ROBUSTNESS MARGINS FOR GLOBAL ASYMPTOTIC STABILITY IN INDIRECT FIELD-ORIENTED CONTROL OF INDUCTION MOTORS

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RESUMO

A estabilidade de acionamentos por orientação de campo indireta de motores de indução quando sujeitos a erros de estimativa da constante de tempo do rotor é analisada. O artigo se concentra na estabilidade global, estendendo os resultados de (Bazanella and Reginatto, 1998). Margens de robustez são determinadas e diretivas para projeto do laço PI de velocidade são obtidas. A metodologia de análise é aplicada a dois sistemas distintos a fim de ilustrar as estimativas de margens de estabilidade.

PALAVRAS-CHAVE: Margens de robustez, estabilidade global assintótica, controle indireto de orientação de campo, motores de indução.

ABSTRACT

The influence of the rotor time constant mismatch on the stability of induction motors under indirect field oriented control is analyzed. The paper focuses on the global asymptotic stability property and extends the results of (Bazanella and Reginatto, 1998). Robustness margins for global asymptotic stability with respect to rotor time constant mismatches are obtained. The effect of the PI settings in such robustness margins is clarified allowing to derive design guidelines. The methodology is applied to two different induction motor drive systems illustrating the stability margin estimates.

KEYWORDS: Robustness margins, global asymptotic stability, indirect field-oriented control, induction motors.

1 INTRODUCTION

Indirect Field Oriented Control (IFOC) is a well established and widely applied control technique when dealing with high performance induction motor drives (Novotny and Lorenz, 1986; Leonhard, 1985; Bose, 1987). The commissioning of an IFOC requires the knowledge of the rotor time constant, a parameter that can vary widely in practice (Krishnan and Doran, 1987; Marino et al., 1993) and is known to cause performance and stability problems.

Most results in the literature address this problem from the application point of view focusing on the performance issue without providing any guarantees about stability. Only the recent works (Bazanella and Reginatto, 2000; Bazanella et al., 1999b; Bazanella and Reginatto, 1998; Ortega et al., 1996; Ortega et al., 1993; De Wit et al., 1996) have aimed at filling in this gap by providing IFOC with a firm theoretical foundation.

In (Bazanella and Reginatto, 1998; De Wit et al., 1996),

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a complete characterization of the equilibria with respect to rotor time constant mismatches has been given. Local stability properties of the equilibrium point have been investigated in (Bazanella and Reginatto, 1998; Bazanella et al., 1999b) where conditions for non-existence of both saddle-node and Hopf bifurcations were provided. Guidelines for setting the PI gains in order to guarantee a certain local stability margin with respect to rotor time constant mismatches for a practical loading range were also given (see also (Bazanella et al., 1999a)).

The robustness of IFOC against mismatches in the rotor time constant has also been analyzed in (De Wit et al., 1996), where the robust global stability of the operating point has been established from a qualitative standpoint. These results were generalized in (Bazanella and Reginatto, 1998) where explicit formulae to conclude about robust global asymptotic stability were derived. A passivity based analysis has been used in (Espinosa-Perez et al., 1998) to conclude about robust global asymptotic stability in the special case of zero load operation.

In this paper we concentrate on the robust global stability property of IFOC with respect to rotor time constant mismatches. We extend the results of (Bazanella and Reginatto, 1998) and provide a deeper insight into the characterization of stability margins for global asymptotic stability of IFOC. Section 2 formulates the problem and provides the complete model for the induction motor with IFOC and PI rotor speed regulation. The model is parameterized in the rotor time constant mismatch, thus allowing for the robustness analysis. In section 3 the main tool for robustness analysis is developed. An explicit condition to conclude about global asymptotic stability is provided. This tool is explored in section 4 to obtain robust global asymptotic stability margins with respect to rotor time constant mismatches. Numerical results are provided for two actual drive systems. Finally, section 5 provides a discussion on the results obtained.

2 PROBLEM STATEMENT

We consider the current fed induction motor model expressed in a reference frame rotating at synchronous speed. In terms of state variables, this model can be written as (Novotny and Lorenz, 1986)

$$\dot{x}_1 = -c_1 x_1 - u_1 x_2 + c_2 u_3 \quad (1)$$

$$\dot{x}_2 = -c_1 x_2 + u_1 x_1 + c_2 u_2 \quad (2)$$

$$\dot{w} = -c_3 w + c_4 [c_5 (x_2 u_3 - x_1 u_2) - T_m] \quad (3)$$

where x_1 and x_2 represent the q-axis and d-axis rotor fluxes, respectively, w is the rotor speed, u_1, u_2 and u_3 stand for the inputs - the slipping frequency, the d-axis and q-axis stator current components, respectively; T_m is the load torque, which is assumed constant, and the "c"parameters are all positive. In particular, c_1 represents the inverse of the rotor time constant, which is a critical parameter for indirect field oriented control. For more information regarding the induction motor modeling the reader is refered to (Leonhard, 1985; Bose, 1987; Krause, 1986; Reginatto, 1993).

In speed regulation applications the indirect field oriented control strategy is usually applied along with a PI speed loop as described by the following equations (Novotny and Lorenz, 1986; De Wit et al., 1996):

$$u_1 = \hat{c}_1 \frac{u_3}{u_2} \quad (4)$$

$$u_2 = u_2^0 \quad (5)$$

$$u_3 = k_p\, e_w(t) + k_i \int_0^t e_w(\zeta) d\zeta \quad (6)$$

where $\hat{c}_1$ is an estimate for the inverse rotor time constant c_1, k_p and k_i are the gains of the PI speed controller, w_{ref} is the constant reference velocity, $e_w = w_{ref} - w$ is the rotor velocity error, and u_2^0 is some constant which defines the flux level.

The knowledge of c_1 is the key issue in IFOC. If $\hat{c}_1 = c_1$, that is, if we have a perfect estimate of the rotor time constant, we say that the control is tuned, otherwise it is said to be detuned. Accordingly, we define

$$\kappa \triangleq \frac{\hat{c}_1}{c_1} \quad (7)$$

as the degree of tuning. It is clear that $\kappa > 0$ and the control is tuned if and only if $\kappa = 1$.

We parameterize the system (1)-(3) in closed-loop with the control (4)-(6) (see Figure 1) in terms of the degree of tuning κ, yielding a fourth-order system that can be described as:

$$\dot{x}_1 = -c_1 x_1 + c_2 x_4 - \frac{\kappa c_1}{u_2^0} x_2 x_4 \quad (8)$$

$$\dot{x}_2 = -c_1 x_2 + c_2 u_2^0 + \frac{\kappa c_1}{u_2^0} x_1 x_4 \quad (9)$$

$$\dot{x}_3 = -c_3 x_3 - c_4 [c_5 (x_2 x_4 - u_2^0 x_1) - T_e] \quad (10)$$

$$\dot{x}_4 = k_c x_3 - k_p c_4 [c_5 (x_2 x_4 - u_2^0 x_1) - T_e] \quad (11)$$

where we have defined the new state variables $x_3 \triangleq w_{ref} - w$ and $x_4 \triangleq u_3$ and the new parameters

$$k_c \triangleq k_i - k_p c_3, \qquad T_e \triangleq T_m + \frac{c_3}{c_4} w_{ref} \quad (12)$$

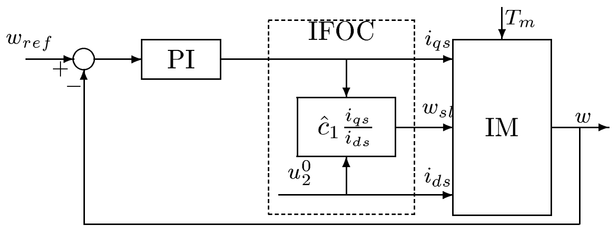

Figure 1: Block diagram of IFOC.

Throughout the paper, both T_m and w_{ref} are assumed constant. Let $x^e = [x_1^e \; x_2^e \; x_3^e \; x_4^e]^T$ represent a generic equilibrium point for the closed loop system (8)-(11) and define the change of coordinates $z \triangleq x - x^e$. In this new coordinates, the system is given by

$$\dot{z}_1 = -c_1 z_1 + (c_2 - \frac{x_2^e \kappa c_1}{u_2^0}) z_4 - z_2 (z_4 + x_4^e) \frac{\kappa c_1}{u_2^0} \quad (13)$$

$$\dot{z}_2 = -c_1 z_2 + \frac{\kappa c_1}{u_2^0} [x_1^e z_4 + z_1 (z_4 + x_4^e)] \quad (14)$$

$$\dot{z}_3 = -c_3 z_3 - c_4 c_5 [z_2 (z_4 + x_4^e) - z_1 u_2^0 + x_2^e z_4] \quad (15)$$

$$\dot{z}_4 = k_c z_3 - k_p c_4 c_5 [z_2 (z_4 + x_4^e) - z_1 u_2^0 + x_2^e z_4] \quad (16)$$

The first issue for the stability analysis of (13)-(16) is the characterization of equilibria. This characterization has been completely provided in (Bazanella and Reginatto, 1998; De Wit et al., 1996). The system (13)-(16) has a unique equilibrium point for any κ in the interval (0, 3), regardless of the PI settings, load conditions, and other parameters of the induction machine. Local stability properties of such equilibrium point have been investigated in (Bazanella and Reginatto, 1998) and (Bazanella et al., 1999b). In (Bazanella et al., 1999b) it was shown that for κ in the interval (0, 3), local stability of the equilibrium point may be lost due to a Hopf bifurcation that may take place for certain loading conditions and PI settings.

In this paper we concentrate on the global asymptotic stability property of the closed-loop system (13)-(16). We look for establishing allowable margins for the degree of tuning κ which preserve global asymptotic stability for given PI settings and loading conditions.

3 A TOOL FOR ROBUST GLOBAL STABILITY ANALYSIS

Let us define, for convenience of notation:

$$\alpha \triangleq \frac{\kappa c_1}{u_2^0 c_4 c_5} > 0$$

$$k_2 \triangleq \frac{\alpha^2 k_i}{c_2} > 0$$

$$k_3 \triangleq \alpha^2 c_3 \frac{k_p}{k_i} > 0$$

and consider the quadratic function

$$V(z) = \frac{1}{2} z^T (P_1 + m P_2) z \quad (17)$$

where

$$P_1 = \begin{bmatrix} k_p^2 + \frac{k_2}{\alpha} & 0 & -k_2 & -k_p \alpha \\ 0 & 0 & 0 & 0 \\ -k_2 & 0 & k_p^2 k_3 + \alpha k_2 & -k_p k_3 \\ -k_p \alpha & 0 & -k_p k_3 & k_3 + \alpha^2 \end{bmatrix} \quad (18)$$

$$P_2 = \begin{bmatrix} I_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & 0_{2\times2} \end{bmatrix} \quad (19)$$

and m is a positive scalar to be assigned. The matrix $P_1 + mP_2$ is a function of the degree of tuning κ. It is symmetric positive definite for any $m > 0$ and any possible operating condition.

The time derivative of (17) along trajectories of the closed-loop system (13)-(16) can be calculated as

$$\begin{aligned} \dot{V}(z) = & -\alpha_1 z_1^2 - \alpha_3 z_3^2 - \alpha_4 z_4^2 - 2\beta_{13} z_1 z_3 \\ & + 2\beta_{14} z_1 z_4 - m c_1 z_1^2 - m c_1 z_2^2 \\ & + 2m\eta_{14} z_1 z_4 + 2m\eta_{24} z_2 z_4 \end{aligned} \quad (20)$$

where

$$\alpha_1 \triangleq \frac{c_1}{c_2} (\kappa + 1)(k_p^2 c_2 + k_i \alpha) > 0$$

$$\alpha_3 \triangleq \frac{c_3}{c_2} \alpha^2 (k_p^2 c_2 + k_i \alpha) > 0$$

$$\alpha_4 \triangleq c_2 k_p \alpha > 0$$

$$\beta_{13} \triangleq -\frac{\alpha}{2} [\frac{c_1}{c_2} (\kappa + 1) k_i \alpha + \frac{c_3}{c_2} (k_p^2 c_2 + k_i \alpha) - k_p k_i]$$

$$\beta_{14} \triangleq \frac{1}{2} [k_i \alpha + k_p^2 c_2 + k_p \alpha c_1 (\kappa + 1)] > 0$$

$$\eta_{14} \triangleq \frac{1}{2} [c_2 - \frac{x_2^e c_1 \kappa}{u_2^0}]$$

$$\eta_{24} \triangleq \frac{x_1^e c_1 \kappa}{2 u_2^0}$$

The expression in (20) can be put into a quadratic form

$$\dot{V}(z) = -z^T Q z \quad (21)$$

where

$$Q = \begin{bmatrix} \alpha_1 + c_1 m & 0 & \beta_{13} & -\beta_{14} - m\eta_{14} \\ 0 & c_1 m & 0 & -m\eta_{24} \\ \beta_{13} & 0 & \alpha_3 & 0 \\ -\beta_{14} - m\eta_{14} & -m\eta_{24} & 0 & \alpha_4 \end{bmatrix}$$

Even though the system is nonlinear, it is possible to obtain such a quadratic form for the Lyapunov derivative thanks to the choice made for the Lyapunov function, which implies cancelation of the nonquadratic terms in equation (20). Let the second order polynomial $p(m)$ be defined by

$$p(m) \triangleq p_2 m^2 + p_1 m + p_0 \qquad (22)$$

with coefficients given by

$$\begin{aligned} p_2 &= -c_1\alpha_3(\eta_{14}^2 + \eta_{24}^2) \\ p_1 &= -\alpha_1\alpha_3\eta_{24}^2 - 2c_1\alpha_3\beta_{14}\eta_{14} + \beta_{13}^2\eta_{24}^2 + c_1^2\alpha_3\alpha_4 \\ p_0 &= c_1(\alpha_1\alpha_3\alpha_4 - \alpha_3\beta_{14}^2 - \beta_{13}^2\alpha_4) \end{aligned}$$

The following result (Bazanella and Reginatto, 1998) provides a robust stability test and establishes the robust global stability of the tuned condition.

Theorem 1 Let $p(m)$ have distinct real roots m_1, m_2 ordered such that $m_2 > m_1$. If

$$m_2 > \frac{\beta_{13}^2 - \alpha_1\alpha_3}{c_1\alpha_3} \triangleq m_0 \qquad (23)$$

then the origin of the system (13)-(16) is globally asymptotically stable.

◇

The proof of Theorem 1 has been provided in (Bazanella and Reginatto, 1998). It is included in this paper for completeness.

Proof: It is clear from (21) that if there exists an m such that all the leading minors of the symmetric matrix Q are positive then the origin will be globally asymptotically stable. The leading minors of first and second order are always positive, whereas the third and fourth order minors are given respectively by

$$\Delta_3(m) = mc_1[(\alpha_1 + mc_1)\alpha_3 - \beta_{13}^2] \qquad (24)$$

$$\Delta_4(m) = mp(m) \qquad (25)$$

If the roots of $p(m)$ are real and distinct, then $p(m) > 0$, $\forall m \in (m_1, m_2)$, since p_2 is negative; then $\Delta_4(m)$ is positive in this interval. On the other hand, $\Delta_3(m) > 0$, $\forall m > m_0$, so that all the leading minors will be positive for $m \in (\max\{m_0, m_1\}, m_2)$. Condition (23) guarantees that this interval is not empty.

□

The condition (23) establishes a simple verification procedure to conclude about global asymptotic stability for any particular IFOC based induction motor drive system. Indirectly, it can also provide a way to characterize robust global asymptotic stability margins with respect to κ for such systems, i.e., for a given PI setting and loading condition find, if possible, a range of κ for which global asymptotic stability of system (13)-(16) is guaranteed. That such a range does exist is a direct consequence of Theorem 1, as stated in the next Corollary.

Corollary 1 The equilibrium of the system (13)-(16) is globally asymptotically stable provided that $\kappa = 1$ or sufficiently close to this value.

◇

4 ANALYSIS OF ACTUAL DRIVE SYSTEMS

Theorem 1 provides a test for global stability. The criterion (23) that determines the global stability depends on all the parameters of the driving, which can be divided into four sets:

- the physical parameters of the motor (the 'c' parameters);
- the setting of the PI (k_p and k_i);
- the load (r^*);
- the mismatch in the rotor time constant (κ).

In order to get a better insight to the problem and establish typical robustness margins, we apply the global stability test to data taken from real induction motors. We aim to study, for a given motor and a given setting of the PI speed loop, what is the region of the parameter plane $\kappa \times r^*$ for which global stability is achieved. To this end, we apply the test (23) with the c parameters, k_p and k_i fixed, varying κ and r^* in the set of practical significance $(\kappa, r^*) \in (0, 3] \times [0, 2]$. In order to make a representative practical assessment of the global stability test, we apply it to two very different motors: a small motor (1hp) and a large motor (500 hp). The parameters for each motor are given in the Appendix.

Furthermore, we perform this procedure for different settings of the PI speed loop, in order to verify its influence on the robustness of the global stability. The PI parameters are usually set in order to provide a desired performance to the system under the assumption of perfect tuning ($\kappa = 1$), under which conditions the system becomes a second-order linear system (Bazanella

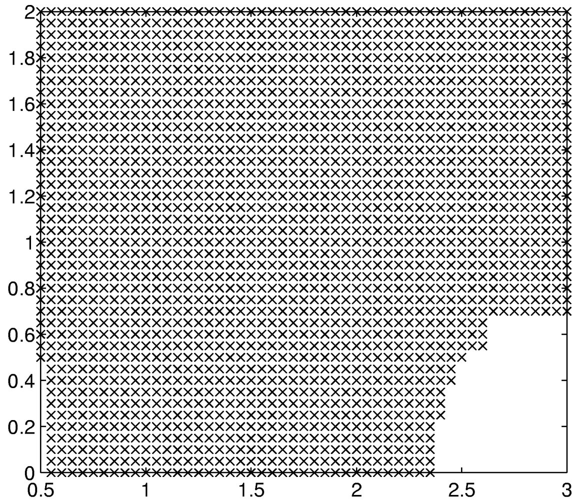

Figure 2: Parameter range of global asymptotic stability for the 1HP motor, $\eta = 0.5$.

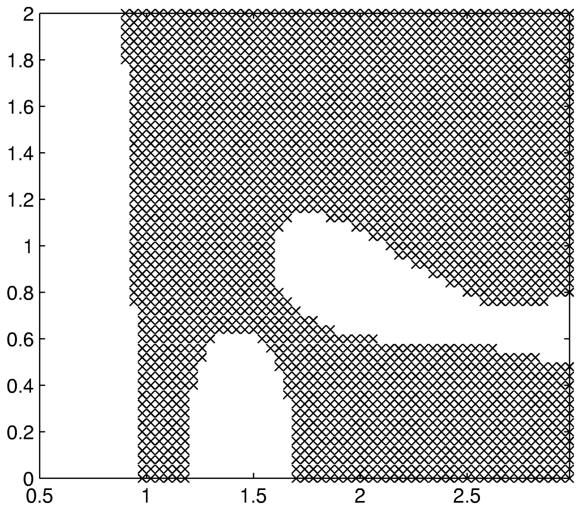

Figure 3: Parameter range of global asymptotic stability for the 1HP motor, $\eta = 5$.

et al., 1999b). We assume that the PI is always set so that the tuned system's transient response is over damped and is η times faster than the rotor time constant c_1, that is, the eigenvalues of the tuned system are assigned as $\lambda_1 = \lambda_2 = -\eta c_1$. Then the parameter η is used to represent the PI setting.

4.1 A small motor

We take data from a three phase induction motor, with 1 HP nominal power output and 220 V nominal line voltage. Let the parameters of the PI be chosen such that the transient response of the tuned system is over damped and dominated by a time constant which is half the rotor time constant, that is, $\eta = 0,5$. Then we apply the global stability test for κ and r^* varying in steps of 0.1. Figure 2 shows the region of the parameter space $\kappa \times r^*$ for which the test gives a positive answer. That is, the operating point is guaranteed to be globally asymptotically stable for all load and parameter mismatch in the dotted region.

Figures 3 and 4 show the parameter values with guaranteed global asymptotic stability for $\eta = 5$ and $\eta = 10$ respectively.

It is clear that for moderate values of η, which means that the PI settings are such that the closed-loop system is not too fast, the region of global stability encompasses a large range of mismatches in the rotor time constant for all loading conditions. As one tries to make the system faster by re-tuning the PI speed controller, this re-

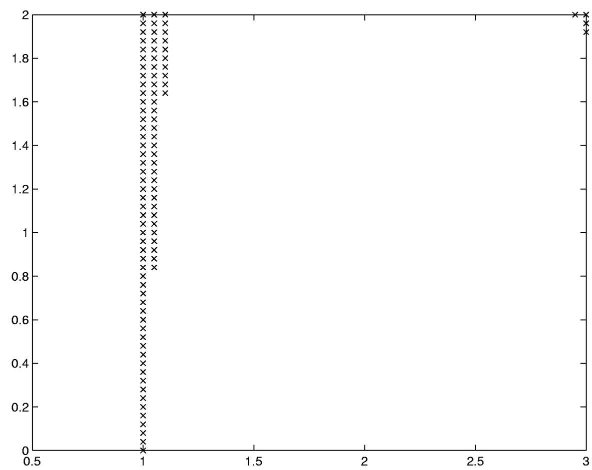

Figure 4: Parameter range of global asymptotic stability for the 1HP motor, $\eta = 10$.

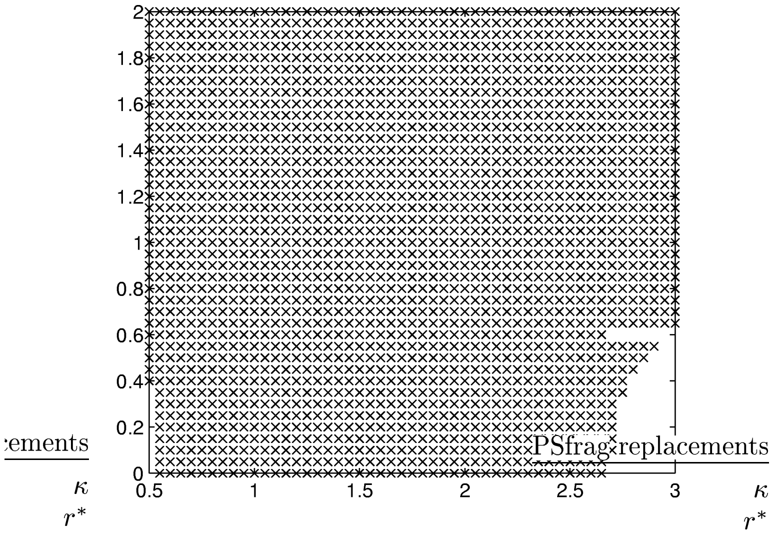

Figure 5: Parameter range of global asymptotic stability for the 500HP motor, $\eta = 0.5$.

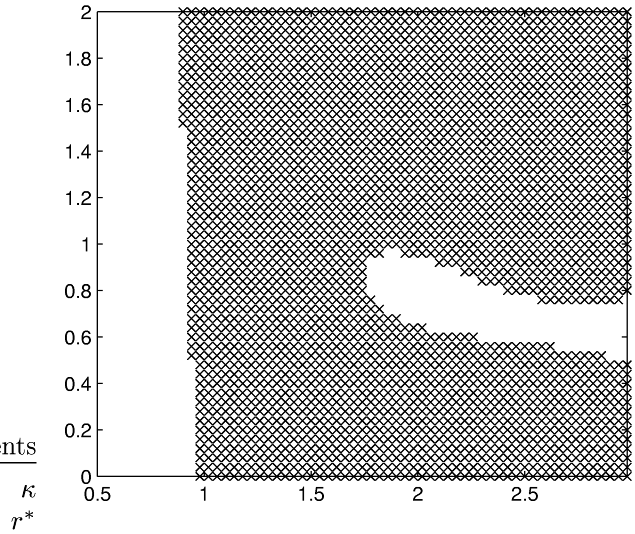

Figure 6: Parameter range of global asymptotic stability for the 500HP motor, $\eta = 5$.

gion shrinks. For values of η larger than ten (that is, closed-loop performance faster than ten times the rotor time constant) global asymptotic stability is guaranteed only if the estimate of the rotor time constant is very good ($\kappa \approx 1$).

4.2 A large motor

We take data from a three phase induction motor, with 500 HP nominal power output and 380 V nominal line voltage. We apply the global stability test for different PI settings defined in the same way as in the $1HP$ motor, again with κ and r^* varying in steps of 0.1. Figures 5, 6 and 7 show the parameter values with guaranteed global asymptotic stability for $\eta = 0.5$, $\eta = 5$ and $\eta = 10$ respectively.

The results for this large motor are more favorable than for the small motor in the previous example, as the normalized range of parameters for which global asymptotic stability is guaranteed are considerably larger. Yet, these ranges still get smaller as one tries to make the system faster. For $\eta = 5$ only overestimates can be tolerated, as the range of κ in the figure starts very close to unity.

5 CONCLUDING REMARKS

We have provided a test for robust global asymptotic stability which can be easily implemented provided the physical parameters of the motor are known. This test provides allowable margins of errors in the rotor time constant for any given IFOC drive.

Rules of thumb for tuning the PI speed loop can also be derived from these results. If global asymptotic stability is required (as in drives subject to large disturbances and/or large set-point variations) then the speed loop should not be made too fast, as the robustness margins would become too small. These results and rules reinforce the results on local asymptotic stability provided in previous works (Bazanella et al., 1999b; Bazanella and Reginatto, 1998; Espinosa-Perez et al., 1998; De Wit et al., 1996).

A DATA FOR THE 1 HP MOTOR

c_1	13.7 s^{-1}	c_2	1.56 Ω
c_3	0.59 s^{-1}	c_4	1.18 $kg^{-1}.m^{-2}$
c_5	2.86	u_2^0	4 A

B DATA FOR THE 500 HP MOTOR

c_1	1.28 s^{-1}	c_2	0.183 Ω
c_3	0.0904 s^{-1}	c_4	0.181 $kg^{-1}.m^{-2}$
c_5	2.93	u_2^0	70 A

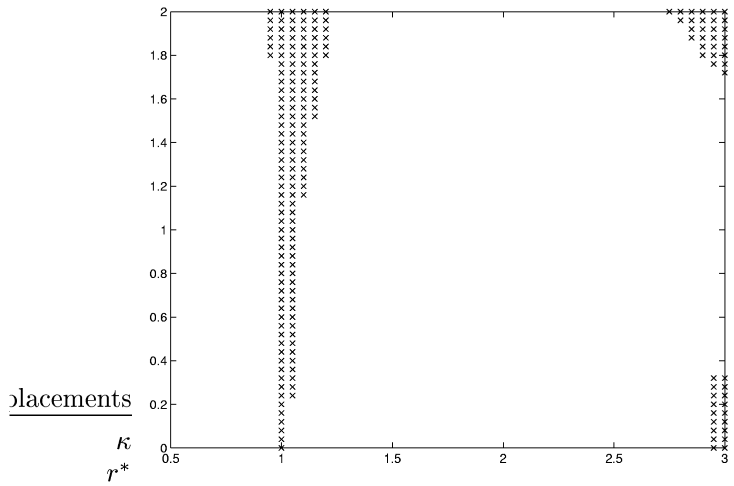

Figure 7: Parameter range of global asymptotic stability for the 500HP motor, $\eta = 10$.

REFERENCES

Bazanella, A. and Reginatto, R. (1998). Robustness margins for indirect field-oriented control of induction motors, *37th CDC*, Tampa, Florida, pp. 1001–1006.

Bazanella, A. and Reginatto, R. (2000). Robustness margins for indirect field-oriented control of induction motors, *IEEE Trans. Aut. Cont.* p. to appear.

Bazanella, A., Reginatto, R. and Valiati, R. (1999a). Instability mechanisms in indirect field-oriented control of induction motors: Mathematical analysis, *Brazilian Power Electronic Conference*, Foz do Iguaçú, BR.

Bazanella, A., Reginatto, R. and Valiati, R. (1999b). On Hopf bifurcations in indirect field oriented control of induction motors: Designing a robust PI controller, *38th Conference on Decision and Control*, Phoenix-AZ, USA.

Bose, B. (1987). *Power Electronics and AC Drives*, Prentice-Hall, Englewood Cliffs, New Jersey.

De Wit, P., Ortega, R. and Mareels, I. (1996). Indirect field-oriented control of induction motors is robustly globally stable, *Automatica* **32**(10): 1393–1402.

Espinosa-Perez, G., Chang, G., Ortega, R. and Mendes, E. (1998). On field-oriented control of induction motors: Tuning of the PI gains for performance enhancement, *Conference on Decision and Control*, Tampa, Florida, pp. WM15–2.

Krause, P. (1986). *Analysis of Electric Machinery*, Series in Electrical Engineering, McGraw-Hill, Usa.

Krishnan, R. and Doran, F. C. (1987). Study of parameter sensitivity in high-performance inverter-fed induction motor drive systems, *IEEE Trans. Ind. Applic.* **IA-23**(4): 623–635.

Leonhard, W. (1985). *Control of Electrical Drives*, Springer-Verlag, Berlin.

Marino, R., Peresada, S. and Valigi, P. (1993). Adaptive input-output linearizing control of induction motor, *IEEE Trans. Aut. Cont.* **38**(2): 208–221.

Novotny, D. and Lorenz, R. (eds) (1986). *Introduction to Field Orientation and High Performance AC Drives (2nd ed.)*, IEEE Industry Applications Society, IEEE, New York.

Ortega, R., Canudas, C. and Seleme, S. (1993). Nonlinear control of induction motors: Torque tracking with unknown load disturbance, *IEEE Trans. Aut. Cont.* **38**(11): 1675–1680.

Ortega, R., Nicklasson, P. and Péres, G. (1996). On speed control of induction motors, *Automatica* **32**(3): 455–460.

Reginatto, R. (1993). Induction motor modeling, in portuguese, *Technical Report RT 93-3*, Federal University of Santa Catarina, Florianópolis, Brazil.