I. Fusion and environmental coupling

Potential barriers are ubiquitous in physics. Surmounting and tunneling such barriers constitutes a problem of general and fundamental importance. In a few cases, such as alpha decay, this problem can be treated with success one-dimensionally by considering only the principal motion of the system over or through the potential barrier. In general, however, many more degrees of freedom are present. These may be taken into account as an environment, which couples to the principal motion [1].

Apart from alpha decay, the tunneling of the nitrogen atom of ammonia through the plane defined by the three hydrogen atoms is a classic example of a one-dimensional tunneling phenomenon. In contrast, chemical reactions, where an activation barrier has to be surmounted, or impurity tunneling in crystals, which couples to lattice vibrations, are considerably affected by environmental degrees of freedom.

Environmenal coupling may be categorized into three extreme cases, which are (i) the trivial case of no coupling, (ii) weak coupling to a large number of additional degrees of freedom, and (iii) the dominance of a few important degrees of freedom which couple strongly. Systems in category (ii) may be described using semi-classical, macroscopic theories based on transport equations, whereas those in category (iii) can be successfully treated by truncating the full many-particle Hamiltonian and including only the strongest couplings in the description. The fusion of the nuclear binary system is unique in nature, since the three extremes and also transitional cases can be studied within the same physical system. This can be achieved by choosing appropriate projectile-target combinations [1].

The diversity of the nuclear binary system may be illustrated with the two isotopes $^{40}\text{Ca}$ and $^{90}\text{Zr}$. The fusion of the double-magic nucleus $^{40}\text{Ca}$ with itself can be well described with a one-dimensional calculation, since in this case environmental degrees of freedom are of no significance (i). In contrast, the fusion of $^{90}\text{Zr}$ with itself is an excellent example of a system where the relative motion of the two nuclei couples to many weak channels (ii), resulting in a considerable shift of the fusion barrier to higher energies. Models based on transport theory, which assume that transfer processes initiate fusion, can explain this shift. Most of the transfer channels have negative Q-values, so that kinetic energy associated with the relative motion of the nuclei is dissipated to open these channels. Additional kinetic energy is thus required to surmount the fusion barrier, which produces the apparent shift of the fusion barrier.

The third extreme (iii), where a few couplings are dominant, is well represented by the fusion reaction $^{40}\text{Ca} + ^{90}\text{Zr}$. In order to describe this system the Hamiltonian for the relative motion may be augmented with Hamiltonians for the various coupling interactions. By limiting the number of coupling interactions to the few dominant degrees of freedom, the wavefunction of the system can be developed in a small number of of states. This leads to a system of coupled equations, which can be solved numerically.

It is instructive, however, to analytically decouple the system of coupled equations by making simplifying assumptions about the coupling interactions. Within the limits of this simplified coupled channels model it is found that the single fusion barrier of the one-dimensional model is re-
placed by a normalized distribution of fusion barriers [2]. The weight of each barrier in this distribution indicates the probability that this particular fusion barrier is encountered by the binary system. It can be shown that the barrier heights in this distribution span the one-dimensional fusion barrier, with at least one of the barriers being lower in energy than the one-dimensional barrier. This has the consequence that, relative to the one-dimensional model, the low energy fusion yield is enhanced. Indeed, as for many other systems [3, 4], for the reaction $^{40}$Ca + $^{90}$Zr a considerable fusion enhancement is observed at energies well below the one-dimensional fusion barrier [5].

II Identification of strong couplings in near-barrier fusion

In order to study environmental couplings in fusion, precision measurements of the total fusion yield as a function of centre-of-mass energy have to be performed. From such data fusion excitation functions can then be obtained. In general, this involves measuring both decay channels of the compound nucleus, i.e. particle evaporation and fission. In the case of $^{40}$Ca + $^{90}$Zr, which may be presented here as an example, particle evaporation is dominant and fission can be neglected without affecting the result.

Measurements of the evaporation residue yield following the fusion of this system were carried out at the XITU Tandem accelerator facility of the Laboratori Nazionali di Legnaro, Italy [5, 6]. The targets were 50 μg/cm$^2$ of isotopically enriched zirconium evaporated onto 15 μg/cm$^2$ carbon foils. In order to measure the fusion cross-sections above and well below the Coulomb barrier, which is about 140 MeV for both systems, the beam energy was varied between 125 and 160 MeV.

The beam intensity was monitored continuously by four silicon surface barrier detectors which detected Rutherford scattering from the target. The monitor detectors were located at the scattering angles $\theta_{lab} = 22^\circ$, above, below, to the left, and to the right of the beam. Recoiling zirconium nuclei could also be clearly identified in the energy spectra of these detectors.

The evaporation residues at $\theta_{lab} = 0^\circ$ were separated from most of the intense flux of beam-like particles using an electrostatic deflector. Following the deflector the evaporation residues and the remaining beam-like particles passed through a channel-plate detector before their energy was measured with a silicon surface-barrier detector. Over the distance of 40 cm between these two detectors the time-of-flight of the evaporation residues was measured. The combined time-of-flight and energy information enabled a clean separation of the evaporation residues from other particles. For each centre-of-mass energy the number of evaporation residue events was divided by the Rutherford scattering yield detected by the monitor detectors. The detection solid angle, the transmission of the electrostatic deflector, and fusion angular distributions were also measured, so that total fusion cross-sections $\sigma^{fus}(E)$ could be derived from the data as a function of centre-of-mass energy $E$. In a similar fashion the fusion excitation function for the heavier system $^{40}$Ca + $^{96}$Zr was also measured in these experiments.

At low energies the measured fusion excitation function for $^{40}$Ca + $^{90}$Zr is enhanced with regard to one-dimensional calculations [5, 6]. While this is a manifestation of environmental coupling in this system, it is not straightforward to identify the important degrees of freedom which contribute. However, based on the simplified coupled channels model a method has been suggested [7], which can aid in the identification of the strong couplings. With this method a representation $D_f^{fus}(E)$ of the fusion barrier distribution is extracted from the fusion excitation function by double-differentiation of the function $E\sigma^{fus}$ according to

$$D_f^{fus}(E) = \frac{d^2(E\sigma^{fus})}{dE^2}$$  (1)

With experimental data this differentiation is typically carried out using a point-difference formula [8]. It is clear that this approach relies on detailed measurements of the fusion excitation function to high precision. While fusion excitation functions tend to be smooth, the representations $D_f^{fus}(E)$ often display pronounced structure which may be correlated to specific environmental couplings. By comparing $D_f^{fus}(E)$ as obtained from experiment and theory, respectively, the success or failure of specific coupling interactions in explaining the fusion dynamics can often be unambiguously demonstrated.

Without environmental coupling the representation $D_f^{fus}(E)$ is a narrow, symmetric peak. For the system $^{40}$Ca + $^{90}$Zr it has been found [5, 6], however, that $D_f^{fus}(E)$ displays two, possibly three, distinct peaks. By including multi-phonon excitations of projectile and target in the Hamiltonian, the full, numerical solution of the coupled channels equations gives a fusion excitation function, which agrees with the experimental excitation function. This calculation also reproduces the structure of the barrier distribution representation $D_f^{fus}(E)$.

Interestingly, the equivalent calculation for the heavier system $^{40}$Ca + $^{96}$Zr fails completely in this regard [5, 6]. This implies that in this system additional degrees of freedom couple to the relative motion of projectile and target nucleus. Since the low-lying collective states in the two zirconium isotopes have very similar excitation energies and deformation parameters, the main differences between these two systems are in their Q-values for neutron transfer. In the heavier system the calcium nucleus can pick-up as many as eight neutrons in transfer reactions with positive Q-value, whereas the equivalent channels in the lighter system all have negative Q-values. The difference in the fusion dynamics of these systems may thus be attributed to strong coupling to multi-neutron transfer in the case of $^{40}$Ca + $^{96}$Zr. Indeed, simplified coupled channels calculations suggest that this multi-neutron transfer may be expected to proceed sequentially [6]. A conclusive theoretical description of the heavier system is still lacking, since it is not clear how to
correctly include the various neutron pick-up channels in a full coupled channels calculation.

III Alternative representations of the fusion barrier distribution

From the previous section it is apparent that the experimental identification of strong environmental couplings in fusion from direct measurements of fusion excitation functions is complex. Alternative techniques requiring less experimental effort would therefore be advantageous.

In a coarse, classical description of fusion, which neglects the effects of internal excitations, transfer, tunneling, and quantum-mechanical interference, barrier transmission may be associated with fusion, and reflection at the barrier may be associated with scattering. In this model the normalised differential scattering cross section \( d\sigma^{sc}/d\sigma^{R}(E) \) for head-on collisions \( (\ell = 0) \) is identical to the reflection coefficient \([1]\):

\[
R(E) = \frac{d\sigma^{sc}}{d\sigma^{R}(E, \ell = 0)} \quad (2)
\]

Here \( E \) is the centre-of-mass energy and \( d\sigma^{R} \) is the Rutherford cross section. Since flux conservation requires that the sum of transmitted and reflected flux is constant, features in the transmission function \( T(E) \) resulting from a distribution of fusion barriers are therefore also present in the energy dependence of the reflection coefficient \( R(E) \), and consequently also appear in the scattering excitation functions. If this correlation between scattering excitation function and transmission function is retained in actual, measured scattering excitation functions, differentiation according to

\[
D^{qel}(E) = -\frac{d}{dE} \left[ \frac{d\sigma^{qel}}{d\sigma^{R}(E)} \right] \quad (3)
\]

would yield an alternative representation \( D^{qel}(E) \) of the fusion barrier distribution \([9]\). In this equation \( d\sigma^{qel}/d\sigma^{R} \) refers to all reflected flux, i.e. the differential quasi-elastic scattering cross section, which includes elastic scattering, inelastic scattering and transfer reactions. It is clear that this approach assumes that energy and angular momentum changes due to inelastic excitations and transfer reactions are small, and that quantum-mechanical effects are negligible, so that the scattered nuclei essentially follow Rutherford trajectories. Barrier distribution representations \( D^{qel}(E) \) measured at different scattering angles may thus be compared by subtracting the appropriate centrifugal energy from the centre-of-mass energy \([1]\).

The technique has been tested \([6]\) by extracting the quasi-elastic excitation functions at \( \theta_{\text{cm}} = 136^\circ \) for the two systems \(^{40}\text{Ca} + ^{90,96}\text{Zr} \) from the monitor detector spectra measured as part of the experiments detailed in the previous section. Indeed, the barrier distribution representations \( D^{qel}(E) \) obtained from these data using a point difference formula are remarkably similar to the representations \( D^{fus}(E) \). This has demonstrated that the function \( D^{qel}(E) \), while not identical to \( D^{fus}(E) \), is an alternative representation of the fusion barrier distribution.

The sensitivity of the representation \( D^{qel}(E) \) to the fusion barrier distribution at energies below the average fusion barrier has been confirmed by experiments for a variety of systems with widely different fusion dynamics and coupling interactions \([9, 10, 11, 12, 13, 14, 15]\). It has, however, also become clear that at energies above the average fusion barrier the representation \( D^{qel}(E) \) rapidly looses this sensitivity with increasing energy. This can be attributed to the fact that at these higher energies the quasi-elastic scattering cross section is dominated by inelastic and transfer channels for which the assumption of classical Rutherford trajectories is poor \([9]\).

The successful identification of signatures of environmental coupling in quasi-elastic scattering excitation functions has motivated the search for these signatures in specific channels of the reflected flux. It has been shown \([16]\) that within the simplified coupled channels model the total elastic scattering amplitude can be expressed as a weighted sum of the partial elastic scattering amplitudes, which are associated with each barrier of the fusion barrier distribution. It then follows that the function

\[
D^{el}(E) = -\frac{d}{dE} \left[ \frac{d\sigma^{el}}{d\sigma^{R}(E)} \right]^{1/2} \quad (4)
\]

is a third representation of the fusion barrier distribution, if phase difference between the partial elastic scattering amplitudes can be neglected. While this latter assumption is not necessarily valid, it has been demonstrated \([16]\) that non-zero phase differences in fact may enhance any structure present in \( D^{el}(E) \).

Experiments \([16]\) using \(^{16}\text{O} \) beams in combination with several target nuclei have shown that \( D^{el}(E) \) reflects the fusion barrier distribution at energies below the average fusion barrier similar to \( D^{qel}(E) \), although features in \( D^{el}(E) \) tend to be broader than in \( D^{qel}(E) \). This additional broadening is due to the many inelastic and transfer channels which only couple weakly to the relative motion, however, absorb incident flux thus lost from the elastic channel \([16]\). It may therefore be presumed that at energies above the average fusion barrier, similar to quasi-elastic scattering, the dominance of inelastic and transfer channels in the reflected flux is responsible for the loss of sensitivity to the fusion barrier distribution in \( D^{el}(E) \).

It is interesting to observe that for the heaviest system studied thus far, \(^{32}\text{S} + ^{208}\text{Pb} \), the representation \( D^{el}(E) \) is shifted downwards in energy by about 4 MeV relative to \( D^{fus}(E) \), whereas such a shift is not observed for any of the lighter systems for which \( D^{el}(E) \) has been measured \([10]\). It is not clear, if this effect, which in contrast is not evident for \( D^{qel}(E) \), may reveal aspects of the fusion dynamics of heavier systems, and further studies would be useful.

A fourth representation \( D^{trans}(E) \) of the fusion barrier distribution may be introduced on the basis of qualitative ar-
so that the integral of \( \frac{d\sigma^{\text{trans}}}{dE}(E) \) is, however, poor and its use as an analytical tool is thus limited.

\[ D^{\text{trans}}(E) = \frac{1}{E_0} \int \frac{d\sigma^{\text{trans}}}{dE}(E) \frac{dE}{R}(E) \]  

(5)

where \( d\sigma^{\text{trans}} \) is the differential cross section for a particular transfer channel and \( E_0 \) a normalization energy, chosen, so that the integral of \( D^{\text{trans}}(E) \) equals 1 MeV\(^{-1}\). This approach relies on the argument that \( (d\sigma^{\text{trans}}/d\sigma^R)(E) \) generally peaks in the vicinity of the barrier energy. If there exists a distribution of such barriers, this distribution should be reflected in \( D^{\text{trans}}(E) \).

By measuring the excitation functions for neutron transfer reactions, for reaction channels involving proton stripping, and for those involving alpha stripping, it has been demonstrated [10] for a series of systems that the shape of \( D^{\text{trans}}(E) \) is correlated to that of the other three representations \( D^{\text{fus}}(E) \), \( D^{\text{qel}}(E) \) and \( D^{\text{el}}(E) \). The sensitivity of \( D^{\text{trans}}(E) \) is, however, poor and its use as an analytical tool is thus limited.

### IV Fusion studies with quasi-elastic scattering measurements

Among the three alternative representations of the fusion barrier distribution \( D^{\text{qel,el,trans}}(E) \), the representation \( D^{\text{qel}}(E) \) stands out, since the experimental effort required in its determination is smallest. It is also the most sensitive of these three representations, albeit limited to energies below the average fusion barrier. The determination of \( D^{\text{qel}}(E) \) is therefore useful in the investigation of environmental coupling, which produces signatures in the low energy part of the barrier distribution. This is the case when the relative motion of the two nuclei couples to positive Q-value channels [4]. Apart from the systems \(^{40}\text{Ca} + ^{90,96}\text{Zr} \), which have already been discussed, the fusion reactions of the sulphur projectiles \(^{32,34,36}\text{S} \) with \(^{208}\text{Pb} \) and those of the oxygen projectiles \(^{16,18}\text{O} \) with \(^{58}\text{Ni} \) may fall into this category. For the three reactions \(^{32,34,36}\text{S} + ^{208}\text{Pb} \) and \(^{208}\text{Pb} + ^{58}\text{Ni} \) the Q-values progressively favour the neutron pick-up channels with decreasing projectile mass. In the case of the reactions \(^{16,18}\text{O} + ^{58}\text{Ni} \), for the \(^{18}\text{O} \) projectile the one and two neutron stripping channels have positive Q-values, whereas for the \(^{16}\text{O} \) projectile they have large negative Q-values.

The representations \( D^{\text{qel}}(E) \) for these systems have recently been measured, with the exception of the reaction \(^{36}\text{S} + ^{208}\text{Pb} \) [14, 15]. In both cases the system with favourable Q-values for neutron transfer shows additional barrier strength at low energies when compared to its partner system. The considerable difference between the two representations \( D^{\text{qel}}(E) \) for the light fusion reactions \(^{16}\text{O} + ^{58}\text{Ni} \) and \(^{18}\text{O} + ^{58}\text{Ni} \) is particularly striking.

While the new data support an important role of positive Q-value neutron transfer channels in the fusion of \(^{32,34}\text{S} + ^{208}\text{Pb} \) and \(^{16,18}\text{O} + ^{58}\text{Ni} \), such an interpretation is only unique, if the properties of the collective states in \(^{16}\text{O} \) and \(^{18}\text{O} \), and in \(^{32,34}\text{S} \) and \(^{34}\text{S} \) are identical, or at least can be assumed to be very similar. Recent results for the fusion of the two sulphur nuclei \(^{32,34}\text{S} + ^{89,90}\text{Y} \) [17] show that in that case the different collectivity of their quadrupole excitations results in a broader fusion barrier distribution for \(^{32}\text{S} \) than for \(^{34}\text{S} \), not unlike what is observed for \(^{32,34,58}\text{S} + ^{208}\text{Pb} \). Thus the observed differences may not be solely due to coupling to the positive Q-value neutron transfer channels. This could also be the case for \(^{16,18}\text{O} + ^{58}\text{Ni} \) where the nuclear structures of the projectiles are considerably different. It is remarkable though that the new quasi-elastic scattering data clearly reveal the important differences between the fusion barrier distributions of these systems. A conclusive picture of the fusion dynamics in these systems may emerge from full coupled channels calculations for quasi-elastic scattering and detailed fusion measurements.

### V Conclusions

The fusion of the nuclear binary system is a unique realisation of the fundamental barrier problem, since the nature of the coupling between environmental degrees of freedom and the principal motion over the barrier can be changed dramatically by selecting different projectile-target combinations. By extracting a representation of the fusion barrier distribution from precise fusion excitation functions specific strong couplings can often be identified. This can be aided through the determination of alternative representations based on measurements of quasi-elastic scattering, elastic scattering, or transfer excitation functions. The three alternative representations have been shown to reflect the fusion barrier distribution at energies below the average fusion barrier with the quasi-elastic representation being the most sensitive. Since quasi-elastic scattering experiments are generally not as complex as fusion measurements, they are well suited to survey a number of reactions to determine good candidates for detailed studies. The representations of the barrier distribution extracted from quasi-elastic scattering excitation functions can be indicative of important coupling interactions, however, the conclusive identification of these couplings may require the measurement and interpretation of the corresponding fusion excitation functions.

### Acknowledgements

The author would like to thank Dr Mahananda Dasgupta, Dr David Hinde, Dr Jack Leigh, Prof. Alberto Stefanini and Ms Tanja Schuck for their contributions to this work.

### References


