Additivity of tree biomass components using ratio estimate

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Abstract: This study deals with the subject biomass estimation. The objective was to achieve the additivity of tree biomass components, which is defined as the compatibility among the component predictions and total tree biomass, using ratio estimates. The biomass estimation model was applied to black wattle trees in forest stands, which include a sample of 670 trees in an age range of 1 to 10.75 years. The adjusted model, in which the total biomass, or sum of predicted components, is a function of the stem volume multiplied by the Scalar Coefficients Proxy of Density, proved to be of great interest for biomass estimation and consistent when compared to the results obtained by WNSUR estimates (traditional method). The natural additivity of the tree biomass components was fully achieved, when modeling them by means of ratio estimation. Equations developed from the proportional behavior of the biomass components at different ages did not require the use of linear regression models and were obtained from calibration with the experimental data. The estimators resulting from these equations proved to be appropriate to make a generic model for correction of ratios coefficients at different ages.

Key words: scalar coefficients proxy of density, ratio estimate, stem volume, quadratic mean diameter, WNSUR.

INTRODUCTION

Biomass in native and planted forests has been increasingly estimated for several purposes in Brazil, highlighting the diverse demands in the energy context, for the assessment of carbon neutralization and to deepen the understanding of the dynamic processes that occur in forest ecosystems over a productive cycle.

Worldwide two schools have contributed substantially to the improvement of biomass modeling in trees. The first one has been working to overcome the innumerable screenings resulting from the incompatibility
of additivity and acceptable precision resulting from the systems of equations adjusted by means of linear and nonlinear regressions.

The second one evolved trying to overcome also the problems of precision of the estimates obtained for each of the components of the trees by means of segmented modeling, that is, trying to improve consistent methodologies to have the best geometric volume of the components, a consistent model to correct shape of these volumes and a model for obtaining density of each component to generate their respective biomasses.

This study deals with the subject biomass estimation which is an indirect goal in the forest inventories and ensures, in essence, the reduction of costs by avoiding the direct measurement of the mass of all trees.

It is a fact that researchers usually apply volume models to biomass estimation. Because biometric variables taken from trees, either isolated or combined, do not have the same metric unit as the dependent variable; this type of modeling becomes very complex. The aim of this work is precisely to address this important issue, making an assumption before defining the model, either using regression or segmented modeling to estimate biomass in a forest area: The metric units of independent variables in a model must be composed to keep the same metric unit of the dependent variable.

If we strictly hold to this assumption, we can easily deduce that the variables \( d \) (stem diameter at breast height) and \( h \) (tree height), as they are used in the volumetric models, alone or combined, do not present metric units compatible with the dependent variable for biomass estimation, which should be expressed in kg.

A second assumption must also be considered in the modeling of biomass: the additivity, which corresponds to the compatibility among tree component estimates (leaves, branches, bark, stem and root), i.e., the sum of the masses of the components must be equal to the total tree mass.

Although biological diversity is surprising and complex, many of the fundamental biological processes manifest themselves in an extraordinarily simple way when equated according to their size, independent of the class or taxonomic group they belong to (West and Brown 2005). This scaling, according to Niklas (1994), follows a power-law behavior, as defined in (1).

\[
Y = \alpha X^\beta
\]

Where: \( Y \) = some quantifiable biological variable; \( \alpha \) = normalization constant; \( X \) = mass of the organism; \( \beta \) = scale exponent.

An additional simplification can be achieved, since the exponent \( \beta \) assumes a limited set of values, which are typically simple multiples of 1/4 (West et al. 1999). Many biological variables follow this scale as they were presented and discussed by West et al. (1997, 1999), Enquist et al. (1998, 1999), Enquist and Niklas (2001), Enquist (2002), Niklas (2004) and West and Brown (2005).

The theme allometry was consolidated by Huxley (1932), who synthesized the theoretical conception about growth and its scales. He assumed that the growth rate of an organ was simultaneously proportional to a constant characteristic of the organ itself \( (\xi) \), given its size at the time \( (X) \), and to a general factor, common to all parts of the organism, representing the effects of its age and of the environmental conditions \( (G) \). Taking the size of the organism as \( Y \) with constant feature \( r_i \), the specific growth rates of organs and of the organism may be defined in (2), with solution of the derivatives in (3).

\[
\frac{dY}{dt} = r_Y G \quad \text{and} \quad \frac{dX}{dt} = \xi X G
\]
\[
\frac{dY}{dX} = \frac{dY}{dt} \frac{dt}{dX} = \frac{\gamma Y}{\xi X} = \frac{\gamma Y}{\xi X}
\]

The expression defined in (3) has solution in (4) and (5), explaining the origin of the model presented in (1).

\[
\frac{dY}{Y} = \frac{\gamma}{\xi} \frac{dX}{X}
\]

\[
\log Y + C_1 = \frac{\gamma}{\xi} \log X + C_2 \quad \text{and} \quad \log Y = \frac{\gamma}{\xi} \log X + (C_2 - C_1)
\]

Considering \((C_2 - C_1) = \log a\), then:

\[
\log Y = \frac{\gamma}{\xi} \log X + \log a
\]

Recomposing the function on its original form we have:

\[
Y = a X^{\beta}, \quad \beta = \frac{\gamma}{\xi}
\]

Allometry is also associated to properties of such changes in the organism, whose characteristics are related to its size. Thus, Niklas (2004) also defines allometry as the study of size and its consequences, according to the model presented in (6), in which \(Y\) is the size of the body's component (measured by its size or mass), \(X\) is a comparable measure of the body’s size, excluding the size of the body part of interest, \(a\) is the allometric constant and \(\beta\) the allometric exponent (scale).

In Forest Science, the model presented in (6) is one of the most common allometric forms (Chan et al. 2013). Although it is a simple allometric model, it has been widely used to estimate the biomass components stem volume and height of the trees, especially considering the variable diameter at breast height \((d)\) as a predictor.

The most appropriate method to evaluate the biomass of a tree is to obtain it separately for each component, implying in its cutting and resulting in a time-consuming task and high cost. An alternative approach is to employ allometric equations, i.e., trees biometrically measured are converted into biomass values by means of empirical allometric models (Brown 1997). There is a vast literature regarding the development and use of allometric biomass models, which was revised by Picard et al. (2012).

Thus, the allometry in the context of the estimation of tree biomass refers to mathematical equations related to the components and total biomass based on characteristics that are easily measured such as diameter at breast height \((d)\), height of the tree \((h)\), wood density \((\rho)\), and crown diameter \((D_c)\). It is obvious that the equations must be based on destructive sampling and then applied generically (Chave et al. 2014), and the quality of these allometric models constitutes one of the most important limitations in the assessment of biomass stocks (Chave et al. 2004, Sanquetta et al. 2015).

The objective of this work is to establish a simple and consistent biological model, testing its performance statistically precise, which estimates biomass accurately. Our proposal was conceived by deriving a new segmented model. Ratio estimator’s performance was compared with those obtained by regression modeling via \textit{SUR} and \textit{NSUR} methodologies. Application of other segmented models to the present data set was not possible because they were not collected to attain all the required information to perform the estimates.

The goal of this research is to develop a segmented model based on the following hypothesis: “The biomass performed by a simple model, using ratio estimates, does not require additional methodological
stratagem to attain additivity and achieving competitive precisions with those obtained by the regression models”.

MATERIALS AND METHODS

APPROACHES FOR BIOMASS MODELING

Estimation using regression modeling

When the model in (6) is adjusted using empirical data, both biomass and predictor variables often faces heteroscedasticity of errors, i.e., a condition in which the variance of errors is not constant. This directly affects the estimator, because when (6) is linearized and the ordinary least squares is applied to estimate the parameters, the estimator does not ensure the smallest variance, i.e., ceases to be unbiased (the best linear unbiased estimator - BLUE). This will introduce bias in the conclusions drawn from the results of the significance tests of the equations and confidence intervals for the coefficients and for the estimates of mean biomasses and their prediction.

Parresol (1999) demonstrated that the heteroscedasticity of errors can be explained by the modeling - which follows a normal distribution with increasing variance as a function of $X_i$, expressed by an exponential relation of $X_i$, allowing the establishment of weights. Thus, the model (6) can be adjusted by generalized least squares, resulting in a BLUE estimator and, consequently, validating the tests of significance of the equation and the construction of confidence intervals. This method has been preferable, because there is no need to transform the parameter estimates of the model.

Considering the theory of allometric partition (TAP), it can be affirmed that the separation of plants into components is regulated mainly by their total size, but keeps a scale relationship between these parts and the whole (West et al. 1999, Savage et al. 2008). West et al. (2000) presented a model in which they sought to provide a theoretical basis for allometric relationships in trees, considering the model presented in (1). The objective was to clarify an explanatory theory for the empirical relationships between the characteristics of vascular plants and not just to observe them. The model is based on principles of hydrodynamics, biomechanics, and tree architecture and provides many scales multiple of 1/4 for various attributes of the tree, which are presented in Enquist (2002).

According to Ducey (2012), generality is often not observed and this has led to questions as to how the allometric scales relations are apparently variant for species and between large spatial domains, especially when applied to tree biomass equations. Niklas (1995) has shown that the allometric relationships change within the same individual throughout its life and in the growth conditions to which it is subjected to, and also referred to this "size-dependent allometry as evidence that no universal allometric scaling coefficients could be estimated for trees".

Although it is common to use the model (6), Jenkins et al. (2003) and Weiskittel et al. (2015) point out that there are substantial evidences that the diameter ($d$) can often be a poor variable to estimate the biomass of the components. This is also referenced by Goodman et al. (2014), when they demonstrated that trees with disproportionate crown diameters may have a relationship between $d$ and total biomass different from trees with proportional crown diameter.

Chave et al. (2009) also showed a high degree of variability in the relationship between tree size and wood density, thus reinforcing that simple dimensional measurements may be insufficient to obtain accurate
estimates of tree biomass due to the variability of wood density. According to MacFarlane (2015), these important differences are related to the dimensioning between the parts relative to the whole, known as allometry.

Brown et al. (2004) noted that the power-law model allowed the definition of other models for allometric scaling in biology. Therefore, the basic form of the model in (6) can be extended to other forms. Instead of using $X_i$ to represent the value of only one feature, it can be extended to represent values of more than one feature, for example, for biomass: $d, h$ and $d^2h$.

In Europe, a comprehensive review on biomass equations was performed by Zianis et al. (2005), in which they reported a total of 607 equations adjusted for 39 species distributed in all European countries. This review highlights that the vast majority of researches (127) used the model $\log(M) = A + B \log(D)$, however the volumetric models of Schumacher and Hall (1933) and Spurr (1952), and their variations, were prevalent in other citations.

Kettering et al. (2001) showed how the function in (6), making $X=d$, could be generalized to refer explicitly to tree height ($h$) and wood density ($\varphi$). Assuming that the biomass $Y$ is initially considered only as a function of the diameter $d$, then $B = ad^b$, in which the coefficients $a$ and $b$ vary between sites. This variation is probably the major source of uncertainty if the biomass estimations are obtained using equations that are not calibrated for different sites. However, if the calibration is performed by collecting biomass data $B$ and diameters $d$ for each site, this procedure is costly and requires destructive actions. Methods to obtain the values of $a$ and $b$ must, therefore, be proposed such that destructive procedures are not required. The parameter $b$ can be estimated by means of a specific relationship of the site between tree height $h$ and diameter $d$, i.e., $h = kd^c$. The parameter $a$ may be estimated from the average of wood density $\varphi$ in the site as $a = r\varphi$, where $r$ is a constant relatively stable among sites. As biomass is proportional to $d^2h$, then they have assumed that $B = ad^2kd^c = d^{2+c}$. There is a critical point in this model when they considered the constant $r$ to be relatively stable among sites, because the site variations have been not satisfactorily treated in most of the biomass models, owing to their complexity.

The model presented in (6) was also enhanced by MacFarlane (2015), which applied it to a system of specific equations for the proposed components. Considering what was explained by Ketterings et al. (2001), we can maintain the form associated with equation (6), expressing it in a generalized way and with a more consistent biological meaning to estimate biomass of the tree components.

$$Y = F\frac{\pi}{4}d^2h$$

Where: $Y =$ biomass of the tree above the ground (kg); $d =$ diameter at breast height (cm); $h =$ height of the tree (m); $\varphi =$ wood density (kg.m$^{-3}$, measured in $d$); $F =$ form factor of the "whole-tree"- defined by Gray (1966) and Cannell (1984) for indexing the tree biomass, relative to the proxy mass, which is a proxy of the tree volume $\left(\frac{\pi}{4}d^2h\right)$ multiplied by the density of the tree, i.e. $(\varphi\frac{\pi}{4}d^2h)$, which he termed proxy biomass of the tree.

Thus, MacFarlane (2015), in view of the possibility of no proportionality between the biomass of the components and the proxy biomass of the tree, proposed the solution presented in (8), in which, when $B \neq 1$, the relationship does not keep proportionality, indicating that the form changes when the tree biomass changes:

$$Y = F(\frac{\pi}{4}D^2H)^B$$

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In this way, he proposes the preliminary segmentation for $B = 1$ in the equation (8), for each of the components of the tree, formalizing them as are presented in (9), (10) and (11):

$$Y_1 = F_1 \left[ \frac{\pi}{4} D^2 (H_B) \right]$$

(9)

$$Y_2 = F_2 \left[ \frac{\pi}{4} D^2 (H_C - H_B) \right]$$

(10)

$$Y_3 = F_3 \left[ \frac{\pi}{4} D^2 (H - H_C) \right]$$

(11)

Where: $Y_1$, $Y_2$ and $Y_3$ are respectively the biomass of the neiloide, considered to a height of 1.30m ($H_B$); of the commercial stem, considered from 1.30m of the tree until the beginning of the crown ($H_C - H_B$) and of the living crown, considered the beginning of the first branch alive until the top of the tree ($H - H_C$).

He also proposed a further separation of the crown biomass, wherein there is a dominant central segment which is separated from its other branches. The mass of these other branches is associated with a differentiated form factor, which he named as abstract form factor ($F_4$) and is applied to an equally abstract volume of this component, calculated by ($\frac{\pi}{4} D^2 W$). $W$ is the average crown width which, when multiplied by its density, will result in the biomass of this component ($\frac{\pi}{4} D^2 W$), and is named as $Y_4$, such that $Y_4 = F_4 \left[ \frac{\pi}{4} D^2 W \right]$. Consequently, the total biomass of the tree is obtained by the sum of these components, i.e.: $Y_T = Y_1 + Y_2 + Y_3 + Y_4$.

When the occurrence of non-proportionality is detected in the form component, as is shown in (8), we can experimentally identify the respective values of the coefficient $B$ for each of the components, i.e., $B_1$, $B_2$, $B_3$ and $B_4$. This system of equations was solved using the *Seemingly Unrelated Regression* (SUR) model.

However, given the difficulty of obtaining the volume of components $v_j$ experimentally in the sampled population, researchers have proposed to apply regression models to identify the biomass of the components using biometric variables more accessible in the forest, leading, as a consequence, to the loss of additivity or the use of the segmented models previously presented in (8), (9), (10), and (11). The concept of additivity of the biomass components was developed from the 1970s, when several researchers (Kozak 1970, Chiyenda and Kozak 1984, Cunia and Briggs 1984, 1985, Reed and Green 1985) began to formulate methods to compose the biomass equations in order to force them to additivity. In this condition, the two variables, form factor $F$ and wood density $\rho_j$ are associated to the coefficients of the proposed regression models, but without configuring a balanced participation in each of them when adjusted for the components, which generates loss of additivity.

Some assumptions need to be defined to ensure accuracy in the prediction of forest biomass on a large scale. Genet et al. (2011) state that a good set of equations for this purpose must meet the following characteristics: i) *consistency* - referring to the separation of the tree total biomass in standard components, however, ensuring its additivity, ii) *robustness* - referring to the composition of an operating system that works correctly for wide variations in the sample population and with low sensitivity to the used sampling procedure and to the formulated hypothesis, and (iii) *accuracy*.

According to Genet et al. (2011), many recent studies with tropical and temperate species have dealt with the variations of allometric parameters and their relationships (Chave et al. 2005, Saint-Andre et al. 2005, António et al. 2007, Wutzler et al. 2008, Pretzsch 2009). They highlighted that both the allometric factor and the allometric exponent vary between stands of different ages, different levels of fertility and between species. However, there is still no consensus on how to appropriately treat these effects.
Ziani and Mencuccini (2004) consider the model presented in (1) as the most common for biomass studies \((M)\). It takes the diameter measured at 1.30m \((D)\) to express the variable \(X\) and, after having been linearized, results in:

\[
\ln M = \ln a + b \ln D \tag{12}
\]

They explained the behavior of the coefficients "\(a\)" and "\(b\)" by taking advantage of the fractal geometry and the size and form of the trees. These procedures allow them to adjust statistical functions for these coefficients based on the relations:

- If the tree volume is proportional to diameter times height, and height is proportional to diameter, they have:
  \[
  V \propto D^d H^b \tag{13}
  \]
  \[
  H \propto D^{b^*} \tag{14}
  \]
  Then:
  \[
  V \propto D^{d + b^*} \quad \text{or} \quad V \propto D^{d + hb^*} \tag{15}
  \]
  Consequently,
  \[
  M \propto D^{d + hb^*} \tag{16}
  \]
  Under these conditions, "\(b\)" can be estimated by
  \[
  b = d + hb^* \quad \text{or} \quad b = \beta_0 + \beta_1 b^* \tag{17}
  \]
  Where: \(b^*\) varies between \(0 < b^* \leq 1\), obtained experimentally for different species using the relationship presented in (14) and (16).
  
  The coefficient "\(a\)" can be estimated as a function of "\(b\)".
  \[
  a = \beta_0 b^\beta_1 \tag{18}
  \]

**Estimation using segmented models**

Vallet et al. (2006) introduced a new concept to treat biomass and carbon estimators in trees when they worked with the segmented models, that is, each parameter of the model is estimated separately. This approach avoids the problems generated by the regression-adjusted models, in which the parameters form and density integrate all other coefficients and cause loss of the additivity. Consequently, the consistency of the model is reduced, not meeting the first characteristic formalized by Genet et al. (2011). Vallet et al. (2006) proposed a model to estimate the total amount of carbon per tree "\(C\)", as it is shown in (19):

\[
C = V_{IFN} * BRANCH_{EF} * DEN * CAR \tag{19}
\]

Where: \(C\) = total carbon per tree; \(V_{IFN}\) = commercial volume; \(BRANCH_{EF}\) = expansion factor for the branches’ volumes; \(DEN\) = basic density for the species and \(CAR\) = carbon proportion in dry matter.

To avoid the use of an expansion factor to the branches, the total volume for the entire tree, including the branches, was considered as it is presented in (20):

\[
V_{SP(C_{130}, h_{tot})} = F(40,000\pi)^{-1} C_{130}^2 h_{tot} \tag{20}
\]
Consequently,

\[ C = Vsp(C_{130}, h_{tot}) \times \text{DEN} \times \text{CAR} \]  

(21)

Please, note that still remains the estimator for \( F \), as shown in (20), that the author proposes to be equated in (22):

\[ F(C_{130}, h_{tot}) = \left[ \alpha + \beta C_{130} + \gamma (hdn) \right] \left( 1 + \frac{\delta}{C_{130}^2} \right) + \epsilon \]  

(22)

To avoid the correlation between \( C_{130} \) and \( h_{tot} \), the variable hardness of the tree \( (hdn) \) was proposed and equated in (23):

\[ hdn = \frac{C_{130}^{0.5}}{h_{tot}} \]  

(23)

Note that the higher the value of \( hdn \) more robust is the tree and the lower this value is, slenderer it will be. By replacing (22) in (20) they have:

\[ Vsp(C_{130}, h_{tot}) = \left[ \alpha + \beta C_{130} + \gamma (hdn) \right] \left( 1 + \frac{\delta}{C_{130}^2} \right) (40,000\pi)^{-1} C_{130}^2 h_{tot} \]  

(24)

The total variance of the estimator can be obtained in (25):

\[ \sigma^2 = \beta^2 \text{var}(C_{130}) + 2\beta \gamma \text{cov}[C_{130}(hdn) + \gamma^2 \text{var}(hdn)] + \sigma^2_\epsilon \]  

(25)

Where: \( \sigma^2_\epsilon \) = variance of the residuals and \( \text{var}(C_{130}) \), \( \text{cov}[C_{130}(hdn) + \gamma^2 \text{var}(hdn)] \) are respectively the variances and covariance of the variables.

Genet et al. (2011) proposed the allometric model in (26):

\[ DM = \alpha + \beta (d^2 h)^q + \epsilon \]  

(26)

Where: \( DM \) = dry mass for certain component; \( d \) = diameter taken at 1.30m; \( h \) = total height; \( \alpha, \beta \) and \( \gamma \) = model parameters to be estimated and \( \epsilon \) = error attributed to the not explained variability.

As the tree biomass results from the characteristics: volume, wood density and form, the scalar value \( \beta \) represents the product of the form factor \( F \) by the wood density \( \varphi \), i.e., \( \beta = \varphi \)\( F \). The form factor was calculated using the estimator proposed by Vallet et al. (2006), as it is shown in (22).

Vallet et al. (2006) also took the pre-existing biomass equation for the species Beech, in Europe, i.e., \( DM = \alpha + b^c (d^2 h)^e \) \( (d \text{ in m and } d^2 h \text{ in m}^3) \) and calibrated it to be adapted to the model in (27). Considering the constant \( c \neq 0 \), the calibration was applied to the sample data from France and equated the variable \( d \) in such a way as to make it equal to \( d^2 h \) in the equation, with the following function:

\[ d = \left( \frac{d^2 h}{k} \right)^{q} \]  

(27)

Genet et al. (2011) found, calibrating the equation (27), the following values for \( k \) and \( q \): \( k = 60.5 \) \( q = 0.368 \) for \( d < 0.25 \) m, and \( k = 34.7 \) \( q = 0.439 \) for \( d \geq 0.25 \) m.

Therefore, they have in the biomass model:

\[ \alpha = a, \quad \beta = \frac{b}{k^{cq}} \quad \text{ and } \gamma = e + cq \]  

(28)
They also proposed to adjust the biomass estimate as a function of age ($A$) and obtained their final models in (29) for the tree components and in (30) to the total biomass above ground:

$$
DM = \alpha + \left( \beta_1 + \beta_2 A + \beta_3 e^{\beta_4 A} \right) (d^2 h) (\gamma_1 + \gamma_2 A) + \epsilon
$$

(29)

$$
DM = \alpha + \left( \beta_1 + \beta_2 A + \beta_3 e^{\beta_4 A} \right) F_{C130}h(d^2 h) (\gamma_1 + \gamma_2 A) + \epsilon
$$

(30)

In the present work, we will maintain the same concept of the previously mentioned authors, that is, the use of the segmented model, and we will introduce a new concept for the additivity of the tree components, which is the compatibility of the biomass estimates of the components with the total stem biomass without using any method to force it, as it occurs in these variables naturally. In addition, it differs from methods that force the additivity, because each biomass component and the total biomass are represented by their own equations. The biomass additivity, as proposed in this work, will only be achieved when using a model as simple as possible: linear and of first order, i.e., $Y = \beta_j X + \epsilon_i$. The application of models of other orders requires the transformation of the data and this leads to loss of additivity, i.e., there is an incompatibility of the biomass component estimates relative to the total biomass.

CONCEPTION FOR BIOMASS MODELING IN THIS RESEARCH

Consider first the theoretical conception presented by MacFarlane (2015) in (7), then proceed as follows.

Form factor ($F$)

The first phase of this design will be to propose a facilitating solution concerning the form factor ($F$) to make the biomass estimation model operationally less complex.

Consider the methodology proposed by Hohenadl (1924) (as cited in Prodan 1965), which divides the tree stem in five or more parts of equal length. However, according to Prodan (1965), the smaller the length of the segments, the closer the volume estimate is to the real value. Thus, dividing the bole of the tree in ten segments, whose length is proportional to the total tree height, will increase the accuracy of these estimates (Figure 1).

To obtain the relationship between the natural and artificial form factor, Ko (1968) introduced the following concept on the quadratic diameter ($d_{q}^{2}$), considering the natural form factor calculated by the Hohenadl’s method:

$$
F_{0.05} = \frac{1}{d_{q}^{2}} \left( \frac{d_{0.05}^2 + d_{0.15}^2 + d_{0.25}^2 + d_{0.35}^2 + d_{0.45}^2 + d_{0.55}^2 + d_{0.65}^2 + d_{0.75}^2 + d_{0.85}^2 + d_{0.95}^2}{10} \right)
$$

(31)

Where: the mean quadratic diameter of the series can be obtained inside the parentheses, i.e.:

$$
d_{q}^{2} = \frac{d_{0.05}^2 + d_{0.15}^2 + d_{0.25}^2 + d_{0.35}^2 + d_{0.45}^2 + d_{0.55}^2 + d_{0.65}^2 + d_{0.75}^2 + d_{0.85}^2 + d_{0.95}^2}{10}
$$

(32)

The Hohenadl’s natural form factor ($\lambda_{0.05}$) will be then obtained as:

$$
\lambda_{0.05} = \frac{d_{q}^{2}}{d_{0.05}^{2}}
$$

(33)
This concept of form factor was enlarged by Ko (1968) using the expression (34), in which the diameter taken at any relative height of the series is considered as the reference to obtain any natural desired form factor:

$$F_{0,x} = \frac{0.1 \sum d_{0,i}^2}{d_x^2} = \frac{d_q^2}{d_x^2}$$  \hspace{1cm} (34)

Where: $d_x$ = reference diameter and $d_{0,i}$ = measured diameters in the middle of each segment.

Taking the mean quadratic diameter as reference, we get the form factor equal to one (Prodan 1965).

$$F_q = \frac{d_q^2}{d_q^2} = 1$$  \hspace{1cm} (35)

The form of the tree, in this case, is equivalent to a cylinder whose diameter is coincident with the mean quadratic diameter. Consequently, the volume is obtained in (36):

$$V = F_{0,x} \frac{\pi}{4} d_x^2 h$$ \hspace{1cm} (36)

Using the form factor equal to one in (36), there is a simplification to obtain the volume of the cylinder $V_s$ equivalent to the stem volume, whose reference is in $d_q$:

$$V_s = \frac{\pi}{4} d_q^2 h = g_q h$$ \hspace{1cm} (37)

Additivity of biomass components

Equating the relationship of the various biomass estimates of the tree components with the cylinder volume in (37) will make this estimator a ratio with specific meaning, constituting the central objective of this work.

Consider that the variable biomass of the components will be called $Y_{ij}$, such that $j$ indicates the component to which the estimator $i$ refers, i.e., it will be equal to $c$ if the estimator is for the crown of
the tree; equal to \( s \) if the estimator is for the stem; \( b \) for the bark; \( r \) for root and \( t \) for the total. The volume of each component will be called \( X_{ij} \) in much the same way as if configured for the variable \( Y_{ij} \), which will allow the specification of the following model:

\[
Y_{ij} = \tilde{\rho}_{ij} X_{ij} + e_{ij}
\]  

(38)

i) Theorem 1: Additivity of components

If the biomass components of a tree are obtained by the product of scalars of their densities equated as a function of the stem volume, then the sum of the resulting biomass will be equal to the biomass of the whole.

ii) Proof

Let \( Y_j \) denote the biomass of the components of a tree, \( \rho_j \) their densities and \( v_j \) the volume of their respective components, such that \( j = 1, 2, 3, \ldots, k \) components, then:

\[
\sum_{i=1}^{k} Y_j = \rho_1 v_1 + \rho_2 v_2 + \rho_3 v_3 + \ldots + \rho_k v_k = Y_t
\]

(37)

Consequently, if we take the volume of each component multiplied by its density, we will obtain the respective biomass and the results of these estimates are additives.

However, given the difficulty of obtaining the experimental volumes of the components \( v_j \), except for the stem volume \( v_s \), which is the easiest component to be obtained, and in the present case will be calculated as defined in (37), then dividing all the terms of the equality in (39) by this estimator we have:

\[
\sum_{i=1}^{k} \frac{Y_j}{v_s} = \frac{\rho_1}{v_s} v_1 + \frac{\rho_2}{v_s} v_2 + \frac{\rho_3}{v_s} v_3 + \ldots + \frac{\rho_k}{v_s} v_k = \frac{Y_t}{v_s}
\]

(40)

If the proportions are denominated \( \frac{v_j}{v_s} = \gamma_j \), then the densities of the components are multiplied by a weight that expresses proportions of the stem volume. Rewriting the equation in (40) we have:

\[
\sum_{i=1}^{k} \frac{Y_j}{v_s} = \rho_1 \gamma_1 + \rho_2 \gamma_2 + \rho_3 \gamma_3 + \ldots + \rho_k \gamma_k = \frac{Y_t}{v_s}
\]

(41)

Note that \( \rho_j \gamma_j = R_j \), which expresses exactly the ratio between the biomass components and the stem volume \( \frac{v_j}{v_s} \), then such ratios are called Scalar Coefficients Proxy of Density - SCPD, which will be the actual density only in the case of the stem biomass estimator. Consequently,

\[
\sum_{i=1}^{k} \frac{Y_j}{v_s} = R_1 + R_2 + R_3 + \ldots + R_k = \frac{Y_t}{v_s}
\]

(42)

As observed, if the denominator \( v_s \) is constant for all components, then the equation (42) can be rewritten as in (43).

\[
\sum_{i=1}^{k} Y_j = R_1 v_s + R_2 v_s + R_3 v_s + \ldots + R_k v_s = Y_t
\]

(43)
As a consequence, the obtaining of these ratios experimentally per component in each tree in the forest, as a function of the stem volume, continues to be additive, because SCPDs are proportions of this volume. This completes the proof.

iii) Corollary 1: Additivity for a sample

If a sample of size \( n \) is taken from a forest population, such that the sample size sufficiency is ensured, then the biomass of the components of the \( n \) sampled trees can be added and their respective means can be obtained.

\[
\sum_{i=1}^{n} \frac{Y_j}{n} = \sum_{i=1}^{n} \frac{R_{1i}v_s}{n} + \sum_{i=1}^{n} \frac{R_{2i}v_s}{n} + \sum_{i=1}^{n} \frac{R_{3i}v_s}{n} + \ldots + \sum_{i=1}^{n} \frac{R_{ki}v_s}{n} = \frac{n}{n} \sum_{i=1}^{n} Y_t
\]

(44)

Consequently,

\[
\sum_{i=1}^{k} \bar{y}_j = \bar{R}_1\bar{v}_s + \bar{R}_2\bar{v}_s + \bar{R}_3\bar{v}_s + \ldots + \bar{R}_k\bar{v}_s = \bar{y}_t
\]

(45)

As can be noted, the additivity of the biomass means, for a sample of \( n \) trees obtained by means of ratio estimates still holds.

Consequently, in the segmented models, it is also possible to establish a sample structure to directly obtain estimates of the tree component basic density using fitted models as a function of the variables diameter at 1.30 m (\( d \)), height of the component (\( h \)), and age of stands (\( A \)), as a second option.

**Total height estimate**

At the moment, the estimate of the stem volume presupposes the measurement of all heights of the sampled trees in the plots. In the case in which only a sample of them is taken from each plot, the remaining tree heights should be estimated. The authors propose the setting of a hypsometric function for this purpose (46).

\[
\frac{\ln h}{\ln d} = \frac{R_h}{\alpha} = \beta_1 + \beta_2 \ln d
\]

(46)

This ratio behaves linearly with respect to the transformed diameters.

**General estimates for different ages**

Vallet et al. (2006), when proposing a generic model for estimating biomass, decided to include in the segmented model a corrective term fitted based on age. In the same way, a new concept will be introduced to obtain estimates of the Scalar Coefficients Proxy of Density - SCPD (\( \hat{R}_j \)), considering the variation in the stand ages.

The additivity of the components is obtained by ratio estimates, as shown in Theorem 1. Naming as \( B_c \) the crown biomass (branches only), \( B_s \) the stem biomass and \( B_t \) the total biomass, then:

\[
B_c + B_s = B_t
\]

(47)

Dividing both terms in equation (47) by the stem volume, we have:

\[
\frac{B_c}{v_s} + \frac{B_s}{v_s} = \frac{B_t}{v_s}
\]

(48)
Thus, we obtained the ratios adopted for solving the additivity, i.e., the Scalar Coefficients Proxy of Density - SCPD ($\bar{R}_j$):

$$R_c + R_s = R_t \quad (49)$$

If proportions of these scalars are taken as a function of the scalar of the stem, we have:

$$\frac{R_c}{R_s} = \frac{B_c v_s - 1}{B_s v_s - 1} = \frac{B_c}{B_s} = Q_1 \quad (50)$$

$$\frac{R_s}{R_s} = \frac{B_s v_s - 1}{B_s v_s - 1} = \frac{B_s}{B_s} = 1 = Q_2 \quad (51)$$

$$\frac{R_t}{R_s} = \frac{B_t v_s - 1}{B_s v_s - 1} = \frac{B_t}{B_s} = \frac{B_c + B_s}{B_s} = \frac{B_c}{B_s} + \frac{B_s}{B_s} = \frac{B_c}{B_s} + 1 = Q_1 + 1 = Q_3 \quad (52)$$

If it is assumed that in the first year of this stand of *Acacia mearnsii* occur a time in which the crown biomass will be equal to the stem biomass, and this will be the starting point, then:

$$B_c = B_s \quad (53)$$

Consequently,

$$Q_1 = 1 \quad (54)$$

As can be noted, the two equations (50) and (51) will start at the same point, i.e., the value equal to one.

To obtain the starting point of the scalar proportion of the total, we have from (52) $Q_3 = Q_1 + 1$ and, therefore,

$$Q_3 = 1 + 1 = 2 \quad (55)$$

If the tree grows, $Q_1$ decreases, hence $R_c R_s = Q_1 \leq 1$ and, applying the same concept to (53) $Q_3 = Q_1 + 1 = 2$.

The adjustment of $Q_j$ as a function of the stand’s age ($A$) can be made using the model (56).

$$Q_j = \beta_1 + \beta_2 \quad (56)$$

Then, by specifying such an estimator for the crown Scalar Coefficient Proxy of Density, we have:

$$Q_1 = \beta_1 + \beta_2 \quad (57)$$

For the total Scalar Coefficient Proxy of Density, we have:

$$Q_3 = \beta_1 + \beta_2 + 1 \quad (58)$$

Letting now $A$ tend to infinity in models (57) and (58), we can obtain their respective asymptotes.

$$Q_{1a} = \beta_2 \quad \text{Asymptote of the model (57)} \quad (59)$$

$$Q_{3a} = \beta_2 + 1 \quad \text{Asymptote of the model (58)} \quad (60)$$
Since $Q_2$ is constant and equal to one, and the other two components are additives, consequently their asymptotes will also be, therefore:

$$\beta_{2e} = \beta_{2i} = \beta_2$$  \hspace{0.5cm} (61)

Then, the two equations converge also to additive asymptotes, i.e.:

$$Q_{1a} = \beta_2$$ \hspace{0.5cm} (62)

$$Q_{3a} = \beta_2 + 1$$ \hspace{0.5cm} (63)

It is clear that the crucial estimator for obtaining the others is stem biomass, which in turn depends on the estimators of stem basic density ($r_s$) and volume ($v_s$). For this estimator, it is suggested that $r_s$ be obtained by dividing the stem biomass by its volume.

**Statistical formulation for ratio estimates**

To obtain ratio estimates applied to quadratic mean diameter, height and biomass, as the best linear trend-less estimates, consider the linear additive model $Y_i = bX_i + e_i$, where $Y_i$ expresses biomass of a given component and $X_i$ the stem volume, i.e., for which two conditions are more likely to occur: (a) the variance of the component biomass estimates is proportional to the stem volume obtained in the sample plots; and b) the variance of the component biomass estimates is proportional to the square of the stem volume obtained in the sample plots (Cochran 1963).

Such weight, if the variance is proportional to the stem volume, is given by $w_i = \frac{1}{\lambda x_i}$ and, if the variance is proportional to the square of the stem volume, is given by $w_i = \frac{1}{\lambda x_i^2}$. As the second option is not the most likely to express the biomass relationship, since such experimental results are not compatible with those used in our sample approach, then opting for the first condition and taking the sum of squares of the errors ($Q$), we have:

$$Q = \sum_{i=1}^{n} w_i e_i^2 = \sum_{i=1}^{n} w_i (y_i - bx_i)^2$$ \hspace{0.5cm} (64)

Taking the derivative of the function in relation to $b$ we get:

$$\frac{\partial Q}{\partial b} = -2\sum_{i=1}^{n} x_i w_i (y_i - bx_i)$$

To minimize the sum of squares we make the derivative equal to zero and solve for the constant $b$, therefore:

$$b = \frac{\sum_{i=1}^{n} x_i w_i y_i}{\sum_{i=1}^{n} x_i^2 w_i}$$ \hspace{0.5cm} (65)

If

$$w_i = \frac{1}{\sigma_{e_i}^2} = \frac{1}{\lambda x_i}$$ \hspace{0.5cm} (66)
Then:

\[
b = \frac{\sum_{i=1}^{n} x_i \frac{y_i}{x_i}}{\sum_{i=1}^{n} x_i^2 \frac{1}{x_i}} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} = \overline{y} / \overline{x} = \hat{R} \quad (67)
\]

The ratio, if specified by tree components, is then defined in (68).

\[
R_j = \frac{\sum_{i=1}^{N} y_{ij}}{\sum_{i=1}^{N} x_{ij}} \quad (68)
\]

In this conception, \( R_j \) is the parameter that expresses the mean Scalar Coefficients Proxy of Density, which may be estimated by simple random sampling and its statistic is presented in (69).

\[
\hat{R}_j = \frac{\sum_{i=1}^{n} y_{ij}}{\sum_{i=1}^{n} x_{ij}} = \overline{y}_j / \overline{x}_j \quad (69)
\]

These estimates generally present trend, since both the numerator \( \overline{y}_j \) as well as the denominator \( \overline{x}_j \) of the ratio varies among the sampling units, which leads \( \hat{R}_j \), in certain situations, to present an asymmetric distribution. If, however, it is obtained with a large sample size, their distributions will tend to normality and, in this case, this tendency becomes very small and, therefore, negligible (Cochran 1963, Sukhatme et al. 1984).

The variance of the ratio estimate can be obtained in (70).

\[
S^2_{\hat{R}_j} = \frac{1}{N} \frac{\sum_{i=1}^{N} (y_{ij} - R_j x_{ij})^2}{N - 1} \quad (70)
\]

Where \( f = n \cdot \frac{N}{n} \)

The mean biomass of the components can be obtained by \( \overline{Y}_j = \overline{X} R_j \), then the total biomass of the components shall be obtained by \( Y_j = N \overline{X} R_j \), and their respective variances in (71) and (72):

\[
S^2_{\overline{Y}_j} \approx \frac{1}{n} \frac{\sum_{i=1}^{N} (y_{ij} - R_j x_{ij})^2}{N - 1} \quad (71)
\]

\[
S^2_{\hat{Y}_j} \approx \frac{N^2}{n} \frac{\sum_{i=1}^{N} (y_{ij} - R_j x_{ij})^2}{N - 1} \quad (72)
\]

According to Cochran (1963), if the parameter variance of the quadratic deviations is obtained by means of ratio estimates \( S^2_d \) (73) and estimated by sampling \( s^2_d \) in (74), therefore the variances of means can be obtained in (75) for the mean biomass of the components and in (76) for the respective totals:

\[
S^2_{\overline{Y}_j} \approx \frac{N (1 - f) \sum_{i=1}^{N} (y_{ij} - R_j x_{ij})^2}{N - 1} \quad (75)
\]

\[
S^2_{\hat{Y}_j} \approx \frac{N^2 (1 - f) \sum_{i=1}^{N} (y_{ij} - R_j x_{ij})^2}{N - 1} \quad (76)
\]
\[ s^2_{\hat{R}_j} = \frac{\sum_{i=1}^{n} (y_{ij} - \hat{R}_j x_{ij})^2}{n - 1} \]  
\[ s^2_{\bar{\hat{Y}}_j} = \frac{(1 - nf)}{n(n - 1)} \sum_{i=1}^{n} (y_{ij} - \hat{R}_j x_{ij})^2 \]  
\[ s^2_{\hat{Y}_j} = \frac{N(N - n)}{n(n - 1)} \sum_{i=1}^{n} (y_{ij} - \hat{R}_j x_{ij})^2 \]

The estimates of the standard errors for the basic densities \( \hat{R}_j \), for the mean of the biomass components \( \bar{\hat{Y}}_j \) and for the total biomass components \( \hat{Y}_j \) can be obtained in (77), (78) and (79) (Cochran 1963):

\[ s_{\hat{R}_j} = \frac{1}{\sqrt{n \bar{x}}} \sqrt{\frac{\sum_{i=1}^{n} (y_{ij} - \hat{R}_j x_{ij})^2}{n - 1}} \]  
\[ s_{\bar{\hat{Y}}_j} = \frac{\sqrt{(1 - nf)}}{n(n - 1)} \sqrt{\frac{\sum_{i=1}^{n} (y_{ij} - \hat{R}_j x_{ij})^2}{n - 1}} \]  
\[ s_{\hat{Y}_j} = \frac{\sqrt{N(N - n)}}{n(n - 1)} \sqrt{\frac{\sum_{i=1}^{n} (y_{ij} - \hat{R}_j x_{ij})^2}{n - 1}} \]

As the population mean for cylindrical volumes is not known, it can be replaced by its estimate \( \bar{x} \) in (77), and the standard error is obtained with the expanded and simplified sum of squares, i.e., using the so-called operational formula in (80), (81) and (82):

\[ s_{\hat{R}_j} = \frac{\sqrt{1 - f}}{\sqrt{n \bar{x}}} \sqrt{\frac{\sum_{i=1}^{n} y_{ij}^2 - 2\hat{R}_j \sum_{i=1}^{n} y_{ij} x_{ij} + \hat{R}_j^2 \sum_{i=1}^{n} x_{ij}^2}{n - 1}} \]  
\[ s_{\bar{\hat{Y}}_j} = \frac{(1 - nf)}{n(n - 1)} \sqrt{\frac{\sum_{i=1}^{n} y_{ij}^2 - 2\hat{R}_j \sum_{i=1}^{n} y_{ij} x_{ij} + \hat{R}_j^2 \sum_{i=1}^{n} x_{ij}^2}{n - 1}} \]  
\[ s_{\hat{Y}_j} = \frac{N(N - n)}{n(n - 1)} \sqrt{\frac{\sum_{i=1}^{n} y_{ij}^2 - 2\hat{R}_j \sum_{i=1}^{n} y_{ij} x_{ij} + \hat{R}_j^2 \sum_{i=1}^{n} x_{ij}^2}{n - 1}} \]

If the distributions are normal, or almost normal, for both the biomass of the components and the Scalar Coefficients Proxy of Density - SCPD, then the confidence intervals can be obtained in (83), (84) and (85). For the biomass’ basic density of the components:

\[ R_j = \hat{R}_j \pm t s_{\hat{R}_j} \]  
\[ \bar{Y}_j = \bar{\hat{Y}}_j \pm t s_{\bar{\hat{Y}}_j} \]  
\[ \hat{Y}_j = \hat{Y}_j \pm t s_{\hat{Y}_j} \]
For the biomass’ total of the components:

$$Y_j = \hat{Y}_j \pm t \frac{s_{\hat{Y}_j}}{\hat{Y}_j}$$  \hspace{1cm} (85)

The details of the respective estimates will be obtained at 95% probability in (86), (87) and (88):

For the biomass’ basic density of the components:

$$E_r = \frac{ts_{\hat{R}_j}}{\hat{R}_j}100$$  \hspace{1cm} (86)

For the biomass’ means of the components:

$$E_{\bar{r}_j} = \frac{ts_{\bar{Y}_j}}{\bar{Y}_j}100$$  \hspace{1cm} (87)

For the biomass’ total of the components:

$$E_r = \frac{ts_{\hat{Y}_j}}{\hat{Y}_j}100$$  \hspace{1cm} (88)

EXPERIMENTAL PROCEDURES IN BLACK WATTLE STANDS

The assumptions made and the methods proposed were evaluated in an experiment conducted in stands of black wattle (*Acacia mearnsii* De Wild.), in the state of Rio Grande do Sul, Brazil.

The field research was conducted in areas where the species is prevalent in the state of Rio Grande do Sul, Brazil, comprising the municipalities of Crystal, Encruzilhada do Sul and Piratini, in June and July 2014. These regions are characterized concerning climate, relief type, and soil in Mochiutti (2007).

The stands were sampled in a sequence of ages after planting to cover all rotation period of the crop (10 years). Four circular plots of 10 m in diameter (78.54 m²) were randomly allocated in each stand. All trees in the plots were felled and measures were taken of diameter at breast height, total height, stem volume, and stem and crown biomass. The study was carried out in 48 temporary plots. Table I shows the central coordinates of the stands and the measurement of the features described above on 670 trees.

**TABLE I**

<table>
<thead>
<tr>
<th>Group of equations</th>
<th>Age of the stand (years)</th>
<th>Age of the stand (qualitative)</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>Too Young</td>
<td>69</td>
</tr>
<tr>
<td>2</td>
<td>1.75 - 1.83</td>
<td>Young</td>
<td>115</td>
</tr>
<tr>
<td>3</td>
<td>2.33 - 3.08</td>
<td>Medium-inicial</td>
<td>163</td>
</tr>
<tr>
<td>4</td>
<td>5.00 - 5.75</td>
<td>Medium-advanced</td>
<td>153</td>
</tr>
<tr>
<td>5</td>
<td>9.83 - 10.75</td>
<td>Mature</td>
<td>170</td>
</tr>
</tbody>
</table>

(i) diameter at breast height was measured with a dendrometric tape and the overall heights with a measuring tape;
(ii) stem volume (with bark) was calculated with the Huber’s method and the stem scaling with the Hohenadl’s method. The scaling was performed with a dendrometric tape and measured along the stem in the positions 5%, 15%, 25%, 35%, 45%, 55%, 65%, 75%, 85% and 95% of the total height. The stem volume was obtained by the equation in (37).

(iii) biomass measurement was performed for the stem component (stem wood + bark) and the crown (live branches, dead branches, leaves, flowers and fruits). For each tree, these components were separated and weighted with a digital scale (Portable Electronic Scale) to get the fresh biomass with 5g of accuracy. The stump biomass was not included in the measurement. The total biomass was defined as the sum of the stem biomass with the crown components.

Crown and stem dry biomass were obtained taking wood samples and immediately weighting them with a digital balance (Hoyle) with a 1g of accuracy. The samples of the crown, approximately 1,500 grams, were taken along the crown at positions 0%, 25%, 50%, 75% and 95% of the total crown length (away from the first branch, regardless of being alive or dead, to the apex of the crown). In each stem, 5 discs of 2cm thick were removed at the positions 0%, 25%, 50%, 75% and 95% of the total height.

The samples were dried in an oven with air-circulation and air renewal at 100 °C to constant mass and weighed with a digital scale with 1g of accuracy. The dry biomass was obtained in (89):

$$B_i = \frac{M_{U_i} M_{S_i}}{M_{U_{Ai}}}$$  \hspace{1cm} (89)

where:

- $B_i$: dry biomass of the stem or crown components, in kg;
- $M_{U_i}$: fresh mass of stem or crown component, in kg;
- $M_{S_i}$: dry mass of the stem or crown sample, in kg;
- $M_{U_{Ai}}$: fresh mass of the stem or crown sample, in kg.

ADDITIVITY OF TREE BIOMASS COMPONENTS USING RATIO ESTIMATE: GENERAL PROCEDURES

The modeling of the biomass components was performed using the general model proposed in (38). Descriptive statistics were obtained for all variables, as well as the correlations between them.

$$Y_{ij} = R_j X_{ij} + e_{ij}$$  \hspace{1cm} (90)

where:

- $Y_{ij}$: biomass of a given component: stem, crown or total, in kg;
- $R_j$: Scale Coefficients Proxy of Density, kg.m$^{-3}$;
- $X_{ij}$: cylinder volume which is based in $dq$, as shown in (37), in m$^3$;
- $e$: error associated to the estimate, in kg.

Four procedures were tested referring on how to obtain $X_{ij}$, which can be directly measured or estimated, and how to generalize $R_j$. For each procedure, we provided the explanations on each coefficient estimated.

(i) Procedure 1: dependent and independent variables are measured

In this procedure, the biomass of the components crown ($Y_c$), stem ($Y_{is}$) and the total biomass ($Y_{it}$), as well as the stem volume ($X_{ij}$) are measured. Modeling is performed in (91), (92) and (93):

$$Y_{is} = R_s X_{is} + e_{is}$$  \hspace{1cm} (91)

$$Y_{ic} = R_c X_{ic} + e_{ic}$$  \hspace{1cm} (92)

$$Y_{it} = R_t X_{it} + e_{it}$$  \hspace{1cm} (93)
(ii) Procedure 2: biomass components and height are measured, and $d_q^2$ is estimated

In this procedure, the biomass of the components crown ($Y_{ic}$), stem ($Y_{is}$) and the total biomass ($Y_{it}$), as well as the height of the tree ($h$) are measured. The variable $d_q^2$ is estimated to enable the calculation of the stem volume. Modeling is performed in (94) to (98).

\begin{align*}
    d_q &= \alpha d \quad (94) \\
    X &= \frac{\pi}{4}d_q^2h \quad (95) \\
    Y_{is} &= R_s \left(\frac{\pi}{4}d_q^2h\right) \quad (96) \\
    Y_{ic} &= R_c \left(\frac{\pi}{4}d_q^2h\right) \quad (97) \\
    Y_{it} &= R_t \left(\frac{\pi}{4}d_q^2h\right) \quad (98)
\end{align*}

(iii) Procedure 3: biomass of the components is measured, $d_q^2$ and height are estimated

In this procedure, the biomass of the components crown ($Y_{ic}$), stem ($Y_{is}$) and the total biomass ($Y_{it}$) are measured. The variables $d_q^2$ and height ($h$) are estimated to enable the calculation of the volume. Modeling is performed in (99) to (103).

\begin{align*}
    d_q &= \alpha d \quad (99) \\
    \ln \frac{h}{d} &= R_h = \beta_1 + \beta_2 \ln d \quad (100) \\
    Y_{is} &= R_s \left(\frac{\pi}{4}d_q^2h\right) \quad (101) \\
    Y_{ic} &= R_c \left(\frac{\pi}{4}d_q^2h\right) \quad (102) \\
    Y_{it} &= R_t \left(\frac{\pi}{4}d_q^2h\right) \quad (103)
\end{align*}

The hypsometric model presented into (100) is suggested by the authors after a casual evidence that the ratio of logarithmic transformation of height $h$ and diameter $d$ fits well with a straight line as a function of logarithm of $d$.

(iv) Procedure 4: generalization of $R_j$

As one of the procedures described previously has been chosen, then $R_j$ will be generalized to cover the range of stand ages, as described in the section CONCEPTION FOR BIOMASS MODELING IN THIS RESEARCH (General Estimates for Different Ages).

ADDITIVITY OF TREE BIOMASS COMPONENTS USING NONLINEAR WEIGHTED-SEEMLINGLY UNRELATED REGRESSIONS

For the same database, Behling et al. (2018) proposed systems of equations for the estimation of crown, stem and total biomass. For the group one, the equations were adjusted through seemingly unrelated regressions and for the other groups using weighted-nonlinear seemingly unrelated regressions (WNSUR). The equations and procedures used for the fitting are detailed in Behling et al. (2018).
COMPARISON OF THE ESTIMATES’ PERFORMANCE USING RATIO AND REGRESSION MODELS

The results from ratio estimates (procedure 1, 2 and 3) and regressions estimates were compared using the following statistics:

i) Bias: $\text{Bias} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)$

ii) Mean absolute error: $\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|$

iii) Mean squared error: $\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$

iv) Root mean squared error: $\text{RMSE} = \left[ \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \right]^{0.5}$

v) Pearson correlation

RESULTS

DESCRIPTION OF THE VARIABLES AND CORRELATIONS

Table II shows the descriptive statistics for the biomass components, stem volume, height and diameter at breast height.

<table>
<thead>
<tr>
<th>Grupo</th>
<th>$\bar{d}$ (cm)</th>
<th>$\bar{h}$ (m)</th>
<th>$\bar{y}_S$ (kg)</th>
<th>$\bar{y}_C$ (kg)</th>
<th>$\bar{y}_T$ (kg)</th>
<th>$\bar{y}_S$ ($m^3$)</th>
<th>$\bar{d}_2^q$ (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.09</td>
<td>2.98</td>
<td>0.67</td>
<td>0.69</td>
<td>1.36</td>
<td>0.00113</td>
<td>0.45</td>
</tr>
<tr>
<td>2</td>
<td>8.22</td>
<td>9.82</td>
<td>15.94</td>
<td>6.23</td>
<td>22.17</td>
<td>0.03375</td>
<td>2.76</td>
</tr>
<tr>
<td>3</td>
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<td>11.75</td>
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<td>0.04585</td>
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</tr>
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<td>11.48</td>
<td>59.92</td>
<td>0.08918</td>
<td>6.88</td>
</tr>
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<td>74.08</td>
<td>15.04</td>
<td>89.12</td>
<td>0.12498</td>
<td>8.98</td>
</tr>
</tbody>
</table>

Total biomass and biomass of the components showed a positive linear trend with the stem volume. The correlations were strong, ranging from 0.78 to 0.98, and all were significant at 95% probability. The smallest correlations were found between the crown biomass and the stem volume, while the largest ones were found for the total and stem biomass with the stem volume (Table III).

<table>
<thead>
<tr>
<th>Biomass Components</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>Stem</td>
<td>94.21% 94.42% 97.56% 96.88% 97.96%</td>
</tr>
<tr>
<td>Crown</td>
<td>88.87% 78.89% 93.10% 90.10% 88.73%</td>
</tr>
<tr>
<td>Total</td>
<td>94.32% 92.67% 97.53% 97.09% 97.72%</td>
</tr>
</tbody>
</table>
APPLICATION OF THE PROCEDURES

(i) Procedure 1: Dependent and independent variables are measured

Results of the ratios and accuracy obtained by means of the first procedure are shown in Table IV.

<table>
<thead>
<tr>
<th>Group</th>
<th>Ratio</th>
<th>Precisions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bs</td>
<td>BC</td>
</tr>
<tr>
<td>1</td>
<td>591.1901</td>
<td>611.9743</td>
</tr>
<tr>
<td>2</td>
<td>472.3366</td>
<td>184.6601</td>
</tr>
<tr>
<td>3</td>
<td>476.7228</td>
<td>148.4614</td>
</tr>
<tr>
<td>4</td>
<td>476.7228</td>
<td>148.4614</td>
</tr>
<tr>
<td>5</td>
<td>543.2471</td>
<td>128.6886</td>
</tr>
</tbody>
</table>

(ii) Procedure 2: Measurement of biomass and height, and estimation of $d_q^2$

Results of the ratios and accuracy obtained by means of the second procedure are shown in Table V.

<table>
<thead>
<tr>
<th>Group</th>
<th>Ratio</th>
<th>Precisions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_q$</td>
<td>$B_q$</td>
</tr>
<tr>
<td>1</td>
<td>1.0032</td>
<td>537.6030</td>
</tr>
<tr>
<td>2</td>
<td>0.7394</td>
<td>448.6395</td>
</tr>
<tr>
<td>3</td>
<td>0.7226</td>
<td>467.6376</td>
</tr>
<tr>
<td>4</td>
<td>0.7019</td>
<td>534.9480</td>
</tr>
<tr>
<td>5</td>
<td>0.7059</td>
<td>584.6860</td>
</tr>
</tbody>
</table>

(iii) Procedure 3: Measurement of biomass and estimation of $d_q^2$ and height,

Results of the ratios and accuracy obtained for the variable $d_q$ has already been presented in Table V. The equations of tree height models are presented in (104) to (108).

\[
\ln h = (3.0558 - 1.8878 \ln d) \quad R^2 = 72.51 \\
\ln h = (1.9150 - 0.4343 \ln d) \quad R^2 = 71.35 \\
\ln h = (1.8646 - 0.3333 \ln d) \quad R^2 = 83.86 \\
\ln h = (1.9455 - 0.3310 \ln d) \quad R^2 = 87.85
\]
\[
\ln h = (1.7584 - 0.2542 \ln d) \quad R^2 = 84.93
\]  
(108)

Results of the ratios and accuracy obtained by means of the second procedure to model the biomass of the components and total biomass are presented in Table VI.

<table>
<thead>
<tr>
<th>Group</th>
<th>Ratio</th>
<th>Precisions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B_s</td>
<td>B_c</td>
</tr>
<tr>
<td>1</td>
<td>570.0388</td>
<td>590.0794</td>
</tr>
<tr>
<td>2</td>
<td>458.0025</td>
<td>270.0873</td>
</tr>
<tr>
<td>3</td>
<td>474.7222</td>
<td>147.8384</td>
</tr>
<tr>
<td>4</td>
<td>542.3116</td>
<td>128.4670</td>
</tr>
<tr>
<td>5</td>
<td>585.7339</td>
<td>118.9215</td>
</tr>
</tbody>
</table>

(iv) Procedure 4: Generalization of \( R_j \)

Taking experimental data of \( Acacia mearnsii \) at different stand ages and making the calibration to get estimators for the asymptotes defined in (62) and (63), which common value was 0.1, we have:

\[
Q_{1a} = 0.1
\]  
(109)

\[
Q_{3a} = 0.1 + 1 = 1.1
\]  
(110)

Considering the age of 1 year, the start points for both curves are equal to one, then:

\[
\beta_{1c} + \beta_{2c} = 1
\]  
(111)

Therefore,

\[
\beta_{2c} = 1 - \beta_{1c} = 1 - 0.1 = 0.9
\]  
(112)

In the same way,

\[
\beta_{1t} + \beta_{2t} + 1 = 2
\]  
(113)

Therefore,

\[
\beta_{2t} = 2 - \beta_{1t} - 1 = 2 - 1.1 = 0.9
\]  
(114)

The equations (57) and (58) are, therefore, adjusted in (113) and (114), and presented in Figure 2.

\[
Q_1 = 0.9 \frac{1}{A} + 0.1
\]  
(115)

\[
Q_3 = 0.9 \frac{1}{A} + 0.1 + 1 = 0.9 \frac{1}{A} + 1.1
\]  
(116)

Explanations for the estimates obtained by means of this procedure are the same as those given for the first procedure presented in methods.
Figure 2 - Variation of the ratio between the biomass of the components and the total biomass with the stem volume in stands of black wattle in the state of Rio Grande do Sul, Brazil.

### TABLE VII

Statistics obtained for comparison of the estimates through ratio and WNSUR procedures of stem biomass in stands of black wattle in the state of Rio Grande do Sul, Brazil.

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Procedure 1</th>
<th>Procedure 2</th>
<th>Procedure 3</th>
<th>WNSUR procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ME</td>
<td>-1.92E-08</td>
<td>2.00E-05</td>
<td>0.34</td>
<td>0.01</td>
</tr>
<tr>
<td>1</td>
<td>MAE</td>
<td>0.10</td>
<td>0.19</td>
<td>0.34</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>MSE</td>
<td>0.02</td>
<td>0.06</td>
<td>0.13</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.13</td>
<td>0.24</td>
<td>0.36</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>0.94</td>
<td>0.93</td>
<td>0.89</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>ME</td>
<td>-0.32</td>
<td>-1.65E-04</td>
<td>0.71</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>0.86</td>
<td>0.79</td>
<td>1.07</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>MSE</td>
<td>1.89</td>
<td>1.25</td>
<td>2.03</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>1.37</td>
<td>1.12</td>
<td>1.43</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>0.94</td>
<td>0.96</td>
<td>0.94</td>
<td>0.96</td>
</tr>
<tr>
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<td>8.21E-04</td>
<td>1.00</td>
<td>0.13</td>
</tr>
<tr>
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<td>MAE</td>
<td>1.98</td>
<td>2.25</td>
<td>2.18</td>
<td>1.89</td>
</tr>
<tr>
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<td>MSE</td>
<td>8.85</td>
<td>12.82</td>
<td>9.59</td>
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</tr>
<tr>
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<td>2.97</td>
<td>3.58</td>
<td>3.10</td>
<td>2.90</td>
</tr>
<tr>
<td></td>
<td>r</td>
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<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>ME</td>
<td>2.37E-06</td>
<td>-1.43E-03</td>
<td>-0.56</td>
<td>0.32</td>
</tr>
<tr>
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<td>MAE</td>
<td>4.75</td>
<td>4.76</td>
<td>5.24</td>
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</tr>
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<td>53.47</td>
<td>47.05</td>
<td>51.00</td>
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</tr>
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<td>r</td>
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<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
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<td>ME</td>
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<td>-4.01E-03</td>
<td>-4.01E-03</td>
<td>-0.27</td>
</tr>
<tr>
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<td>6.27</td>
<td>7.17</td>
<td>7.82</td>
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</tr>
<tr>
<td></td>
<td>MSE</td>
<td>92.92</td>
<td>104.39</td>
<td>119.03</td>
<td>95.42</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>9.64</td>
<td>10.22</td>
<td>10.91</td>
<td>9.77</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
</tr>
</tbody>
</table>

COMPARISON OF ESTIMATES USING RATIO AND REGRESSION ESTIMATES

Tables VII, VIII and IX shows the results of the statistics for comparison of the estimates through ratio and WNSUR procedures.
TABLE VIII
Statistics obtained for comparison the estimates trough ratio and WNSUR procedures of crown biomass in stands of black wattle in the state of Rio Grande do Sul, Brazil.

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Procedure 1</th>
<th>Procedure 2</th>
<th>Procedure 3</th>
<th>WNSUR procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ME</td>
<td>-0.02</td>
<td>2.08E-05</td>
<td>0.35</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>0.10</td>
<td>0.23</td>
<td>0.35</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>MSE</td>
<td>0.02</td>
<td>0.08</td>
<td>0.15</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.14</td>
<td>0.29</td>
<td>0.38</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>0.94</td>
<td>0.85</td>
<td>0.82</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>ME</td>
<td>1.46</td>
<td>-9.61E-05</td>
<td>0.42</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>1.60</td>
<td>1.19</td>
<td>1.09</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>MSE</td>
<td>4.57</td>
<td>2.33</td>
<td>2.14</td>
<td>2.17</td>
</tr>
<tr>
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<td>RMSE</td>
<td>2.14</td>
<td>1.53</td>
<td>1.46</td>
<td>1.47</td>
</tr>
<tr>
<td></td>
<td>r</td>
<td>0.79</td>
<td>0.81</td>
<td>0.82</td>
<td>0.80</td>
</tr>
<tr>
<td>3</td>
<td>ME</td>
<td>5.39E-08</td>
<td>2.55E-04</td>
<td>0.31</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>1.50</td>
<td>1.32</td>
<td>1.33</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
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<td>4.23</td>
<td>2.96</td>
<td>3.49</td>
<td>2.83</td>
</tr>
<tr>
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</tr>
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<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
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</tr>
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<td>3.39</td>
</tr>
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<td>0.91</td>
<td>0.90</td>
</tr>
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<td>6.53</td>
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</tr>
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<td>r</td>
<td>0.89</td>
<td>0.90</td>
<td>0.90</td>
<td>0.91</td>
</tr>
</tbody>
</table>

DISCUSSION

Because most biomass studies are based on fitted volumetric models, i.e., using empirical data of biomass such as the predictor variables diameter at breast height (d) and the height of the tree (h), alone or combined, they often face heteroscedasticity of errors. Then, after the model is linearized, if appropriate, and the "ordinary least squares" is applied to estimate the parameters, this estimator ceases to be BLUE. This will affect the conclusions drawn on the significance tests of the equations used and the construction of confidence intervals for the coefficients, as well as the mean estimates and their prediction.

This heteroscedasticity of errors can be explained by its modeling, which follows a normal distribution with increasing variance in relation to $X_i$ and is expressed by an exponential behavior of $X_i$, allowing the establishment of weights. In this way, the regression model can be adjusted by “generalized least squares”, resulting in a BLUE estimator and, consequently, will validate the significance tests of the equation proposed and the construction of confidence intervals (Parresol 1999).

The two variables form factor $F$ and wood density $r$ are associated with the coefficients of the regression models, however not setting up a balanced participation in the coefficients of each model when adjusted.
<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Procedure 1</th>
<th>Procedure 2</th>
<th>Procedure 3</th>
<th>WNSUR procedure</th>
</tr>
</thead>
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<td>1.70</td>
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<td>0.95</td>
<td>0.94</td>
<td>0.95</td>
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<td>4.06</td>
<td>3.81</td>
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</tr>
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<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
</tr>
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<td>70.77</td>
<td>61.09</td>
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<td>8.41</td>
<td>7.82</td>
</tr>
<tr>
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<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
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<td>12.90</td>
<td>12.29</td>
</tr>
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for the components, which leads to the loss of additivity. To solve this problem, several researchers have developed, since the 1970s, complex methodologies to force the additivity of components, known in the literature as SUR and NSUR (Kozak 1970, Chiyenda and Kozak 1984, Cunia and Briggs 1984, 1985, Reed and Green 1985, Parresol 1999, 2001).

The model presented in this paper uses ratio estimates, in which the total biomass and the biomass of the components are function of the stem volume multiplied by the respective Scalar Coefficients Proxy of Density - SCPD. In this case, the assumption that the metric unit of the independent variables will assume the same metric unit of the dependent variable was assured and allowed to maintain the desired property in the biomass modeling: the additivity.

Considering the whole experience with biomass estimation presented in this work, especially those developed in Europe, it became apparent that the biomass estimators and carbon sequestration by trees were obtained with segmented models, i.e., each parameter of the model is estimated separately, avoiding problems with complex methodologies peculiar of linear regression models (Zianis and Mencuccini 2004, Chave et al. 2005, Saint-Andre et al. 2005, Vallet et al. 2006, Antônio et al. 2007, Wutzler et al. 2008, Pretzsch 2009, Genet et al. 2011, MacFarlane 2015). For the same reason, the conceptual modeling proposed and
performed in this work was obtained with a rational model, because it does not require extra methodological
devices for additivity, as it was evidenced by the proof of Theorem 1 and its corollary. Even though the
European experience has sought creative solutions to get a consistent estimator for the form factor ($F$)
(Vallet et al. 2006, Genet et al. 2011), including the new variable called hardiness ($h_{nd}$) to correct the effect
of density in the form of trees, our experience has shown another way to deal with this estimator. We use the
knowledge of the school of Freiburg, i. Br., Germany, which applies the concept of volume equivalence, in
which, using the quadratic mean diameter ($d^2_q$) of stems, the volume taken at this reference shall be a cylinder
equivalent to the stem volume referenced at the $DBH$ (Prodan 1965, Ko 1968). Under these circumstances,
the tree form factor shall be always equal to one and, therefore, enabling a solution for the relationship
between ($d^2_q$) and ($DBH$), for which the accuracy of the mean estimate can be less than 4%. Such a solution is
easy to implement in Brazilian companies, since they all have to collect sample data for adjusting volumetric
equations and the database has been gradually standardized with the Hohenadl’s methodology (Hohenadl
1924), as presented in this work.

Therefore, the results obtained by ratio estimates and WNSUR were similar, evaluated by the statistics
ME, MAE MSE and RSME were also similar between the four different procedures and thus it is confirmed
the second part of the proposed hypothesis.

The variable basic densities ($r_j$) were equated to be included in the proposed biomass models,
considering the advantages of convenient properties of ratio estimates and because they are exactly the
estimators per component. However, given the difficulty of obtaining experimentally the volumes of
the components of each tree, we proposed to solve the problem relativizing them as a function of the
stem volume. This transformation has converted the volumes of the components of each tree into Scalar
Coefficients Proxy of Density - SCPD and, as it has been demonstrated in Theorem 1 and in its corollary,
the additivity was maintained. Then, all the biomass estimators can be evaluated by taking only the stem
volume and the respective ratio estimates already obtained experimentally.

Vallet et al. (2006) proposed the inclusion of a corrective term, adjusted as a function of age, to obtain a
generic model for estimating biomass. To do so, they considered separately the estimates for the components
and for total biomass. In the same way, a generic solution for the correction of the components and total
biomass estimators for different ages was proposed in this research, taking advantage of a hyperbolic
function fitted to the proportions of the SCPD as a function of the Scalar Proxy of Density of the stem
to facilitate the obtaining of the other estimators that, despite changing their values, continue to maintain
additivity with advancing of stand ages. This occurs because the biomass of components maintains a
proportional behavior as age increases.

As we can see in the proposal presented in this work, the variable height was maintained to calculate
the stem volume ($V_s$), considering its significant additional contribution in obtaining the stem and total
volumes. However, it is evident the worldwide difficulty of measuring it in all trees. Normally, it is worth
fitting hypsometric functions to provide all height values for the biomass equations. Vallet et al. (2006)
emphasizes the importance of maintaining the height in the estimators for the additional contribution that it
makes to obtaining the volumes. We consider appropriate to adjust a hypsometric model per plot, although
in the present work it was done per forest area. If adjusting it per plot, it will be possible to indirectly mediate
the effects of fertility or site variations, and stand density.

The model performed better for stem and total biomass than for crown biomass. Typically, the
crown component is more difficult to model, since it is affected by several relational effects on its final
configuration, mainly from the environment, and its inclusion in the biomass models is complex. MacFarlane (2015) dealt with this issue, although with restrictive aspects, as he assumed the existence of a central axis in the crown, a mean of crown diameter and an abstract form factor in order to obtain its volume. In the circumstances presented in this work, using the ratio estimate and a form factor equal to one, many of these problems were reduced to obtain a good estimate of the crown biomass.

The correlations observed between the variables included in the biomass model, using ratio estimates, indicated that the stem volume is an excellent independent variable for shaping it. The confidence intervals resulted in errors of less than 11% of the mean value in all cases. The adjustment of the mean quadratic diameter of the stem \( d^2_q \) as a function of \( d \) also proved entirely satisfactory, with a high correlation between the two variables and high accuracy, as already presented above. This shows that the adjustment of \( d^2_q \) as a function of \( d \) by means of ratio estimate is appropriate. Balbinot et al. (2017) have compared direct and indirect methods to estimate tree biomass of native species. The authors have concluded that direct method is the most effective way to quantify stem biomass, but indirect methods, that use estimated volume, did not differ statistically for the estimated and observed stem biomass.

The application of the adapted model, in which the total biomass or the biomass of the components is fitted as a function of the stem volume, estimated by diameter \( d \) and multiplied by the Scalar Coefficients Proxy of Density - SPDC, is also very appropriate, because additivity of the biomass components is maintained, a very desirable characteristic. Depending on the procedure, the model becomes indirectly dependent on \( d \), a less costly and easier variable to be measured in the forest inventory.

All proposed and tested procedures resulted in acceptable explanations for the estimated coefficients and mean values, even with little variation between them. The estimation of coefficients to model the biomass of the components and total biomass as a function of the stem volume by ratio estimation method also proved very promising, because it is easy to solve, with only one division operation between two values and can be done quickly. Therefore, these estimators showed to be accurate and, especially, managed to preserve the natural biomass additivity, proving the hypothesis established in this study.

Ratio estimation may contribute beyond the biomass modeling, since it is possible to reduce the operational costs when applying it to forest inventory. It is known that the coefficients obtained by means of ratio estimates are dependent only on the means of the two variables involved, then the inventory should prioritize the evaluation of individuals from classes close to the means of the selected variables, thereby greatly reducing the sampling and, consequently, the costs.

The ratio estimates generate additivity for the tree components, in the same way they do for the tree data, whereas if they are modeled via linear or non-linear regressions, it is necessary to force it through SUR or NSUR methodologies to achieve additivity. In the proposed segmented estimators, the metric unit of the ratios are the same as those of the resulting coefficient, which was called "Scalar Coefficient Proxy of Density", i.e., the metric unit of the ratios is kg.m\(^{-3}\) and, when multiplied by the value of the variable \( X \) will result in biomass in kg, which does not happen in almost all the volumetric models adapted to obtain biomass estimates. This confirms the first part of the proposed hypothesis.

The restriction on metrics of the dependent or independent variables in linear and non-linear regression models for biomass evaluation was applied because this is one of the significant reasons for not achieving additivity, i.e., it is modeling for indirect obtaining of biomass. In the case of our proposal, by the application of the ratio estimates, this condition is satisfied and, therefore, generates the natural additivity. This concept is not, of course, the real value of the basic density in the case of crown biomass, bark, or possibly only of
the branches, because the cylindrical volume obtained experimentally is only for the stem. The variables arising in these cases, which were called "Scalar Coefficient Proxy of Density", have characteristics of the metric unit of the basic density variable (kg.m⁻³). For the reasons given above, these variables were also obtained as a function of the cylindrical stem volume, owing to the high correlation that the tree biomass components have with it, as it is commonly used in allometric procedures in linear regression modeling. This reinforces the confirmation of the first part of the proposed hypothesis.

The approximate nature of the estimators, due to the actual values of density, has enabled us to ensure them consistency and absence of bias. This is acceptable in view of the difficulty in achieving the total volume of the tree with inclusion of branches, leaves or bark of the branches for precisely obtaining the density value of these components.

Additionally, even though assuming a linear or non-linear relationship of biomass with the variables diameter taken at 1.30 m \((d)\) and tree height, it is implicitly accepting that this relationship is also being carried out based on an incomplete stem volume, as in the case of the Schumacher-Hall’s model or the Spurr’s model, being the only conceptual difference the presence of intercession in these models, which implies a solution only via linear or nonlinear regression.

The proposed method is simple and, as evidenced by the results obtained experimentally, are consistent and accurate. We emphasize that in all forest stands of Brazilian companies, because of the extraordinary genetic improvement achieved by them, tree volumes are used in the same way for adjusting volumetric equations and to obtain biomass estimates, as well as for the additional adjustment of a function to estimate the mean square diameter per tree, which can be easily done experimentally, therefore is quite appropriated for the purposes used in this work.

The generalization of the use of fitted functions for other components will be the result of a mere complement of statistical procedures for assessing the adequacy of the adjustments per forest area or, more generally, for an annual unit of planting (Figure 2).

Studies on the application of the proposed model should be expanded. It is also suggested the application of this modeling per diameter class.

**CONCLUSIONS**

The proposed model, using ratio estimates, is appropriate and promising for modeling of tree biomass.

The adjustment of the mean quadratic diameter of the stem \((d_q^2)\) as a function of \(d\) also proved entirely satisfactory, with a high correlation between the two variables and high accuracy. This shows that the adjustment of \(d_q^2\) as a function of \(d\) by means of ratio estimate is appropriate and is a key point for obtaining volumetric equivalence of tree components.

In the application of the adapted model, the total biomass, or biomass of components, are obtained as a function of the stem volume, which is estimated as a function of \(d\) and multiplied by the Scalar Coefficients Proxy of Density – SCPD. These integrated procedures are quite appropriate.

The use of a form factor equal to one for all the biomass components was a quite significant improvement in modeling biomass components. This approach means to transform their estimated volumes to equivalent real volumes avoiding, consequently, to have the adverse variation of this variable when it is present in the model as a random variable.
The natural additivity of the tree biomass components was fully achieved, when modeling them by means of ratio estimation.

Equations developed from the proportional behavior of the biomass components at different ages did not require the use of linear regression models and were obtained from calibration with the experimental data. The estimators resulting from these equations proved to be appropriate to make a generic model for correction of ratios coefficients at different ages.

The results obtained by ratio estimates and WNSUR were quite similar, evaluated by the statistics ME, MAE MSE and RSME, which were also similar between the four proposed procedures.

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AUTHOR CONTRIBUTIONS

Both authors contributed equally on this paper, on the development of the theoretical conception for the evaluation of tree biomass by ratio estimators. It was also applied the modeling of simultaneous regression equations, which allowed the comparison between the results obtained by these two methodologies. The research was conducted with data of tree biomass of the species Acacia mearnsii, provided by the company TANAGRO S.A. of Rio Grande do Sul, Brazil.

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