Is it Possible to Test Brane-World Scenarios by Observation of Quasars and Microquasars?

C. H. Coimbra-Araújo, R. da Rocha, IFGW, Universidade Estadual de Campinas, CP 6165, 13083-970, Campinas, SP, Brazil
and I. T. Pedron
Universidade Estadual do Oeste do Paraná, 85960-000 Marechal Cândido Rondon, Brazil
(Received on 15 October, 2005)

The aim of this work is to present a possible way to estimate observational electromagnetical clues in the spectrum of quasars and microquasars due to the presence of extra dimensions. Here we analyze two possible ways to detect these electromagnetic signals: by the brane corrected accretion mechanism and by production of electromagnetic KK modes. We show that for the static black hole accretion case, the brane corrections cause a extremely small variation in the luminosity of the quasar. For the electromagnetic KK modes case, apparently the answer for the biggest quasars is positive and on the other hand, the possibility that microquasars can present constraints on extra-dimensional gravity in their spectra is less probable.

I. INTRODUCTION: BLACK HOLES ON THE BRANE

For many reasons, brane-world models present a rich way to explain our universe [1]. The Einstein equations in the brane can be written as the projection of Einstein equation in the high-dimensional bulk. The bulk is an Anti-de Sitter space and has warp properties symbolized by a negative cosmological constant. Assuming $Z_2$ symmetry, the Israel-Darmois junction conditions, and the Bianchi identities, Einstein equations are written as

$$G_{\mu\nu} = - \frac{1}{2} \Lambda_5 g_{\mu\nu} + \frac{1}{4} \kappa_5 \left[ TT_{\mu\nu} - T^a_{\mu} T_{a\nu} + \frac{1}{2} g_{\mu\nu} (T^a - T^a_{\mu\nu}) \right] - E_{\mu\nu},$$

where $T = T^a_{\mu}$ denotes the trace of the momentum-energy tensor $T_{\mu\nu}$. $G_5$ denotes the 5-dimensional Newton gravitational constant, and $E_{\mu\nu}$ denotes the ‘electric’ components of the Weyl tensor. The constant $\kappa_5 = 8\pi G_5$, where $G_5$ denotes the 5-dimensional Newton gravitational constant, that can be related to the 4-dimensional gravitational constant $G$. A vacuum on the brane, where $T_{\mu\nu} = 0$ outside a black hole (for a review, see e.g. [3]), implies, by Bianchi identities, that $E_{\mu\nu} = 0$. A particular manner to express the vacuum field equations in the brane is $E_{\mu\nu} = -\rho_{\mu\nu}$. One can use a Taylor expansion in order to probe properties of a static black hole on the brane [2]. Thus, for a static spherical metric on the brane given by

$$g_{\mu\nu} dx^\mu dx^\nu = -F(r) dt^2 + \frac{dr^2}{H(r)} + r^2 d\Omega^2,$$

where $d\Omega^2$ denotes the spherical 3-volume element related to the geometry of the 3-brane. The metric is led to the Schwarzschild one, if $F(r)$ equals $H(r)$. The exact determination of these radial functions remains an open problem in BH theory on the brane [1, 3].

II. ACCRETION EFFICIENCY IN SCHWARZSCHILD QUASARS WITH CORRECTIONS

Defining $\psi(r)$ as the deviation from a Schwarzschild form for $H(r)$ [1, 3]

$$H(r) = 1 - \frac{2GM}{c^2r} + \psi(r),$$

where $M$ is constant, and where, for a large BH with horizon scale $r \gg \ell$, $\psi(r) \approx -\frac{4GM^2}{3c^2\ell^2}$ [1]. Now we can explore the idea that, if black holes and specially the supermassive black holes (SMBHs) present in the nucleus of galaxies and quasars, do cause deviations from the 4D general relativity, these corrections should cause a small deviation in all SMBH properties [4]. First we need to estimate the corrected Schwarzschild brane radius $R_{\text{brane}}$. This is possible doing $H = 0$:

$$R_{\text{brane}}^3 = \frac{2GM}{c^2} R_{\text{brane}}^2 - \frac{4GM^2}{3c^2} = 0.$$  

Using Cardano’s formulation it follows that

$$R_{\text{brane}} = \left( a + \sqrt{b} \right)^{1/3} + \left( a - \sqrt{b} \right)^{1/3} + \frac{2GM}{3c^2},$$

where

$$a = \frac{R_S}{3} \left( \ell^2 + \frac{R_S^2}{9} \right),$$

$$b = \frac{R_S^2 \ell^2}{9} \left( \ell^2 + 2R_S^2 \right),$$

where $R_S$ is the classical Schwarzschild radius. Now, substituting the values of $G$ and $c$ in the SI, and adopting $\ell \sim 0.1$ mm and $M \sim 10^6 M_\odot$ (where $M_\odot \approx 2 \times 10^{33}$ g) denotes solar mass, corresponding to the mass of a SMBH, it follows from eq.(4) that the correction in the Schwarzschild radius of a SMBH by brane-world effects is given by $R_{\text{brane}} - R_S \sim 100$ m, and since the Schwarzschild radius $R_S$ is defined as $\frac{2GM}{c^2}$ =
2.964444 × 10^{12} \text{m}, the relative error concerning the
brane-world corrections in the Schwarzschild radius of a SMBH is
given by $\frac{\Delta R_{\text{brane}}}{R_{\text{SMBH}}} \sim 10^{-10}$. These calculations show
that there exists a correction in the Schwarzschild radius of
a SMBH caused by brane-world effects, although it is negligible.
Then, for the static case we must look for another kind
of observational mechanism.

III. ELECTROMAGNETIC KK MODES?

A natural manner to detect extra dimensions is by detection
of gravitational waves due to extradimensional or Kaluza-
Klein (KK) modes. Here on we will use the method developed
by [5]. Our assumption is to consider the conversion of
these gravitational KK modes in electromagnetic radiation. For
this we will assume a generic black string background with-
out charge producing a static Reissner-Nordström black hole
on the brane. This background is given by

$$ds^2 = e^{-2\eta/\ell} \left[-(1-2GM/r-f)dt^2 + \frac{dr^2}{1-2GM/r-f} + r^2d\Omega^2 + dy^2\right]. \quad (7)$$

where $f$ is a function independent of the charge. Using the
field equations found in eq. (1), it is possible to find the per-
tubative equations, assuming that the bulk cosmological
constant is given by $\Lambda_5 = -6\ell^{-2}$ [1]. Here we must also assume
the etro-vacuum energy-momentum tensor. The perturbed
metric is $\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$. Then we found the follow metric perturbations equations

$$k_{\nu\lambda} = \left. \frac{\partial}{\partial \lambda} \right| \left[ h_{\mu\nu} k_{\lambda} - k_{\nu\lambda} k_{\mu\nu} - \frac{1}{2} g_{\mu\nu} k_{\lambda} \right] - \frac{6}{f^2} h_{\lambda\nu} \right| + \frac{1}{2} \delta F_{\mu\nu} F_{\lambda}^{\lambda} - \frac{1}{2} h\delta F_{\mu\nu} F_{\lambda}^{\lambda} \right| T_{\mu\nu}: \right.$$

$$+ h_{\lambda\nu} \left[ \delta F_{\mu\nu} \lambda + \delta F_{\nu\lambda} \mu \right] - \frac{1}{2} \left( \delta F_{\mu\nu} \lambda \right)^{\lambda} + \frac{1}{2} h_{\lambda\nu} \left( \delta F_{\lambda\lambda} \nu \right) T_{\mu\nu}.$$