OPTIMIZATION OF A PULTRUSION PROCESS USING FINITE DIFFERENCE AND PARTICLE SWARM ALGORITHMS

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Abstract - Pultrusion is one of several manufacturing processes for reinforced polymer composites. In this process fibers are continuously pulled through a resin bath and, after impregnation, the fiber-resin assembly is cured in a heated forming die. In order to obtain a polymeric composite with good properties (high and uniform degree of cure) and a process with a minimum of wasted energy, an optimization procedure is necessary to calculate the optimal temperature profile. The present work suggests a new strategy to minimize the energy rate taking into account the final quality of the product. For this purpose the particle swarm optimization (PSO) algorithm and the computer code DASSL were used to solve the differential algebraic equation that represents the mathematical model. The results of the optimization procedure were compared with results reported in the literature and showed that this strategy may be a good alternative to find the best operational point and to test other heat policies in order to improve the material quality and minimize the energy cost. In addition, the robustness and fast convergence of the algorithm encourage industrial implementation for the inference of the degree of cure and optimization.

Keywords: Polymer-matrix composites; Degree of cure; Computational modeling; Particle swarm optimization.

INTRODUCTION

Polymer composites are materials made of a cured thermosetting or thermoplastic resin and reinforcement elements (Santos et al., 2009). These materials have several good properties, such as corrosion resistance, low density and high compressive and flexural strength, depending on the resin-fiber blending. Composite materials can be manufactured by different processes, such as pultrusion, resin transfer molding and hand lay-up. Among these technologies, pultrusion stands out because of its ability to join high production rate with high quality of produced parts. In this process, reinforced fibers are saturated in a resin tank and continuously pulled through a heated die, where the resin gradually cures and solidifies, resulting in a composite part with the same cross-sectional profile as the die cavity (Srinivasagupta and Kardos, 2004). The die may be heated by electrical heaters, strip heaters, hot oil or by steam, although electrical heaters are the most common. Outside the die, the composite part is pulled by a continuous pulling system and then a cutoff saw cuts the part into a desired length (Liu and
Hillier, 1999). Figure 1 shows a simplified diagram of a pultrusion process.

Cure is characterized by a polymeric exothermic reaction. A non-uniform or low degree of cure implies a product of unreliable quality. Owing to the heat released during the reaction, the reaction system can be overheated and resin degradation can occur (Han et al., 1986). However, the resin needs to be heated at a minimum value of temperature at which the cure reaction may be started. By considering these constraints, an optimization policy must be developed in order to find the minimum energy rate necessary to produce a composite with high degree of cure at the die exit.

Li et al. (2002) optimized the pultrusion process based on the uniformity of the degree of cure at the die exit as the objective. An iterative procedure based on the steepest descent method and a finite element method (LUSAS FE package) were used. The effects of the element aspect ratio and different initial heater conditions were studied. The results of the case studies showed that the algorithm developed is numerically stable and provides optimal die temperatures for producing a uniformly cured pultrudate.

Srinivasagupta et al. (2003) used a multi-objective optimization procedure that involves control, environmental and quality aspects for pultrusion optimization. The procedure was able to determine the optimal values of the processing parameters, such as heating zone temperatures, as well as the equipment specifications (die and heater dimensions) that satisfy multiple objectives.

In Carlone et al. (2006) two different three-dimensional models which take into account thermo-chemical aspects were compared. Both models considered heat transfer due to heating platens, convective boundary conditions, die-cooler at the die entrance, composite material anisotropy, mass transport effects, and the heat generated by the resin cure reaction. A remarkable accuracy was achieved using the proposed finite element method; which required less computational time with respect to the finite difference model.

Carlone et al. (2007) used a hybrid method based on genetic algorithms and the simplex method for minimizing the same function proposed by Li et al. (2002) (The objective function considered was based on the variance of the degree of cure, evaluated in the exit cross-section, with respect to a target value). The optimization technique was applied taking into account only the heater temperatures as optimization variables; all the other parameters, such as pull speed or cooling channel temperature were assumed as constants. The optimization procedure was applied to obtain a part characterized by a good quality, i.e., a uniform distribution of the degree of cure. A uniform distribution of the degree of cure, with a satisfactory mean value at the exit of the forming die, could be achieved.

In Santos et al. (2012) the pultrusion of an “I” beam composite was simulated by the Finite Volume Method. It was possible to verify that the energy requirements can be reduced by changing the heating configuration of the pultrusion die. Hence, an alternative configuration with internal heaters inside the die body was simulated. The sum of all heating power was considered as the objective function in order to find the minimal energy consumption. Internal
heaters were capable of increasing the thermal efficiency of the process by improving the thermal distribution on the die body. Moreover, this heating arrangement also allowed insulating the die surface.

Stroher et al. (2013) applied spatial discretization in bar cross section using the Galerkin finite element method while time integration used a second-order implicit backward difference formula, also known as the Gear method. The computational domain was discretized using an unstructured mesh with triangular elements and an adaptive refinement. Iterative algorithms were used to solve the algebraic equation system. Results showed that, as the temperature and degree of cure along the die extension increased, the volumetric heat capacity and the thermal conductivity also increased. The influence of the pulling speed and the die temperature on the thermal property variation were also analyzed.

More recently, Silva et al. (2014) investigated new relative positions for the heaters in order to optimize the heat distribution process and energy consumption. Finite Elements Analysis was applied to identify the best relative position of the heaters in the die. The analysis was firstly developed based on eight cylindrical heaters located in four different location plans. In a second phase, in order to refine the results, a new approach was adopted using sixteen heaters with the same total power. The authors concluded that the correct positioning of the heaters could reduce around 10% the energy consumption, decreasing the production costs.

Our previous research (Santos et al., 2012), nicely demonstrated that the CFD-PSO algorithm can simulate and optimize pultrusion of composite parts with very irregular shapes. Moreover, different heating strategies can be designed. However, one of the drawbacks is that the computational performance can be excruciatingly slow for difficult problems. Roughly speaking, for design purposes, the strategy presented in Santos et al. (2012) enjoys good properties. On the other side, if the goal was to provide a fast optimization, i.e., for real time optimization, CFD-PSO would not be recommended. Hence, in this paper we are interested in constructing a more simple and robust algorithm in order to provide a fast simulation-optimization. Our method is based on a mathematical model which joins the code DASSL (Petzold, 1982) and a particle swarm optimization (PSO) method (Kennedy and Eberhart, 1995). The goal is to minimize the energy rate with a constraint of a minimum degree of cure to be achieved at the die exit. A cylindrical composite part, also studied by Santiago et al. (2003) and Hunter et al. (1986), was used as case-study.

This paper is organized as follows. First, a mathematical formulation of the pultrusion process is presented. Secondly the optimization procedure is explained. In the sequel, the numerical simulation and optimization are realized and discussed. Finally, conclusions of the obtained results are made.

**MATHEMATICAL MODEL**

In this section, the theoretical background of pultrusion is described (Coelho and Calado, 2002; Santos et al., 2012). Heat transfer in the pultrusion process, with a cylindrical die, can be modeled using the mass and energy balance equations described by Equations (1) and (2), respectively.

\[
\frac{d\alpha}{dt} = \frac{r_a^2}{C_{a0}} \cdot \frac{d\alpha}{dz} = \frac{r_a}{u_z} \cdot C_{a0}
\]

\[
\rho_c \cdot c_{pc} \cdot u_z \cdot \frac{\partial T}{\partial z} = \frac{1}{r'} \frac{\partial}{\partial r'} \left[ r' \left( k_c \frac{\partial T}{\partial r'} \right) \right] - c_{a0} \cdot r'_a \cdot (1 - \phi_f) \cdot (\Delta H_{re})
\]

where \( r' \) and \( r'' \) are die and radius length (m), respectively, \( t \) is time (s), \( T \) is temperature (K), \( u_z \) is the pulling speed \((m \cdot s^{-1})\), \( c_{pc} \) is the composite specific heat \((J \cdot kg^{-1} \cdot K^{-1})\), \( c_{a0} \) is the resin concentration \((kg \cdot m^{-3})\), \( \alpha \) is the degree of cure, \( \phi_f \) is the fiber volume fraction, \( \Delta H_{re} \) is the energy released by cure reaction \((J \cdot kg^{-1})\), \( k_c \) is the composite thermal conductivity \((J \cdot m^{-1} \cdot kg^{-1} \cdot K^{-1})\) and \( \rho_c \) is the composite specific mass \((kg \cdot m^{-3})\).

Equation (1) describes the mass balance, which quantifies the reaction inside the pultrusion die. Equation (2) describes the energy balance. As seen, the energy has a convective term in the \( z \) direction and a diffusive term in the radial direction; according to the literature (Coelho and Calado, 2002, Santos et al., 2012), the conductive heat transfer along the \( z \)-direction can be neglected.

The physical properties of the composite are calculated by the following equations:

\[
\rho_c = \phi_c \rho_{pc} + \phi_f \rho_f
\]

\[
\rho_c c_{pc} = \rho_c \phi_c c_{pc} + \rho_f \phi_f c_{pf}
\]
where the subscripts \( r \), \( f \) and \( c \) represent resin, fiber and composite, respectively.

The cure rate of epoxy resin is written as (Santiago et al., 2003):

\[
\frac{d\alpha}{dz} = D_a \cdot r_a
\]  

where \( k(T) \) is the reaction order and \( k(T) \) is the Arrhenius equation, written as:

\[
k(T) = A \cdot e^{\frac{-E_a}{RT}}
\]  

In this equation, \( A \) is a kinetic pre-exponential factor, \( E_a \) is the activation energy and \( R \) is the universal gas constant. The boundary conditions may be written as:

\[
\alpha(r, z') = 0
\]  

which is the degree of cure at die entrance and

\[
T(r, z') = T_0
\]

where Equations (9), (10) and (11) are the temperatures at the die entrance, die center and die surface, respectively, and \( R_c \) is the composite radius.

In this work, mass and energy equations are normalized:

\[
\frac{1}{k_c} = \frac{1}{k_e} \cdot \frac{\phi_e}{k_e} + \phi_f \cdot \frac{1}{k_f}
\]

and \( \beta \) are dimensionless constants, defined by following equations:

\[
D_a = \frac{L}{u_z} \cdot e^{\frac{-E_a}{RT_0}}, \quad \delta = \frac{k_e \cdot L}{\rho_c c_p u_z R_c^2},
\]

\[
\sigma = \frac{-E_a}{RT_0} \cdot \frac{\Delta H_e}{\rho_c c_p T_0}
\]

where \( L \) is the die length.

The normalized cure reaction of epoxy resin (Santiago et al., 2003) is written as:

\[
r_a = e^{\frac{-1}{\sigma}(1 - \alpha)^n}
\]

where \( n \) is the order of reaction.

The fiber-resin feed temperature \( \theta_o \) is regarded as the ambient temperature: \( \theta(r, z = 0) = 1 \) and the feed degree of cure is zero, \( \alpha(r, z = 0) = 0 \). Normalized boundary conditions used to simulate this process are:

I) Symmetry condition at \( r = 0 \):

\[
\frac{d\theta(r, z)}{dr} \bigg|_{r=0} = 0
\]

II) Temperature in the die-composite interface:

\[
\theta(r = 1, z) = \theta(z)
\]

Taking into account that a second-order derivative can be written using a second-order central finite difference formulation, the partial differential equations were discretized in the radial direction as:

\[
\frac{\partial \theta}{\partial r} \bigg|_{r=0} = \frac{\theta_{t+1} - \theta_{t-1}}{2\Delta r}
\]

resulting in a system of algebraic equations:

\[
\frac{\partial \theta}{\partial r} \bigg|_{r=0} = \frac{\theta_{t+1} - \theta_{t-1}}{2\Delta r}
\]

for the normalized die length and normalized part radius, respectively, \( D_a \) is the Damköhler number, \( \sigma \) and \( \beta \) are dimensionless constants, defined by following equations:

\[
D_a = \frac{L}{u_z} \cdot e^{\frac{-E_a}{RT_0}}, \quad \delta = \frac{k_e \cdot L}{\rho_c c_p u_z R_c^2},
\]

\[
\sigma = \frac{-E_a}{RT_0} \cdot \frac{\Delta H_e}{\rho_c c_p T_0}
\]
\[ \frac{d\alpha_i}{dz} = D^r \alpha_i \]  \hfill (21)\\

where \( i = 1, n_p - 1 \), and \( n_p \) is the number of nodal points in the radial direction. Then, at \( r = 0 \), \( \theta_o = \theta_1 \).

Figure 2 shows the grid domain of the finite difference technique. These equations were integrated in the \( z \) direction by a version of the integration routine DASSL (Petzold, 1982). The simulation of pultrusion with this procedure is a part of the optimization study that is explained in the next topic.

\[ \text{OPTIMIZATION PROCEDURE} \]

In order to optimize the pultrusion process, an objective function similar to that proposed by Coelho and Calado (2002), which represents the energy rate, was used:

\[ F(T) = Q = \int_{z=0}^{z=L} k_c S \cdot \frac{dT}{dr'} \cdot \rho_{R_c} \cdot dz' \]  \hfill (22)\\

where \( S = (2\pi R_c) \cdot L \) is the superficial area of the cylindrical composite surface and \( Q \) is the energy rate needed to cure the system. The objective function can be re-formulated in a dimensionless form as:

\[ f(\theta) = \eta \cdot \int_{z=0}^{z=L} \frac{d\theta}{dr} \cdot dz \]  \hfill (23)\\

in which \( \eta = 2 \cdot L \cdot \pi \cdot k_c \cdot T_o \).

Considering the objective function written in Equation (23), the purpose of this work was to reduce the energy rate, or minimize \( f(\theta) \). In this paper the quality aspect is considered as a constraint. Thus, the constrained non-linear optimization problem is written as:

\[ \min_{\theta, \theta_0} f(\theta) \]

subject to: \( g(\alpha) \leq 0 \)

\[ \theta_L \leq \theta \leq \theta_U \]  \hfill (24)\\

where \( \theta_L \) and \( \theta_U \) are the lower and upper limits of dimensionless temperature, respectively and:

\[ g(\alpha) = \left[ \frac{\sum_{i=1}^{n_p} (\alpha_i - \alpha_d)^2}{n_p} \right] - \varepsilon \]  \hfill (25)\\

In Equation (25), \( \varepsilon \) (tolerance factor) measures the maximum allowed sum of square deviation within the degree of cure at a nodal point \( i \) to be achieved at the die exit and the desired value of degree of cure, \( \alpha_d \). The choice of this value was made based on the algorithm experiments to evaluate the sensitivity of \( g(\alpha) \) by manipulating this parameter. Therefore, constraint \( g(\alpha) \) represents the least square deviation function, which evaluates the uniformity of cure for all nodal points at die exit. In order to incorporate the constraints in optimization formulation, a square function was added:

\[ f(\theta) = \left( \eta \int_{z=0}^{z=L} \frac{d\theta}{dr} \cdot dz \right) + \psi \left[ \max \left( 0, g(\alpha) \right) \right]^2 \]  \hfill (26)\\

where \( \psi \) is a positive penalty parameter. In Equation (26), when the constraint is violated, there is a positive contribution to the penalty term (if \( g(\alpha) \geq 0 \)).

The optimization method applied to minimize the objective function is the particle swarm optimization, PSO, developed by Kennedy and Eberhart (1995). In this method, a set of points, denominated particles, is
configured to find the optimal value of the objective function.

PSO estimates values of temperature between the lower and upper limits of temperature of four heating zones \( (\theta_1, \theta_2) \) of the composite surface, as suggested by other authors (Srinivasagupta and Kardos, 2004, Coelho and Calado, 2001) who studied the same type of problem. The assumption of four heating zones implies that the pultrusion die has two heating plates coupled on the die surface. In order to achieve the temperature profile, according to estimated values of \( \theta_1, \theta_2 \), these values are interpolated by a linear function, resulting in an expression that represents the boundary condition, \( \theta(r = 1, z) = \theta(z) \), necessary to solve the DAE system.

The PSO model may be written as:

\[
\begin{align*}
\mathbf{v}^{k+1}_i & = w \mathbf{v}^k_i + c_1 \cdot r_1 \cdot (\mathbf{p}^k_i - \mathbf{x}^k_i) \\
& \quad + c_2 \cdot r_2 \cdot (\mathbf{p}_{global}^k - \mathbf{x}^k_i) \\
\end{align*}
\]

where \( k \) is the PSO iteration, \( d \) is the search direction, \( v \) is the particle velocity, \( x \) is the position in search space, \( c_1 \) and \( c_2 \) are positive constants, called cognitive and social parameters, respectively, \( r_1 \) and \( r_2 \) are random numbers that vary at each iteration between the interval \([0 1]\), \( p_i \) is the best point of an individual particle \( i \) and \( P_{global} \) is the best value found for all particles. The choice of parameters \( c_1, c_2 \) and inertia weight \( w \) depends on the problem formulation. According to our experience, 30 particles were sufficient for optimization. In addition, a maximum of 20 iterations (this value is the maximal allowable iteration for PSO to find the best solution) was used. The choice of PSO parameters was made according to Schwab et al. (2008). Therefore, no further study was realized to investigate the influence of these parameters. The algorithm was written in FORTRAN 90 code as sketched in Figure 3(a). Figure 3(b) illustrates the PSO flowsheet.

\[
\mathbf{x}^{k+1}_i = \mathbf{x}^k_i + \mathbf{v}^k_i 
\]

(28)
RESULTS: SIMULATION

Before the optimization study, the proposed model was validated by comparing our results with experimental results from Hunter et al. (1986), according to the processing conditions in Tables 1 and 2.

Figure 4 shows the temperature profiles of the composite center for \( np = 40 - 10 \). As observed, the temperature profile at the composite center presents a very similar behavior when compared with the results obtained by Hunter et al. (2006) at the same location. Santiago et al. (2003) also carried out the same comparative study. As seen, the error is more pronounced between the interval \( 0.4 \leq z \leq 0.7 \), where the temperature reaches the maximum value, approximately at \( z = 0.55 \). The reason lies in the fact that experimental errors occurred during the study, as highlighted also by Santiago et al. (2003).

Table 1: Kinetic parameters (Santiago et al., 2003).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( s^{-1} )</td>
<td>pre-exponential factor</td>
<td>( 2.918x10^6 )</td>
</tr>
<tr>
<td>( E_a )</td>
<td>( kJ \cdot mol^{-1} )</td>
<td>activation energy</td>
<td>( 25.57x10^3 )</td>
</tr>
<tr>
<td>( n )</td>
<td>-</td>
<td>order of reaction</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 2: Parameters of resin, fiber and composite and physical properties of the pultrusion model (Santiago et al., 2003).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>-</td>
<td>volume fraction</td>
<td>0.72</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>kg.m$^{-3}$</td>
<td>fiber specific mass</td>
<td>2580</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>kg.m$^{-3}$</td>
<td>resin specific mass</td>
<td>1234</td>
</tr>
<tr>
<td>$C_{p_r}$</td>
<td>J.kg$^{-1}$.K$^{-1}$</td>
<td>resin specific heat</td>
<td>1833</td>
</tr>
<tr>
<td>$C_{p_f}$</td>
<td>J.kg$^{-1}$.K$^{-1}$</td>
<td>fiber specific heat</td>
<td>920</td>
</tr>
<tr>
<td>$k_r$</td>
<td>J.m$^{-1}$.kg$^{-1}$.K$^{-1}$</td>
<td>resin thermal conductivity</td>
<td>1693</td>
</tr>
<tr>
<td>$k_f$</td>
<td>J.m$^{-1}$.kg$^{-1}$.K$^{-1}$</td>
<td>fiber thermal conductivity</td>
<td>1967</td>
</tr>
<tr>
<td>$T_o$</td>
<td>K</td>
<td>feed resin-fiber temperature</td>
<td>300</td>
</tr>
<tr>
<td>$C_a$</td>
<td>kg.m$^{-3}$</td>
<td>feed resin concentration</td>
<td>1100</td>
</tr>
<tr>
<td>$R_c$</td>
<td>m</td>
<td>composite radius</td>
<td>6.352 x 10$^{-3}$</td>
</tr>
<tr>
<td>$L$</td>
<td>m</td>
<td>die length</td>
<td>1.254</td>
</tr>
<tr>
<td>$u_z$</td>
<td>m.s$^{-1}$</td>
<td>speed in pull direction</td>
<td>3.0 x 10$^{-3}$</td>
</tr>
<tr>
<td>$\Delta H_{r}$</td>
<td>J.kg$^{-1}$</td>
<td>released energy</td>
<td>-398.44 x 10$^3$</td>
</tr>
</tbody>
</table>

Figure 4: Comparison between theoretical and experimental results (Hunter et al., 1986) for the temperature profile at the composite center ($r = 0$).

In this present study, the following equation is considered as an error estimation of the temperature profile:

$$ e_\theta = \frac{\sum_{i=1}^{N_e} (\theta_{i}^{\exp} - \theta_{i}^{\text{sim}})^2}{N_e} \tag{29} $$

in which $N_e$ is the number of experimental points and $\theta_{i}^{\text{sim}}$ is the simulated temperature. A statistical analysis was carried out for error evaluation. In this light, the same procedure was realized for $n_p = 10, 20, 30$ and 40 points, resulting in errors equal to 0.31, 0.25, 0.23 and 0.23, respectively. Clearly, the algorithm performs better for finer grids, in our case, $n_p = 40$. Then, $n_p = 40$ was established for the following numerical experiments.

The degree of cure profile is sketched at four regions of the composite part in Figure 5. As observed, the degree of cure achieves the maximum value at the die exit. According to the results, the composite surface achieves a degree of cure of 1.0 at approximately $z = 0.2$, while the composite center achieves this condition at $z = 0.6$. Obviously, after this point, there is no need to heat the material because the resin is completely cured. It is easy to figure out that a reduction of temperature would not affect the material quality (degree of cure).

RESULTS: OPTIMIZATION STUDY

The case study of Santiago et al. (2003) was chosen as the reference for a comparative study between optimized and experimental results realized by Hunter et al. (1986). According to the proposed objective function (Equation (22)), the energy rate equals to 138 W. In the next topic, the proposed optimization procedure will be applied in order to minimize this value and keep the material quality.
also the energy rate at each iteration of the optimization algorithm. The minimum objective function is equivalent to 94.13 W, 31% smaller than the total energy rate spent in the reference operational conditions of Santiago et al. (2003). In addition, the computation performance of the PSO-DASSL routine is very fast, around 2 min implemented on a 2.3 GHz – 4 GB PC.

Table 3: Optimization parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_L$</td>
<td>-</td>
<td>lower limit of dimensionless temperature</td>
<td>1.1</td>
</tr>
<tr>
<td>$\theta_U$</td>
<td>-</td>
<td>upper limit of dimensionless temperature</td>
<td>1.7</td>
</tr>
<tr>
<td>$\psi$</td>
<td>-</td>
<td>penalty parameter</td>
<td>100</td>
</tr>
<tr>
<td>$\alpha_d$</td>
<td>-</td>
<td>degree of cure set point</td>
<td>0.95</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>-</td>
<td>tolerance factor</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 4: Temperature and objective function.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>Energy rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.410</td>
<td>1.168</td>
<td>114.564</td>
</tr>
<tr>
<td>2</td>
<td>1.475</td>
<td>1.100</td>
<td>98.855</td>
</tr>
<tr>
<td>3</td>
<td>1.475</td>
<td>1.100</td>
<td>98.855</td>
</tr>
<tr>
<td>4</td>
<td>1.430</td>
<td>1.100</td>
<td>94.388</td>
</tr>
<tr>
<td>5</td>
<td>1.430</td>
<td>1.100</td>
<td>94.388</td>
</tr>
</tbody>
</table>

In Figure 6 the predicted optimal temperature profile is depicted. It is interesting to note that the temperature at the composite surface increases at the die entrance and reaches its maximum value equal to 430 K (dimensionless value of 1.43). After this point, the temperature is reduced to the value of 330 K (dimensionless value of 1.1). Clearly, the proposed strategy is to heat the material during the first stage of the cure reaction, while reusing the released heat during the second stage $\theta_2$. This configuration can be explained by the fact that a considerable amount of energy is released by the cure reaction and, therefore, the material does not need to be excessively heated after this stage. This peak can be identified in Figure 6, where the temperature profiles are inverted around $z = 0.7$.

The optimal degree of cure profile is depicted in Figure 7. It is clear that the cure profile presents a smoother curve, if compared with the cure profile of the reference case (Figure 5). Here, the rate of degree of cure in relation to the die length is considerably smaller than the same results obtained in Santiago et al., (2003); however, the degree of cure constraint (Equation (25)) is kept.

In order to evaluate the influence of $\alpha_d$, a comparative study was carried out with different values of $\alpha_d$: 0.7, 0.8, 0.95 and 1.0. The results are displayed in Table 5. It can be seen that the energy rate presents higher values for higher values of $\alpha_d$, as expected. It is easy to check that, in the last case, for $\alpha_d = 1$, the energy rate necessary to cure the material is higher. On the other hand, for small values of $\alpha_d$, less amount of energy is needed.

Table 5: Influence of $\alpha_d$.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Energy rate in W for $\alpha_d = 0.7$</th>
<th>Energy rate in W for $\alpha_d = 0.8$</th>
<th>Energy rate in W for $\alpha_d = 0.95$</th>
<th>Energy rate in W for $\alpha_d = 1.00$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>88.166</td>
<td>87.193</td>
<td>114.564</td>
<td>97.145</td>
</tr>
<tr>
<td>2</td>
<td>83.175</td>
<td>87.193</td>
<td>98.855</td>
<td>97.145</td>
</tr>
<tr>
<td>3</td>
<td>82.010</td>
<td>87.193</td>
<td>98.855</td>
<td>97.145</td>
</tr>
<tr>
<td>4</td>
<td>82.010</td>
<td>87.193</td>
<td>94.388</td>
<td>97.145</td>
</tr>
<tr>
<td>5</td>
<td>82.008</td>
<td>87.193</td>
<td>94.388</td>
<td>97.123</td>
</tr>
</tbody>
</table>

Additional studies were realized in order to compare the influence of some optimization parameters. Table 6 shows the influence of the penalty parameter, $\psi$, on the objective function, regarding
\( \alpha_d = 0.95 \). Obviously, when the value of \( \psi \) is zero, there is no constraint on the optimization problem. Then, the computed energy rate is very small. Contrary to this, as \( \psi \) increases, the penalty term becomes larger for any value of \( g(\alpha) \), which violates the equality constraints. Hence, the computed energy rate becomes higher.

Table 7 shows the influence of the tolerance \( \varepsilon \) on optimization results. The choice of \( \varepsilon \) is a measure of the importance of the constraint written in Equation (25). In other words, if one desires a high degree of cure uniformity, a small \( \varepsilon \) must be required.

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>Energy rate in W</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27.59238</td>
</tr>
<tr>
<td>0.001</td>
<td>27.94128</td>
</tr>
<tr>
<td>0.01</td>
<td>31.08143</td>
</tr>
<tr>
<td>0.1</td>
<td>57.86185</td>
</tr>
<tr>
<td>1</td>
<td>57.86185</td>
</tr>
<tr>
<td>10</td>
<td>94.38852</td>
</tr>
<tr>
<td>100</td>
<td>94.38954</td>
</tr>
</tbody>
</table>

Table 7: Influence of the parameter \( \varepsilon \).

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>Energy rate in W</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>94.47151</td>
</tr>
<tr>
<td>0.01</td>
<td>94.38852</td>
</tr>
<tr>
<td>0.1</td>
<td>94.38852</td>
</tr>
<tr>
<td>1</td>
<td>94.38852</td>
</tr>
</tbody>
</table>

CONCLUSIONS

This paper describes an algorithm to optimize the pultrusion process. The model was solved by the DASSL routine with very few points and the results show a good prediction of temperature profile and degree of cure. A PSO-DASSL routine was written for pultrusion optimization and several optimization runs were done in order to evaluate the effect of constraints on optimal operation strategy.

The present optimization algorithm can reduce the temperature at the second heating zone, taking into account the released heat of cure reaction. The obtained results encourage us to apply the optimization code to improve industrial performance of pultrusion. The main aspect is the relative facility of calculating the optimal temperature profile and consequently the minimum energy rate. It is important to emphasize that the optimal temperature profile depends on the control strategy implemented to warm the pultrusion die. In other words, the assumption of four heating zones is a restriction that can be changed to obtain different optimal control strategies. Moreover, the high computational performance of our strategy allows possible industrial implementations for industrial monitoring and optimization.

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REFERENCES