Study of Dynamical Dipole Moment in an Asymmetric Double Quantum Well

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A study of the dynamical behavior of the charge in an asymmetric double quantum well is presented. In particular, the time behavior of intrasub-band dipole moment which is induced by intersub-band optical excitation is analyzed in two aspects: first, design of the quantum wells in order to spatially separate the charge and store it and second study the dynamic tunneling effect between the two wells. Since both conduction band levels are optically populated by transitions from a single band level, they are coherently coupled. The intrasub-band density matrix completely characterizes the dynamics of the conduction band electrons. These dipole moment oscillations are entirely equivalent to the tunneling oscillations between the two wells. The time dependent dipole moment produced by these real-space charge oscillations is obtained as a source of dipole electromagnetic radiation in THz region, which has been detected experimentally. The calculations show the geometry dependence of the emitted radiation. Based on the reported decoherence time for this process a coherent control mechanism of the THz radiation is introduced by varying the phase of a second laser.

Semiconductor heterostructures are a perfect laboratory to study coherent optical effects in condensed matter systems due to their high quality and the possibility to engineer the structure of energy levels. Tunneling is a fundamental quantum mechanically phenomenon of interest in many branches of physics specially in quantum wells. In order to study tunneling without the influence of transport, one should study an isolated structure in which transport effects are not important: an asymmetric double quantum well structure (a-DQW) Coherent oscillations of a wave packet in a semiconductor double quantum well structure were detected using time resolved degenerate four wave mixing and pump and probe spectroscopy [1]. The early observation of THz radiation from an oscillating wave packet in a semiconductor nanostructure, using an asymmetric quantum well structure (a-DQW) was reported in [2]. In this work a systematic study of the dynamic behavior of the charge in an a-DQW is presented, in which particularly the dynamic tunneling effect between the two wells is examined in the following way: An appropriated design of heterostructure can show coherence between two conduction band levels, coupled to one and the same ground state level (a valence band level or another conduction sub-band). Such structure behave qualitatively like a three level system. In the present calculation we choose the three levels in the conduction band. The intrasub-band density matrix completely characterizes the dynamics of the conduction band electrons, the total dipole moment of conduction electrons distributed between both upper states can be expressed through their density matrix [3]. The time dependent dipole moment produced by these real-space charge oscillations is obtained and a detailed calculation of the time evolution process as function of the barrier width is presented. Such time dependent dipole moment is a source of dipole electromagnetic radiation, which can be detected [2] and can be coherently controlled by phase-shift of two pump pulses as shown in [4]. This technique has also been suggested for controlling decoherence effects [5] and [6].

![Figure 1](p) Transition diagram of a three level system excited from the lower level to both upper levels simultaneously with a pulse whose spectrum overlaps both transition frequencies.

The charge dynamic behavior of an a-DQW designed as a three level system is studied. A double quantum well consists of two wells separated by a nar-
row barrier whose width \( b \) is comparable to the tunneling decay exponent. The wave functions of the two levels are now coupled and strong modification of them is expected if the energies of the confined levels in both wells are close to each other, which means tunneling resonance. The two resonance levels hybridize and form new wave functions for the asymmetric quantum well. The wave functions of the a-DQW are obtained from a linear superposition of the wave functions of the two wells entering in the structure [7].

The electronic density of the a-DQW exhibits temporal oscillations with the frequency of the splitting in the tunnel-coupled system. Fig. 2 shows the charge probability in both wells with the electron initially localized in the wide well (WW). The transfer is complete only in exact tunneling resonance case, as can be seen in Fig. 2 for narrow barriers the charge is equally distributed in both wells, but as barrier width increases the charge will prefer to be in the WW. In this way a charge storage in the WW is reached, which can be controlled varying the barrier width.

\[
\rho_{01}(t) = \frac{i}{\hbar} (E_0 \tau \rho_{01}) e^{i \hbar (\epsilon_1 - \epsilon_0) t} \tag{1}
\]

\[
\rho_{02}(t) = \frac{i}{\hbar} (E_0 \tau \rho_{02}) e^{i \hbar (\epsilon_2 - \epsilon_0) t} \tag{2}
\]

where \( \rho_{01} \) and \( \rho_{02} \) are the corresponding dipole transition matrix elements and \( \epsilon_1 \) and \( \epsilon_2 \) are the energies of the upper levels. These two polarizations interfere giving a quantum beat and coupling the two upper states through a new polarization obtained from the equation of motion of the intrasubband density matrix component. The intrasubband non diagonal element of the density matrix \( f_{ij} \) is

\[
f_{12}(t) = \rho_{01} \rho_{02} \left( \frac{E_0 \tau}{\hbar} \right)^2 e^{i \frac{\hbar}{\tau} (\epsilon_2 - \epsilon_1) t} \tag{3}
\]

The two final states are coherently coupled to the same initial state, it means, when both interband transition matrix elements \( \rho_{01} \) and \( \rho_{02} \) are nonzero, ultrafast intersubband optical polarization creates coherent coupling between the two final states, it means an intraband coupling. In particular we can express the total dipole moment of conduction electrons distributed between states 1 and 2 through their density matrix. Denoting \( D_{i,j} \) the matrix elements of the dipole moment between conduction band states i and j, we can write the total dipole moment of the conduction electrons as

\[
D(t) = \sum_{j=1,2} D_{i,j} f_{ij} \tag{4}
\]

so that the intraband coherence causes the dipole moment of the system to oscillate with the quantum beat frequency \( (\epsilon_2 - \epsilon_1) t / \hbar \). In the case of a DQW system, these dipole moment oscillations are entirely equivalent to tunneling oscillations because an electron initially localized in one of the two wells of a DQW structure will tunnel into the other well and then back into the first in a periodic fashion. The time dependent dipole moment is produced by these real space charge oscillations. The tunneling time as function of the barrier width is shown in Fig. 3, where it is observed that tunneling time is a non-linear function of the barrier width. This time increases fast for narrow barriers and very slow for wide barriers.

![Figure 2](image2.png)

Figure 2. Charge probability as function of time in both wells. The well parameters are WW width 140 A, NW width 60 A for several barrier widths with electronic mass of GaAs.

**Coherent coupling**

Intersub-band transitions in the wide well (WW) occur at lower energy than in the narrow well (NW). A short laser pulse can excite an electronic wave packet that is mostly localized in the WW between the final states (1 and 2). The laser spectrum should be wider than the energy splitting of the upper states. Since both excited levels are optically populated by transitions from the same ground state (0) of the wide well, they are coherently coupled. Moreover, the ground state of the wide well is optically coupled to both excited levels. The temporal evolution of the polarization components \( \rho_{01} \) and \( \rho_{02} \) is described by the Bloch equations. Quantum beats occur when the electrons from the lower level are simultaneously excited into both upper levels. Assuming the excitation pulse to be a \( \delta \) function in the time domain \( E(t) = E_0 \tau \delta(t) \), the polarizations \( \rho_{01} \) and \( \rho_{02} \) for positive times are:

\[
\rho_{01}(t) = \frac{i}{\hbar} (E_0 \tau \rho_{01}) e^{i \hbar (\epsilon_1 - \epsilon_0) t} \tag{1}
\]

\[
\rho_{02}(t) = \frac{i}{\hbar} (E_0 \tau \rho_{02}) e^{i \hbar (\epsilon_2 - \epsilon_0) t} \tag{2}
\]

where \( \rho_{01} \) and \( \rho_{02} \) are the corresponding dipole transition matrix elements and \( \epsilon_1 \) and \( \epsilon_2 \) are the energies of the upper levels. These two polarizations interfere giving a quantum beat and coupling the two upper states through a new polarization obtained from the equation of motion of the intrasubband density matrix component. The intrasubband non diagonal element of the density matrix \( f_{ij} \) is

\[
f_{12}(t) = \rho_{01} \rho_{02} \left( \frac{E_0 \tau}{\hbar} \right)^2 e^{i \frac{\hbar}{\tau} (\epsilon_2 - \epsilon_1) t} \tag{3}
\]

The two final states are coherently coupled to the same initial state, it means, when both interband transition matrix elements \( \rho_{01} \) and \( \rho_{02} \) are nonzero, ultrafast intersubband optical polarization creates coherent coupling between the two final states, it means an intraband coupling. In particular we can express the total dipole moment of conduction electrons distributed between states 1 and 2 through their density matrix. Denoting \( D_{i,j} \) the matrix elements of the dipole moment between conduction band states i and j, we can write the total dipole moment of the conduction electrons as

\[
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so that the intraband coherence causes the dipole moment of the system to oscillate with the quantum beat frequency \( (\epsilon_2 - \epsilon_1) t / \hbar \). In the case of a DQW system, these dipole moment oscillations are entirely equivalent to tunneling oscillations because an electron initially localized in one of the two wells of a DQW structure will tunnel into the other well and then back into the first in a periodic fashion. The time dependent dipole moment is produced by these real space charge oscillations. The tunneling time as function of the barrier width is shown in Fig. 3, where it is observed that tunneling time is a non-linear function of the barrier width. This time increases fast for narrow barriers and very slow for wide barriers.
Moreover these oscillations are the source of electromagnetic radiation in the THz region. Coherent charge oscillations can be controlled by a sequence of two pulses because the driven term in the density equation is the product of the polarization created by the first pulse with the field of the second pulse. When both excitation pulses are present, the amplitude and phase of the charge oscillations become dependent on the relative phase-shift between the two pulses as shown in figure 4. The driving term in the density matrix equation for a two-pulse excitation contains the product of optical polarization of the first pulse and the electric field of the second pulse, which is a second order effect in nonlinear susceptibility $\chi^2$ of the three level system. The time separation of the phase locked pulse pair must be shorter than the time it takes for the THz signal to decay completely. This effect shows the coherent control of THz waves produced by an a-DQW by simply changing the phase between the pulses. The time separation of the phase locked pulse pair must be shorter than the time it takes for the THz signal to decay completely.

In this work the dynamic dipole moment for a special heterostructure, an asymmetric double quantum well, as a three level system in the coherent regime, is studied. The charge wave function packet is described as a superposition of the eigenstates of both wells. The time dependent charge probability shows the relevant role played by the barrier width in this system, which can be used to control the charge distribution, for example to store it. In order to deeply understand this process the polarizations (non diagonal matrix elements) are studied in the simple case that they are produced by a $\delta$-like excitation pulse by calculating all dipole moment transitions $d_{01}, d_{02}$ and $D(t)$ in this process. In particular the temporal variation of the dipole moment $D(t)$ is the source of the THz radiation. The corresponding polarizations $p_{01}$ and $p_{02}$ give rise to a quantum beat and to $f_{12}$ which is responsible for the coherent coupling between states 1 and 2.

Figure 3. Tunneling time as function of barrier width. It shows a linear region for widths between 80 Å and 120 Å for the chosen WW width of 140 Å and NW width of 60 Å.

Figure 4. The signals from a-DQW excited by a sequence of two mutual coherent optical pulses with phase difference zero and $\pi/2$.

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