Exciton Trapping in a Periodically Modulated Magnetic Field

J.A.K. Freire¹, V.N. Freire¹, G.A. Farias¹, and F.M. Peeters²

¹Departamento de Física, Universidade Federal do Ceará, Centro de Ciências Exatas,
Campus do Pici, Caixa Postal 60890, 60455-760 Fortaleza, Ceará, Brazil
²Departement Natuurkunde, Universiteit Antwerpen (UIA),
Universiteitsplein 1, B-2610 Antwerp, Belgium

Received on 23 April, 2000

The behavior of excitons in spatially modulated magnetic fields is described taking into account the exciton spin contribution. The results show that the exciton trapping in periodic magnetic fields is possible and dependent on the modulation profile.

I Introduction

In view of recent developments in nanotechnology fabrication, the study of charged particles under the influence of nonhomogeneous magnetic fields have been the subject of a great number of theoretical [1] and experimental [2] investigations. Among these systems, the magnetotransport through spatially periodic magnetic fields has recently yielded very interesting results. Several works have been performed on the possibilities of exciton trapping by using nonhomogeneous stress [3] and spatially varying electric fields [4].

Recently, circular magnetic fields profiles were used to induce exciton trapping [5]. In these magnetic systems, the low-field gradient region creates a minimum in the effective confinement potential for the excitons. Thus, they can be trapped in the magnetic field inhomogeneity. To the best of our knowledge, the confinement of excitons in periodic magnetic fields has not been investigated. The purpose of this work is to present results on the behavior of excitons in spatially modulated magnetic fields.

II The modulated magnetic field structure

One dimensional periodic magnetic profiles can be created, e.g., by the deposition of ferromagnetic films or stripes on top of a semiconductor heterostructure, with a homogeneous magnetic field applied perpendicularly to the plane of the structure [6]. These films are patterned in such a way that the magnetic domains consists of parallel stripes with magnetization perpendicular or parallel to the film, changing sign from one stripe to the next, giving rise to a periodically modulated magnetic field in the plane of the semiconductor structure (see sketch in Fig. 1(a)).

Assuming a magnetization perpendicular to the xy plane, with boundaries $x \in (-a/2, a/2)$, $y \in (-\infty, \infty)$, and $z \in (-h/2, h/2)$, where $a$ and $h$ is the stripe width and thickness, respectively, the corresponding equation for the $z$ component of the magnetic field emerging from one stripe can be written as follows [1, 6]:

$$B_z(x) = B_a + B_s(x, z + h/2) - B_s(x, z - h/2), \quad (1)$$

where $B_s(x, z) = (\mu_0 M / 2\pi) \left[ \arctan (c_+) - \arctan (c_-) \right]$, $c_{\pm} = (x \pm 0.5a) / z$, $M$ is the stripe magnetization, $z$ is the distance of the center of the stripe to the quantum well, and $B_a$ the uniform magnetic field. The resulting periodic magnetic field is a superposition of those due to the individual magnetized stripes, in such a way that:

$$B_{z \; \text{total}}(x) = \sum_{N=-\infty}^{\infty} (-1)^N B_z(x - Nl). \quad (2)$$

where $N$ is the number of stripes and $l$ is a constant related to the lattice periodicity. The magnetic field profile for $N = 4$ is shown in Fig. 1 (b) (see dashed-dotted line).

The Hamiltonian describing the exciton motion in a nonhomogeneous magnetic field can be written as [6]:

$$H = H^z(z_i) + W(r, z_i, z_h) + H^{2D}(R, r) + H^{mv}(X), \quad (3)$$

where $H^z(z_i) = -(\hbar^2 / 2m^*_e) (\partial^2 / \partial z_i^2) + V_i(z_i)$ is the Hamiltonian describing the electron and heavy-hole confinement in the quantum well; $W(r, z_i, z_h) = (\epsilon^2 / \varepsilon) \left\{ \gamma / r - [r^2 + (z_e - z_h)^2]^{-1/2} \right\}$ is related to the
difference between the 2D and 3D Coulomb interaction and is treated as a perturbation:

\[ H^{2D}(\mathbf{r}, r) = H^{CM}(\mathbf{R}) + H^{cd}(\mathbf{r}, \mathbf{R}, \nabla R), \]

with \( H^{CM}(\mathbf{R}) = -\left( k^2 / 2M \right) \nabla_\mathbf{R}^2 \) and \( H^{cd}(\mathbf{r}, \mathbf{R}, \nabla R) = -\left( k^2 / 2\mu \right) \nabla_\mathbf{r}^2 - (\gamma e^2 / \epsilon r) + u_1 + u_2 \), describes the exciton motion in the \( xy \) plane subjected to the periodic magnetic field \( B_z(X) \), where \( \mathbf{r} \) and \( \mathbf{R} \) are the exciton relative and center-of-mass motion coordinates, \( \mathbf{r} = r - r_b \) and \( \mathbf{R} = (m_r^* r_r + m_h^* r_h) / M \), respectively, with the total exciton mass \( M = (m_r^* + m_h^*) \); \( u_1 \) (\( u_2 \)) is related to the first (second) order dependence of the relative coordinates with the nonhomogeneous magnetic field [6], and \( \mu = m_r^* m_h^* / M \) is the exciton reduced mass.

![Figure 1](image_url)

**Figure 1.** (a) Experimental setup showing the stripe parameters: \( a \) (\( b \)) is the stripe width (thickness), \( d \) its distance to the middle of the quantum well, \( M \) the magnetization, \( B_z \) the applied magnetic field, and \( L \) is the quantum well width. (b) Effective potential and respective magnetic field for the exciton ground state as a function of the \( X \) coordinate. In this figure, \( N = 4 \), \( a = 0.5 \) \( \mu \)m, \( h = 0.2 \) \( \mu \)m, \( d = 0.08 \) \( \mu \)m, \( B_z = 0 \) T, and the iron magnetization \( M = 1740 \) emu/cm\(^3\).

Finally, the last term in equation (3), i.e.,

\[ H^{m_z}(X) = \mu \left[ g_{e,z} S_{e,z} - 1 / 3 g_{h,z} J_{h,z} \right] B_z(X), \]

describes the exciton spin interaction with the nonhomogeneous magnetic field. In the above equation, \( m_z = \pm 1, \pm 2 \) is the exciton quantum spin number which is related to the electron \( (S_{e,z} = \pm 1 / 2) \) and heavy-hole \( (J_{h,z} = \pm 3 / 2) \) spin numbers, and \( g_{e,z} \) \( (g_{h,z}) \) is the electron (heavy-hole) \( g \) factor.

We used the adiabatic approach because the motion in the \( z \) direction, the exciton relative motion and the spin motion are fast as compared to the center-of-mass motion [6]. We also assume that all displacements are decoupled from each other in such a way that one can write the total exciton wave function as:

\[ \Psi^{m_z}(\mathbf{r}, \mathbf{r}, z_e, z_h) = \phi(\mathbf{R}) \Phi(\mathbf{r}) F(z_e, z_h) \mathcal{L}^{m_z}(\mathbf{R}), \]

with \( \phi(\mathbf{R}) = \exp(i Q_Y \mathbf{r}) / \psi(\mathbf{r}) \), where \( Q_Y \) is the wave vector of the center-of-mass motion in the \( Y \) direction (exciton free direction).

The eigenvalues of the confinement in the quantum well, the exciton relative and spin motion can be obtained by applying the exciton Hamiltonian (Eq. (3)) to the above wave function [6]. For the exciton center-of-mass motion, we obtain the following Schrödinger-like equation:

\[ \left\{ \frac{d}{dX} \frac{h^2}{2 M e^{2I}(X)} \frac{d}{dX} - V^{eff}(X) + E \right\} \psi(X) = 0, \]

with

\[ M^{eff}(X) / M = \left[ 1 - \frac{e^2 \mu}{h^2 M^{eff}(X)} \frac{\alpha^{m_z} B_z(X)^2}{\gamma^2} \right]^{-1}, \]

\[ V^{eff}(X) = \frac{h^2}{2 M^{eff}(X)} Q_Y^2 + \frac{e^2 \gamma^2 \mu^2 B_z(X)^2}{2 h^2 \gamma^2} \]

\[ + \frac{e \hbar}{2 \mu \gamma} \zeta B_z(X) m_r \pm \frac{1}{2} \mu B g_{e,z} + B_z(z). \]

The above equation includes the different eigenvalues for the fast motion that contribute as an effective potential and also results in an effective mass. Here, \( \alpha^{m_z} \) and \( \beta^{m_z} \) (in units of \( \alpha_B^{m_z} \) and \( \alpha_B^{m_z} \), respectively) are constants related to the relative radial \( (n_r) \) and angular \( (m_r) \) quantum numbers [5], \( \zeta = (m_r^* + m_h^*) / M \), and \( g_{e,z} \) \( (g_{h,z}) \). The effective potential as a function of the \( X \) coordinate is shown in Fig. 1(b).

### III The exciton trapping energy

In order to estimate the exciton confinement, we have defined the exciton trapping energy as the difference in energy between an excitonic state in the homogeneous applied field \( B_z \) and its corresponding state in the periodic magnetic field. As a consequence of this definition, the \( Q_Y \) term that is not dependent on the magnetic field \((h^2 / 2M) Q_Y^2 \) is not included in the calculation of \( E_r \), which greatly decreases the exciton dependence on the wave vector \( Q_Y \). Indeed, the influence of the first term in Eq. (9) on the exciton confinement is so small that it will not be further considered. In all our numerical calculations, we have considered \( a = 0.5 \mu \)m, \( h = 0.2 \mu \)m, \( d = 0.08 \mu \)m, and \( M = 1740 \) emu/cm\(^3\) [3].

The trapping energy of the exciton ground state as a function of the applied magnetic field \( B_z \) is shown in Fig. 2 for several number of stripes. The energies related to the magnetic field created by one stripe are completely different from the ones of the situation when
$N \neq 1$, but they are very similar when $N \geq 2$ for both spin orientation, i.e., they are independent of the stripe numbers. The exception is the $s = -1$ spin state case, which shows different behaviors when $N$ is even and when it is odd.

![Figure 2](image_url)  

Figure 2. Trapping energy of the exciton ground state as a function of the applied field $B_n$ for the number of stripes (a) $N = 1$, (b) $N = 2$, (c) $N = 11$ and (d) $N = 20$, with spin $m_s = 0$ (solid), $m_s = +1$ (dashed), and $m_s = -1$ (dotted).

Notice that the exciton is always trapped in the periodic magnetic potential when $N$ is even, independent of the spin orientation, which is not true when $N$ is odd. In this situation, the exciton is not trapped for $s = -1$ and $B_n > 0$.

The angular momentum and spin interaction with the magnetic field give the stronger contribution to the confinement potential (see Eq. 11). The trapping energy for the exciton excited states $2p^-, 2s$, and $2p^+$, as a function of the applied field $B_n$ is shown in Fig. 3. Notice that the $2s$ state is an even function of the applied magnetic field, and that the $2p^-$ and $2p^+$ states are symmetric in $B_n$ with respect to each other. This can be explained as follows: the $\beta_0^2$ term (see Eq. (9)) gives a stronger contribution as compared to $\beta_1^2$, but even in this situation of zero angular momentum, the exciton spin should dominate the effective potential equation.

This does not occur because the exciton $g$ factor for a 80 Å well width is very small ($g_{exc} \approx -0.3$). Also due to the small $g$ factor, the angular momentum gives a bigger contribution than the spin, and the angular term is symmetric in $B_n$. The $2p^\pm$ states flip $B_n(X)$ in signal, which explains such behavior. Also notice that the trapping energies of the excited states are about 1 order of magnitude larger than those of the ground state (Fig. 2).

![Figure 3](image_url)  

Figure 3. Trapping energy of the exciton excited states as a function of the applied magnetic field $B_n$ for relative quantum states (a) $2p^-$, (b) $2s$, and (c) $2p^+$, with spin numbers $m_s = 0$ (solid), $m_s = +1$ (dashed) and $m_s = -1$ (dotted).

### IV Final remarks

The exciton trapping energy in GaAs/Al$_{0.2}$Ga$_{0.7}$As quantum wells in a periodically modulated magnetic field which we have obtained are not within the actual energy detection limit of photoluminescence (PL) experiments. However, our work undoubtedly shows that the trapping in periodic magnetic structures exists, and gives further information on its characteristics that could guide future experiments. We would like to suggest that to increase $E_T$ one can confine the exciton in wide gap semiconductors structures. In these systems, the exciton energy can be higher than 100 times that of the corresponding GaAs situation. Notice that an increase of 10 times should make the exciton trapping energy detectable by PL measurements.
Acknowledgments

This work was partially supported by CNPq, CAPES and FINEP. F. M. Peeters was supported by FWO-Vl, IMEC, and IUAP-IV.

References