# STRUCTURAL DEFORMATIONS ANALYSIS BY MEANS OF KALMAN-FILTERING

*Analise da deformação estrutural com filtros de Kalman*

ENGIN GULAL

Yildiz Technical University
Faculty of Civil Engineering
Department of Geomatic Engineering
Davutpasa Campus, 34220 - Esenler- Istanbul, Turkey
engingulal@gmail.com

**ABSTRACT**

The surveillance of engineering structures like dams is an interdisciplinary task and mainly focused on the assessment of stability and reliability of the objects to be monitored. To show the co-operation of the disciplines involved in a comprehensible manner, it is suitable to use system analysis approaches. Structural deformations analysis by means of system analysis is explained in the following with an example of a dam. The determination of the dam deformations is demonstrated by an integration of computed and measured data by using Kalman Filtering.
**Keywords:** Dam; Deformation; Geodetic Monitoring; Kalman Filtering.

**RESUMO**

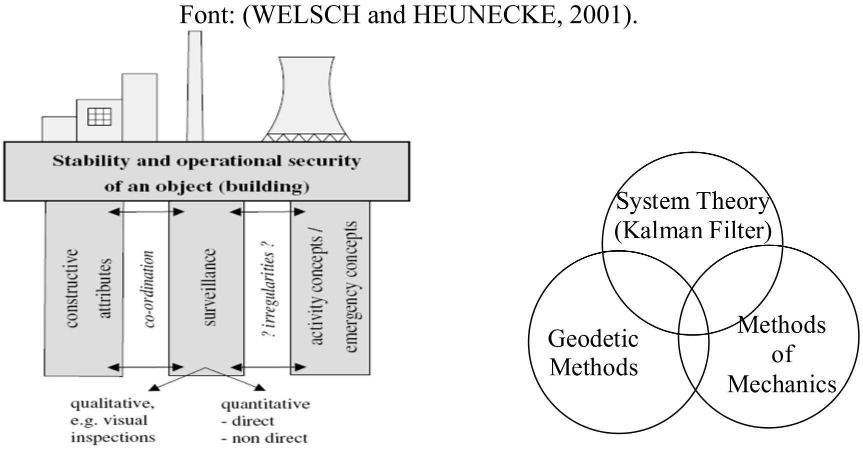
O monitoramento e controle de estruturas de engenharia como barragens, é uma tarefa multidisciplinar e focada principalmente na avaliação da estabilidade e confiabilidade do objeto a ser monitorado. Para mostrar a colaboração das disciplinas envolvidas de maneira compreensível, é conveniente usar a naálise de sistemas aproximados. A análise da deformação de estruturas através da análise de sistemas é explanada com um exemplo de barragem como a seguir. A determinação da deformação da barragem é mostrada pela integração de cálculos e de dados medidos usando Filtros de Kalman.
**Palavras-chave**: Barragem; Deformação; Monitoramento Geodésico; Filtros de Kalman.

## 1. INTRODUCTION

Over the past 20 years there has been significant progress in the development of new methods for the analyses of deformation measurements. International Federation of Surveyors (FIG) has been leading in the developments, particularly in the areas of integrated geometrical analysis of structural deformations and combined integrated analysis. The activity of FIG Working Group 6.1 has been focused on developing new methods, which could be applied to monitoring and analysis of the structures (CHRZANOWSKI et al., 2003, CHRZANOWSKI 2006, CHRZANOWSKI and WHITAKER 2008). In 1992, an FIG WG 6.1 ad hoc Committee on Terminology and Classification of Deformation Models was created. The work of the committee was completed in 2001 and the results were published by FIG as a publication No. 25 (WELSCH and HEUNECKE, 2001).

Stability and operational safety of a structure consist of three components; constructional safety, surveillance and emergency concept, (Figure 1). Aim and purpose of any surveillance is the earliest possible detection of damage, failure or an injury to the safe operation of a construction in order to be able to react appropriately and in time. In frameworks of the stability and operational safety of a structure, geodetic monitoring measurements must be seen as a part of surveillance concept. Herein, the measurements are not only restricted to the determination of the geometrical changes but also to the determination of the causal input quantities (external and internal loads). The activity and emergency concepts include but are not limited to aspects of normative and budgetary constraints; integrity, material and structure damage assessment; intensified surveillance and diagnostic technologies; lifetime and utilization evaluation; maintenance and repair, out-of-service and replacement decisions etc.

Figure 1 - Interdisciplinary approach of stability and operational safety of structures Font: (WELSCH and HEUNECKE, 2001).



Determination of stability and operational safety of a structure require interdisciplinary approach depending on the developments of measurement and analysis techniques. .

*Engineering geodesy* deals with planning, setting out, recording and the monitoring of structures and other objects. The task of geodetic monitoring measurements generally exists of determination of deviations from expected changes on selected object points. Widespread measurement techniques are terrestrial measurements, leveling and Global Navigation and Satellite System (GNSS) and other structural measurement techniques (inclinometers, pendulum etc.).

*Structural mechanics* is the computation of deformations, deflections and internal forces or stresses within structures, either for design or for performance evaluation of existing structures. The Finite element method (FEM) is a powerful technique originally developed for numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems.

As an aspect of *systems theory* is a method for understanding the dynamic behavior of complex systems. In system theory, the set-up of an appropriate mathematical-physical representation of the transfer function of a dynamic system is called system identification.

Setting up a model for the transfer function is important for the choice of a parametric or a non-parametric identification, (HEUNECKE and WELSCH 2000). A suitable tool for parametric identification is provided by the application of Kalman-filtering. Until now, Kalman-filtering was successfully used in civil engineering in combination with FE-models. Heunecke 1996 was determined the diurnal variation due to solarization of a suspension bridge pylon, Hesse 2002 analyzed a shell structure under varying loads, Eichhorn 2006 calculated the thermal deformations of bar-shaped machine elements and Teskey 1988 and Gulal 1997 used this technique for dam monitoring.

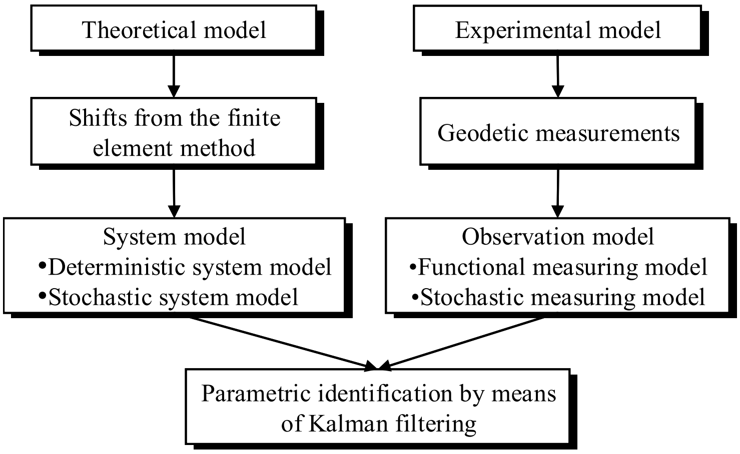
Figure 1 likes to show that there is an intersection of interdisciplinary approaches between system theory, engineering geodesy and mechanics for the stability and operational safety of structures. This intersection is explained and demonstrated in this paper with the analysis of the dam.

## 2. METHOD

Kalman 1960 published his famous paper describing a recursive solution to the discrete-data linear filtering problem. Since that time, due in large part to advances in digital computing; the Kalman filter has been the subject of extensive research and application. Kalman filter is used for the optimal estimation of the state of complex systems.

The Figure 2 gives basic idea of the Kalman filtering. The qualitative conceptions about the behavior of an object are concretized on theoretical and on experimental methods. A quantitative system model is set up by means of finite elements as method. The observation model is formed depending on the basis the geodetic measuring techniques and process methods. Identification by means of Kalman filtering provides optimal estimation of both geometrical state and material parameters of the object.

Figure 2 - Identification by means of Kalman filtering.




The finite element method, briefly as FEM designation, is an effective numeric procedure for the computation of complex constructions in civil engineering. Basic idea of FEM applications in structural mechanics problems is analyzing the structure by dividing it into geometrical small elements. After the definition of the structure, $\Delta X$ shifts on knots can be obtained dependent on the rigidity matrix K and the affected forces $\Delta f$.

$$\Delta X = K^{-1}\Delta f \tag{1}$$

This relationship is called strength shift relation. This quantity can be derived from the coordinate changes between two different states (k+1/k) which are obtained by geodetic methods.

$$\Delta x_{k+1,k} = \hat{x}_{k+1} - \hat{x}_k = \Delta X \tag{2}$$

If one releases the correcting variables $u_k$ in the force vector F, filter adapted notation can be indicated in that to the Kalman filter.

$$\Delta x_{k+1,k} = K^{-1}\Delta f = (K^{-1}F)_{k+1,k}\, u_k = B_{k+1,k}\, u_k \tag{3}$$

The current object state can be computed for the static load case under a load on time k+1 by means of the shift vector ΔX.

$$\overline{x}_{k+1} = T_{k+1,k}\, x_k + B_{k+1,k}\, u_k + C_{k+1,k}\, w_k \tag{4}$$

State vector of dam which is supplemented with the material parameter can be obtained with consideration of water pressure is $u_w$ and change of temperature $u_T$. Material parameters are young module *E*, the transverse contraction ratio $\mu$ and the thermal expansion coefficients $\alpha$ for an example application on dam, (GULAL 1997).

$$\overline{x}_{k+1} = T_{k+1,k}\, \hat{x}_k + B_{k+1,k}\, u_k + C_{k+1,k}\, w_k$$

$$\begin{bmatrix} \overline{x}_{R,k+1} \\ \overline{x}_{O,k+1} \\ \overline{x}_{M,k+1} \end{bmatrix} = \begin{bmatrix} I_R & 0 & 0 \\ 0 & I_O & T_{O,M} \\ 0 & 0 & I_M \end{bmatrix} \begin{bmatrix} \hat{x}_{R,k} \\ \hat{x}_{O,k} \\ \hat{x}_{M,k} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{W,k+1,k} & B_{T,k+1,k} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{W,k} \\ u_{T,k} \end{bmatrix} + \begin{bmatrix} I_R & 0 & 0 \\ 0 & I_O & T_{O,M} \\ 0 & 0 & I_M \end{bmatrix} \begin{bmatrix} w_{R,k} \\ w_{O,k} \\ w_{M,k} \end{bmatrix}$$

(5)

The sub vectors $x_R$, and $x_O$, contain the information about the coordinates of the reference and object points and the sub vector $x_M$ represents the material parameters, which are used in the model. The transition matrix $T_{k+1,k}$ is the derivative of the predicted shifts toward the material parameters at the time $t_k$.

$$T_{O,M,k+1,k} = \frac{\partial \psi \text{dyn}}{\partial x_M} = \frac{\Delta \psi \text{dyn}}{\Delta x_M} \tag{6}$$

$B_{k+1,k}$ is the derivative of the predicted shifts toward the correcting variables of the filter process.

$$B_{k+1,k} = \frac{\partial \psi \text{dyn}}{\partial u_k} = \frac{\Delta \psi \text{dyn}}{\Delta u_k} \tag{7}$$

The product $C_{k+1,k}\, w_k$ remains here unconsidered, because the expectation value is $E\{w_k\}=0$. The application of the covariance propagation law supplies the cofactor matrix of the predicted state vector.

$$Q_{\overline{x}\overline{x},k+1} = T_{k+1,k} Q_{\hat{x}\hat{x},k} T^T_{k+1,k} + B_{k+1,k} Q_{uu,k} B^T_{k+1,k} + C_{k+1,k} Q_{ww,k} C^T_{k+1,k} \tag{8}$$

Thus the system model is defined. The functional model of the observation model can be formed with a mathematical relationship between the true values of the observations and the unknown parameters.

$$
\begin{aligned}
l_{k+1} + v_{k+1} &= A_{x,k+1}\,\hat{x}_{k+1} \\
l_{k+1} + v_{k+1} &= \begin{bmatrix} A_{R,k+1} & A_{O,k+1} & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_{R,k+1} \\ \hat{x}_{O,k+1} \\ \hat{x}_{M,k+1} \end{bmatrix}
\end{aligned} \tag{9}
$$

Accuracy conditions of the observations are expressed by the stochastic model.

$$
\sum\nolimits_{ll,k+1} = E\{\varepsilon\varepsilon^T\} = \sigma_0^2 Q_{ll,k+1} \tag{10}
$$

Unification of system equation (5) and observation equation (9) is performed in the Gauss-Markov model.

$$
\begin{matrix} l & + & v & = & A & \hat{x} \end{matrix}
$$

$$
\begin{bmatrix} \bar{x} \\ l \end{bmatrix}_{k+1} + \begin{bmatrix} v_x \\ v_l \end{bmatrix}_{k+1} = \begin{bmatrix} I \\ A_x \end{bmatrix}_{k+1} \hat{x}_{k+1}, \quad \sum\nolimits_{k+1} = \sigma_0^2 \begin{bmatrix} Q_{\overline{xx}} & O \\ O & Q_{ll} \end{bmatrix}_{k+1} \tag{11}
$$

The unification of system equation and observation equation constitutes the nature of the Kalman filtering, whose results the filtered state vector $\hat{x}_{k+1}$ with its covariance matrix $\Sigma\hat{x}\hat{x}_{k+1}$ is.

Using the gain-matrix $K_{k+1}$ (Eq.12) that minimizes the a posterior error covariance the output quantities of main interest can be comprehended by a vector Y for time $t_{k+1}$ Eq.(13).

$$
K_{k+1} = Q_{\overline{xx},k+1} A_{k+1}^T (Q_{ll,k+1} + A_{k+1} Q_{\overline{xx},k+1} A_{k+1}^T)^{-1} = Q_{\overline{xx},k+1} A_{k+1}^T D_{k+1}^{-1} \tag{12}
$$

$$
Y = \begin{bmatrix} d \\ \hat{x} \\ v_{\bar{x}} \\ v_l \end{bmatrix}_{k+1} = \begin{bmatrix} -A & I \\ I - KA & K \\ -KA & K \\ Q_{ll} D^{-1} A & -Q_{ll} D^{-1} \end{bmatrix}_{k+1} \begin{bmatrix} \bar{x} \\ l \end{bmatrix}_{k+1} \tag{13}
$$

The difference $d_{k+1}$ in Eq. 13 is called the innovation. The innovation reflects the discrepancy between the predicted measurement and the actual measurement. A residual from zero means that the system and observation equations are in complete agreement (HEUNECKE 1995).

## 3. MATERIAL

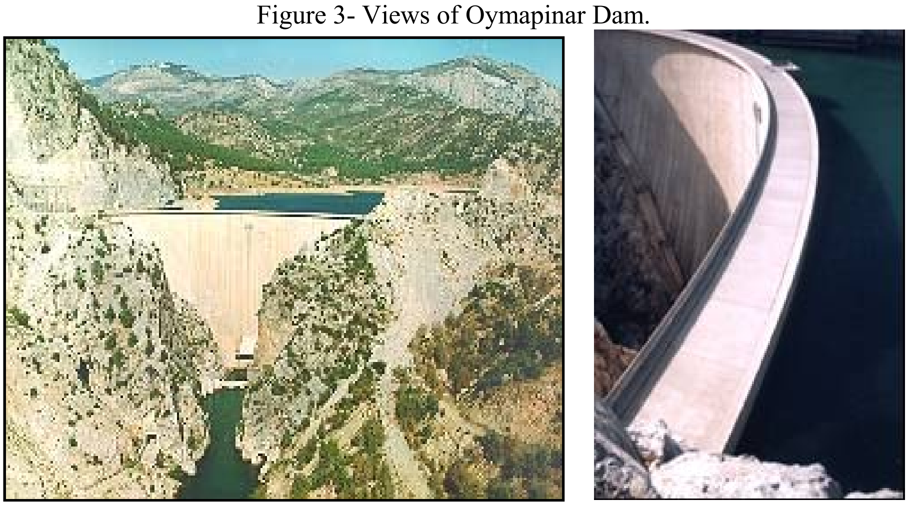
### 3.1 Presentation of Oymapinar Dam

The hydroelectric plant Oymapinar is located on Turkey's southern coast near the provincial capital of Antalya at the southern edge of the Taurus Mountains, approximately 18km inland from Mediterranean.

The construction site was opened in May 1977 and the work on the concrete dam in February 1983 was locked. Planning and consultation was undertaken by the consulting firms Coyne and Bellier of Paris-France. The execution of all construction work was in the hands of Bilfinger and Berger Company Wiesbaden-Germany.

The main body of Oymapinar Dam consists of 24 vertical concrete blocks, each of them is 15.2 m long. The concrete dam is 185m high, 25m wide at base, 5m wide at the crest and has a total crest length of 365m. The double curved arch dam transfers the water pressure as a three dimensional structure on the rock abutments which rise up from the valley at an incline of 600. The volume of concrete in the dam itself is 650.000$m^3$. Altogether the project contains 900.000$m^3$ of concrete.

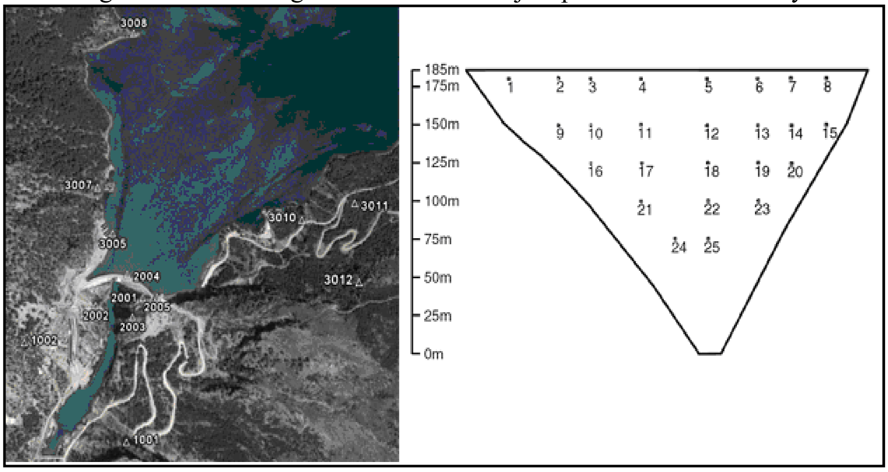
Figure 3- Views of Oymapinar Dam.



### 3.2 Monitoring of Oymapinar Dam

At the inception of the construction a control network was established for setting-out purposes. However, when the construction was finished in 1983 and the lake behind the dam began to be filled, the monitoring net was used to obtain statistically reliable information about the displacement of the concrete dam. The monitoring network consisted of 12 observation pillars on the surroundings of the dam and 25 targets (object points) on the surface of the dam (Figure 4). Due to problems of limited sight only 22 targets are suitable for intersection observations.

Figure 4- Monitoring network and the object points at the dam body.




The task of the net is to supply statistically secured statements about the deformation behavior of the concrete dam. The deformation of the concrete dam can be determined by measuring the object points geodetically, which present the dam.

During lake filling (February 1983 to July 1984) the construction company (Bilfinger&Berger) was responsible for the monitoring and the network was observed 24 periods as the reservoir water level rose. When the operational phase began in 1984, Yildiz Technical University, Istanbul took charge of the observations. From 1984 till 2010 20 epochs were undertaken. In 1994 and 1996 observations were made by Yildiz Technical University in corporation with the University of Darmstadt and the University of Hannover, Germany (HOSBAS et al.1994, GRABOWSKI et al. 1997).

For the numerical application, two special cases were selected as representative epochs, in which the water levels are 176m and 184m in February 1984 and March 1984 respectively and the significant deformations are expected. The geodetic measurements in these epochs were conducted by Bilfinger Berger Co. by using Wild T2 theodolite.

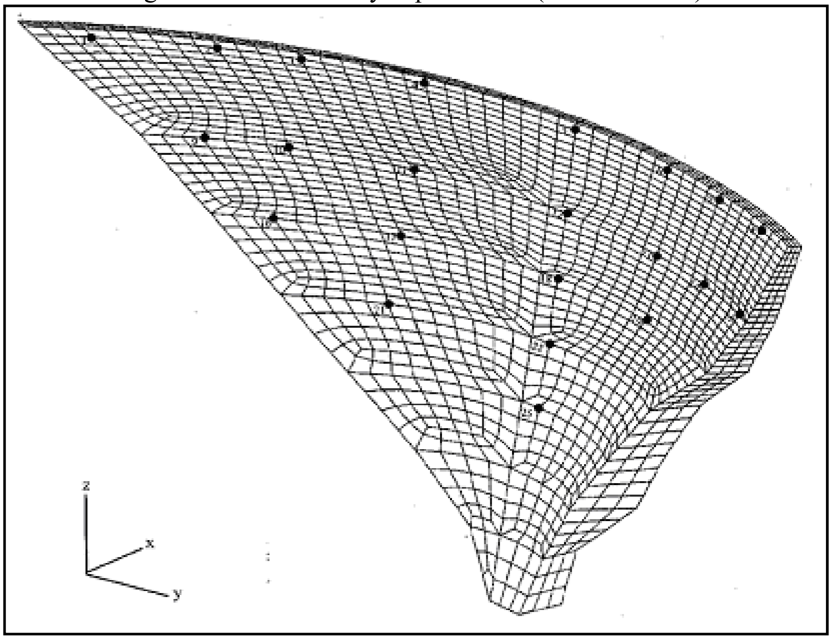
## 4. RESULTS

### 4.1 The static behavior of Oymapinar Dam

Static behavior of Oymapinar Dam was calculated by using Finite Elements Method (FEM). The three-dimensional FE computation of Oymapinar Concrete

Dam was carried out by using the FE-Net represented in Figure 5 and the program system PATRAN (PATRAN 2012). The FE-Net was divided into 4221 volume elements with 5888 knots.

Figure 5- FE-Net for Oymapinar Dam (GULAL 1997).




The FE-Net is so developed that some knots at the wall surface on the downstream face are identical with the geodetic measuring points (Figure 4). The deformation of the concrete dam was computed under water pressure and consideration of the temperature differences. The material parameters for the Oymapinar concrete dam the report of the company Coyne and Bellier inferred. Young module, the transverse contraction ratio and the thermal expansion number is accepted as 20.000MN/m$^2$, 0.2 and 8.10$^{-6}$ 1/$^o$C, (COYNE ET BELLIER 1970). In Figure 6 the horizontal shifts for the water pressure (water level is 176m) are represented. When the water level is 176, the largest horizontal shifts in the centre and in the upper third of the body arise with 42mm. With increasing distance from the body axle and the crest the horizontally shifts become smaller. With full back up (water level is 184m) the body of dam in the centre bends over additionally 14mm, (Figure7). The temperature differences between air and waterside are from the average absolute temperatures approx. 3.7°C. The process of the temperature-dependent shifts is largest in the body centre and in the range of the crown with 2mm. One recognizes the fact that the deformations are substantially stronger by water pressure than by changes of temperature of the Mediterranean climate caused.

Figure 6- The horizontal shifts computed using FEM for the load case water level of 176m.

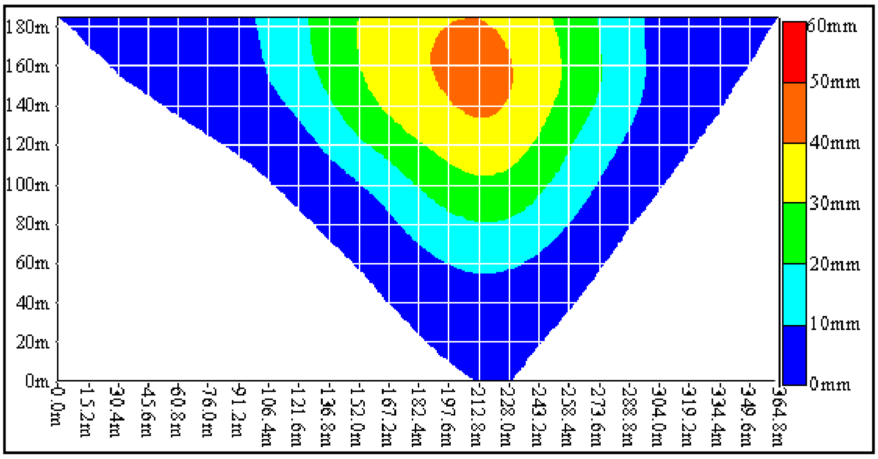
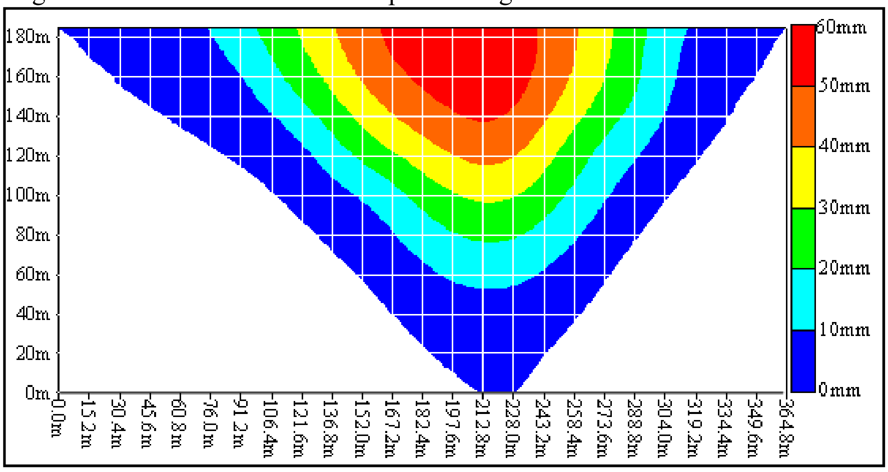



Figure 7- The horizontal shifts computed using FEM for the water level of 184m.

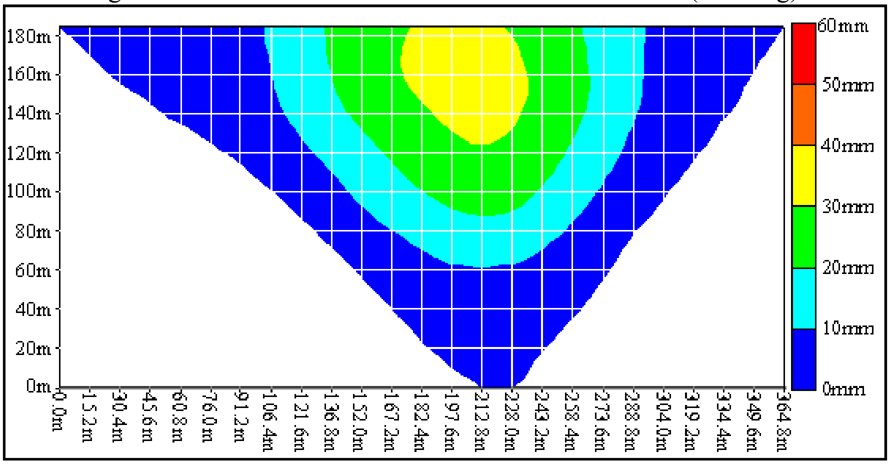



### 4.2 Identification of dam body by means of Kalman filtering

Current state of the system is predicted by using Eq. 5 after the deformations on the dam body are calculated. In the $\overline{x}_{k+1}$ predicted state vector; $x_R$ defines the observation points coordinated around the dam, $x_O$ defines the 22 object points coordinates on the dam body and $x_M$ defines the material parameters such as young module, the transverse contraction ratio and the thermal expansion number. $T_{O,M}$ matrix component (Eq. 6) in the transition matrix $T_{k+1,k}$, which defines the effect of the material parameters on the system state, was calculated for only object points on dam body. $B_{k+1,k}$ matrix (Eq. 7) defines the effects of the water load and temperature on the system state $\overline{x}_{k+1}$. Young module, the transverse contraction ratio and the thermal expansion number was accepted as 20.000MN/m$^2$, 0.2 and 8.10$^{-6}$ 1/°C respectively (COYNE et BELLIER, 1970) and system state was predicted for the time point on February 1984 when the water level $u_w$ is 176m. Observation equation which is the second component of the filtering process was determined by processing the geodetic observations on February 1984 and by using Eq. 9. The system equation and the measuring equation were summarized in Gauss-Markov model (Eq. 11). After the filtering of the deformation behavior (Eq. 13) current state of the concrete dam (both geometrical condition and material parameter) was determined.

Deformation values determined for dam body after the filtering process is given in Figure 8. Deformations take the biggest values (31mm) on the middle axis of the dam and on the upper quarter of the dam.

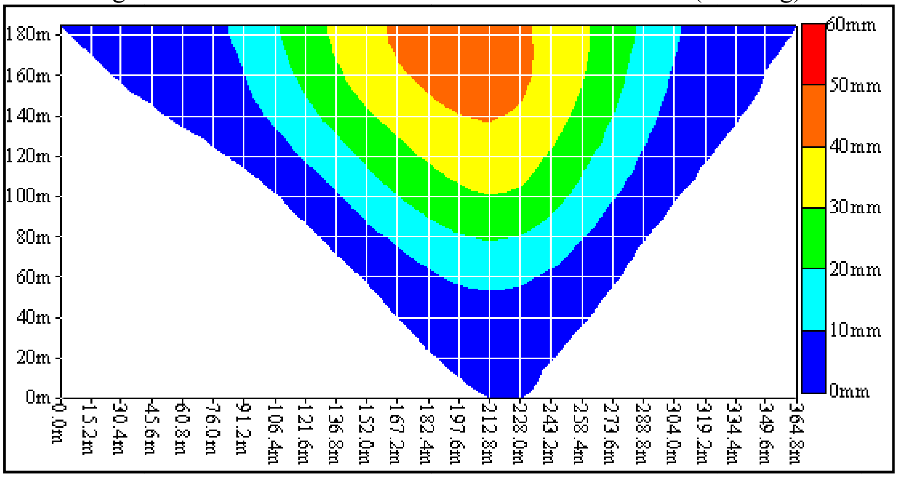
Figure 8- Horizontal deformations for water level 176m (Filtering).




After the filtering process, young module was increased 3.252 MN/m$^2$ and has 23251 MN/m$^2$ value, the transverse contraction ratio was decreased 0.01 and has 0.19 value. There is no significant change on thermal expansion number. Second filtering process was executed for the March 1984 (k+1) period when the water level is 184m by accepting the February 1984 period as reference. After the filtering process, the behavior of Oymapinar concrete dam was represented in Figure 9 with a 184m water level.

For the horizontal shifts of the crest with full water a maximum amount was determined by 43mm. As from Figure 9 to be recognized, the horizontal deformations with increasing distance from the wall centre and the crest become smaller.

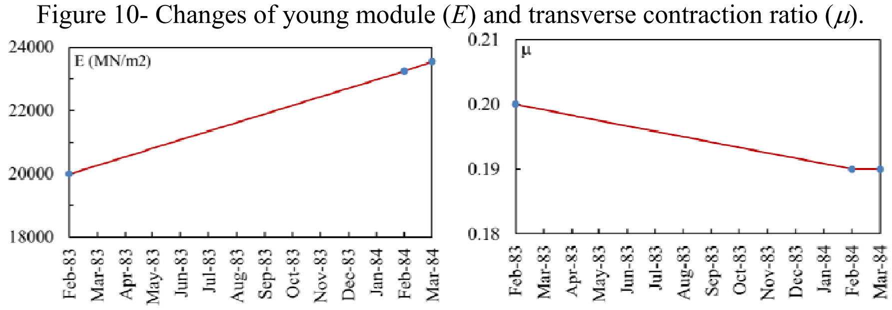
Figure 9- Horizontal deformations for water level 184m (Filtering).




After the diagram of the deformations arising at the downstream face of the wall in the following figure the changes arising with the material parameters are illustrated with the time.

As is to be seen in Figure 10, the young module increased and converged 3.550MN/m$^2$ due to the computations accomplished here after approximately one year for instance according to amount to the value of for instance E=23.550MN/m$^2$. The transverse contraction ratio was not changed after the second filtering and decreased 0.01 at the end of the one year and takes 0.19 values. Because of the smaller temperature difference in Mediterranean during the epochs, the thermal expansion number does not change significantly.

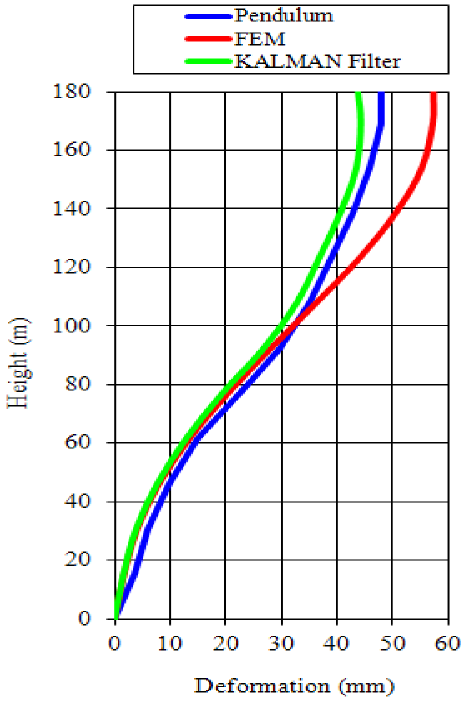
Figure 10- Changes of young module (*E*) and transverse contraction ratio ($\mu$).



## 5. DISCUSSION

Figure 11 illustrates the deformation on the middle axis of the dam when the reservoir is full and the water level is 184m. Kalman filtering results and pendulum measurement results using FEM is given in figure. Results from the every three methods on the lower 2/3 section (120m) of the dam are well-matched with each other. Standard deviation of the three values is 2mm. This value is also the standard deviation of the deformations resulting from the filtering process.

Figure 11 – Deformation the middle axis of dam.




Results on the crest section are different from each other. The differences between the deformations resulting from the filtering process and pendulum measurements and FEM results are 4mm and 13mm respectively. While taking the Kalman filtering results as reference, there is 10% difference with pendulum results and 30% differences with FEM results. It can be said that filtering results and the pendulum results are well-matched. Differences of the FEM results on crest section are originated from the border conditions in the calculation method and changes in time domain in the Young module which is defined in the next paragraph.

After the filtering process Young module Oymapinar dam is increased 3.550MN/m$^2$ after 14 mount and is reached 23.550MN/m$^2$. It can be said that Young Module is increased as 18% yearly by increasing 250MN/m$^2$ monthly. Depending on the increase on the Young Module, displacement values resulting from FEM computations on the dam crest are decreased 7mm. It can be said that the FEM results and the filtering results are well-matched each other. In Simmler 1984 was stated that on the Spullersee, Weissee and Tauernmoos dams in Europe, young module was increased as 40% and 60% during 12 years depending on quality experiments. On the Kölnbrein dam in Austria which has similar characteristics with Oymapinar dam, young module was determined as 18000-23000MN/m$^2$ during the planning process. In Herzog 1988, after the experiments and calculations this young module was determined as 28000MN/m$^2$. This condition is similar with the conditions for Oymapinar dam.

Poison number was decreased 5% one year later from the construction was completed. This means that, with the demand of the building a proportional deformation in load direction develops and perpendicularly to the load direction. It offers itself therefore to take the transverse contraction ratio out of the filter process to reduce the model and the counting expenditure for to avoid. The transverse contraction ratio is to be regarded more than one auxiliary variable for the computation of the shifts in the FEM.

Because of the little temperature changes between the periods, thermal expansion number has no significant changes like transverse contraction ratio. From this consideration the thermal expansion number, with small temperature differences between air and waterside, can remain unconsidered during the filter process.

## 6. CONCLUSIONS

Already in the year 1920 probably first times in the world in Switzerland geodetic deformation measurements at dams were accomplished. With the development of the electronic data processing, the geodetic measuring procedures and the analysis procedures change in particular.

The presented procedures make a comprehensive analysis and interpretation possible of deformation processes. The adaptive Kalman filter in dynamic models presents itself as extensive complex subsection in the deformation analysis. The large advantage of the filter process is apart from the computation of the geometrical partition the examination and adjustment of the material parameters to the actual object behavior.

The task of the geodetic monitoring of buildings may not be any longer alone the statement of movements and deformations, but must be seen it in the context of stability building than key information for the evaluation of the condition of the building. The analysis of deformation processes is regarded no longer alone as geodetic or geometrical problem, but as an integral component of a specialized spreading evaluation and interpretation.

**REFERENCES**

CHRZANOWSKI A.; DING X.; ROBERTS G.; WHITAKER C. Goal and Achievements of FIG Working Group WG6.1 Deformation Measurements and Analysis. *11th FIG Symposium on Deformation Measurement*, May 25-28 2003, Santorini, Greece.

CHRZANOWSKI A. Task and Achievements of FIG Working Group on Deformation Measurements and Analysis, *12th FIG Symposium*, May 22-24 2006, Baden.

CHRZANOWSKI A.; WHITAKER C. Activity of the FIG Working Group 6.1 on Deformation Measurements-Progress Report 2006-2008, *13th FIG Symposium on Deformation Measurementand Analysis*, May 12-15 2008, Lisbon.

COYNE ET BELLIER, Oymapinar Dam and Hydroelectric Project, Dam Analysis Report, Paris, 1970.

EICHHORN A. Analysis of Dynamic Deformation Processes with Adaptive Kalman-Filtering, *3rd IAG / 12th FIG Symposium*, May 22-24 2006, Baden.

GRABOWSKI J.; GULAL E.; HEER R.; HEUNECKE O.; PELZER H.; UZEL T. Deformation surveys at the Oymapinar Dam - A Review on Some Results, *Second Turkish-German Joint Geodetic Days,* 28-30 May 1997, Berlin.

GULAL E. Geodaetische Uberwachung einer Talsperre; eine Anwendung der Kalman Filtertechnik, Dissertationen, *Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universitaet Hannover*, 1997.

HESSE C. Deformation analysis of a shell structure under varying loads with Kalman-filter techniques, *2nd Symp. for Geotech. and Structural Engineering*, May 21-24 2002 Berlin.

HERZOG M. Die Baugrundverformung bei Gewölbestaumauern, *Bautechnik,* Heft 65,1988.

HEUNECKE O. Zur Identifikation und Verifikation von Deformationsprozessen mittels adaptiver Kalman-Filterung (Hannoversches Filter). *Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universitaet Hannover,* No. 208, 1995.

HEUNECKE O. Einige Gedanken zur fachübergreifenden Untersuchung von Deformationsvorgängen, dargestellt am Beispiel der Filterung der Biegelinie eines Pylons, *Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universitaet Hannover*, pp. 75-92, 1996.

HEUNECKE O. WELSCH W. A Contribution to Terminology and Classification of Deformation Models in Engineering Surveys, *Journal of Geospatial Engineering*, vol.2. No.1, pp.35-44, 2000,Hong Kong.

HOSBAS G.; KARTAL F.; ERSOY N.; ERKUCUK G.; UZEL T.; EREN K. Surveillance of Oymapinar Dam Deformations by Means of Geodetic Control Network, *1st Turkish International Symposium on Deformations,* 5-9 September, 1994, Istanbul.

KALMAN R. E. A New Approach to Linear Filtering and Prediction Problems, *Journal of Basic Engineering*, ASME , pp. 35-45,1960.

PATRAN 2012 Reference Manual, Part 3: Finite Element Modeling, http://www.mscsoftware.com/

SIMMLER H. Langzeitverhalten Österreicher Talsperren und Fluss-Stauwerke, *Die Talsperren Österreichs,* Heft 28, 1984

TESKEY W. F. Integrierte Analyse von geodätischen, geotechnischen und physikalischen Modelldaten zur Beschreibung des Deformationsverhaltens großer Erddämme unter statischer Belastung, *DGK Reihe C*, 1988, Nr. 341, München.

WELSCH W.; HEUNECKE O. Models and Terminology for the Analysis of Geodetic Monitoring Observations – Official Report of the Ad-Hoc Committee of FIG Working Group 6.1. *The 10th FIG International Symposium on Deformation Measurements*, March 19-22 2001, California.

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