Volume 1, Number 1 (March, 2008) p. 1 - 16 • ISSN 1983-4195
REVISTA IBRACON DE ESTRUTURAS E MATERIAIS
IBRACON STRUCTURES AND MATERIALS JOURNAL

Similarities between punching and shear strength of steel fiber reinforced concrete (SFRC) slabs and beams

Similaridades entre resistência à punção em lajes-cogumelo e ao cisalhamento em vigas de concreto armado com fibras de aço


J.B. DE HANAI[a]
jbhanai@sc.usp.br

K.M.A. HOLANDA[b]
accetti@cnpq.br

Abstract

This paper discusses the influence of steel fiber in both punching strength of flat slabs and shear strength of concrete beams. Similarities in the structural behavior were observed in an experimental analysis of analogous slabs and beams. The analogous slabs and beams had the same height, longitudinal reinforcement ratio and concrete properties. Concrete mixtures were designed to attain different strength levels, from ordinary to high-strength range. Hooked-end steel fibers with circular section and rectangular section and different aspect ratios were used. Ultimate load capacity and ductility of analogous slabs and beams showed the same performance tendencies as the fiber content varied from 0% to 2%. Analytical investigations were developed to evaluate the ultimate load capacity of slabs and beams. The main conclusions are: a) shear tests on prismatic beams provide useful information for steel fiber reinforced concrete (SFRC) mixture design for slab application; b) theoretical strength models that have a linear dependence on fiber content can be used to predict the effect of fiber addition.

Keywords: punching shear; shear strength; flat slab; beam; steel fiber reinforced concrete (SFRC).

Resumo

O artigo analisa a influência da adição de fibras de aço ao concreto, tanto na resistência à punção em lajes-cogumelo quanto na resistência ao cisalhamento em vigas de concreto armado. Pela análise dos resultados de ensaios de modelos estruturais de lajes e vigas em condições análogas, foram identificadas similaridades de efeitos no seu comportamento estrutural. As lajes e vigas análogas testadas tinham a mesma altura, taxa de armadura longitudinal e propriedades do concreto. Os concretos aplicados foram dosados para atingir diferentes níveis de resistência, inclusive aquelas consideradas elevadas. As fibras empregadas eram de fios de seção transversal circular e retangular, com ganchos nas extremidades e diferentes relações de aspecto. A capacidade resistente última e a ductilidade das lajes e vigas análogas mostraram as mesmas tendências, conforme o volume de fibras de aço variou entre 0% e 2%. Desenvolveu-se uma análise teórica para avaliar a capacidade resistente última das lajes e vigas, sendo que as principais conclusões foram: a) os ensaios de flexão sobre prismas de concreto fornecem informações muito úteis para a dosagem de concretos com fibras de aço e sua aplicação em lajes sujeitas à punção; b) modelos teóricos que assumem uma dependência linear entre capacidade resistente (à punção e ao cisalhamento) e o volume de fibras oferecem resultados satisfatórios para dimensionamento.

Palavras-chave: concreto com fibras; laje-cogumelo; vigas; punção; cisalhamento.

[a] *Escola de Engenharia de São Carlos da Universidade de São Paulo, Departamento de Engenharia de Estruturas, e-mail: jbhanai@sc.usp.br, Av. Trabalhador Sao-Carlense, 400, São Carlos – SP, 13566-590, Brasil.*

[b] *Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), e-mail:accetti@cnpq.br, SEPN 509, Bloco A, Ed. Nazir I, Brasilia – DF, 70750-501, Brasil.*

1. Introduction

1.1 Brief review

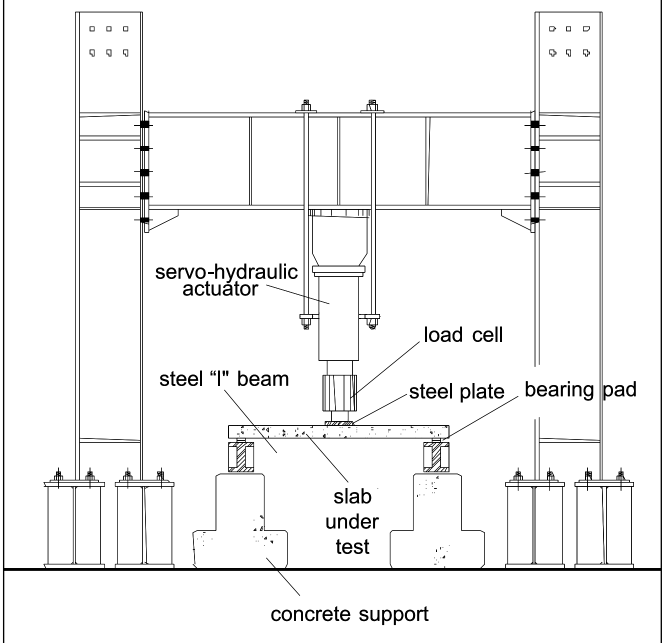
Flat slab structural systems have widely been used in building construction due to several advantages that include architectural design and fast construction methods. Technological advances in this field came with new prestressing materials and techniques, also the application of special concretes such as the self-compacting concrete and high-strength concrete.
Punching shear strength of flat slabs is the most important limitation of this particular structural system due to its fragile nature. This kind of failure must be avoided by providing the slab-column connection of efficient mechanisms of strength and ductility.
Several investigations have shown the performance improvement of the slab-column connection by using steel fiber concrete. Both strength and ductility enhancement were observed by Swamy & Ali [1], Alexander & Simmonds [2], Theodorakopoulos & Swamy [3], Tan & Paramasivam [4], Shaaban & Gesund [5], Harajli et al. [6], Hughes & Xiao [7], Prisco & Felicetti [8] and McHarg et al. [9].
The combination of high-strength concrete and steel fibers shows interesting results because punching strength can be raised without loss in ductility. Zambrana Vargas [10], Azevedo [11] and Holanda [12] performed punching tests in many combinations of ordinary and high strength concrete, steel fibers and shear reinforcement. The test results confirmed the expectations, also revealing a good efficiency of steel fibers in high-strength matrices.
It is well known that phenomena of punching in slabs and shear in beams can be represented by strength mechanisms of the same nature. In both cases the ultimate load is affected by the intensity of diagonal tension, arching effect, friction action in cracks and dowel action.
A tentative analysis of the similarities between punching strength in slabs and shear strength in beams is reported in this paper. Slab-column connections and analogous simply supported beams were tested, mainly to observe the effect of steel fiber addition to concrete on the ultimate load and ductility. The variables in the tests were concrete strength, type and volume fraction of steel fibers,

Table 1 – Characteristics of the concrete mixtures

Materials	Series S1	Series S2 and S3	Series S4	Series S5
Mixture proportions[(1)]	1:1.94:2.06:0.65	1:0.76:1.24:0.34	1:1.8:2.5:0.5	1:1.33:2.33:0.34
Portland cement (kg/m^3)	424.8 (Type IS)	718.6 (Type IS)	423.2 (Type IS)	470.6 (Type III)
River sand (kg/m^3)	824	546	761	626
Coarse aggregate (kg/m^3)	875	891	1056	1096
Water (kg/m^3)	276.1	244.3	211.3	160
Superplasticizer (kg/m^3)	0 0.66% 0.99%	1.0% 1.7% 2.0%	0.5% 1.0%	0 3% 3%
Silica fume (kg/m^3)	-	-	-	47.1
Steel fiber (kg/m^3)	0 78.5 157 Dramix ZP-305[(2)]	0 78.5 157 Dramix ZP-305[(2)]	59.85 119.70 Dramix RL-45/50[(3)]	0 59.85 119.70 Harex HSCF25[(4)]

[(1)] Proportions by weight (cement:sand:coarse aggregate: water)
[(2)] ℓ=30 mm (fiber length); D = 0.55 mm (diameter); ℓ/D = 54.5; f_y = 1150 MPa
[(3)] ℓ=50 mm (fiber length); D = 1.05 mm (diameter); ℓ/D = 48; f_y = 1000 MPa
[(4)] ℓ=25 mm (fiber length); D = 0.667 mm (equivalent diameter); ℓ/D = 37.48; f_y = 770 MPa

Figure 1 – Scheme of the slab testing system



height of slabs and beams and dimension of the loaded area.
The ACI 318M-02 [13] design method was adapted to consider the steel fiber action. The theoretical results are compared to the experimental ones and a final discussion is presented.

1.2 Justification

Slabs and beams without transversal reinforcement may reach an ultimate limit state without yielding of the flexural reinforcement. A fragile failure may occur and this kind of ruin must be avoided.
Strength mechanisms in slab punching and beam shear are similar, both in relation to the main mechanisms (diagonal tension, contribution of the compressed struts) and to complementary mechanisms (dowel effect of the longitudinal reinforcement, friction between crack surfaces, arching effect, strut flexural stiffness).
The rising of concrete strength increases the punching strength and shear strength, but high-strength concrete presents lower toughness. When fracture occurs, its surface is smoother and this leads to decreasing of the positive effects of friction and interlocking of aggregates. The addition of steel fibers affords higher toughness to concrete and provides bridging effects across cracks, both in the fracture surface and around the reinforcement.
The test results of this research showed the feasibility of shear tests on prismatic beams to choose the type and amount of steel fibers to be used in flat-slabs. These tests would be simpler and cheaper than the complex slab-column connection tests.
There are several theoretical approaches that explain the shear transfer in a slab-internal column connection, with or without punching reinforcement. However, none of the design codes considers fiber's contribution in punching strength of flat slabs.

2. Materials and experimental program

Five series of analogous slabs and beams (S1 to S5) were tested. In Series 1, concrete compressive strength was kept around 25 MPa and steel fiber volume fraction was considered equal to 0%, 1% and 2%.
In Series 2 and 3, concrete strength was about 56 MPa and steel fiber volume fraction varied in the same way. In Series 3, only beams with 170 mm height were tested, to observe any size effect.
In Series 4 punching test results were taken from Azevedo [11] in which a different type of steel fiber was used (0%, 0.75% and 1.5% volume fraction). Complementary slabs and beams were tested to complete the series and concrete strength was kept around 40 MPa.
Finally, in Series 5 only beams were tested to complement the test results obtained by Zambrana Vargas [10]. Concrete strength was about 73 MPa and a third type of steel fiber (0%, 0.75% and 1.5% volume fraction) was added.

2.1 Materials

Concrete mixtures were designed to attain different strength levels, from ordinary to high-strength range. ASTM Type IS Portland cement (blended blast furnace slag cement), river sand (fineness modulus equal to 2.37), basalt coarse aggregate (maximum size equal to 6.3 mm) and a superplasticizer admixture were used in concrete mixtures for test series S1 to S4. For the series S5, ASTM Type III Portland cement (high early strength cement) and silica fume were also applied. Hooked-end steel fibers with circular section and rectangular section and different aspect ratios were used. Table 1 summarizes the concrete mixtures composition.

Figure 2 – Plan view of the slab reinforcement

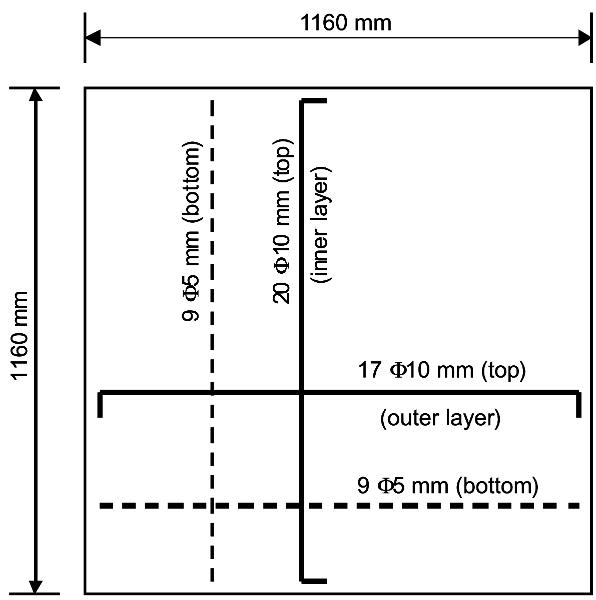
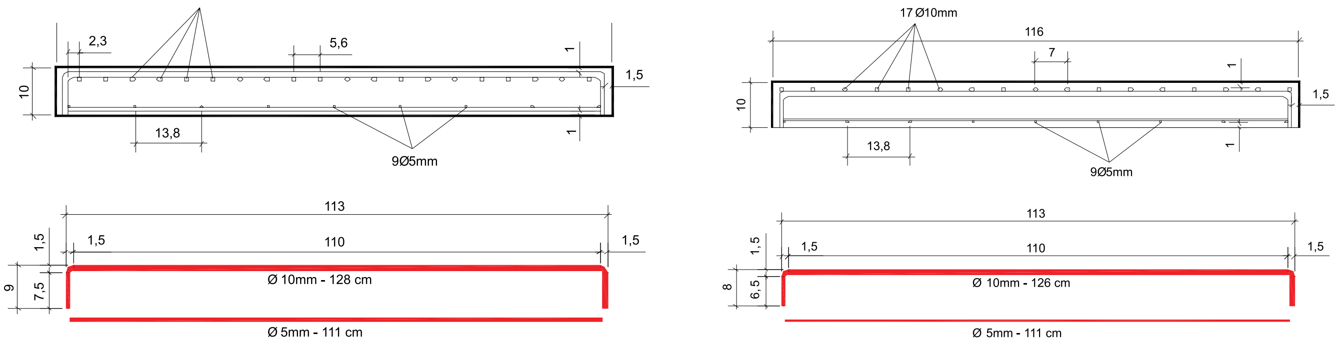


Figure 3 – Slab reinforcement details



a) outer top and bottom layers

b) inner top and bottom layers

2.2 Punching Tests on Slabs

In Series 1, 2 and 4 punching tests were carried out on 1160 mm square and 100 mm thickness slabs. All the slabs were supported along their perimeter and tested by pushing down on the center of the slab through a hydraulic actuator. This means that, referring to a real situation in a building, the load was applied upside down on the tests. Figure 1 illustrate the loading test system.

Figures 2 and 3 show the slab reinforcement characteristics. The drawings show the reinforcing bars as they were positioned for slab casting, in the same way that occurs in a real situation. The top flexural reinforcement consisted of 10 mm diameter bars, 17 in one direction (effective depth of 85 mm) and 20 in the other (effective depth of 75 mm), in order to obtain the same resistant moment in both directions. The bottom flexural reinforcement consisted of nine 5 mm diameter bars in each direction.

Some steel reinforcement bars were instrumented with electrical strain gages to monitor the strains and forces during the tests and to better evaluate the structural behavior of the slabs. Figure 4 shows the location of the strain gages and displacement transducers.

The test force was introduced by a servo-hydraulic actuator over a square steel plate (80 mm side), in a 0.005 mm/s displacement-controlled mode.

2.3 Shear Tests on Beams

In series 1, 2 and 4 shear tests were carried out on 600 mm length, 100 mm height and 120 mm width beams. The height and the flexural reinforcement ratio of these beams were the same of the analogous slabs. The beams were simply supported at their ends and tested by pushing down on the center through a hydraulic actuator.

The beams bottom flexural reinforcement of Series 1, 2 and 4 consisted of two 10 mm diameter bars (effective depth of 85 mm) and the top flexural reinforcement consisted of two 5 mm diameter bars. Also in the beams some steel reinforcement bars were instrumented with electrical strain gages to monitor the strains and forces during the tests. Figure 5 shows the reinforcement and instrumentation details for these series.

In Series 3 the beam's height was enlarged to 170 mm to observe eventual size effect in the search of similarities amongst beams and slabs. The beam's length was 1100 mm and the beam's width was 130 mm. The flexural reinforcement ratio was maintained.

In Series 5 the width of the beams was reduced to 110 mm to adjust the flexural reinforcement ratio to be the same of the main reinforcement of the slabs tested by Zambrana Vargas [10]. The beams bottom flexural reinforcement of Series 5 consisted of four 10 mm diameter bars (effective depth of 155 mm) and the top flexural reinforcement consisted of two 5 mm diameter bars.

Table 2 summarizes the main characteristics of the experimental program.

3. Experimental results and discussions

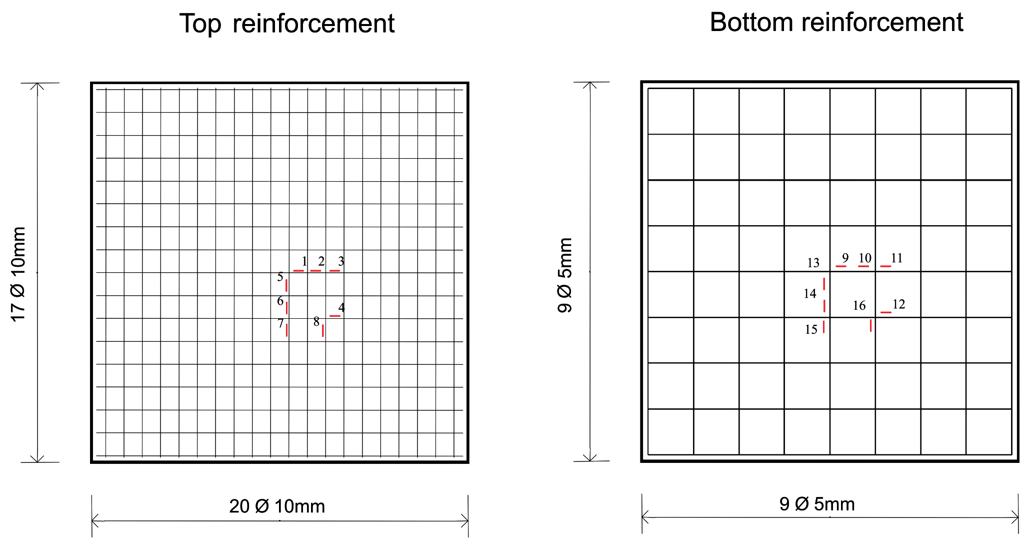
Table 3 presents the ultimate load values recorded on the test of slabs and beams performed by Holanda [12] and Table 4 presents test data from Azevedo [11] and Zambrana Vargas [10].

The graphic in Figure 6 shows a relation between the concrete tensile strength (normalized by the square root of the concrete compressive strength) with the fiber volume fraction added to concrete slabs. The relation showed to be almost linear. Since concrete strength varied on each test series, normalization was applied to reduce the influence of the concrete tensile strength on the test result analysis. Square root of concrete compressive strength was used because it better represents the tensile strength than the plain compressive strength.

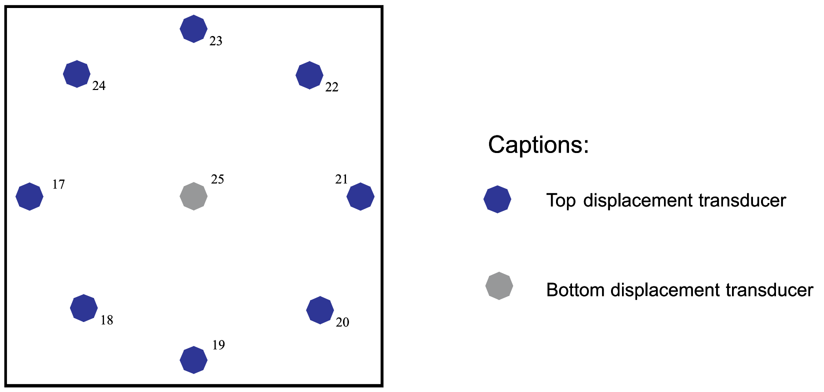
All slabs were designed to fail by punching shear, what really happened. In the steel fiber reinforced slabs, there was a tendency of ultimate loads to be closer to the estimated load for flexural failure, as more fibers were added to concrete. This means that the addition of steel fibers contributed to the ductility of the connection in the rupture. From the test results, it can be seen that the best performance both in strength and ductility was obtained in Series 2, using high strength concrete and 2% of steel fibers.

In all test series, both in slabs and beams, it was observed that strength and ductility increased as the volume of steel fibers increased (Figures 7 and 8). Load-deflection diagrams observed

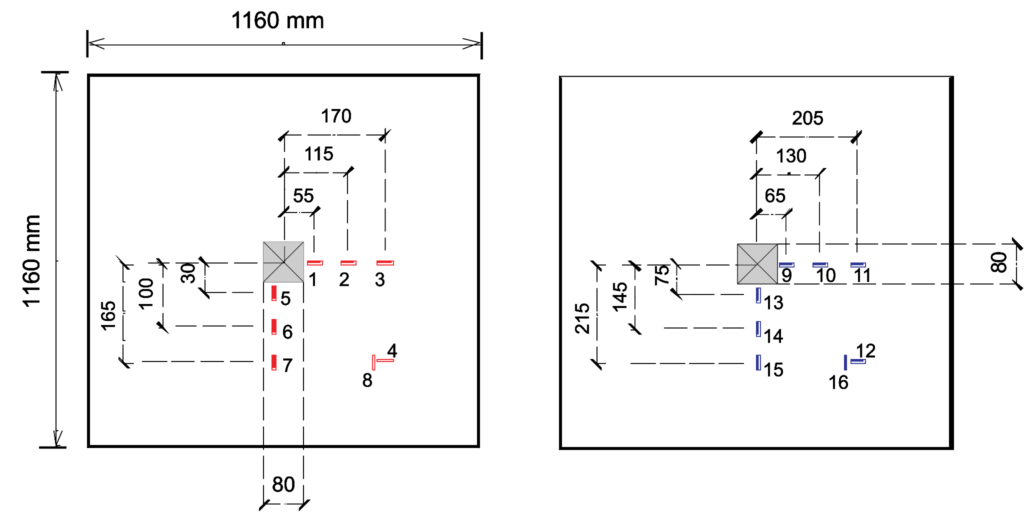
Figure 4 – Location of strain gage and displacement transducer measurement points



a) Location of the electric strain gages in stee lreinforcement bars

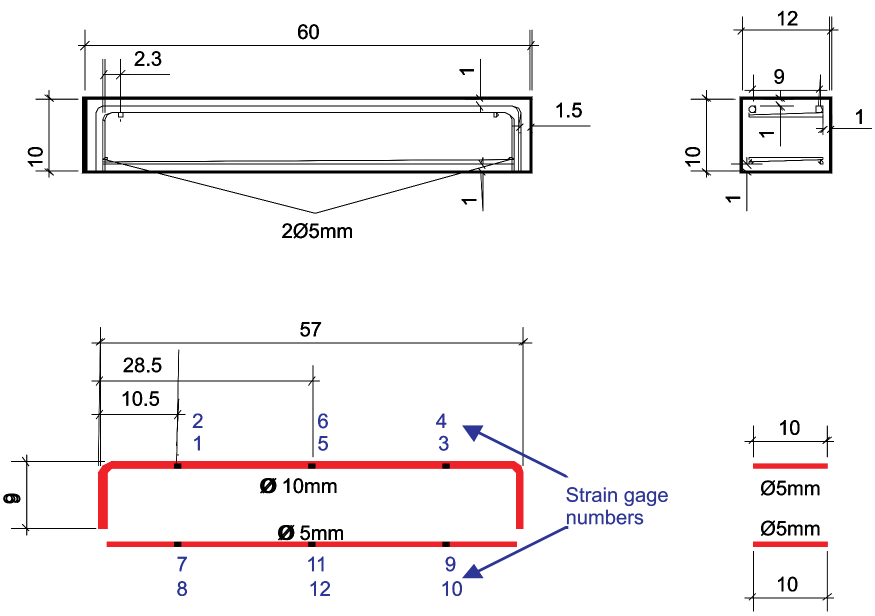


b) Location of the vertical displacement transducers



c) Location of the electric strain gages

Figure 5 – Reinforcement and instrumentation details of the beams of Series 1, 2 and 4



in slabs and beams were similar, mainly looking at the variation of fibers volume. Therefore, a previous performance analysis of beams may offer useful data to design the type and amount of fibers to be used in slabs. The main advantage of this approach is that several small beams can be tested at low cost and, with help of Fracture Mechanics methods, a confident design of fiber reinforced concrete can be designed for slab punching strength and ductility.

The largest normalized punching strength of slabs without fibers was observed in Series 1 slabs ($f_{c14} \cong 25$ MPa), followed by Series 4 slabs ($f_{c14} \cong 40$ MPa), and at last by Series 2 slabs ($f_{c14} \cong 60$ MPa), due to the brittleness of high strength concrete. By adding fibers, one can notice an inversion of this order for amounts higher than 1.5%. The largest punching strength was obtained in Series 2 slabs, followed by Series 4 slabs and by Series 1 slabs. Fibers were more efficient in the high-strength concrete.

In Series 4, the conclusion is that fiber RC 65/30 BN (aspect ratio 66.7) produced greater ductility in slabs than fiber RL 45/50 BN (aspect ratio 48). The modification of the aspect ratio had little influence in the strength of structural elements (Figure 9).

Beams without fibers presented a sudden failure by shear after the formation of inclined cracks, as illustrated in Figure 10. The beams reinforced with high ratios of fibers (Figure 11) had a shear failure concurrently with the yielding of flexural reinforcement.

As occurred with slabs, steel fiber reinforced beams showed a more ductile failure, with a higher stress level in the flexural reinforcement, also showing cracks at the maximum bending moment region.

In all beams of Series 1 to 4, the failure occurred by fiber pull-out, while in the Series 5 beams fiber rupture occurred. The fibers used in Series 5 did not provide high flexural toughness level.

Many authors mentioned the linear relationship between punching strength and specific fiber volume, at least in an interval of fiber volume fraction between 0 and 2%. This fact was also confirmed by Brazilian researchers. Holanda [12] demonstrates that there is a linear relation in the study of shear in beams. Figures 12 to 14 show diagrams of normalized ultimate load versus fiber volume fraction for specimens of Series 2 to 5. Normalization was made dividing the ultimate loads by the square root of the concrete compressive strength, to reduce the influence of concrete strength variation.

Figure 15 shows graphics of relative load (load divided by ultimate load) versus relative displacement (displacement divided by ultimate displacement) of slabs and beams, with the same concrete strength and the same amount of fibers. It is noticed that in both graphics, the pre-peak section of the slab and of both beams is almost coincident.

4. Analytical investigation

4.1 Analysis of beams according to ACI 318M-02

The ACI 318M-02 [13] adopts a relation between concrete tensile strength (split test) and the square root of the compressive strength, as shown in Equation 1.

$$f_{sp} = 0.5563\sqrt{f_c} \qquad (1)$$

Table 2 – Main characteristics of the experimental program

Series	Slab	Beam	$h^{(1)}$ (mm)	$\rho^{(2)}$ (%)	$f_{c14}^{(3)}$ (MPa)	$f_{t14}^{(4)}$ (MPa)	$V_f^{(5)}$ (%)
S1	L1	V1A V1B	100	1.57	23.1	2.14	0
	L2	V2A V2B	100	1.57	24.4	2.59	1
	L3	V3A V3B	100	1.57	28.1	2.98	2
S2	L4	V4A V4B	100	1.57	57.0	3.81	0
	L5	V5A V5B	100	1.57	59.7	5.45	1
	L6	V6A V6B	100	1.57	52.4	6.59	2
S3	-	V7A V7B	170	1.59	57.0	3.81	0
	-	V8A V8B	170	1.59	59.7	5.45	1
	-	V9A V9B	170	195	52.4	6.59	2
S4	-	VP1A VP1B	100	1.57	36.1	3.42	0
	L7	V10A V10B	100	1.57	36.6	3.97	0.75
	L8	V11A V11B	100	1.57	46.1	5.17	1.50
S5	-	V12A V12B	100	1.71	75.3	4.46	0
	-	V13A V13B	100	1.71	73.5	5.65	0.75
	-	V14A V14B	100	1.71	73.1	7.96	1.50

(1) Beam's height or slab thickness
(2) Flexural reinforcement ratio
(3) Concrete compressive strength at 14 days (test age, cylindrical samples)
(4) Concrete tensile strength at 14 days (test age, split test on cylindrical samples)
(5) Fiber volume fraction

being f_c the concrete strength to axial compression and f_{sp} the concrete tensile strength (split test), both in MPa.

Many authors demonstrate a strong correlation between shear strength and the square root of concrete compressive strength. This corresponds to an indirect correlation of shear strength to the concrete tensile strength.

It is well known that when a low volume fraction of steel fibers are added to concrete, the first cracking strength is not significantly increased. However, ductility and post-cracking resistance can be increased, thus providing shear resistance by bridging diagonal cracks. In addition, because fibers would control the opening of these cracks, shear resistance through aggregate interlock would likely be increased.

Equation 1 from ACI Code is very simple and affordable for practical calculations. Therefore, a tentative analysis was done to verify if it also is able to represent by means of a single parameter, the tensile strength of fiber reinforced concrete, the shear load capacity increase in situations of diagonal tension rupture.

At first, to correlate the concrete tensile strength to the compressive strength of the steel fiber reinforced concrete, a linear regression of experimental results was made. Equation

Table 3 – Ultimate test loads on slabs and beams – Holanda(12)

Series	f_{c14} (1) (MPa)	f_{t14} (2) (MPa)	Slab	P_u (3) (kN)	Beam	F_u (4) (kN)
S1	23.1	2.14	L1	137.20	V1A V1B	24.86 29.65
	24.4	2.59	L2	139.55	V2A V2B	43.65 47.17
	28.1	2.98	L3	163.62	V3A V3B	55.14 51.05
S2	57.0	3.81	L4	192.86	V4A V4B	36.26 36.35
	59.7	5.45	L5	215.14	V5A V5B	72.78 66.60
	52.4	6.59	L6	236.17	V6A V6B	57.17 53.85
S3	57.0	3.81	-	-	V7A V7B	54.82 46.44
	59.7	5.45	-	-	V8A V8B	68.32 80.06
	52.4	6.59	-	-	V9A V9B	81.17 104.93
S4	36.1	3.42	-	-	VP1A VP1B	28.42 27.01
	36.6	3.97	L7	182.85	V10A V10B	42.71 38.99
	46.1	5.17	L8	210.90	V11A V11B	49.99 61.79
S5	75.3	4.46	-	101	V12A V12B	6.[illegible]9 50.80
	73.5	5.65	-	112	V13A V13B	62.64 51.43
	73.1	7.96	-	136	V14A V14B	67.50 55.09

(1) Concrete compressive strength at 14 days (test age, cylindrical samples)
(2) Concrete tensile strength at 14 days (test age, split test on cylindrical samples)
(3) Ultimate test loads on slabs
(4) Ultimate test loads on beams

Table 4 – Ultimate test loads on slabs of other authors

Series	Slab	h[(1)] (mm)	ρ[(2)] (%)	f_{c14}[(3)] (MPa)	f_{t14}[(4)] (MPa)	V_f[(5)] (%)	P_u[(6)] (kN)	Reference
S4	OSC.S1	100	1.57	43.7	3.76	0	176.48	Azevedo(11)
S5	L07	60	1.73	88.7	5.3	0	101	Zambrana Vargas(10)
	L08	60	1.73	79.0	6.3	0.75	112	Zambrana Vargas(10)
	L09	60	1.73	93.0	7.6	1.5	136	Zambrana Vargas(10)

(1) Beam's height or slab thickness
(2) Flexural reinforcement ratio
(3) Concrete compressive strength at 14 days (test age)
(4) Concrete tensile strength at 14 days (test age)
(5) Fiber volume fraction
(6) Ultimate test load on slabs

2 express this relationship in a similar format of Equation 1.

$$f_{sp} = (0{,}15\,V_f + 0{,}51)\sqrt{f_c} \qquad (2)$$

being f_{sp}, f_c given in MPa and V_f in %.
The ACI 318M-02 [13] prescribes Equation 3 to evaluate the ultimate shear force for beams without stirrups:

$$V_u = (0{,}166\sqrt{f_c}\ b\,d)/10 \qquad (3)$$

where:
f_c: concrete axial compressive strength;
d: effective slab depth;
b: beam length;

Figure 6 – Concrete tensile strength versus fiber volume fraction for slabs specimens

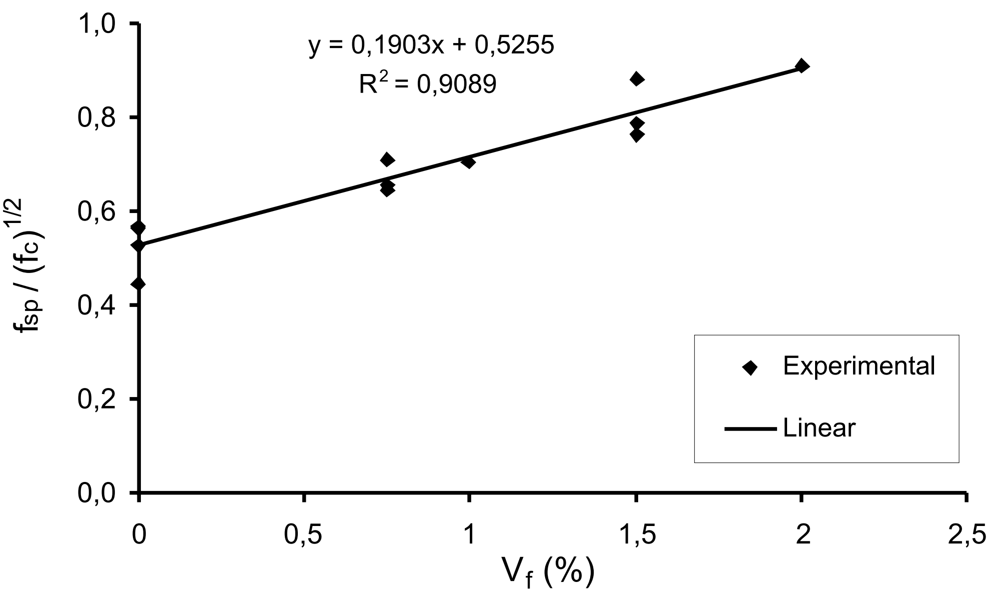


Figure 7 – Punching shear versus fiber volume fraction for slabs specimens

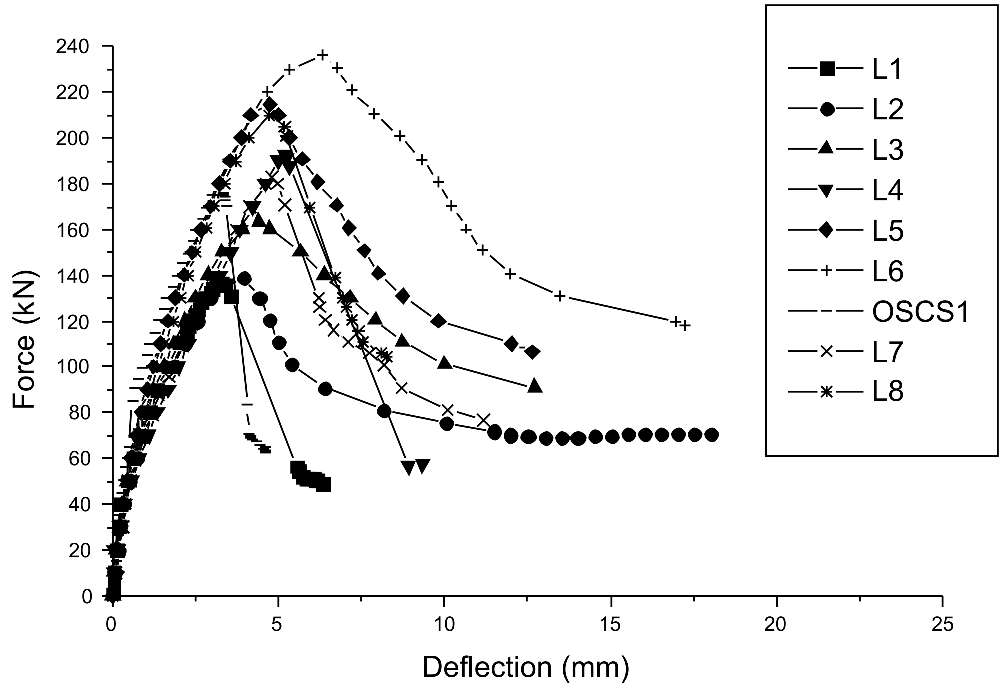


Figure 8 – Shear strength versus fiber volume fraction for beams specimens

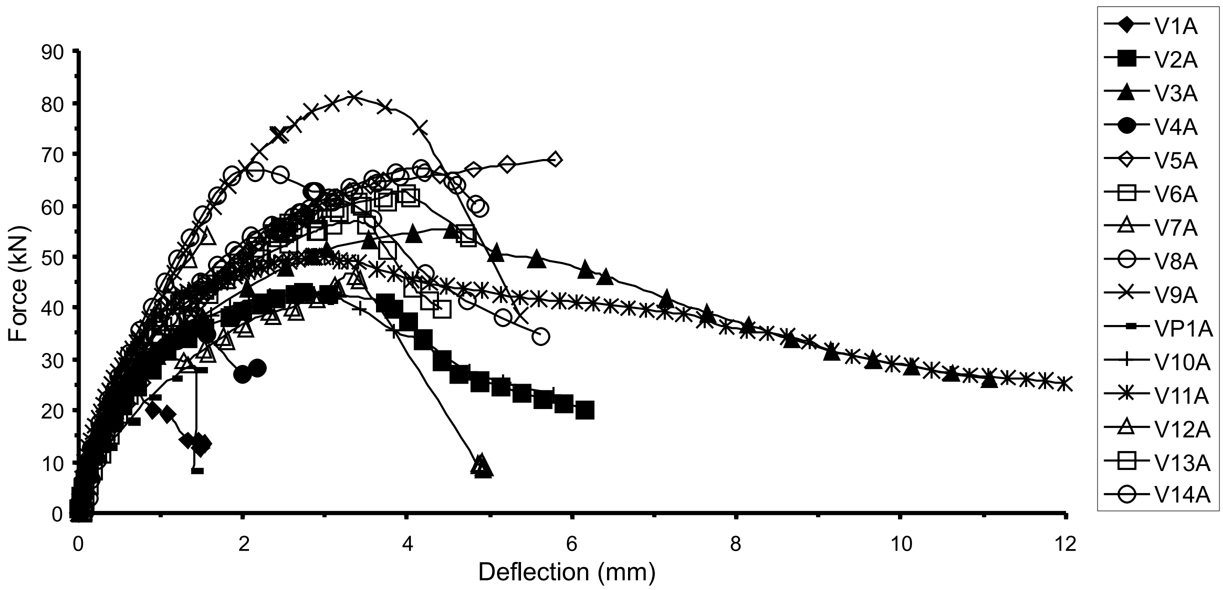


Figure 9 – Normalized load versus midpoint deflection for slabs

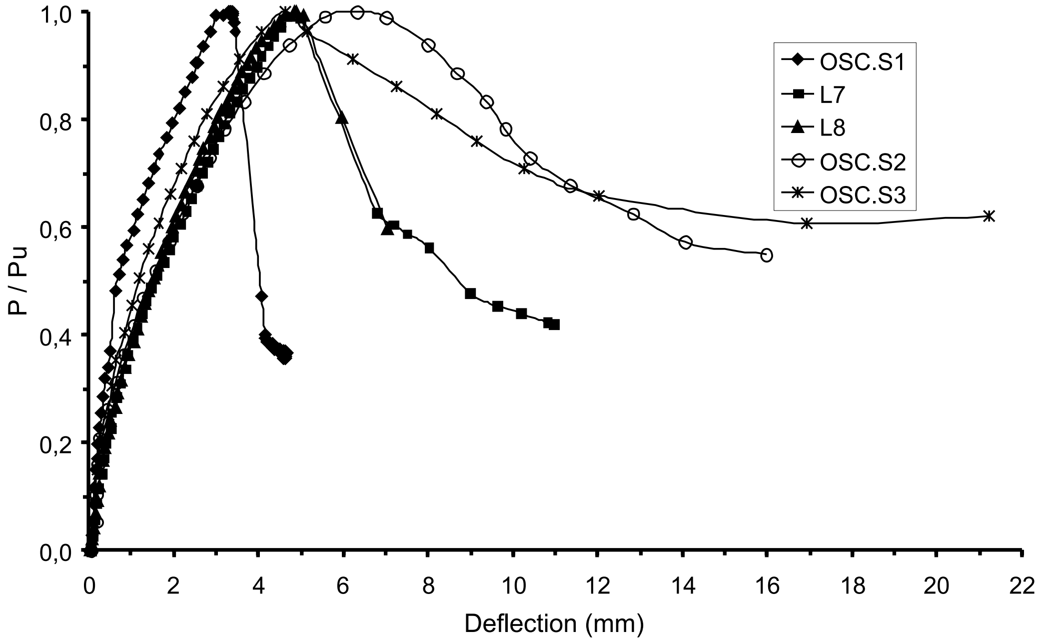
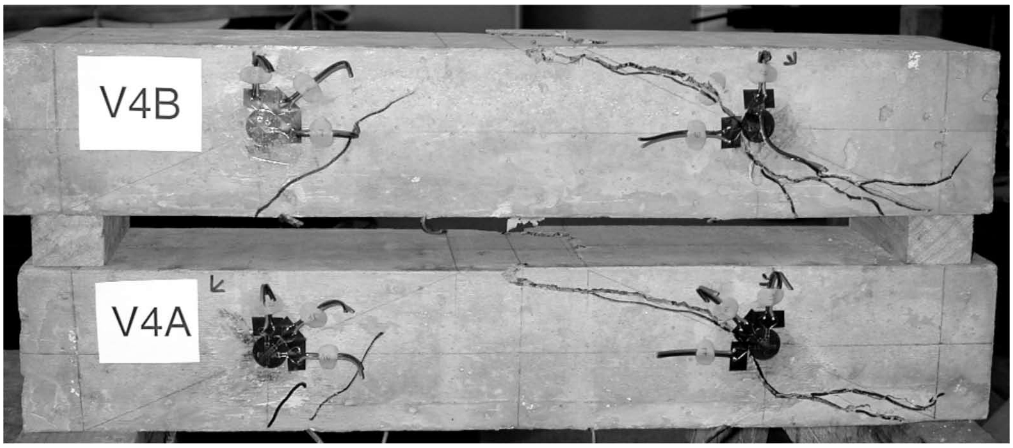


Figure 10 – View of beams of Series 2 (without fibers) after testing



being f_c given in MPa; and b and d in cm.

To consider the steel fiber effect, a certain way the value of f_{sp} obtained from Equation 2 can be introduced in Equation 3, thus a modified equation from ACI 318M-02 [13] can be stated (Equation 4). Some adaptations were done to preserve the adjustment and safety factors that are implicit in the coefficient 0.166 of Equation 3.

$$V_u = [\frac{0,166}{0,51}(0,15\,V_f + 0,51)\sqrt{f_c}\;b\,d]/10 = [0,3255\,(0,15\,V_f + 0,51)\sqrt{f_c}\;b\,d]/10 \qquad (4)$$

being f_c given in MPa; b, d in cm and V_f in %.

Theoretical values obtained by using Equation 4 showed reasonable correlation with the beam test results, as demonstrated in Figure 16. It can be seen that the tested equation represents the experimental tendencies, since the linear regression results in an almost parallel line to the V_f axis and on the safe side, as general.

4.2 Analysis of slabs according to ACI 318M-02

For the analysis of the slab tests, another relationship between SFRC tensile strength and compressive strength was found for the

Figure 11 – View of beams of Series 2 (with fibers) after testing

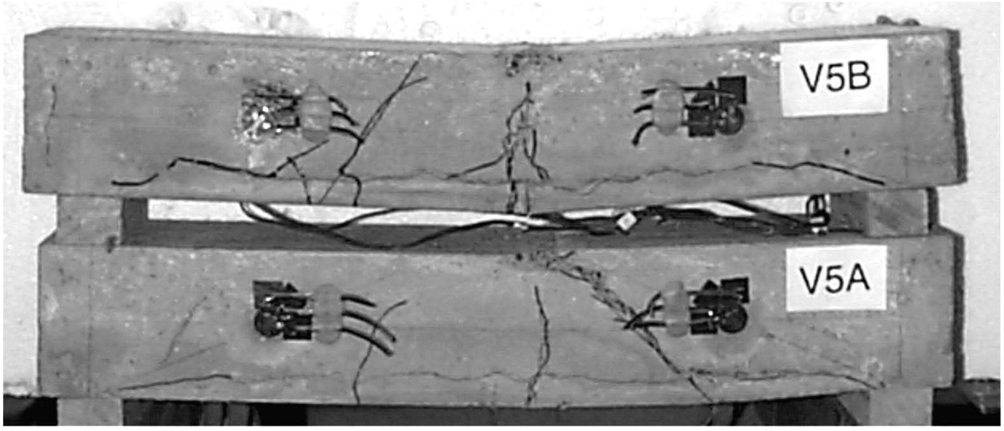
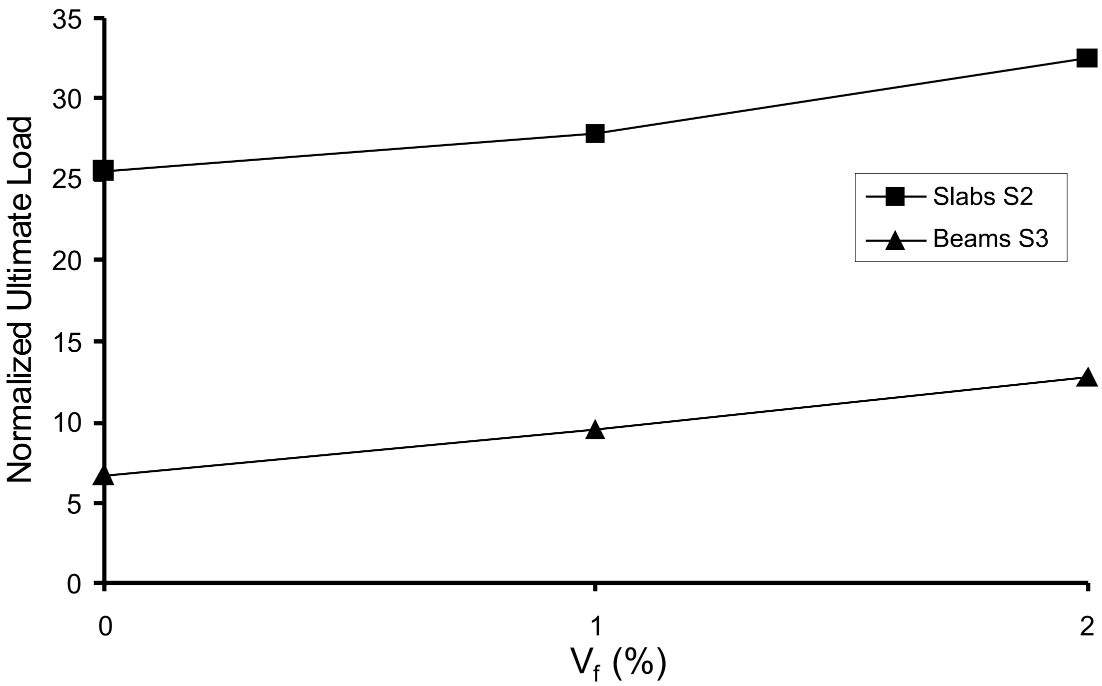


Figure 12 – Normalized ultimate load versus fiber volume fraction for specimens of Series 2 and 3



test results. Equation 5 express this correlation, in a similar format of Equation 2.

$$f_{sp} = (0{,}19\,V_f + 0{,}53)\sqrt{f_c} \quad (5)$$

being f_{sp} and f_c given in MPa and V_f in %.
The ACI 318M-02 [13] prescribes Equation 6 to evaluate the ultimate punching load for slabs without transversal reinforcement and square section columns.

$$P_u = (0{,}3321\sqrt{f_c}\,b_o d)/10 \quad (6)$$

where:
f_c: concrete axial compressive strength;
b_o = 4(c+d) → perimeter where punching occurs;
d: effective slab depth;
c: column length;
being f_c given in MPa; and b_o, d in cm.

Figure 13 – Normalized ultimate load versus fiber volume fraction for specimens of Series 4

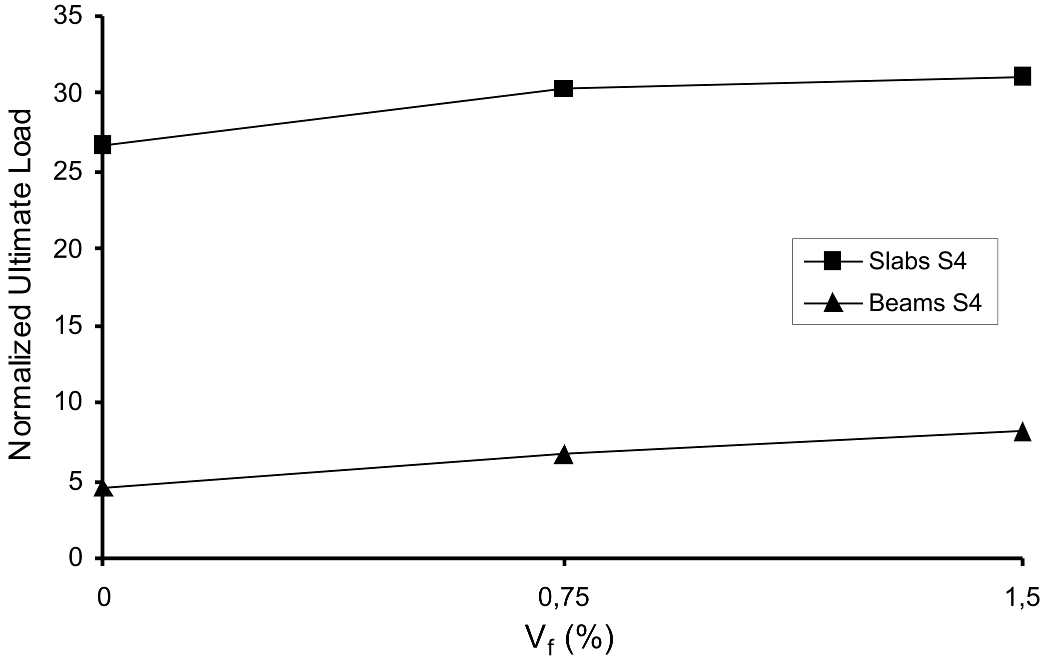
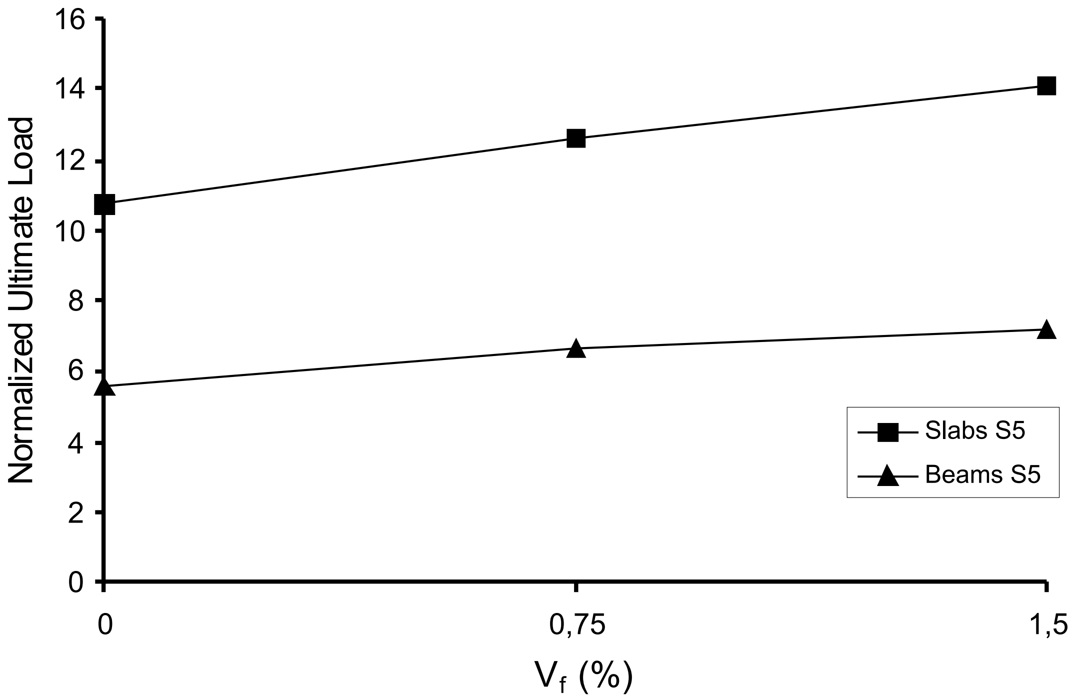


Figure 14 – Normalized ultimate load versus fiber volume fraction for specimens of Series 5



Alike in case of beams, the value of f_{sp} (from Equation 5) is introduced in Equation 6 to consider the effect of steel fibers in punching strength. Adaptations were done to preserve the adjustment and safety factors that are implicit in the coefficient 0.3321 of Equation 6. By this procedure Equation 7 was obtained.

$$P_u = [\frac{0,3321}{0,53}(0,19\,V_f + 0,53)\sqrt{f_c}\,b_o d]/10 = [\,0,6266\,(0,19\,V_f + 0,53)\sqrt{f_c}\,b_o d]/10 \qquad (7)$$

being f_c given in MPa; b_o, d in cm and V_f in %.

Figure 17 shows an evaluation of the response of Equation 7 when compared to the experimental data obtained in this research, together with some of Zambrana Vargas [10] and Azevedo [11]. It can be seen that Equation 7 offers a reasonable response to the experimental tendencies, as the fiber volume increases. However, since the linear trend is not parallel to the V_f axis, an additional adjustment must be done. Considering the particular data of this research, an empirical coefficient of about 0.3102 was applied to reduce the effect of V_f in Equation 7. This procedure leads to Equation 8, here called ACI 318 Modified Equation. The performance of Equation 8 is also shown in Figure 18.

$$P_u = 0,6266(0,19 \times 0,3102\,V_f + 0,53)\sqrt{f_c}\,b_o d = 0,6266(0,06\,V_f + 0,53)\sqrt{f_c}\,b_o d \qquad (8)$$

5. Conclusions

Based on the experimental results, it can be concluded that there are unequivocal similarities between punching shear strength in flat slabs and shear strength in analogous beams. The analogous slabs and beams must have the same height, longitudinal reinforcement ratio and concrete properties. Therefore, shear tests on small prismatic beams can be performed to get useful indicators for the steel fiber reinforced concrete mixture design, looking at its application in flat slab-column connections.

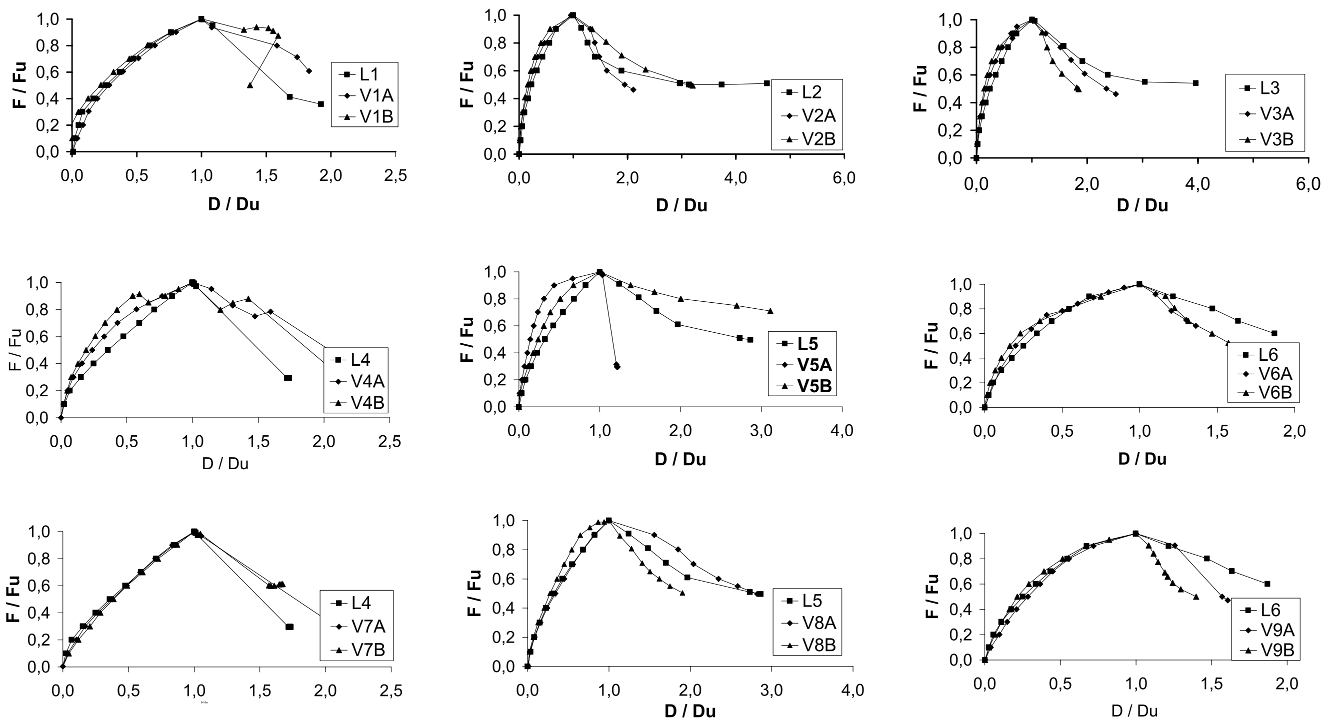
In addition to load capacity information, the analysis of these similarities can give information about ductility of the connection, as shown in load versus deflection diagrams of analogous slabs and beams.

In this experimental program only three types of fibers were used, since it aimed at evaluating other parameters, such as concrete strength, fiber volume, effective depth and column side length.

ACI Code gives very simple equations to estimate shear strength of beams and punching strength of slabs. In these equations, the square root of concrete compressive strength corresponds to an indirect correlation of shear strength to the concrete tensile strength.

Theoretical analysis based on ACI Modified Equations, adapted to steel fiber reinforced concrete, both for slabs and for beams, provided good indicators of shear strength of beams without stirrups and punching shear strength of slabs without transversal reinforcement. Test results also showed that theoretical strength models, based on linear dependence on fiber content, can be used to predict the effect of fiber addition.

Figure 15 – Relative load versus relative deflection for slabs and beams specimens



Future research works in this way can lead to good knowledge improvements, by detailing other aspects such as the application of other kinds of fibers, with different aspect ratios; the test of analogous slabs and beams of larger dimensions, to check the scale effect; the extension of the experimental study to slabs and beams with transversal reinforcement; and the variation of the flexural reinforcement ratio in the test series.

6. Acknowledgements

The authors thankfully acknowledge the financial support of CNPq – National Council for Scientific and Technological Development and FAPESP –State of Sao Paulo Research Foundation, also to Belgo Mineira Bekaert Arames S/A, Holdercim Brasil S/A and Jeruel Artefatos Plasticos Ltda., for supplying the materials used in the research.

Figure 16 – Comparison of experimental and theoretical (Equation 4) beam test results

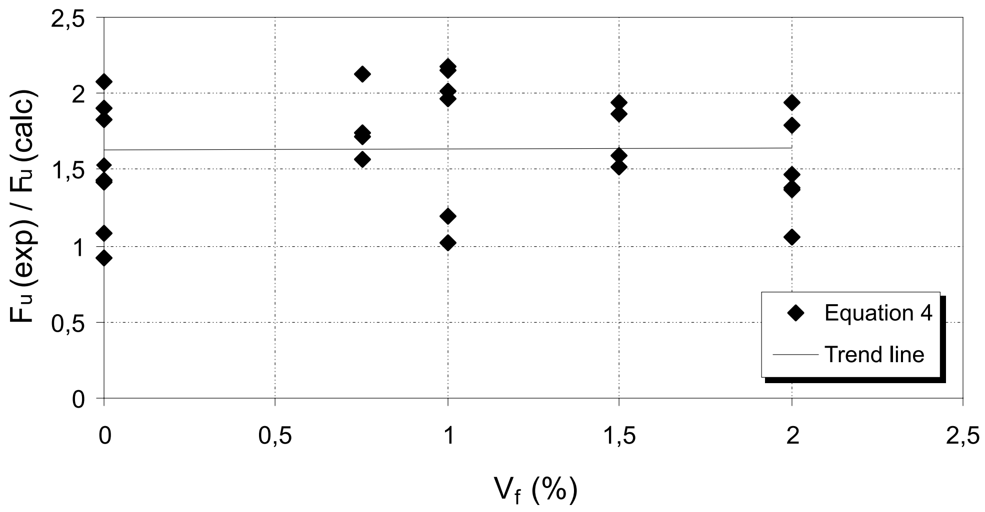
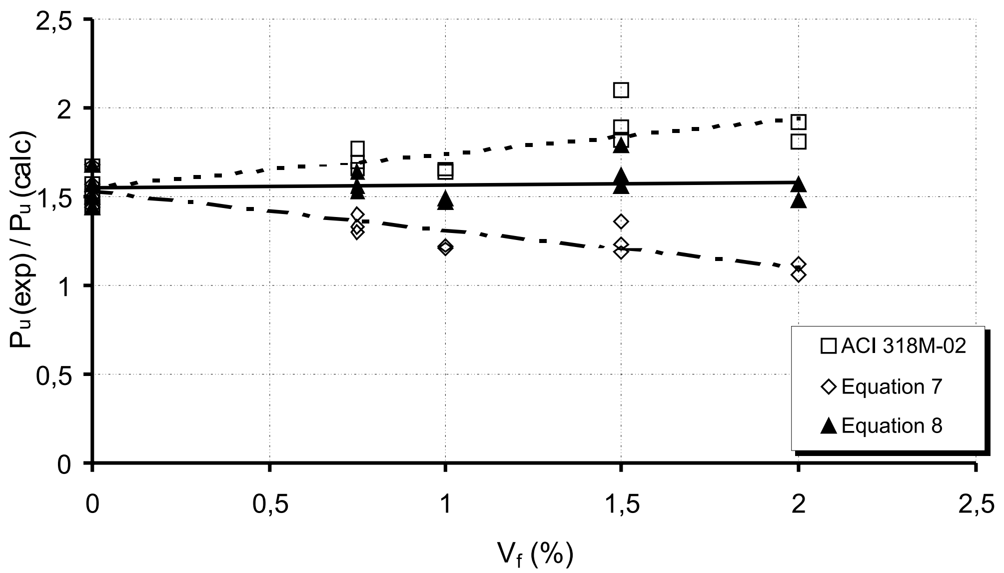


Figure 17 – Comparison of experimental and theoretical (Equations 7 and 8) slab test results



7. References

[01] Swamy R.N., Ali S.A.R. Punching shear behavior of reinforced slab-column connections made with steel fiber concrete. Journal of the American Concrete Institute 1982;79(5):392-406.
[02] Alexander S.D.B., Simmonds S.H. Punching shear tests of concrete slabs-column joints containing fiber reinforcement. ACI Structural Journal 1992;89(4):425-432.
[03] Theodorakopoulos D.D., Swamy N. Contributions of steel fibers to the strength characteristics of lightweight concrete slab-column connections failing in punching shear. ACI Structural Journal 1993;90(4):342-355.
[04] Tan K.H., Paramasivam P. Punching shear strength of steel fiber reinforced concrete slabs. Journal of Materials in Civil Engineering 1994;6(2):240-253.
[05] Shaaban A.M., Gesund H. Punching shear strength of steel fiber reinforced concrete flat plates. ACI Structural Journal 1994;91(4):406-414.
[06] Harajli M.H., Maalouf D., Khatib H. Effect of fibers on the punching shear strength of slab-column connections. Cement & Concrete Composites 1995;17(2):161-170.
[07] Hughes B.P., Xiao Y. Flat slabs with fibre or link reinforcement at slab-column connections. Institution of Civil Engineers Structures & Buildings 1995;110:308-321.
[08] Prisco M., Felicetti R. Some results on punching shear in plain and fibre-reinforced micro-concrete slabs. Magazine of Concrete Research 1997;49(180):201-219.
[09] McHarg P.J., Cook, W.D., Mitchell D., Yoon Y.S. Benefits of concentrated slab reinforcement and steel fibers on performance of slab-column connections. ACI Structural Journal 2000;97 (2):225-234.
[10] Zambrana Vargas E.N. Punching shear strength of high strength concrete flat slabs with steel fibers. MSc dissertation. University of Sao Paulo at Sao Carlos, 1997 (in Portuguese).
[11] Azevedo A.P. Resistance and ductility of slab-column connections on high strength concrete flat slabs with steel fibers and shear reinforcement. MSc dissertation. University of Sao Paulo at Sao Carlos, 1999 (in Portuguese).
[12] Holanda K.M.A. Analysis of strength mechanisms and similarities between punching shear in flat slabs and shear in beams made of steel fiber reinforced concrete. PhD thesis. University of Sao Paulo at Sao Carlos, 2002 (in Portuguese).
[13] ACI 318M-02. Building code requirements for structural concrete. American Concrete Institute, Farmington Hills, Michigan, 2002.