1 Introduction

With the advent of radioactive nuclear beams and the discovery that nuclear matter, under certain conditions, may present a halo structure, a renewed interest has surged in the investigation of sizes and radial shapes of nuclei. In the case of light neutron-rich nuclei, this new halo structure is composed of an extended low-density distribution of loosely bound valence neutrons (halo) surrounding a core consisting of the majority of the nucleons.

From the experimental point of view, the study of proton-nucleus scattering at intermediate energies is a useful method for determining accurate nuclear matter distributions in stable nuclei [1, 2, 3] mainly due to the fact that the Glauber multiple-scattering formalism can correlate the differential cross sections and the matter distribution [2]. Recently, it has also been used extensively to determine the matter distribution [1] and the size of stable nuclei [2, 3, 4]. As done in Refs. [7, 8], or through relativistic models [9, 10], usually with about the same good results. An advantage of a relativistic optical model is that its two potentials simultaneously determine both the central and spin-orbit interactions. When the relativistic impulse approximation (RIA) is used [11, 12], these interactions may be determined in terms of the corresponding scalar and vector nuclear densities [14].

In a previous work [15] we have used a relativistic nuclear optical potential, constructed from a Lorentz scalar, \( V_s \), and the time-like component of the four-vector potential, \( V_0 \), with Gaussian form factors to calculate elastic scattering differential cross sections and analyzing powers for \( p + {}^6\text{He} \) at intermediate energies, using the available experimental data in the range \( 0 \leq t \leq 0.22 \text{(GeV/c)}^2 \).

2 The Dirac optical potential

In relativistic optical model analysis of intermediate energy scattering [9], the Dirac equation is usually used in the form,

\[
\{\gamma^\mu(p-u) + \beta[m + V_s(r)] + [V_0(r) + V_c(r)]\} \Psi(r) = E \Psi(r),
\]

where \( V_s \) is an attractive Lorentz scalar potential, \( V_0 \) is the repulsive time-like component of a four-vector potential and \( V_c \) is the Coulomb potential. The choice of the potentials is motivated by meson exchange considerations and simplicity. The simplest meson exchange interaction possessing a certain justification on physical grounds, which is also capable of providing nuclear saturation properties, takes into account the exchange of an attractive intermediate range isoscalar scalar meson and a repulsive shorter range isoscalar vector meson [14, 16, 17, 18]. The two corresponding potentials \( V_s \) and \( V_0 \) play an essential role in the description of both elastic scattering and polarization data at intermediate energies since their sum is the principal contribution to the central potential while their difference determines the spin-orbit interaction [19]. The scalar (vector) optical potentials used in the analyses here can be written as

\[
V_s = U_s f_{V_s}(r) + iW_s f_{V_s}(r),
\]

\[
V_0 = U_0 f_{V_0}(r) + iW_0 f_{V_0}(r),
\]

where \( f_{V_s} \) and \( f_{V_0} \) are the form factors for the scalar and vector potentials, respectively.
with each of the potentials possessing both real and imaginary parts with possibly different radial dependences.

In the late seventies, Arnold, Clark and Mercer [9] measured angular distributions and polarizations for \( p^+{^4}\text{He} \) elastic scattering at intermediate energies, from 0.561 to 1.73 GeV. With a modified Woods-Saxon parametrization for the Dirac optical potential, with different sets of parameters at each energy, they obtained very impressive results for the Dirac optical potential, with different sets of parameters. However, nuclear structure calculations demonstrate that the nuclear densities of \( p^+{^4}\text{He} \) have a radial dependence \( f(r) \) which is very nearly Gaussian. Based on this physical evidence we have therefore chosen a Gaussian parametrization of the form factor [15] to fit the experimental data for \( p^+{^4}\text{He} \) elastic scattering at intermediate energies and we have restricted our attention to energies near those at which experimental data for \( p^+{^6}\text{He} \) elastic scattering exists, viewing an extrapolation of our global parameters to these latter cases. In what follows we will discuss our fitting procedure which is different from that of Ref. [9].

3 Results

3.1 \( p^+{^6,8}\text{He} \) elastic scattering

We consider the elastic scattering of \( p^+{^6,8}\text{He} \) at the laboratory energies for which data exist [5, 13], \( E_{\text{Lab}} = 0.717 \text{ GeV} \) and \( E_{\text{Lab}} = 0.671 \text{ GeV} \), respectively. To perform the RIA calculation for \( p^+{^6}\text{He} \), we followed the same procedure used for \( {^4}\text{He} \) in Ref. [15]. We generated equal scalar and vector matter densities using the appropriate Gaussian harmonic oscillator (HO) geometrical parameters of Ref. [5], together with the relativistic Love-Franey NN t-matrix of Ref. [21]. The results of the RIA calculation (full lines) for the angular distributions, shown in Figs. (3) and (4), agree well with the experimental data in the range of very low values of the momentum transfer but, as this quantity increases, they deviate from the experimental data in form and absolute values. These results are very similar to those obtained for \( p^+{^4}\text{He} \) elastic scattering.

We have attempted to extend our fit of the \( {^4}\text{He} \) data [15] by interpreting them in the context of a simple RIA calculation. To do so, we extract the effective strengths, \( \tilde{V}_{0i} = \frac{1}{2}(U_i W_i)\pi [r_{0i}^2]^{3/2} \), that, when multiplied by the \( {^4}\text{He} \) density, would yield the potentials obtained in our fit to the \( p^+{^4}\text{He} \) data. We have multiplied these strengths by the appropriate \( {^6,8}\text{He} \) densities, based on the occupation of the \( 1s_{1/2} \) and \( 1p_{3/2} \) orbitals, obtained using the Gaussian (HO) parametrization of Ref. [5], to obtain effective Dirac potentials for these systems,

\[
V_i(r) = \tilde{V}_{0i} \left[ \frac{4}{(\pi r_{0i}^2)^{3/2}} \exp \left( -r^2 / r_{0i}^2 \right) \right] + \zeta \frac{2}{3} \frac{1}{(\pi r_{0i}^2)^{3/2}} \frac{r^2}{r_{0i}^2} \exp \left( -r^2 / r_{0i}^2 \right),
\]

where \( \zeta = 2, 4 \) for \( {^6}\text{He} \) and \( {^8}\text{He} \), respectively, with \( r_{0i} \) being the core \((1s_{1/2})\) radial parameter and \( r_{0i} \) the valence \((1p_{3/2})\) one.

### Table I. Calculated rms radii in \( {^4}\text{He} \), \( {^6}\text{He} \) and \( {^8}\text{He} \)

<table>
<thead>
<tr>
<th></th>
<th>( {^4}\text{He} )</th>
<th>( {^6}\text{He} )</th>
<th>( {^8}\text{He} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{\text{rms}}(\text{fm}) )</td>
<td>1.50</td>
<td>2.36</td>
<td>2.49</td>
</tr>
<tr>
<td>( r_{\text{rms}(1)}(\text{fm}) )</td>
<td>1.49(3)</td>
<td>2.30(7)</td>
<td>2.45(7)</td>
</tr>
<tr>
<td>( r_{\text{rms}(10)}(\text{fm}) )</td>
<td>2.38 ± 0.05</td>
<td>2.38 ± 0.05</td>
<td>2.41 ± 0.04</td>
</tr>
</tbody>
</table>

The radii were modified slightly to take into account the differences between the experimental \( {^4}\text{He} \) radius of Ref. [5] and those obtained in our previous fit. In order to check our extrapolations we have used the generated potentials to calculate the effective rms radii of both systems. As we can see from Table I there is a reasonable agreement with the experimental values taken from Refs. [5, 13] in the case of \( {^4}\text{He} \). The resulting angular distribution is shown in Fig. 1 and is also in good agreement with the available experimental data [5, 13], taking into account the larger error bars at higher values of the momentum transfer, given in Ref.[13]. We have also performed a free fit (dotted lines) of the data, taking as a first guess the physical parameters of Ref. [15], in order to measure the quality of our physical extrapolation. As shown in Fig. 1, the free fit (dotted lines) provides slightly better agreement with the experimental data when compared to the physical extrapolation. In the case of \( p^+{^8}\text{He} \) elastic scattering, the physical extrapolation provides only slightly better agreement with the data than the RIA results, both shown in Fig. 2. This is also reflected in a poorer reproduction of the \( r_{\text{rms}} \) radius, given in Table I. Here, we have again performed a free fit of the parameters which yields fairly good agreement with the experimental data. In Figs. 3 and 4 we compare the polarizations for \( {^4}\text{He} \) and \( {^8}\text{He} \), calculated with the physical extrapolation, with the RIA results. Substantial differences exist between the two, as was the case in \( p^+{^4}\text{He} \) scattering [15]. At present no experimental data exist for the polarizations. Although extremely difficult to perform, experimental measurements of these polarizations would be useful for better determining their optical potentials.

4 Conclusions

We have calculated \( p^+{^6,8}\text{He} \) elastic scattering differential cross sections and polarizations using the relativistic impulse approximation and an adjusted Dirac optical potential. We have shown that the RIA results obtained using the parameters of Ref. [5] describe both systems well at low values of the momentum transfer. However, they deviate substantially from the data at higher momentum transfer and, moreover do not reproduce the expected oscillatory structure in the angular distributions and the corresponding polarizations. As an alternative to the RIA, we have used physical motivated Gaussian form factors given in the fit of the \( p^+{^4}\text{He} \) data [15] and, then attempted to extend the parameters obtained to the cases of \( p^+{^6,8}\text{He} \), with quite reasonable results in the first case and poorer agreement in the second.
one. In all cases, we obtained reasonable fits to the data where the parameters were permitted to vary freely. We have also calculated the polarization in $p^{+}{^6}\text{He}$ scattering. As in the case of $p^{+}{^4}\text{He}$, we find large differences between the RIA results and those of our physical extrapolation. Experimental data, which do not exist at present, will be necessary to resolve these discrepancies.

In closing, we should mention that a possible reason for the failure of the RIA in describing the $p^{+}{^4}\text{He}$, $p^{+}{^8}\text{He}$ data, mainly at higher momentum transfer, is that it has been pushed beyond the limit of its applicability in the simple manner in which it is used here. The decomposition of the effective Dirac potential into a scalar potential and the fourth component of vector is valid in the target rest frame. The scattering calculations, however, are performed in the CM frame and the boost that carries one frame to the other will convert the vector fourth component potential into a full vector potential. For proton scattering on a system such as $^{40}\text{Ca}$, the vector components introduced are small and can be neglected. For extremely light systems, such as those studied here, this may not be the case. We plan to examine the importance of this effect in the future.

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References


