</b>
a) Implement the action plan;
b) Assess the effectiveness of these actions, performing a new failure mode assessment considering the severity, occurrence, and detection criteria. If the actions have been effective, the reduction of the RPN value is expected to take place in relation to the initial result.

Source: Adapted from García & Gilabert (2011) and Liu et al. (2013).

In step II, the values resulting from calculating the RPN for each failure are ranked in descending order. RPN classification determines the failure's priority level. The experts and other personnel involved in the process analyzed must develop and implement action plans to eliminate or mitigate the potential causes of priority failures. Finally, in step III, the failure modes are reassessed using Equation 1 to verify the effectiveness of the actions implemented. The new failure mode ranking guides the next process improvement actions (Maleki & Saadat, 2013; Carpinetti, 2016).

Although it is a widely accepted and broadly used method, FMEA has been criticized in several studies that point out some deficiencies in step I, related to the method's risk analysis procedure and failure mode prioritization. Some of these deficiencies are listed below:

- i. Use of deterministic numerical values, which do not allow the quantification of uncertain or imprecise measures, inherent to the risk assessment process (Liu et al., 2011; Bozdog et al., 2015);
- ii. Different sets of "S", "O" and "D" classifications can produce the same RPN value, but their hidden risk implications may be entirely different (Pillay & Wang, 2003; Liu et al., 2011);
- iii. The relative importance among "S", "O", and "D" is not taken into consideration. This may not be the case when one considers a practical implementation of FMEA (Liu et al., 2011; Xiao et al., 2011);
- iv. The mathematical formula for calculating the RPN is questionable. No arguments are found to justify the fact that the RPN is calculated by multiplying the scores of each criterion (Liu et al., 2011; Mahmoodi & Mirzazadeh, 2014);
- v. Small variations in a classification may lead to effects on the RPN, depending on the values of other factors. For example, if the scores of "O" and "D" are 10, the difference of 1 point in "S" will cause a difference of 100 points in the RPN. If the values of "O" and "D" were equal to 2, the same difference of 1 point in "S" would generate a variation of only 4 points in the RPN (Liu et al., 2011);

- vi. The RPN considers only three factors in the analysis, whereas other important aspects, such as economic factors, environmental impacts, and production losses are ignored (Liu et al., 2013; Zhao et al., 2017);
  - vii. Failure scores in the three factors are difficult to determine accurately. A lot of information can be expressed in the form of linguistic judgments, such as “probable”, “important” or “very high” (Pillay & Wang, 2003; Liu et al., 2011).
- To overcome FMEA limitations and improve its performance, studies have increasingly been proposing to combine it with multicriteria decision methods (MCDM), as the following section discusses (Zhao et al., 2017).

### 3.2 FMEA applications combined with MCDM methods

MCDM methods present themselves as a solution for decision problems in which qualitative and quantitative criteria and at least two alternatives are involved (Guarnieri, 2015). According to Ahmadi et al. (2017), MCDM methods can be subdivided into two types: MODM (Multi-Objectives Decision Making) and MADM (Multi-Attribute Decision Making). While MODM methods focus on optimization problems, MADM methods rank predefined alternatives.

Chart 2 shows the preliminary studies found by way of a bibliographic survey, highlighting the decision techniques used in each of them. All these studies use FMEA combined with MCDM methods to support risk analysis based on the ranking of RPN values. Due to the cuts incurred by this research, only studies focused on industrial production processes were included. In these applications, some of the most common objectives are the prevention and/or reduction of accidents, defective parts, production downtime, and waste of resources.

**Chart 2.** Decision models for FMEA risk analysis.

Proposed by:	Technique(s) Used	Application
Ekmekçioğlu & Kutlu (2012)	Fuzzy AHP (Analytic Hierarchy Process) and Fuzzy TOPSIS (Fuzzy Technique for Order of Preference by Similarity to Ideal Solution)	Assembly process of an automotive industry
Kutlu & Ekmekçioğlu (2012)	Fuzzy AHP and Fuzzy TOPSIS	Manufacturing process of an automotive industry
Chang et al. (2013)	GRA (Grey Relational Analysis) and DEMATEL (Decision making and Trial Evaluation Laboratory)	Manufacturing and assembly process of electronic components
Maleki & Saadat (2013)	AHP and REMBRANDT System	Manufacturing process of hydraulic pumps
Mahmoodi & Mirzazadeh (2014)	Fuzzy TODIM (Iterative Multicriteria Decision Making) and FTF (Fuzzy Time Function)	Process of an automotive industry
Bozdag et al. (2015)	Interval Type 2 Fuzzy Sets	Assembly process
Haq et al. (2015)	FST (Fuzzy Set Theory)	Assembly line at the Ford Motor Company
Ahmadi et al. (2017)	TOPSIS	Steel Manufacturing Process
Certa et al. (2017)	ELECTRE TRI (Elimination and Choice Expressing Reality)	Processes at a dairy industry
Chen (2017)	Fuzzy ISM (Interpretive Structural Model), DEMATEL and ANP (Analytic Network Process)	Notebook manufacturing
Hajimolaali et al. (2017)	Fuzzy TOPSIS	Pharmaceutical industry manufacturing
Zhao et al. (2017)	IVIFS (Interval-valued Intuitionistic Fuzzy Set) and MULTIMOORA (Multi-Objective Optimization by Ratio Analysis)	Steel Manufacturing Process
Banduka et al. (2018)	FST	Manufacturing process of an automotive industry

Source: The authors.

Among the 13 studies described in Chart 2, 12 consider only the three criteria traditionally used in FMEA: severity, occurrence, and detection. The only exception is the model proposed by Banduka et al. (2018), which also includes a factor related to the failure's internal and external costs. In relation to decision techniques, eight studies use approaches based on the fuzzy sets theory (FST), which seems to be related to their ability to support decisions under uncertainty. Unlike the studies proposed by Chang et al. (2013), Ahmadi et al. (2017) and Certa et al. (2017), the models based on FST enable the use of linguistic variables to assess the elements of the problem. It is also noted a wide diversity of combinations between decision techniques. Chen (2017) combined FMEA with Fuzzy ISM, DEMATEL and ANP methods. Mahmoodi & Mirzazadeh (2014) applied Fuzzy TODIM with FTF. Ekmekçioğlu & Kutlu (2012) and Kutlu & Ekmekçioğlu (2012) suggest the combined use of Fuzzy AHP with Fuzzy TOPSIS.

Although the models based on fuzzy TOPSIS (Ekmekçioğlu & Kutlu, 2012; Kutlu & Ekmekçioğlu, 2012) have brought significant contributions to the literature on this subject, the combined use with Fuzzy AHP to determine criteria weights may involve limitations, such as: (1) difficulty of guaranteeing consistency by means of comparative judgments, requiring the performance of various consistency tests; (2) the need for a greater number of judgments, which implies investing more data collection efforts; (3) possibility of obtaining null weights for criteria, which makes failure scores in the criterion with zero weight not be considered in calculating the RPN. It is worth mentioning that the first two limitations pointed out are also valid for models based on other techniques that require paired comparisons between criteria and alternatives, such as AHP (Maleki & Saadat, 2013) and ANP (Chen, 2017).

The model proposed by Hajimolaali et al. (2017), which uses only the Fuzzy-TOPSIS method for assessing criteria weight and alternatives scores, does not suffer these limitations. However, alike the models developed by Ekmekçioğlu & Kutlu (2012) and Kutlu & Ekmekçioğlu (2012), Hajimolaali et al. (2017) consider only the three criteria traditionally used by FMEA, without including the failure-related costs. Therefore, the development of the decision model proposed by this study aims to circumvent these limitations.

### 3.3 The Fuzzy-TOPSIS method

#### 3.3.1 Basics of the method

The Fuzzy set theory - FST was created by Zadeh (1965) to allow the modeling of systems with categories of elements whose boundaries are considered uncertain (Lima & Carpinetti, 2015). Chen (2000) was the first to propose the combination of the TOPSIS method with FST with the objective of adapting TOPSIS for decisions in scenarios of uncertainty, characterized by the absence of information, inaccurate data, qualitative variables, and subjective judgments. Lima & Carpinetti (2015) emphasize that the adequacy of the FST to such scenarios is related to the logic that defines the degree of belonging of elements in fuzzy sets. A fuzzy set is modeled by a function of relevance  $\mu_A(x) : X \rightarrow [0.0, 1.0]$ , which allows partial relevance levels. While in the classical sets theory, each set is defined using a characteristic function  $\mu_A(x) : X \rightarrow \{0.0, 1.0\}$ , in the fuzzy logic, function  $\mu_A(x)$  includes values in the continuous interval  $[0.0, 1.0]$ . In this way, it is considered the existence of intermediate levels

between the “false” ( $\mu_A(x) = 0$ ) and the “true” ( $\mu_A(x) = 1$ ) (Zadeh, 1965; Lima & Carpinetti, 2015; Pedrycz & Gomide, 2007).

In the Fuzzy-TOPSIS method developed by Chen (2000), the judgments from experts to quantify the score of the alternatives and criteria weight are modeled by linguistic variables. A linguistic variable is that whose values are defined sentences in natural or artificial language (Zadeh, 1973). Its use implies the choice of a set of linguistic terms to quantify its values appropriately. In this sense, upon analyzing failures using FMEA and Fuzzy TOPSIS, the value of the linguistic variable “Severity” can be measured by means of the linguistic terms “low”, “medium”, and “high”.

Linguistic terms are represented by fuzzy numbers, whose function of relevance can have different formats, such as triangular, sigmoid, or trapezoidal. As shown in Figure 1, a triangular fuzzy number  $\tilde{A}$  can be written by means of its vertices ( $l, m, u$ ), where  $m$  represents a crisp central value,  $l$  is the lower limit, and  $u$  is the upper limit. The triangular numbers are often used due to the greater simplicity in the calculations involved. Moreover, the functions of the triangular type are more sensitive than trapezoidal functions to respond to changes in the values of  $x$  (Pedrycz & Gomide, 2007; Kahraman, 2008; Lima & Carpinetti, 2015). Pedrycz & Gomide (2007) explain that the algebraic operations involving two triangular numbers are made based on the  $l, m$ , and  $u$  values. So, to sum the triangular numbers  $\tilde{A}$  and  $\tilde{B}$ , Equation 2 is used. Equation 3 is applied for subtraction, while Equations 4 and 5 perform the multiplication and division operations, respectively.

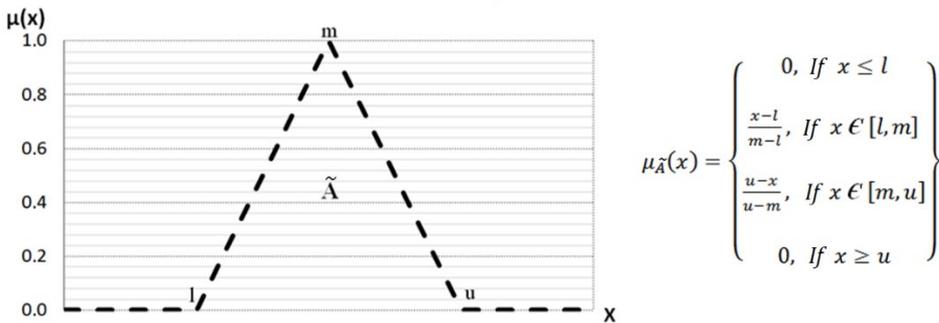


Figure 1. Triangular fuzzy number. Source: Pedrycz & Gomide (2007).

$$\tilde{A} + \tilde{B} = [l_A, m_A, u_A] + [l_B, m_B, u_B] = [l_A + l_B, m_A + m_B, u_A + u_B] \tag{2}$$

$$\tilde{A} - \tilde{B} = [l_A, m_A, u_A] - [l_B, m_B, u_B] = [l_A - u_B, m_A - m_B, u_A - l_B] \tag{3}$$

$$\tilde{A} * \tilde{B} = [l_A, m_A, u_A] * [l_B, m_B, u_B] = [l_A * l_B, m_A * m_B, u_A * u_B] \tag{4}$$

$$\tilde{A} / \tilde{B} = [l_A, m_A, u_A] / [l_B, m_B, u_B] = [l_A / u_B, m_A / m_B, u_A / l_B] \tag{5}$$

### 3.3.2 Fuzzy-TOPSIS steps

Based on Chen (2000) and Lima & Carpinetti (2015), the steps of the Fuzzy-TOPSIS method are described below:

- a) Add the linguistic values of each decision maker ( $DM_r$ ), referring to the scores of the alternatives, by applying Equation 6. In this equation,  $\tilde{x}_{ij}^r$  describes the score of the alternative  $A_i$  ( $i = 1, \dots, n$ ), in relation to the criterion  $C_j$  ( $j = 1, \dots, m$ ), provided by the decision maker  $DM_r$  ( $r = 1, \dots, k$ ). The assessments referring to the criteria weights are aggregated using Equation 7, where  $\tilde{w}_j^r$  describes the weight of  $C_j$  in accordance with  $DM_r$ ;

$$\tilde{x}_{ij} = \frac{1}{K} [\tilde{x}_{ij}^1 + \tilde{x}_{ij}^r + \dots + \tilde{x}_{ij}^k] \tag{6}$$

$$\tilde{w}_j = \frac{1}{K} [\tilde{w}_j^1 + \tilde{w}_j^2 + \dots + \tilde{w}_j^k] \tag{7}$$

- ii) Using the results obtained in the previous step, the decision matrix  $\tilde{D}$  should be assembled, containing the aggregate scores of the alternatives, as well as a vector  $\tilde{w}$  for the criteria aggregated weights, according to Equations 8 and 9, respectively;

$$\tilde{D} = \begin{matrix} & C_1 & C_2 & \dots & C_j & \dots & C_m \\ \begin{matrix} A_1 \\ : \\ A_i \\ : \\ A_n \end{matrix} & \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1j} & \dots & \tilde{x}_{1m} \\ : & : & & : & & : \\ \tilde{x}_{i1} & \tilde{x}_{i2} & \dots & \tilde{x}_{ij} & \dots & \tilde{x}_{im} \\ : & : & & : & & : \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \dots & \tilde{x}_{nj} & \dots & \tilde{x}_{nm} \end{bmatrix} \end{matrix} \tag{8}$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_m] \tag{9}$$

- iii) Normalize the values of the  $\tilde{D}$  matrix. The normalized matrix  $\tilde{R}$  is calculated using Equation 10, where  $\tilde{r}_{ij}$  should be obtained using Equations 11 or 12;

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n} \tag{10}$$

$$\tilde{r}_{ij} = \left( \frac{l_{ij}}{u_j^+}, \frac{m_{ij}}{u_j^+}, \frac{u_{ij}}{u_j^+} \right), \text{ and } u_j^+ = \max_i u_{ij} \text{ (benefit criteria)} \tag{11}$$

$$\tilde{r}_{ij} = \left( \frac{l_j^-}{u_{ij}}, \frac{l_j^-}{m_{ij}}, \frac{l_j^-}{l_{ij}} \right), \text{ and } l_j^- = \min_i l_{ij} \text{ (cost criteria)} \tag{12}$$

iv) Obtain the weighted matrix  $\tilde{V}$ , represented by Equation 13. The component values of this matrix are obtained according to Equation 14, which multiplies the  $\tilde{w}_j$  weights by the  $\tilde{r}_{ij}$  elements of the normalized matrix  $\tilde{R}$ ;

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n} \tag{13}$$

$$\tilde{v}_{ij} = \tilde{r}_{ij} * \tilde{w}_j \tag{14}$$

v) Obtain the Fuzzy Positive Ideal Solution,  $FPIS, A^+$  and the Fuzzy Negative Ideal Solution,  $FNIS, A^-$  as shown in Equations 15 and 16. According to Chen (2000), component values of ideal solutions can be defined as  $\tilde{v}_j^+ = (1,1,1)$  and  $\tilde{v}_j^- = (0,0,0)$ ;

$$A^+ = \{ \tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_m^+ \} \tag{15}$$

$$A^- = \{ \tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_m^- \} \tag{16}$$

vi) Calculate  $D_i^+$ , which indicates the distance between the values of FPIS and the alternatives scores. Similarly, obtain the distance  $D_i^-$  between the values of FNIS and the alternatives scores. To that end, Equations 17 and 18 are applied, in which  $d(., .)$  indicates the distance between two fuzzy numbers according to the *vertex* method. In cases in which fuzzy numbers of the triangular type are adopted, Equation 19 should be applied to obtain the values of  $d(., .)$ ;

$$D_i^+ = \sum_{j=1}^n d_v(\tilde{v}_{ij}, \tilde{v}_j^+) \tag{17}$$

$$D_i^- = \sum_{j=1}^n d_v(\tilde{v}_{ij}, \tilde{v}_j^-) \tag{18}$$

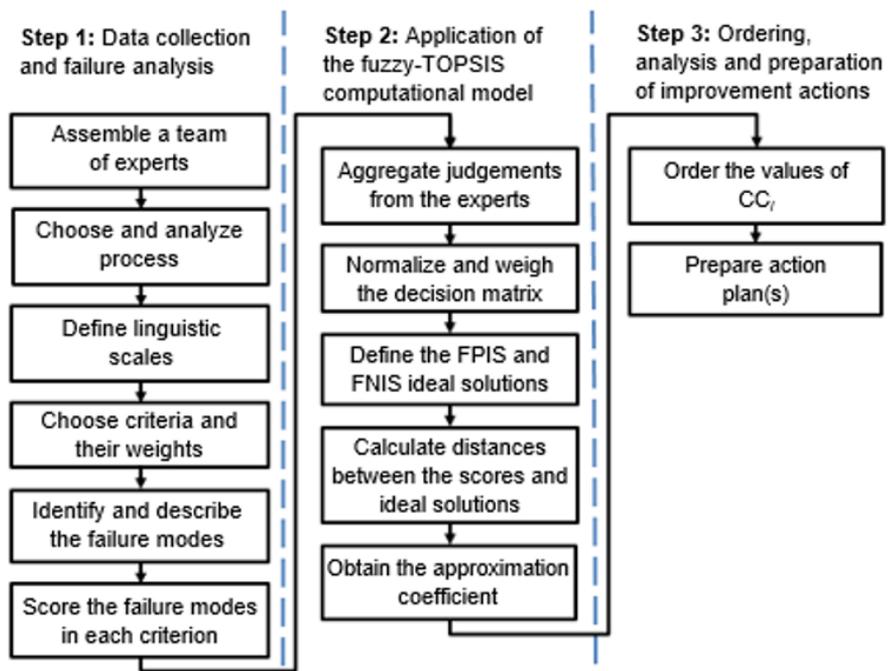
$$d(\tilde{x}, \tilde{z}) = \sqrt{\frac{1}{3} [(l_x - l_z)^2 + (m_x - m_z)^2 + (u_x - u_z)^2]} \tag{19}$$

vii) Using Equation 20 and the values of  $D_i^+ e D_i^-$ , one must calculate the approximation coefficient  $CC_i$ , which represents the overall performance of each of the alternatives assessed. Finally, it is necessary to draw up an alternatives ranking by means of the descending order of the values of  $CC_i$ . The closer the value of  $CC_i$  is to 1.0, the better is the alternative's overall performance.

$$CC_i = \frac{D_i^-}{(D_i^+ + D_i^-)} \tag{20}$$

#### 4 Decision model proposed for prioritizing risks in processes

Figure 2 shows the decision model proposed to support risk prioritization in production processes. The model was developed by combining FMEA (Carpinetti, 2016) with the Fuzzy-TOPSIS technique (Chen, 2000), being composed by three main steps. While step 2 consists of applying Fuzzy TOPSIS, steps 1 and 3 combine activities based on this method and on FMEA. Step 1 begins with the formation of a multidisciplinary team, comprised of experts in the process under study and managers from the organization, who will act as decision makers. Depending on the nature of the process under study, this team may involve professionals from the areas of engineering, quality, supply, manufacturing, among others. The first assignment of the decision makers is to define the scope of the application, i.e., to choose which process will be assessed, to explain its function and to analyze the activities that comprise it. By way of brainstorming sessions and analyzing process documentation and records, the decision makers must also identify and describe the potential or known failure modes.



**Figure 2.** Model to support the analysis and prioritization of risks in processes.  
Source: The authors.

Still in step 1, based on the preferences of the experts and/or in literature studies, it is necessary to build a fuzzy linguistic scale to score failures and another one to define criteria weights. It is the responsibility of the decision makers to choose the failure assessment criteria, as well as to define the relative importance (weight) of each one of them. The chosen criteria must be related to factors capable of measuring the impact of the failures to the end customer and the organization. Such criteria should also be associated with factors that direct the implementation of actions to eliminate or mitigate

the risks involved. After defining the criteria, the decision makers must individually assess each failure mode described and assign the corresponding scores to each criterion. This assessment must be based on the decision makers' experience and other available information.

Step 2 follows the procedure defined by the Fuzzy-TOPSIS method (Chen, 2000). The values assigned to failures and the criteria weights are entered into the Fuzzy TOPSIS computational model. At first, the aggregation of the judgments delivered by the decision makers is made. While the failure score aggregation is performed according to Equation 6, the criteria weight aggregation is performed using Equation 7. The failure scores should be normalized and weighted by applying Equations 13 and 14, respectively, to then define the positive and negative ideal solutions. Subsequently, the distances of each alternatives score in relation to the values of FPIS and FNIS are calculated using Equations 17 and 18. At the end of this step, Equation 20 is applied to obtain the approximation coefficients  $CC_i$  which allow failure mode classification.

Finally, in step 3, the values of  $CC_i$  are classified in descending order, creating a ranking in which the alternatives with the higher overall scores represent the most undesirable failure modes. Therefore, based on this ordering, failures are prioritized to develop the actions necessary to eliminate or mitigate the major risks of the process under analysis. After the implementation of these actions, the model can be once again applied to assess the effectiveness of the solutions developed. To do this, it is necessary to assign new scores to the failure modes, insert them into the computational model and order the results.

#### 4.1 Pilot application

The model was applied to a melting and pouring process of a domestic metalworking company, supplier of gray and nodular cast iron parts for the automotive market. This company adopts FMEA as the main risk analysis tool at the initial stage of its process development. The manager responsible for production was requested to assemble a cross-functional team composed by experts involved with the process in question. There was no restriction as to the number of experts, leaving it up to the manager the definition of this team. In addition to the knowledge of the process in question, they should have prior experience in the application of FMEA. The manager then presented a list with four decision makers, from the areas of engineering, quality, and production.

In relation to the process analyzed, an induction furnace and manual refractory pots suspended on monorails are used as resources. This process is mapped in a flowchart and features four steps, which are described in auditable procedures. From a brainstorming session that involved the analysis of existing documents and records concerning the process, the decision makers identified 13 potential failure modes. Next, they chose the criteria considering their ability to identify and measure the impact of the failure modes. The following criteria (also understood here as risk factors) were defined: Cost (C), Severity (S), Frequency (F) and Control (CO). The Cost (C) criterion was selected based on Banduka et al. (2018). This factor refers to internal costs, associated with rework, scrap, line downtime, among other types of losses, as well as external costs, which are incurred when the failure affects the end customer and results in expenses with warranties, lawsuits, returns and/or loss of market share. The Severity (S), Frequency (F) and Control (CO) criteria derive from the traditional FMEA. The linguistic scales to assess criteria weights and failure mode scores were defined

according to Wang (2011) and Chen (2017). According to Chen (2000), fuzzy triangular numbers were adopted to quantify the linguistic terms. Chart 3 presents the linguistic scales developed.

**Chart 3.** Linguistic scales for the assessment of criteria and alternatives.

Criteria weight assessment				Alternative score assessment			
Linguistic Term	Fuzzy Number			Linguistic Term	Fuzzy Number		
Very Low Importance (VLI)	0.01	0.03	0.25	Very Low (VL)	0.10	0.10	2.50
Low Importance (LI)	0.01	0.25	0.50	Low (L)	0.10	2.50	5.00
Medium Importance (MI)	0.25	0.50	0.75	Moderate (M)	2.50	5.00	7.50
High Importance (HI)	0.50	0.75	1.00	High (H)	5.00	7.50	10.00
Very High Importance (VHI)	0.75	1.00	1.00	Very High (VH)	7.50	10.00	10.00

Source: Adapted from Wang (2011) and Chen (2017).

Using the scales presented, the experts scored each of the failure modes identified. Chart 4 shows the linguistic scores of the failure modes in relation to the criteria assessed, as well as the weights of these criteria. The decision makers attributed the linguistic terms that best represent the actual condition of the process, based on their knowledge, experience, and other available information. The linguistic terms were converted into fuzzy numbers as per Chart 3. These values were aggregated using fuzzy arithmetic mean, in accordance with Equations 6 and 7, respectively. The results are presented in Table 2.

The fuzzy numbers shown in Table 2 were normalized considering the particularities of each criterion. From the modeling point of view, Cost (C), Severity (S) and Frequency (F) were normalized as benefit criteria (Equation 11), since the higher the score of a failure in these criteria, the greater will its RPN be. In the case of the Control (CO) criterion, a high score indicates that there are already mechanisms for risk monitoring and mitigation, which implies a lower RPN. Therefore, Equation 12 was applied. After normalization, the decision matrix was weighted using Equation 14, which multiplies the values normalized by their respective aggregated weights. The normalized and weighted matrix was omitted due to size limitations of this article.

**Chart 4.** Linguistic judgments of the decision makers.

Process step	Failure Mode	Decision maker 1				Decision maker 2				Decision maker 3				Decision maker 4			
		C	S	F	CO												
Melting	(F1) Metallic load composition error	V	L	H	H	V	L	M	M	L	L	M	M	V	L	H	H
Metal treatment	(F2) Insufficient alloys	M	H	L	H	M	H	L	H	M	V	L	M	M	H	L	H
Metal cleaning	(F3) Deficient bath cleanliness	M	M	M	M	M	M	M	M	M	M	L	L	M	M	M	H

Chart 4. Continued....

Process step	Failure Mode	Decision maker 1				Decision maker 2				Decision maker 3				Decision maker 4			
		C	S	F	CO	C	S	F	CO	C	S	F	CO	C	S	F	CO
Pouring	(F4) Bath weighing error	M	H	V L	V H	M	M	V L	H	M	M	M	L	M	M	V L	H
	(F5) Deficient inoculation	M	H	M	L	L	M	L	M	L	M	M	M	M	H	M	L
	(F6) Pouring start delay	M	L	L	H	L	M	L	H	L	M	M	M	L	M	L	H
	(F7) Furnace temperature below the specified	M	H	L	H	M	H	L	V H	M	M	L	M	M	H	L	H
	(F8) Deficient pot heating	L	L	L	H	L	L	L	M	V L	M	L	M	L	L	L	M
	(F9) Pouring time above the specified	M	V H	M	H	M	H	M	M	M	M	L	M	M	V H	M	H
	(F10) Incomplete pouring	M	H	L	H	M	H	L	H	M	M	M	L	M	H	L	H
	(F11) Interrupted pouring	M	M	L	L	M	M	L	L	M	M	L	L	M	M	L	L
	(F12) Chemical composition out of the specified	M	V H	M	M	M	H	M	M	M	V H	L	M	M	H	M	M
	(F13) Fading time exceeded	M	V H	L	L	M	V H	L	M	M	V H	L	M	M	V H	L	L
<b>Criteria weights</b>		V H I	H I	H I	V H I	H I	V H I	V H I	V H I	H I	H I	M I	H I	V H I	V H I	H I	V H I

Source: The authors.

Table 2. Decision matrix with aggregate scores and criteria weight vector.

	Cost (C)			Severity (S)			Frequency (F)			Control (CO)		
	l	m	u	l	m	u	l	m	u	l	m	u
F1	0.10	0.70	3.13	0.10	2.50	5.00	3.75	6.25	8.75	3.75	6.25	8.75
F2	2.50	5.00	7.50	5.63	8.13	10.00	0.10	2.50	5.00	4.38	6.88	9.38
F3	2.50	5.00	7.50	2.50	5.00	7.50	1.90	4.38	6.88	2.53	5.00	7.50
F4	2.50	5.00	7.50	3.13	5.63	8.13	0.70	1.33	3.75	4.40	6.88	8.75
F5	1.30	3.75	6.25	3.75	6.25	8.75	1.90	4.38	6.88	1.30	3.75	6.25
F6	0.70	3.13	5.63	1.90	4.38	6.88	0.70	3.13	5.63	4.38	6.88	9.38
F7	2.50	5.00	7.50	4.38	6.88	9.38	0.10	2.50	5.00	5.00	7.50	9.38
F8	0.10	1.90	4.38	0.70	3.13	5.63	0.10	2.50	5.00	3.13	5.63	8.13
F9	2.50	5.00	7.50	5.63	8.13	9.38	1.90	4.38	6.88	3.75	6.25	8.75
F10	2.50	5.00	7.50	4.38	6.88	9.38	0.70	3.13	5.63	3.78	6.25	8.75
F11	2.50	5.00	7.50	2.50	5.00	7.50	0.10	2.50	5.00	0.10	2.50	5.00
F12	2.50	5.00	7.50	6.25	8.75	10.00	1.90	4.38	6.88	2.50	5.00	7.50
F13	2.50	5.00	7.50	7.50	10.00	10.00	0.10	2.50	5.00	1.30	3.75	6.25
Weight	0.63	0.88	1.00	0.63	0.88	1.00	0.50	0.75	0.94	0.69	0.94	1.00

Source: The authors.

The fuzzy positive ideal solutions (FPIS) and the fuzzy negative ideal solutions (FNIS) were defined according to Chen (2000) for each of the criteria. Using Equations 15 and 16, we get  $A^+ = [(1.0, 1.0, 1.0), (1.0, 1.0, 1.0), (1.0, 1.0, 1.0), (1.0, 1.0, 1.0)]$  and  $A^- = [(0.0, 0.0, 0.0), (0.0, 0.0, 0.0), (0.0, 0.0, 0.0), (0.0, 0.0, 0.0)]$ . The distance between the scores of each alternative in relation to FPIS ( $D^+$ ) was calculated using Equation 17. Next, the relative distance to FNIS ( $D^-$ ) was obtained using Equation 18. The results of the calculations of individual distances in each criterion (columns 2 to 5 and 7 to 10) and the sum of these distances (columns 6 and 11) are presented in Table 3.

**Table 3.** Positive and negative distances in relation to FPIS and FNIS.

	Criteria				$D^+$	Criteria				$D^-$
	C	S	F	CO		C	S	F	CO	
F1	0.49	0.45	0.30	0.57	1.82	0.14	0.14	0.16	0.18	0.62
F2	0.30	0.24	0.45	0.57	1.55	0.39	0.40	0.43	0.43	1.65
F3	0.30	0.35	0.37	0.56	1.58	0.39	0.39	0.37	0.29	1.44
F4	0.30	0.32	0.48	0.57	1.67	0.39	0.39	0.38	0.32	1.48
F5	0.36	0.30	0.37	0.56	1.58	0.32	0.32	0.34	0.35	1.33
F6	0.39	0.37	0.42	0.57	1.75	0.28	0.28	0.28	0.27	1.11
F7	0.30	0.28	0.45	0.57	1.59	0.39	0.40	0.40	0.38	1.57
F8	0.44	0.43	0.45	0.57	1.88	0.21	0.21	0.22	0.21	0.84
F9	0.30	0.24	0.37	0.57	1.48	0.39	0.40	0.43	0.41	1.63
F10	0.30	0.28	0.42	0.57	1.57	0.39	0.40	0.40	0.38	1.57
F11	0.30	0.35	0.45	0.46	1.56	0.39	0.39	0.37	0.29	1.44
F12	0.30	0.22	0.37	0.56	1.45	0.39	0.41	0.44	0.44	1.68
F13	0.30	0.18	0.45	0.56	1.49	0.39	0.42	0.47	0.47	1.75

Source: The authors.

Finally, the approximation coefficient  $CC_i$  of each alternative was generated using Equation 20. These values represent the overall score for each failure, thus indicating the risk priority level. As shown in Table 4, the values of the approximation coefficients were ordered from the largest to the smallest, thus obtaining a classification that allows us to guide the decision-making process about the more harmful risks to the process and support the preparation of action plans.

**Table 4.** Failure mode classification result.

Classification	$CC_i$		Failure Mode
1 <sup>st</sup>	0.540	F13	Fading time exceeded
2 <sup>nd</sup>	0.536	F12	Chemical composition out of the specified
3 <sup>rd</sup>	0.525	F9	Pouring time above the specified
4 <sup>th</sup>	0.514	F2	Insufficient alloys
5 <sup>th</sup>	0.501	F10	Incomplete pouring
6 <sup>th</sup>	0.496	F7	Furnace temperature below the specified
7 <sup>th</sup>	0.481	F11	Interrupted pouring
8 <sup>th</sup>	0.477	F3	Deficient bath cleanliness
9 <sup>th</sup>	0.471	F4	Bath weighing error
10 <sup>th</sup>	0.457	F5	Deficient inoculation
11 <sup>th</sup>	0.387	F6	Pouring start delay
12 <sup>th</sup>	0.308	F8	Deficient pot heating
13 <sup>th</sup>	0.255	F1	Metallic load composition error

Source: The authors.

The results obtained were endorsed by the company's experts. According to them, failure F13 (fading time exceeded), classified as priority, drastically affects the mechanical properties of the parts, causing the batches produced to be discarded. If any defective unit reaches the customer, the associated risks will be very high. Failure F1 (metallic load composition error), on the other hand, received the lowest  $CC_i$  value, thus considered of lower priority. According to the experts, this is justified by the fact that this failure can be detected and corrected during the operation, using mechanisms that already exist in the company. Therefore, experts should primarily focus on the development of actions to mitigate risks relating to fading time. If there are sufficient resources, they should also invest efforts in the elaboration and implementation of action plans related to failures F12 (chemical composition out of the specified), F9 (pouring time above the specified), F2 (insufficient alloys), F10 (incomplete pouring), and so forth.

### 4.1 Sensitivity analysis

With the objective of assessing the effects of criteria weight variation in failure mode classification and checking the consistency of the results obtained in implementing the proposed model, some sensitivity analysis tests were performed. Four scenarios were tested. In all of them, the failure scores shown in Chart 4 were considered. Chart 5 shows the criteria weight values tested in each scenario, assigned according to the linguistic scale from Chart 3. The "very high" importance (VHI) value was alternately assigned to each of the criteria, whereas the others were kept with "very low" importance (VLI). Table 5 shows the results obtained in the four sensitivity tests. To facilitate classification comparison, it also shows the results of the implementation in the company.

**Chart 5.** Weights assigned to criteria in each scenario.

Scenario	Criteria			
	Cost	Severity	Frequency	Control
1	VHI	VLI	VLI	VLI
2	VLI	VHI	VLI	VLI
3	VLI	VLI	VHI	VLI
4	VLI	VLI	VLI	VHI

Source: The authors.

**Table 5.** Results of the sensitivity analysis tests.

Classification	Pilot application		Scenario 1		Scenario 2		Scenario 3		Scenario 4	
	$CC_i$	Failure	$CC_i$	Failure	$CC_i$	Failure	$CC_i$	Failure	$CC_i$	Failure
1°	0.540	F13	0.3906	F12	0.3978	F13	0.1457	F12	0.1344	F13
2°	0.536	F12	0.3897	F13	0.3700	F12	0.1435	F9	0.1343	F12
3°	0.525	F9	0.3891	F9	0.3535	F2	0.1400	F13	0.1334	F2
4°	0.514	F2	0.3891	F2	0.3513	F9	0.1395	F2	0.1321	F9
5°	0.501	F10	0.3887	F11	0.3195	F10	0.1394	F10	0.1316	F10
6°	0.496	F7	0.3881	F10	0.3190	F7	0.1374	F7	0.1313	F7
7°	0.481	F11	0.3876	F7	0.2927	F5	0.1368	F3	0.1311	F11
8°	0.477	F3	0.3848	F3	0.2782	F4	0.1335	F11	0.1270	F4
9°	0.471	F4	0.3840	F4	0.2627	F11	0.1315	F4	0.1261	F3
10°	0.457	F5	0.3354	F5	0.2596	F3	0.1249	F5	0.1153	F5
11°	0.387	F6	0.3014	F6	0.2238	F6	0.1065	F6	0.1004	F6
12°	0.308	F8	0.2397	F8	0.1712	F8	0.0840	F8	0.0802	F8
13°	0.255	F1	0.1783	F1	0.1410	F1	0.0703	F1	0.0621	F1

Source: The authors.

The results obtained in the four scenarios indicate a significant difference in the values of  $CC_i$  in relation to the values of the implementation in the company. The biggest difference between these values is observed in scenario 4, where the “control” criterion receives the greatest weight. Generally, there are some changes in failure mode classification (alternatives) in all scenarios, although some similarities remain. In the four scenarios, failures F13 and F12 alternated between the top positions of the ranking, thus confirming that they are the ones that deserve greater attention. When a greater weight is assigned to the cost and frequency criteria, the priority failure becomes “chemical composition out of the specified”.

Failures F2 and F9 also stood out, because while F9 varies between the 2<sup>nd</sup> and 4<sup>th</sup> position, F2 varies between the 3<sup>rd</sup> and 4<sup>th</sup>. F3 and F4 remain in an intermediate priority level, alternating respectively between 7<sup>th</sup> and 10<sup>th</sup>, and between the 8<sup>th</sup> and the 9<sup>th</sup> position. The most significant change occurred with F5, which jumped from the 10<sup>th</sup> to the 7<sup>th</sup> position in scenario 2. On the other hand, the failure modes of the last three positions in the ranking did not suffer classification changes. Therefore, it appears that the changes in the weights assigned to the criteria may cause some changes in the relative ranking between failures that are in close positions, even though that the failures situated at the extremes of the ranking remain the same.

## 5 Conclusion

This study presented a new approach to support risk prioritization and analysis in industrial production processes. The proposed decision model combines FMEA with the Fuzzy-TOPSIS method developed by Chen (2000). An application of the model was carried out in a metalworking process with the participation of four experts. The results suggest that the “fading time exceeded” and “chemical composition outside of the specified” failures should be treated with the highest priority. The sensitivity analysis tests corroborate these results. The outputs of the model provide subsidies to formulate and implement action plans focused on minimizing or eliminating priority failures. Another contribution of this study to the literature consists in mapping techniques used in risk prioritization models in FMEA based industrial processes.

The proposed approach is an alternative to circumvent all FMEA limitations described in section 3.1. In this sense, some of the contributions of this study towards risk analysis and prioritization in processes are: (1) the possibility of considering the weights (level of importance) of different criteria; (2) it supports decisions in scenarios of uncertainty through the use linguistic terms and fuzzy numbers to express the decision makers’ judgments; (3) it allows the consideration of other risk factors, in addition to those traditionally used in FMEA; (4) it supports group decision making, so as to consider the knowledge and experience of experts from different areas. When compared to the multicriteria models proposed by Ekmekçioğlu & Kutlu (2012) and Kutlu & Ekmekçioğlu (2012), which use Fuzzy TOPSIS for assessing failure modes, but apply Fuzzy AHP to determine weights, the proposed model requires a smaller number of judgments from experts to assess weights. In addition, the weight determination procedure is simpler, easy to understand, does not require consistency tests nor generates null criteria weights. Another contribution is that, unlike most of the models shown in Chart 2, this study considers the internal and external costs of failures as a determinant factor in their prioritization.

Future studies may apply the model in companies from other industrial sectors for prioritizing risks from potential and/or known failures. It can also be adapted to analyze risks

in service companies, to consider criteria which are specific for such environments, such as the existence of customer recovery mechanisms. In addition to process failure analysis, the model can be applied to analyze other types of risks, including work-related accidents. Although the computational implementation in MS Excel contributes towards calculation transparency and facilitates replication, the model can also be implemented in the form of software with graphical interface to promote its usability. Regarding the proposal's limitations, since the model focuses on situations of uncertainty, it is not possible to use exact numerical values (such as number of failures expressed in ppm) as input scores. Another limitation of this study is to have used only four criteria for failure analysis, even though this was a choice made by the decision makers. Future studies may consider criteria related to environmental impacts and other risk factors relevant to the implementation environment.

## References

- Ahmadi, M., Molana, S. M. H., & Sajadi, S. M. (2017). A hybrid FMEA-TOPSIS method for risk management, case study: Esfahan Mobarakeh Steel Company. *International Journal of Process Management and Benchmarking*, 7(3), 397-408. <http://dx.doi.org/10.1504/IJPMB.2017.084913>.
- Almeida, A. T. (2013). *Processo de decisão nas organizações: construindo modelos de decisão multicritério* (1. ed.). São Paulo: Atlas.
- ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS, ABNT. (2018). NBR ISO 31000. Gestão de riscos: diretrizes. Rio de Janeiro. ABNT.
- Aven, T. (2011). Selective critique of risk assessments with recommendations for improving methodology and practise. *Reliability Engineering & System Safety*, 96(5), 509-514. <http://dx.doi.org/10.1016/j.ress.2010.12.021>.
- Aven, T., & Renn, O. (2009). On risk defined as an event where the outcome is uncertain. *Journal of Risk Research*, 12(1), 1-11. <http://dx.doi.org/10.1080/13669870802488883>.
- Banduka, N., Tadić, D., Mačuzić, I., & Crnjac, M. (2018). Extended process failure mode and effect analysis (PFMEA) for the automotive industry: the FSQC-PFMEA. *Advances in Production Engineering & Management*, 13(2), 206-215. <http://dx.doi.org/10.14743/apem2018.2.285>.
- Bertrand, J. W. M., & Fransoo, J. C. (2002). Operations management research methodologies using quantitative modeling. *International Journal of Operations & Production Management*, 22(2), 241-264. <http://dx.doi.org/10.1108/01443570210414338>.
- Bozdag, E., Asan, U., Soyer, A., & Serdarasan, S. (2015). Risk prioritization in Failure Mode and Effects Analysis using interval type-2 fuzzy sets. *Expert Systems with Applications*, 42(8), 4000-4015. <http://dx.doi.org/10.1016/j.eswa.2015.01.015>.
- Carpinetti, L. C. R. (2016). *Gestão da qualidade: conceitos e técnicas* (3. ed.). São Paulo: Grupo Gen-Atlas.
- Certa, A., Enea, M., Galante, G. M., & La Fata, C. M. (2017). ELECTRE TRI-based approach to the failure modes classification on the basis of risk parameters: an alternative to the risk priority number. *Computers & Industrial Engineering*, 108, 100-110. <http://dx.doi.org/10.1016/j.cie.2017.04.018>.
- Chang, K. H., Chang, Y. C., & Tsai, I. T. (2013). Enhancing FMEA assessment by integrating grey relational analysis and the decision making trial and evaluation laboratory approach. *Engineering Failure Analysis*, 31, 211-224. <http://dx.doi.org/10.1016/j.engfailanal.2013.02.020>.
- Chen, C. T. (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems*, 114(1), 1-9. [http://dx.doi.org/10.1016/S0165-0114\(97\)00377-1](http://dx.doi.org/10.1016/S0165-0114(97)00377-1).

- Chen, J. K. (2017). Prioritization of corrective actions from utility viewpoint in FMEA application. *Quality and Reliability Engineering International*, 33(4), 883-894. <http://dx.doi.org/10.1002/qre.2064>.
- Ekmekçioğlu, M., & Kutlu, A. C. (2012). A fuzzy hybrid approach for fuzzy process FMEA: an application to a spindle manufacturing process. *International Journal of Computational Intelligence Systems*, 5(4), 611-626. <http://dx.doi.org/10.1080/18756891.2012.718104>.
- García, A., & Gilabert, E. (2011). Mapping FMEA into Bayesian networks. *International Journal of Performability Engineering*, 7(6), 525-537.
- Guarnieri, P. (2015). Síntese dos principais critérios, métodos e subproblemas da seleção de fornecedores multicritério. *Revista de Administração Contemporânea*, 19(1), 1-25. <http://dx.doi.org/10.1590/1982-7849rac20151109>.
- Hajimolaali, M., Kebriaeezadeh, A., Abdollahiasl, A., Safari, H., & Yektadoost, A. (2017). A new approach in identifying and evaluating quality risks in the pharmaceutical industry. *Journal of Pharmacoeconomics and Pharmaceutical Management*, 3(1-2), 17-20.
- Haq, I. U., Izhar, K., Anwar, S., Khan, M. T., Ahmed, B., & Maqsood, S. (2015). Fuzzy logic based failure mode and effect analysis of automotive powertrain assembly systems. *Technical Journal*, 20, 57-64.
- Kahraman, C. (Ed.). (2008). *Fuzzy multi-criteria decision making: theory and applications with recent developments* (Vol. 16). Boston: Springer Science & Business Media. <http://dx.doi.org/10.1007/978-0-387-76813-7>.
- Kutlu, A. C., & Ekmekçioğlu, M. (2012). Fuzzy failure modes and effects analysis by using fuzzy TOPSIS-based fuzzy AHP. *Expert Systems with Applications*, 39(1), 61-67. <http://dx.doi.org/10.1016/j.eswa.2011.06.044>.
- Lima, F. R., Jr., & Carpinetti, L. C. R. (2015). A comparison between TOPSIS and Fuzzy-TOPSIS methods to support multicriteria decision making for supplier selection. *Gestão & Produção*, 22(1), 17-34. <http://dx.doi.org/10.1590/0104-530X1190>.
- Liu, H. C., Liu, L., & Liu, N. (2013). Risk evaluation approaches in failure mode and effects analysis: a literature review. *Expert Systems with Applications*, 40(2), 828-838. <http://dx.doi.org/10.1016/j.eswa.2012.08.010>.
- Liu, H. C., Liu, L., Bian, Q. H., Lin, Q. L., Dong, N., & Xu, P. C. (2011). Failure mode and effects analysis using fuzzy evidential reasoning approach and grey theory. *Expert Systems with Applications*, 38(4), 4403-4415. <http://dx.doi.org/10.1016/j.eswa.2010.09.110>.
- Magalhães, W., & Lima, F. R., Jr. (2019). Model for risk prioritization based on FMEA and fuzzy-TOPSIS. *Mendeley Data*, 1. <http://dx.doi.org/10.17632/d76ddk3zmk.1>.
- Mahmoodi, A. M., & Mirzazadeh, A. (2014). A New Analysis of Failure Models and Effects by Fuzzy Todim with using Fuzzy Time Function. *International Journal of Fuzzy Logic System*, 4(2), 7-21. <http://dx.doi.org/10.5121/ijfls.2014.4202>.
- Maleki, H., & Saadat, Y. (2013). Comparison of failure mode and effects analysis by using AHP vs. REMBRANDT system. *International Journal of Industrial and Systems Engineering*, 14(1), 5-19. <http://dx.doi.org/10.1504/IJISE.2013.052918>.
- Pedrycz, W., & Gomide, F. (2007). *Fuzzy systems engineering: toward human-centric computing*. Hoboken: John Wiley & Sons. <http://dx.doi.org/10.1002/9780470168967>
- Pillay, A., & Wang, J. (2003). Modified failure mode and effects analysis using approximate reasoning. *Reliability Engineering & System Safety*, 79(1), 69-85. [http://dx.doi.org/10.1016/S0951-8320\(02\)00179-5](http://dx.doi.org/10.1016/S0951-8320(02)00179-5).
- Tixier, J., Dusserre, G., Salvi, O., & Gaston, D. (2002). Review of 62 risk analysis methodologies of industrial plants. *Journal of Loss Prevention in the Process Industries*, 15(4), 291-303. [http://dx.doi.org/10.1016/S0950-4230\(02\)00008-6](http://dx.doi.org/10.1016/S0950-4230(02)00008-6).
- Wang, C. H. (2011, december). A novel approach to conduct risk analysis of FMEA for PCB fabrication process. In *2011 IEEE International Conference on Industrial Engineering and*

*Engineering Management* (pp. 1275-1278). New York: IEEE.  
<http://dx.doi.org/10.1109/IEEM.2011.6118121>.

- Xiao, N., Huang, H. Z., Li, Y., He, L., & Jin, T. (2011). Multiple failure modes analysis and weighted risk priority number evaluation in FMEA. *Engineering Failure Analysis*, 18(4), 1162-1170. <http://dx.doi.org/10.1016/j.engfailanal.2011.02.004>.
- Zadeh, L. A. (1973). Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-3(1), 28-44. <http://dx.doi.org/10.1109/TSMC.1973.5408575>.
- Zhao, H., You, J. X., & Liu, H. C. (2017). Failure mode and effect analysis using MULTIMOORA method with continuous weighted entropy under interval-valued intuitionistic fuzzy environment. *Soft Computing*, 21(18), 5355-5367. <http://dx.doi.org/10.1007/s00500-016-2118-x>.