Experimental and numerical evaluation of viscoelastic sandwich beams

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Abstract

Viscoelastic materials can dissipate a large amount of energy when subjected to cyclic shear deformations, but they have low bearing capacity. Therefore they are often employed as a damping layer in sandwich structures. These sandwich structures present a high damping ratio and simple application.

In order to design sandwich structures, many aspects ranging from computer modeling to laboratory testing should be considered. In this study, a test set of experiments were performed and results are compared with a numerical GHM (Golla, Hughes and Mc Tavish method) based model, in order to establish a method to support viscoelastic sandwich beam design.

In this way, starting from the dynamic properties of a viscoelastic material, a numerical model is used to evaluate the behavior of these structures. Comparisons with uncontrolled structures are also presented, showing the dissipative characteristics of this passive control.

Keywords: sandwich structure, viscoelasticity, GHM model.

1. Introduction

Aiming for the reduction of structural vibrations, several techniques were developed to increase structural damping. Among these techniques, the passive control with viscoelastic materials has shown reasonable efficiency. These materials have low bearing properties with a high dissipative capacity when subjected to cyclic deformations. That is the main reason to apply viscoelastic materials in sandwich layers with stiff elastic materials (Felippe *et al.* 2013).

In this way, in order to effectively reduce structural vibrations using viscoelastic materials, it is important to understand the dynamic behavior of the structure and the viscoelastic material (VEM) used. This type of control system has experienced a growth in practical applications due to some benefits related to cost-effectiveness and a high level of dynamic damping (Battista *et al.* 2010, Kim *et al.* 2006, Moliner *et al.* 2012, Saidi *et al.* 2011).

Within this context, this paper will discuss the computational modeling of viscoelastic materials and their use for reducing vibrations in structures, working as a passive control mechanism in sandwich layers. Aiming to numerically simulate the behavior of viscoelastic sandwich beams, an experimental program with rectangular cross-section beams was conducted and the results compared with those obtained with a computational model. In order to build the computational models, a viscoelastic finite element (CST) using the GHM method (Golla, Hughes and Mc Tavish method) was developed.

2. The GHM method for viscoelastic materials modeling

The stress-strain relation on Laplace's domain as mentioned by Golla, Hughes and Mc Tavish (1985) may be written as:

$$\sigma(s) = [E_0 + h(s)]\, \varepsilon(s) \tag{1}$$

where s is the Laplace operator, σ(s) and ε(s) are, respectively, the stress and strain on Laplace's domain, E_0 is the elastic fraction of complex modulus and *h(s)* is the relaxation function. Function *h(s)* can be written using Biot's series with four terms (or two GHM terms):

$$h(s) = \alpha_1 \frac{s^2 + \beta_1 s}{s^2 + \beta_1 s + \delta_1} + \alpha_2 \frac{s^2 + \beta_2 s}{s^2 + \beta_2 s + \delta_2} \tag{2}$$

where *N* is the number of terms, αi, βi and δi are materials constants and (αi, βi, δi) ≥ 0.

Starting from the equation of motion in the Laplace domain:

$$\{ Ms^2 + K^I \}\, q(s) = f^I(s) \tag{3}$$

where M, K^I and $f^I(s)$ are respectively the mass, stiffness and external loading in the Laplace domain, where:

$$K^I = [E_0 + h(s)]k_v \tag{4}$$

where: K_v is the rigidity fraction associated with geometrical characteristics of the model. The GHM model defines the equation of motion in the time domain as:

$$\bar{\mathbf{M}}\ddot{\mathbf{q}} + \bar{\mathbf{C}}\dot{\mathbf{q}} + \bar{\mathbf{K}}\mathbf{q} = \bar{\mathbf{f}}, \tag{5}$$

where:

$$\bar{\mathbf{M}} = \begin{bmatrix} M & 0 & 0 \\ 0 & \frac{\alpha_1}{\delta_1}K_v & 0 \\ 0 & 0 & \frac{\alpha_2}{\delta_2}K_v \end{bmatrix}, \quad \bar{\mathbf{C}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\alpha_1\beta_1}{\delta_1}K_v & 0 \\ 0 & 0 & \frac{\alpha_2\beta_2}{\delta_2}K_v \end{bmatrix}, \tag{6}$$

$$\bar{\mathbf{K}} = \begin{bmatrix} K_v(E_0 + \alpha_1 + \alpha_2) & -\alpha_1 K_v & -\alpha_2 K_v \\ -\alpha_1 K_v & K_v & 0 \\ -\alpha_2 K_v & 0 & K_v \end{bmatrix}, \quad \mathbf{q} = \begin{Bmatrix} q \\ \hat{z}_1 \\ \hat{z}_2 \end{Bmatrix}, \quad \bar{\mathbf{f}} = \begin{Bmatrix} f(t) \\ 0 \\ 0 \end{Bmatrix}, \tag{7}$$

and $\hat{z}_i$ is the auxiliary variable introduced into the problem, called dissipation variable.

Generalizing Eq. (5) for *n* degrees of freedom, Eqs. (6-7) may be written as:

$$\bar{\mathbf{M}} = \begin{bmatrix} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\alpha_1}{\delta_1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{\alpha_2}{\delta_2}\mathbf{I} \end{bmatrix}, \quad \bar{\mathbf{C}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\alpha_1\beta_1}{\delta_1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{\alpha_2\beta_2}{\delta_2}\mathbf{I} \end{bmatrix}, \tag{8a}$$

$$\bar{\mathbf{K}} = \begin{bmatrix} \mathbf{K}_v(E_0 + \alpha_1 + \alpha_2) & -\alpha_1\mathbf{R} & -\alpha_2\mathbf{R} \\ -\alpha_1\mathbf{R}^T & \alpha_1\mathbf{I} & \mathbf{0} \\ -\alpha_2\mathbf{R}^T & \mathbf{0} & \alpha_2\mathbf{I} \end{bmatrix}, \quad \bar{\mathbf{q}} = \begin{Bmatrix} \mathbf{q} \\ \hat{\mathbf{z}}_1 \\ \hat{\mathbf{z}}_2 \end{Bmatrix}, \quad \bar{\mathbf{f}} = \begin{Bmatrix} \mathbf{f(t)} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{8b}$$

where:

$$K_v = T^T \Lambda T, \tag{9}$$

and Λ is a diagonal matrix consisting of the non-zero eigen-values of the stiffness matrix normalized with respect to the elastic modulus; **T** is the matrix of vectors corresponding to the non-zero eigen-values of the matrix $(1/E)K_{elastic}$; $R = T\Lambda 1/2$ and $z_i = R\,\hat{z}_i$.

As shown in Eqs. (2-8) the number of dissipative degrees of freedom associated with viscoelastic elements depends on the number of terms used in the relaxation function and the number of rigid body motions (Barbosa 2000). It should be noted that the greater the number of terms used to write the relaxation function, the more accurate the model will be.

3. Experimental program

A set of tests was performed with viscoelastic sandwich cantilever beams. The beams were divided into four groups in accordance with their layer configuration: a VS1 beam, with two elastic layers (base beam and restraining layer, both clamped) and one viscoelastic layer; a VS1c beam, with two elastic layers (base beam clamped and free restraining layer) and one viscoelastic layer; a VS2 beam, with three elastic layers (one base beam and two restraining layers, all clamped) and two viscoelastic layers; and a VS2c beam, with three elastic layers (one base beam clamped and two free restraining layers) and two viscoelastic layers. The layer configuration of the beams can be seen in Fig. 1.

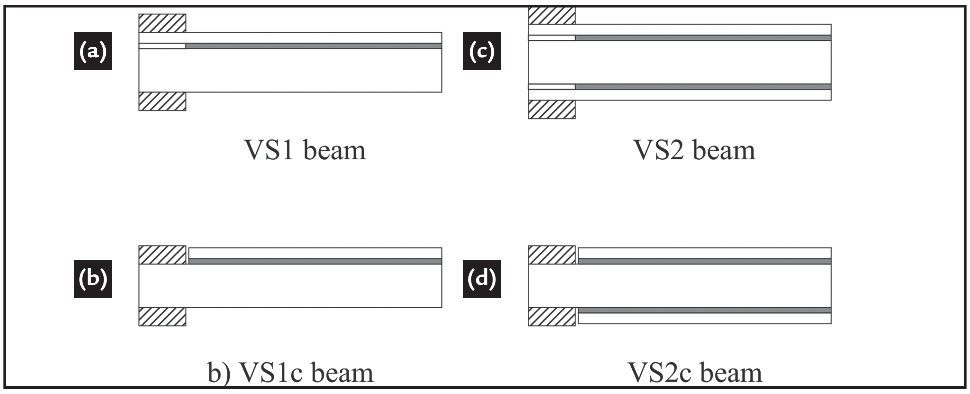

Figure 1
Longitudinal section of the analyzed beams.

These beams have a rectangular cross section and 1140 mm length, working as a base structure (with 16.1mm height) and onto this structure, viscoelastic layers (with 2.0mm height) and elastic constraining layers (with 3,17mm height) were mounted, as shown in Fig. 1. Aluminum was the material used for the elastic layers and VHB 4955 was used as the viscoelastic material.

These beams were stimulated under the action of a hammer impact at 15 cm from the cantilever and at 15, 20 and 25 cm from the cantilever, transversal displacements were observed during the period. In Fig. 2, one can see how the beams were instrumented with LVDT sensors fabricated by Balluff, which could register displacements without touching the structure.

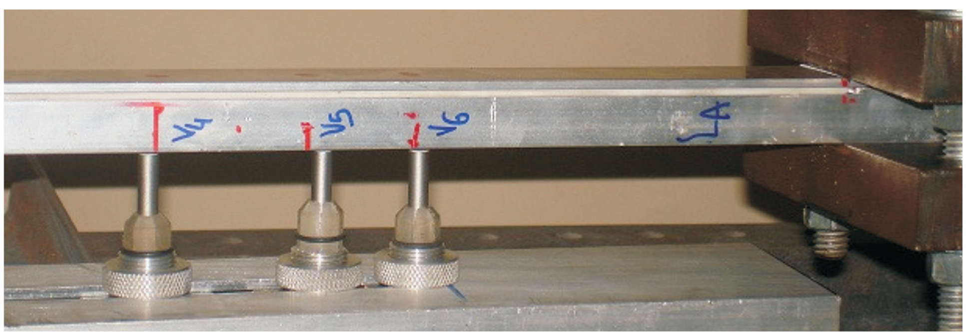

Figure 2
Viscoelastic sandwich beam ready to be tested.

Two elastic beams, A and B, were tested and then the sandwich beam configuration was mounted on these elastic beams, making a total of two specimens. The natural frequencies and damping ratios of the respective beams A and B are shown in Table 1.

Table 1
Natural frequencies and damping ratios of beams A and B.

Vibration mode	Beam A		Beam B	
	Natural frequency (Hz)	Damping ratio (%)	Natural frequency (Hz)	Damping ratio (%)
1	10.25±0.00	0.05±0.00	10.24±0.00	0.05±0.00
2	63.38±0.00	0.03±0.00	63.70±0.00	0.04±0.00
3	179.00±0.00	0.06±0.00	179.26±0.00	0.05±0.00

For the specimens VS1, VS1c, VS2 and VS2c the natural frequencies and their damping ratios are listed in Tables 2-5, respectively.

Vibration mode	Beam A		Beam B	
	Natural frequency (Hz)	Damping ratio (%)	Natural frequency (Hz)	Damping ratio (%)
1	11.31±0.02	4.98±0.11	11.03±0.02	4.44±0.01
2	63.37±0.17	4.90±0.19	61.76±0.14	4.32±0.05
3	175.05±0.12	4.39±0.01	168.08±0.23	3.28±0.06

Table 2
Natural frequencies and damping ratios of VS1 specimens.

Vibration mode	Beam A		Beam B	
	Natural frequency (Hz)	Damping ratio (%)	Natural frequency (Hz)	Damping ratio (%)
1	9.82±0.00	2.74±0.01	8.41±0.01	2.23±0.02
2	63.70±0.04	4.80±0.10	55.09±0.06	3.48±0.06
3	174.05±0.35	4.44±0.04	145.48±0.16	3.86±0.07

Table 3
Natural frequencies and damping ratios of VS1c specimens.

Vibration mode	Beam A		Beam B	
	Natural frequency (Hz)	Damping ratio (%)	Natural frequency (Hz)	Damping ratio (%)
1	-	-	12.34±0.05	7.92±0.11
2	-	-	64.79±0.37	8.65±0.20
3	-	-	173.29±0.90	6.17±0.49

Table 4
Natural frequencies and damping ratios of VS2 specimens.

Vibration mode	Beam A		Beam B	
	Natural frequency (Hz)	Damping ratio (%)	Natural frequency (Hz)	Damping ratio (%)
1	8.26±0.00	4.75±0.31	9.82±0.01	5.14±0.03
2	56.81±0.22	6.67±0.02	6.67±0.31	8.60±0.20
3	146.04±1.44	4.73±0.56	4.73±0.41	5.90±0.97

Table 5
Natural frequencies and damping ratios of VS2c specimens.

3.1 GHM Model's parameters

There are several methodologies for characterizing the Complex Modulus of viscoelastic materials: ASTM Standard Method (ASTM 1993), Direct Method (Faisca 1998) and Indirect Method (Masterson and Miles 1995). These methods basically register the temporal responses at a given temperature, when a specimen is forced into shear or axial deformation.

After the values of Complex Modulus are experimentally determined, one can adjust the curves of the real part of the Complex Modulus and loss factor for the points obtained experimentally. In the case of the formulation GHM, the complex shear modulus is given by:

$$G'(\omega) = G_0 + \alpha_1 \frac{\omega^2\left(\omega^2 - \delta_1 + \beta_1^{\,2}\right)}{\left(\delta_1 - \omega^2\right)^2 + \beta_1^{\,2}\omega^2} + \alpha_2 \frac{\omega^2\left(\omega^2 - \delta_2 + \beta_2^{\,2}\right)}{\left(\delta_2 - \omega^2\right)^2 + \beta_2^{\,2}\omega^2} \quad (10)$$

$$\eta(\omega) = \frac{1}{G'(\omega)}\left(\frac{\alpha_1\beta_1\delta_1\omega}{\left(\delta_1 - \omega^2\right)^2 + \beta_1^{\,2}\omega^2} + \frac{\alpha_2\beta_2\delta_2\omega}{\left(\delta_2 - \omega^2\right)^2 + \beta_2^{\,2}\omega^2}\right) \quad (11)$$

and these functions are used to determine the GHM parameters.

Here it was used the Direct Method, for frequencies between 0 and 200 Hz. Using data from the experiments, the GHM parameters could be determined using the Nonlinear Least Squares Method (Coleman and Li 1994, Coleman and Li 1996). These fitted values are shown in Table 7. Figure 3 shows two graphics comparing the experimental values and the adjusted curves of $G'(\omega)$ and $\eta(\omega)$.

Table 7
GHM parameters
adjusted to the viscoelastic material.

Parameter	Value	
	Term 1	Term 2
E_0	1898260.0 Pa	
α	763774.0 MPa	6873966.0 MPa
β	2.9178×107 s^{-1}	1.2146×107 s^{-1}
δ	3.2408×108 s^{-2}	4.0554×109 s^{-2}

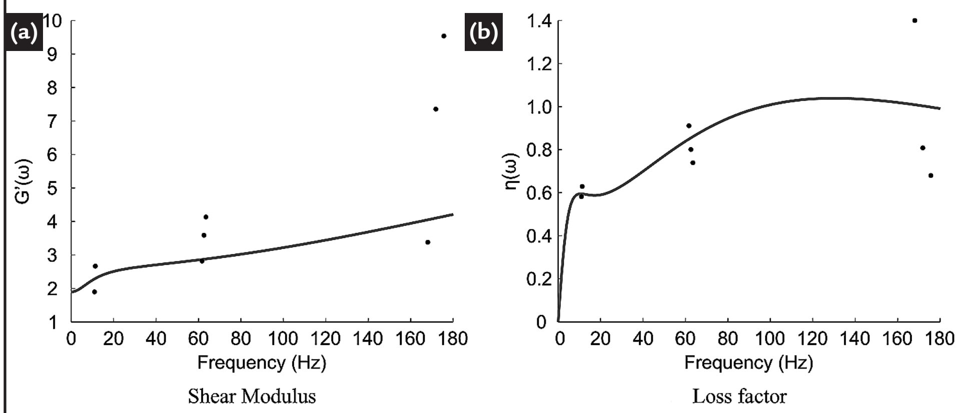

Figure 3
Experimental values
and fitted curves of $G'(\omega)$ and $\eta(\omega)$.

4. Numerical evaluation

Using equations 6-7, it is possible to achieve viscoelastic elements matrices for the linear triangular Finite Element.

The beams were simulated under the action of a hammer impact at 15 cm from the cantilever and, at the same point, transversal displacement was observed along the time. The domain of the structure was discretized with linear triangular element meshes like the one shown in Figure 4.

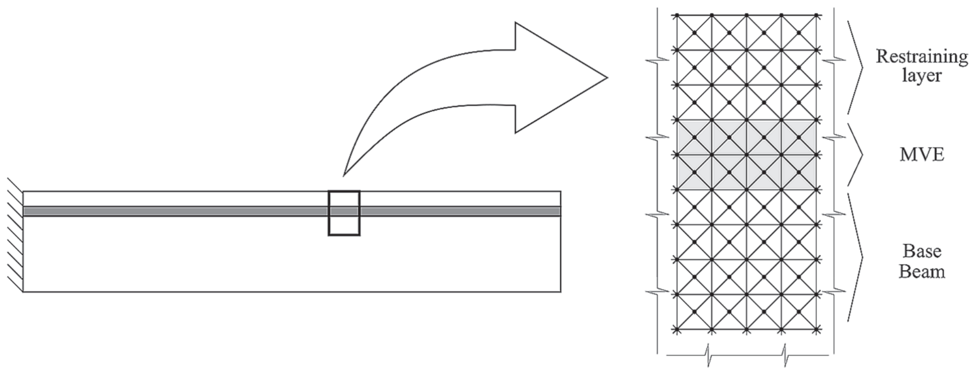

Figure 4
Structural Finite Element discretization.

In order to establish the models, considered were the mechanical properties listed in Table 8.

Table 8
Mechanical properties
adopted for sandwich structure.

Mechanical property	Elastic layer	Viscoelastic layer
Density (kg/m)	8794.0	795.0
Poisson's coefficient	0.30	0.49
Elastic Modulus (GPa)	109.6	-

With the model, meshes and mechanical properties presented, the time response of beams could be obtained. In order to obtain the natural frequencies, a spectral response was constructed and the time response signal filtered around the first three natural frequencies identified; then, to obtain the damping ratios, the obtained filtered signal was used. The relationship between the numerical and experimental data can be seen in Figures 5-8.

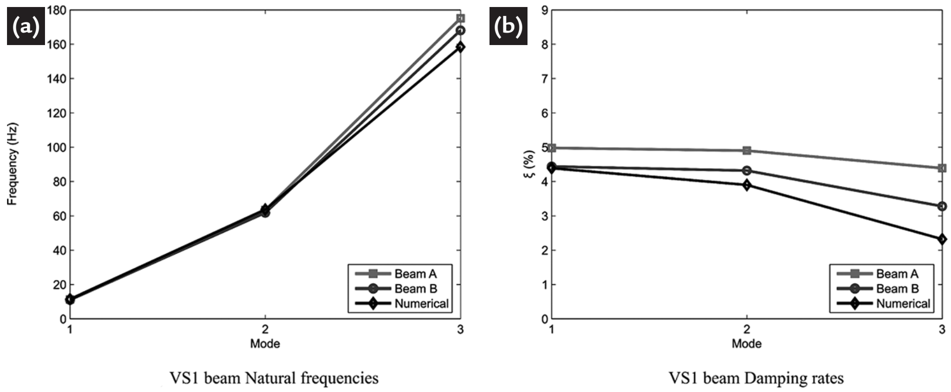

Figure 5
Numerical and experimental frequencies and damping ratios identified for VS1 beam.

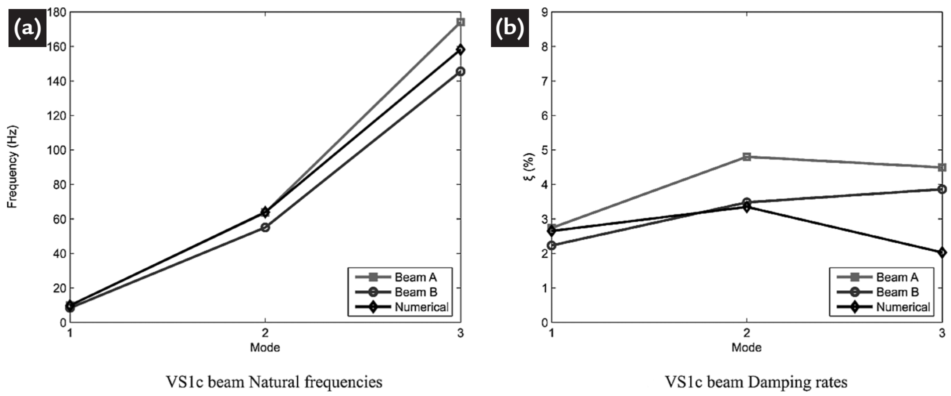

Figure 6
Numerical and experimental frequencies and damping ratios identified for VS1c beam.

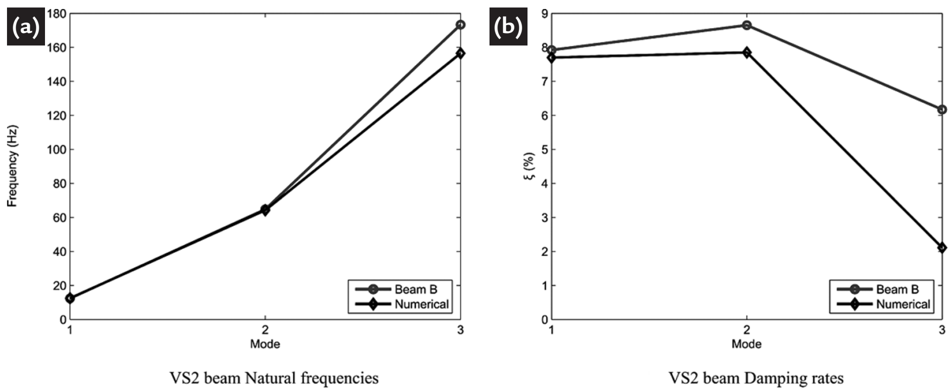

Figure 7
Numerical and experimental frequencies and damping ratios identified for VS2 beam.

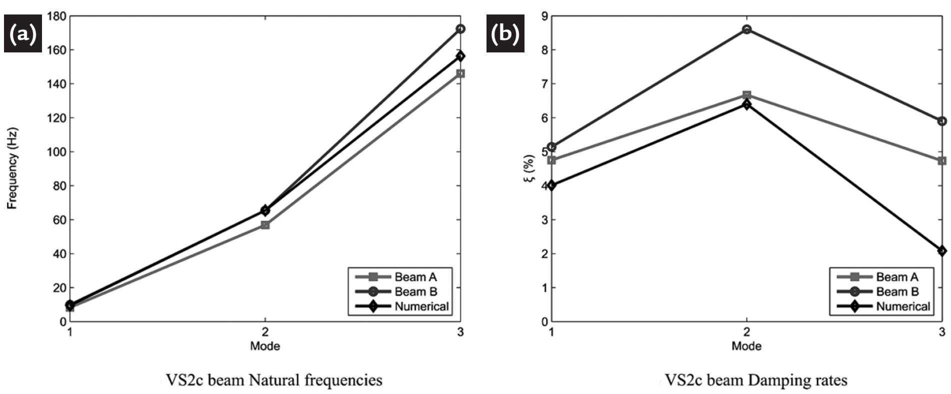

Figure 8
Numerical and experimental frequencies and damping ratios identified for VS2c beam.

As one can see, the natural frequencies predicted with the numerical model show good agreement with those identified experimentally and one can state that the GHM model used provided consistent results in terms of the damping ratio and natural frequencies.

5. Conclusions

This study evaluated some beams with and without sandwich viscoelastic damping treatment. It has been shown that this type of passive control significantly improves the damping ratios of an elastic base beam, from 44.6× (for VS1c layout) to 173× (for VS2c layout), confirming the effectiveness of this treatment. As can be seen, beams with clamped restraining layers presented a higher damping ratio than those where this layer was not restrained. This behavior was expected since, when these layers are constrained, larger shear deformations are imposed on VEM. Consequently, the material dissipates more energy.

Also evaluated was a computational model for viscoelastic materials acting as structural vibration dampers: the GHM Method. Analyzing the obtained responses for the cantilever beams, one can observe that despite the good correlation between the fitted curves and the experimental data, the damping factor obtained through the numerical model was, in general, underestimated. Notwithstanding, there was good agreement between the natural frequencies obtained with the model and the experimental values. Obviously these differences cannot be attributed only to curve fitting. Other factors such as: the methodology used on modal identification; 2) Dispersion of experimental results; and 3) The Finite Element discretization, also play a significant influence on the numerical results.

In previous studies, it was possible to observe the influence of the curve fit over the numerical results. Apparently, this model tends to underestimate the damping ratios.

Therefore it is considered that since the GHM model provided results close to the experimental data and in favor of safety, this model can be a useful tool to project or simulate sandwich beams.

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