ARTICLES

THE MATHEMATICS LABORATORY AS SPACE FOR TEACHER EDUCATION

ZAQUEU VIEIRA OLIVEIRA
LUZIA MAYA KIKUCHI
TRANSLATED BY LUZIA MAYA KIKUCHI

ABSTRACT
The mathematics laboratory is an essential learning space for students of basic education as well as for early teacher education. In addition to the materials and physical space provided, it constitutes a proper space capable of promoting the reflection of future teachers. In this exploratory study, conducted with students in Teaching Methods in Mathematics Education at Universidade de São Paulo, we verified the initial conception of the students about such lectures and how the mathematics laboratory influenced them in the process of critical maturation concerning their roles as teachers. From this study, we noted that the moments of production and reflection on the tasks, in addition to practices with their peers, were crucial in preparing them for their future profession.

TEACHER EDUCATION • TEACHING PRACTICE • MATHEMATICS • LABORATORIES

O LABORATÓRIO DE MATEMÁTICA COMO ESPAÇO DE FORMAÇÃO DE PROFESSORES

RESUMO
O laboratório de Matemática é tido como um importante espaço de aprendizagem tanto dos estudantes do ensino básico quanto na formação inicial de professores. Além dos materiais e da área física que fornece, esse espaço constitui-se como um lugar capaz de suscitar a reflexão dos futuros docentes. Neste estudo exploratório, feito com os alunos da disciplina Metodologia de Ensino de Matemática da Universidade de São Paulo, verificamos a concepção inicial dos estudantes sobre tal disciplina e como o laboratório de Matemática os influenciou no processo de amadurecimento crítico em relação à sua atuação como professor. Por meio deste estudo, notamos que os momentos de produção e reflexão sobre as atividades, além da prática junto aos seus pares, foram cruciais para prepará-los para a sua futura profissão.

FORMAÇÃO DE PROFESSORES • PRÁTICA DE ENSINO • MATEMÁTICA • LABORATÓRIOS
Le laboratoire de mathématiques est considéré comme un espace d’apprentissage important tant pour les étudiants de l’enseignement fondamental que pour la formation initiale des enseignants. Outre les matériaux et l’espace physique qu’il procure, cet espace est propice à susciter la réflexion chez les futurs enseignants. Dans cette étude exploratoire, effectuée auprès des élèves du cours de Méthodologie de l’Enseignement des Mathématiques de l’Universidade de São Paulo, nous avons observé la conception initiale que les étudiants ont de cette discipline et comment le laboratoire de Mathématiques les a influencés dans leur processus de maturation critique à l’égard de leur rôle en tant qu’enseignant. Grâce à cette étude, nous avons pu constater que les moments de production et de réflexion sur les activités, ainsi que la pratique entre pairs, ont été indispensables pour les préparer à leur futur métier.

FORMATION DES ENSEIGNANTS • PRATIQUE D’ENSEIGNEMENT • MATHÉMATIQUE • LABORATOIRE

El laboratorio de Matemáticas es considerado como un importante espacio de aprendizaje, tanto para los alumnos de educación básica como en la formación inicial de los profesores. Además de los materiales y del área física que ofrece, este espacio constituye un espacio capaz de suscitar la reflexión de los futuros docentes. En este estudio exploratorio, realizado con los alumnos de la asignatura Metodología de Enseñanza de Matemáticas de la Universidade de São Paulo, verificamos la concepción inicial de los estudiantes sobre tal disciplina y como el laboratorio de Matemáticas los influenció en el proceso de madurez crítica em relación a su actuación como profesores. Por medio de este estudio, nos damos cuenta que los momentos de producción y reflexión sobre las actividades, además de la práctica junto a sus pares, fueron cruciales para prepararlos para su futura profesión.

FORMACIÓN DE PROFESORES • PRÁCTICA DE ENSEÑANZA • MATEMÁTICA • LABORATORIOS
For a long time, we have noted that there is a breach in the transition from the first to the final years of basic education. In the first five years, children have a teacher who is responsible for teaching all core subjects, whereas in the last years, there are teachers specializing in each subject. Part of this process concerns the previous educational system, which divided schools into primary and secondary education. Moreover, at many Brazilian universities, teacher training programs, such as early years teacher education (called the Pedagogy Teacher Education Program) and middle school teacher education specializing in mathematics (called Mathematics Teacher Education Program) focus on different goals.

On the one hand, there is a Pedagogy Teacher Education Program, the number of pedagogical courses of which is higher than that of specialized subjects, involving widespread classroom practices of all core subjects. There is no focus on the mathematics content itself.

On the other hand, there is a Mathematics Teacher Education Program with fewer pedagogical courses but with a more in-depth training program on mathematical contents. In many universities, some courses in Mathematics Teacher Education Program lectures occur in two departments: mathematics and education. As a result, the connection between these two areas becomes weak and incipient because of a lack of discussion between professors in both departments and the lack of links between theory and practice.
established in fewer courses, usually limited to Teaching Practices or Teaching Methods courses.

Although most of the professors who lectured in these courses in the Department of Education have a Mathematics Teacher Education Degree, there are some differences in conceptions of teaching for mathematicians – a career in which is connected to mathematics studies – to mathematics educators – whose studies focus on teaching and learning processes or other associated subjects. Those different conceptions interfere directly in the quality of the training of future mathematics teachers.

Teaching Practices or Teaching Methods courses occupy an essential place in Mathematics Teacher Education Program because they are located at a dual confluence of subjects: the pedagogical and specific mathematical contents. Courses of this nature provide an opportunity for students to discuss and relate their future practices in the classroom, for instance, to the theory of mathematics and education.

Valente (2014) showed that the contents of Teaching Practice courses are outdated. Inherited by the professors who lecture in these courses, such contents mostly focus on teaching tools or ready-made techniques to teach specific content, as if the teaching practice in the classroom resembled following a recipe book.

However, the institutionalization and dissemination of mathematics education in Brazil have propitiated broader training for professors, allowing them to seek new ways of practicing in recent studies and to stop following an inconvenient and questionable pedagogical practice. Scientific papers on some experiences and reports in teacher education courses and the interchange of ideas during events in the areas of education and mathematics education have played relevant roles in these new professional practices (VALENTE, 2014).

In this paper, we engage in an essential discussion of Teaching Practices or Teaching Methods in teacher education in mathematics: the role that the mathematics laboratory can play in the future practices of these professionals. This study shows an experience that occurred in 2016 in the Mathematics Teacher Education Program of the Institute of Mathematics and Statistics (IME) at Universidade de São Paulo (USP), São Paulo campus, where the Mathematics Laboratory (LabMat) at Faculdade de Educação (FE – School of Education), USP plays a fundamental role in lecturing for the courses mentioned above.

We begin the paper by bringing some theoretical considerations concerning the relevance of the mathematics laboratory for the training of mathematics and then describe the experience that occurred at USP. We present a brief description of the contents of the courses and the tasks developed. In the following section, we discuss the methodology
applied in this study, present and analyze the data and finally offer some considerations about the results achieved.

THE ROLE OF THE MATHEMATICS LABORATORY IN TEACHER EDUCATION

THE IMPORTANCE OF PRACTICE IN TEACHER EDUCATION

In many professions, practice differs little from planning, but this is not the case for teaching, in which the plan can completely change when put into practice. This change occurs due to the creativity of students, who ask questions and express thoughts that could totally transform and change the class the teacher had planned. Nevertheless, planning is essential to achieving the goals of the class, even when not all the steps are followed as planned earlier (LORENZATO, 2009).

Thus, regarding early teacher education, we consider as relevant the courses and environments where students can create tasks and develop activities, produce teaching materials and discuss with their colleagues the possible settings in which they can apply them, as well as the potentialities and difficulties that might be encountered in the classroom.

Simulated practices and mandatory internships are crucial since all of the arsenal of tools created must be tested and implemented. Future teachers are required to plan, practice and evaluate, during their pre-service teacher education, to resort to all the theory available and to critically reflect on their future teaching practice.

CREATIVITY IN THE EDUCATIONAL ENVIRONMENT

The word creativity has been used quite frequently, and the educational environment is no exception. But what defines creativity? According to Csikzentmihaly (1996, p. 23), creativity is a process that occurs not only inside a person’s mind but also through the interaction between his or her thoughts and the sociocultural context. In other words, for a solution to be considered creative within a system or a model, other persons involved in the matter can validate such solution. This means that it is vital to exchange experiences and ideas with one’s peers. Creativity is a systemic process in which the social, historical and cultural context exert influence.

The same process should occur in the educational environment with teachers and, consequently, with their students (e.g., ALENCAR, 1986; FLEITH; ALENCAR, 2005; MARTINS, 2004; NAKANO; WECHSLER, 2007). Therefore,
According to Oliveira and Alencar (2007, p. 224), in higher education, there is “inefficiency in promoting creativity, no encouragement of creative and independence thinking, and emphasis on memorization and the reproduction of knowledge”.

Although these excerpts concern the education and practices of language teachers, they are not different from the education of mathematics teachers. Furthermore, mathematics education has long appreciated creativity, as we can see in texts about problem solving (BROLEZZI, 2013; RUNCO, 1994), mathematical modeling (ARAÚJO, J., 2002; BORBA; MENEGHETTI; HERMINI, 1999), and other studies about the relevance of the use of technologies in the classroom (FERREIRA, 2016; MACHADO, 1999). However, such studies emphasize the creative learning process of the students but not the teacher education that promotes creative classes for students. But how can teachers help promote creative environments if they did not experience one during their education?

In the Mathematics Teacher Education Program, the courses on Teaching Practices or Teaching Methods can provide such creative environment, so that future mathematics teachers can develop their own tasks as a simulation test of their future professional practice, and discuss and share thoughts to reflect on and amend their future practice. In courses of this nature, theory and practice facilitate the promotion of creativity.

THE MATHEMATICS LABORATORY AS A SPACE FOR TEACHER EDUCATION

Studies about the mathematics laboratory have emphasized its importance for the learning of mathematics in basic education. Maschietto and Trouche (2010) quoted a relevant speech by the French mathematician Félix Édouard Justin Émile Borel (1871-1956) during a conference in Paris at the beginning of the last century:

To lead, not only the pupils, but also the teachers, and moreover the society to a more exact notion of what are mathematics and
their actual role in modern life, it will be necessary to do more and to create real mathematics laboratories. I think that this question is very important and should be studied very seriously (BOREL, 1904, apud MASCHIETTO; TROUCHE, 2010, p. 39)

However, what precisely is a mathematics laboratory? Ewbank (1971, p. 559) offered two different conceptions:

The phrase is used to mean a place, a process, a procedure. As a place, it is a room set aside for mathematical experiments and practical activities. [...] This latter use of the term as a process and a procedure is far more critical because not every school could have a mathematics laboratory but every school or individual teacher could use this method of teaching.

Using the first meaning of the term – place –, the mathematics laboratory is a space where there are mathematics learning tools, for example, structured materials (abacuses, pattern blocks, geoboards, Cuisenaire rods, or Montessori materials), mathematics games (mathematical dominos, Tower of Hanoi, tangrams, or pentominoes) and commercial games with the possibility of generating debates and discussions around mathematical contents (Battleship or Mastermind), in addition to other educational materials, such as textbooks, alternative textbooks and movies.

Still regarding the possibilities of the mathematics laboratory and its materials, the advent and spread of digital technology (i.e., computers and smartphones) have resulted in many studies and literature on the relevance of including these technologies in the classroom. However, we emphasize that it is essential to distinguish between the real and potential uses of digital technologies, and that the future teacher can only note such nuance during experiments and reflections on their use (MASCHIETTO; TROUCHE, 2010). This understanding of the use of digital technologies can be amplified to other resources in educational practices since the potential use of some tasks cannot always be achieved in real applications. However, by creating the habit of planning beforehand, future teachers may learn to visualize many aspects that might interfere during the application moment, allowing them to adopt potentially better approaches to tasks.

Nevertheless, the teaching material available in the mathematics laboratory itself does not change the difficulties faced over the years to learn mathematics. It is at this time that a direct intervention of the teacher is required. In this way, Lorenzato (2009) stated:
The action of the teacher is decisive for student success or failure. It is not enough to have a LEM to make students learn in a meaningful way. More important than having a LEM at school is the fact that teachers know how to use the teaching materials correctly since [...] they require specific knowledge from the people who use them (LORENZATO, 2009, p. 23-24, free translation).

Thus, during pre-service teacher education, teacher candidates need to have access to places of creation and practice so that they can experiment and learn how to use teaching materials in the mathematics laboratory.

However, Ewbank (1971) stated that a mathematics laboratory goes beyond the place with such materials, and exceeds the space where students create, experiment and practice. Considered as “a process, a procedure”, the laboratory might mean not a place but an environment of reflection and creativity. In this respect, Bittar and Freitas (2005) stated:

Our conception of the Mathematics Education Laboratory goes beyond the display of teaching material collection that is available to be contemplated. It must be a dynamic space that promotes the exchange of ideas and pedagogical practices in mathematics. To this end, it is fundamental that the intellectual engagement of teachers and students on experimental tasks be developed (BITTAR; FREITAS, 2005, p. 231, free translation).

Ewbank (1971) emphasized that it is possible to transform a classroom, even the simplest and most precarious one, into a mathematics laboratory. Nonetheless, we cannot neglect the complexity of this task, especially when future teachers are trained in courses in which they have no opportunities to create, innovate and dialogue with their peers.

Going beyond the meanings presented by Ewbank (1971), Passos (2009, p. 90) stated that the mathematics laboratory “rather than being restricted to a place or process, must include attitude”. One of the purposes of the mathematics laboratory “is to make students think for themselves, to question, to see patterns – in other words, to develop an attitude of mathematics inquiry” (PASSOS, 2009, p. 90-91). Perhaps this is the main contribution that we hope to gain from the mathematics laboratory. However, we should be aware that it is not easy to attain this goal. First, it is necessary to work on the creativity and criticism of teacher candidates to develop attitudes that help them take, to the mathematics classroom, tasks with the potential to develop among students what the author called and “attitude of mathematics inquiry”.

This is the acronym for Laboratório de Ensino de Matemática, which means Laboratory of Mathematics Education.
Regarding the variety of laboratories according to extensive literature, Rodrigues (2011) presented the following classification:

- **Archive depot:** a space of storing materials that the teacher can access for use in the classroom. The laboratory should be a specific place with a function similar to that of a library.

- **Classroom:** a view similar to that of place described by Ewbank (1971); in other words, any classroom could and must be a laboratory since the teacher can challenge students with questions and discussions to foster reflections focused on the learning of mathematics.

- **Course:** A curriculum component of many courses in the Mathematics Teacher Education Program with a theoretical-practical approach, which means that it is not only a course to show applications of some tasks, teaching skills and techniques for the future teachers but also to discuss studies of current topics in mathematics education.

- **Laboratory of Technology:** Beyond the computer laboratory as a place, the author mentions the laboratory’s potential as a space of collaborative learning and shared knowledge. The mathematics laboratory is also mentioned for distance education.

- **Traditional laboratory or mathematics laboratory:** A place to perform experiments with teaching materials, focused on teaching procedures, in a similar sense to that of a science laboratory for teaching “scientific methods”.

- **Laboratory of Mathematics Teaching or Shared Classroom:** A place to perform tasks focused on living processes, reflections and questions, as well as to encourage attitudes to help students in the construction of mathematical knowledge. It unites the physical space of the traditional laboratory and the classroom, turning it into one place whether for basic education or for teacher education in mathematics.

- **Laboratory of Mathematics Education or Education Agent:** A conception that encompasses the laboratory of mathematics teaching but differs from it because it serves as a space for teaching, research and extension activities for the pre-service and continuing mathematics teacher education. In this way, this laboratory not only provides teacher education, but also includes the future teachers in the academic research environment.

Lorenzato (2009) presented a conception of laboratory that suits the creation of didactic pedagogical situations as well. In other words, beyond being used by students, the laboratory is a place for teacher
planning: “it is a space to encourage both students and teachers to inquire, conjecture, search, experiment, analyze, and conclude, that is, to learn and mainly to learn how to learn (LORENZATO, 2009, p. 7).

Passos, Gama and Coelho (2007) showed that readings and tasks that encourage reflection lead teacher candidates to change their conceptions over time. The simplistic expectation of a laboratory as a tool – a place that provides didactical materials and will be sufficient to overcome learning difficulties – is replaced by the one of a space for reflection on important theoretical and pedagogical aspects of the use of didactical material available in the laboratory (PASSOS, 2009; PASSOS; GAMA; COELHO, 2007).

Turrioni (2004) presented the potentialities of the laboratory as a space for reflection on pre-service teacher education because the laboratory could be a place for professional development, allowing teachers to look into their practices with the purpose of improving them, as well as to search for creative solutions to be developed in the classroom.

The laboratory can be a space for interaction between the university and the school, as well as a place for teacher education in mathematics. In this respect, the mathematics laboratory is a learning space beyond the walls of the university, offering tasks for basic education students and opportunities for the teacher candidates to accomplish their mandatory internships, supervised in the university itself (LOPES; ARAÚJO, J., 2007, CEDRO; MOURA, 2007).

Digital technologies also play a significant role in this environment, as indicated by Miskulín (2009). For her, “the relationship with technology can enhance teachers’ capacity of reflection about their thinking processes”. Digital technologies require future teachers to include themselves in a new professional culture that requires the search for new strategies that promote the mathematics teaching (MISKULIN, 2009, p. 159). From her point of view, the mathematics laboratory “is considered an interactive setting for collaborative learning and shared knowledge” (MISKULIN, 2009, p. 163).

Although there has been an emphasis on the use of the laboratory for years, Rodrigues (2011) stated that the conception of the mathematics laboratory as a place to perform experiments with didactical materials focusing on teaching procedures is the one that prevails among students.

However, none of these objections is new, and they have been discussed for a long time, as shown by the study of Oliveira, A. (1983). In the face of the shortcomings highlighted by students themselves in the Mathematics Teacher Education Program, she studied the influence of the mathematics laboratory as a teaching space. Oliveira, A. (1983) showed that there is potential in the laboratory to improve the learning
of mathematics in basic and higher education, as well to provide space for early and continuing education through “Permanent Education”, and provide integration of the university with the community around it. In this way, we can see that, in the laboratory, the three bases – teaching, research and extension – can occur in an integrated approach.

We consider essential the triple concept of the mathematics laboratory: place, process and attitude (PASSOS, 2009). Undoubtedly, the most important aspect of the quoted excerpts (BITTAR; FREITAS, 2005; EWBANK, 1971; LORENZATO, 2009; PASSOS, 2009) is promoting the laboratory as a process, a place for experimentation by mathematics teacher candidates, a space for creation and practice, which allows for failure and reworking, for creativity and reflection, and for developing attitudes, criticism, ideas that could be reflected on and replicated later in the mathematics classrooms of basic education.

THE MATHEMATICS LABORATORY AT FEUSP AND TEACHING METHODS COURSES

In past decades, the LabMat at FEUSP has been used to provide varied tasks/activities, such as: undergraduate courses– especially in the courses of the Pedagogy Teacher Education Program and Mathematics Teacher Education Program – and for graduate courses; interaction with students, parents and teachers in the Experimental School (EA)9 at USP; continuing education for teachers through extension projects (FRANZONI; PANESSIAN, 1999; CEDRO; MOURA, 2007); and research group meetings, according to the concept of the laboratory of mathematics education introduced by Rodrigues (2011).

Among the undergraduate courses usually provided by LabMat at FEUSP, there are: Teaching Methods in Mathematics Education I and II for the last year of the Mathematics Teacher Education Program at IME. These courses, as presented on the contents roll/ementa do curso, have the following goals:

- a critical reflection about the conceptions of mathematics shared by teacher candidates, as well as of the influence of these conceptions on pedagogical practice;
- the articulation of the topics addressed in pedagogical courses and mathematical content remaining from the contents roll of the Teacher Education Program
- the establishment of linkes between the content of many courses from the curriculum of the Teacher Education Programs and those that the teacher candidates will teach in basic education;
- raising awareness of the situation of mathematics education in Brazil and other countries, through contact with the curricula, programs and other didactical materials;

---

9 This is an acronym for Escola de Aplicação, which means Experimental School.
- the actual practice of mathematics education through supervised internships, simulated classes, guided teaching, as well as other tasks directly related to teaching action (UNIVERSIDADE DE SÃO PAULO – USP, 2010a; USP, 2010b, free translation).

We note the relevance given to the connection that should be established between the knowledge acquired in pedagogical courses and mathematical content, as well as of relationships between theory and practice, both through the mandatory internship – 60 hours in each course – and through the tasks that allow the simulated practice of teacher candidates themselves.

To achieve these goals successfully, during 2016, Teaching Methods I and II lectures occurred at LabMat at FEUSP. The contents of both courses consist of a set of theoretical lectures about the latest topics in education and mathematics education, and another group of classes called workshops, in which, beyond the theoretical aspects, the teacher candidates plan, execute and discuss activities. In these two courses, there was also time not only to prepare and discuss the lesson plans that could be applied during their internships, in their future teaching careers, but also for reports on the internship experience, and, at the end of each semester, a set of lessons was presented in the seminars.

Specifically regarding the Teaching Methods in Mathematics Education I, the theoretical lectures addressed the following topics: conceptions of mathematics; mathematics teaching and implications for teaching; aims and contents in mathematics curricula; psychological perspectives and mathematics teaching; and evaluation in mathematics education. The practical classes brought didactical-pedagogical aspects and workshops on geometry and measurements, algebra, the history of mathematics, teaching and educational technology and crossover projects.

The workshop on geometry and measurements, for instance, was organized around three modules of activities that students should develop in groups. The first addressed the use of geoboards in different levels of tasks, from simple ideas of perimeter and areas, for primary school, to the possibilities of connection with algebra, such as developing the Pythagorean theorem. The second module was composed of activities to be developed with recyclable packages, relating plane and spatial geometry, as well as material economy, in relation to capacity and shape of the packaging. The third module brought a more heterogeneous set of tasks, such as polyominoes, honeycombs and the optimization of shapes.

The lecture on the history of mathematics and teaching was taught by a guest professor, who led a workshop to produce slabs of clay
and to establish relations among teaching, history and mathematical knowledge in ancient Mesopotamia.

The lecture on educational technology and crossover projects – given by the student monitor of this course and offered in the Integrated Laboratory of Education and Technology (LIET)\textsuperscript{10} at FEUSP – consisted of a workshop in which students could address the potentialities of educational platforms, such as Khan Academy and Mangahigh.

In this course, students were still asked to do group research on mathematics contents (i.e., plane and solid geometry, trigonometry, functions, polynomials, statistics, etc.), and offered a historical approach to the content and ideas for the tasks. The works were handed in as a final paper and presented as a seminar at the end of the semester. All oral presentations were divided into two parts: one expository and the other as a practice in which members of the groups exposed one of their activities to have the other students perform a practical simulation of it. In the end, the group, in collaboration with the professor and the student monitor, mediated reflection on different mathematical and pedagogical aspects of each activity proposal.

Regarding Teaching Methods II/Metodologia II, theoretical lectures addressed the following topics: teaching plan and lesson plans; problem solving; mathematics; creativity and spaces of non-formal learning and special needs education; and mathematics education. The practical classes addressed didactical-pedagogical aspects and provided for the students’ workshops and tasks of measurements, probability and statistics and errors and learning difficulties in algebra.

In the errors and difficulties lecture, for example, in addition to discussion of a reading chosen by the professor, all of the teacher candidates brought a report about their experiences during the internship or about some moment in which they noted students had some learning difficulties in algebraic contents.

The Measurements lecture was given by a guest professor who presented a task/an activity, the theme of which encompassed all of the years of elementary, middle and high school to discuss how one can approach the same topic or adapt the same task/activity according to different levels of the students’ schooling.

In this course, students needed to perform three tasks throughout the semester. The first was to be done during their mandatory internship program, which involved teaching a class. The teaching task proposal should include the use of books, videos or films of scientific diffusion, or games and applications – materials with pedagogical potential explored in some classes of the two courses. Although not all of the students were in the internship at the same school or in the same level of education, they worked in groups to perform the tasks so that the topic or content was the same for all of them. This task was handed in
as a paper, and an oral presentation was given showing the lesson plan proposals and the reports of each teaching experience.

There were two versions of the second task: a written version and presentation as a seminar about lesson plans using educational technology. Last, for the third task, the student should visit a museum or other non-formal educational space and prepare a visit guide and lesson plan considering the mathematical content to work on in such places.

Questions were asked throughout the year regarding the dynamics of the classes, particularly how the students were gradually gaining autonomy to develop tasks and how discussions occurred in the classroom. These facts concerned the professor and the student monitor. This study emerged from this environment as presented here, and we intend to discuss some aspects considered relevant in the teacher education of novices. In particular, we shall show how the environment of the mathematics laboratory, besides providing materials, can be a space for reflection and creativity. To this end, we used the opinion of students themselves intending to comprehend their conceptions of teaching practices in mathematics education and the role that the mathematics laboratory and its practical tasks played in their perceptions as future teachers.

Before describing the exploratory study, it is important to mention that there is a course entitled Laboratory of Mathematics (USP, 2006) in the curriculum of the Mathematics Teacher Education Program. This course belongs to the first year of the program and its pedagogical aim is to “present problem situations that challenge and drive the autonomy and thinking of students; to discuss relevant topics of basic education, in order to provide an adequate conceptual basis” and “foster the comprehension of the nature of mathematical thought, mathematical language and mathematical performance”. Such goals are quite different from the goals of Teaching Methods in Mathematics Education I and II courses, which are the focus of this study.

THE EXPLORATORY STUDY

The discussions concerning the importance of the mathematics laboratory for teacher education draw on an exploratory study conducted with the students of two groups (day and night/evening period) of the Mathematics Teacher Education Program, from IME-USP in the course Teaching Methods in Mathematics Education II in the second semester of 2016 regarding the. However, we further detail that the collected data and our discussions are also related to the course Teaching Methods in Mathematics Education I, offered in the first semester of the same year, since it is related to the same group of students in both semesters.
To obtain the opinions of students, the exploratory study was conducted through an online survey (see Chart 1), using a tool called Survey Monkey,\(^\text{11}\) in the last weeks of the Teaching Methods in Mathematics Education II course.

To support and add further information from the online survey, we quoted some excerpts from some tasks and papers developed by the students, mostly in the course held in the second semester of 2016. We chose some samples of tasks for their uniqueness or for their emphasizing the relevance of contributions made through the reflections and exchanges of experiences with their peers during the course.

For the survey, we posed six written questions (see Chart 1) except for the first question. We chose this model to guarantee that all of the answers were faithful to their opinions and not susceptible to be driven by third parties or evaluators. For discussion purposes and due to size limits, we shall analyse only on questions 2, 3 and 4.

CHART 1
SURVEY ADMINISTERED TO THE STUDENTS IN THE TEACHING METHODS IN MATHEMATICS EDUCATION II COURSE

1. Select courses that you attended in 2016 with Prof. [Author].
   ( ) Teaching Methods in Mathematics Education I
   ( ) Teaching Methods in Mathematics Education II
2. What were your conceptions of teaching practice before attending Teaching Methods in Mathematics Education I and II courses?
3. What contents of both courses do you believe contributed the most to your education as a teacher in the future?
4. How have the following tasks (workshops, readings and discussions developed during the course) contributed to your mandatory internship?
5. What kind of difficulties did you find developing:
   a) workshops during classes;
   b) tasks using educational technology.
6. Should you have any critiques, suggestions or comments, please feel free to write them below. This will help improve this course.

Student participation was voluntary and anonymous. The profiles of students were very heterogeneous in terms of age, education and semester in the course; also, most of them were already working as teachers. For this reason, we did not consider possible differences in conceptions of mathematics and mathematics teaching between them, but we valued their expectation from this course.
As we mentioned before, the courses Teaching Methods in Mathematics Education I and II, if attended at the expected time, belong to the last year of the program. However, for many reasons that do not interfere with the results of this study, some students were not in their last year of the program.

It is important to state that there is a possibility of attending two courses in the same year or attending Methods I in one year and Methods II in the subsequent year. In this study, we did not consider these aspects and analyzed the answers from students who attended only Methods II, as well as from those who attended both courses in 2016 with the professor mentioned above.

Thirty out of the 55 enrolled students in both periods took part in the study, which corresponds to approximately 55% of adhesion. Only 4 of them did not attend Teaching Methods in Mathematics Education I in the first semester of 2016.

We are aware that, in a qualitative study, the analysis of answers may create much disagreement since there is no consensus or common methodology – and perhaps this is of great importance for the research environment because it allows analyzing a specific object from different views and perspectives. The qualitative approach has been more and more used as an alternative to the rigidity of positivism, especially in education and social science studies (ALVES; SILVA, 1992). D’Ambrósio (2004) corroborated this opinion, stating that qualitative research is a way to value the opinions and ideas of people, seeking to attribute meaning to discourses and narratives that would be evidenced in a quantitative study.

The use of semi-structured interviews or open-ended questions allows for a more accurate analysis of the opinion of the surveyed individual. Interpretations may stem from the researcher him/herself, according to Alves and Silva (1992), becomes the very tool of the work, based on his/her experience on the theme. However, as these authors reminded us, such methods can be questioned regarding its methodological rigor in data collection. Nevertheless, with a clear view of what one wishes to research about the subjects, with theoretical-methodological support, the researcher can be minimize this problem by means of data systematization and complementation texts (ALVES; SILVA, 1992).

For data treatment, we used the theoretical-methodological proposal of Organizing Models of Thinking (MORENO et al., 2000) since this methodology allows us to abstract a posteriori categories and meanings attributed to the open-ended answers of the surveyed. As we chose open-ended questions for the data collection, this methodology was the most suitable for the analysis of the results, enabling us to represent the categories of graphs 1, 2 and 3 by the students’ answers.
themselves. In educational psychology, many studies, such as Pátaro (2011) and Araújo (2013), have adopted this methodology to analyze qualitative data.

PRESENTING AND DISCUSSING THE RESULTS

We asked students about their conceptions of teaching practices before attending the course Teaching Methods in Mathematics Education I (question 2). Graph 1 shows their expectations.

**GRAPH 1**

**DISTRIBUTION OF THE ANSWERS GIVEN BY THE STUDENTS TO QUESTION 2**

<table>
<thead>
<tr>
<th>What were your conceptions of teaching practice before attending Teaching Methods in Mathematics Education I and II courses?</th>
<th>0%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
<th>40%</th>
<th>45%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expositive lecture supported by blackboard and chalk</td>
<td>10%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching rehearsal</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teaching through projects</td>
<td>3%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Being in contact with didactical methods</td>
<td>17%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Being in contact with teaching-learning</td>
<td>7%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not specified</td>
<td>43%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: The authors.

As we can see, a significant proportion of the students (43%), before attending the courses, did not have in mind any specific conception of what they considered teaching practice. In the answers there were statements such as “There were few” or “Before attending these courses, I believe that my conceptions were very shallow”, and some students stated that they had previous contact with teaching methods different from traditional one, but they did not specify which. We can thus suppose that some students did not know what they would learn in Teaching Methods in Mathematics Education I and II.

The second highest rate (20%) of students answered that they considered teaching practice to be the action of “rehearsing” teaching in the classroom. Although this aspect is considered relevant, we cannot summarize teaching practices to this meaning only. Moreover, one of the proposals of the course is to encourage the teacher candidates to be the protagonists in the classroom and not mere spectators, as it usually occurs in a standard didactic contract:
I wondered whether we would learn specific things about how to teach the content. As a sort of a step-by-step approach to how to teach the topic of functions, for example. I’m very pleased that I was wrong. (Student 20, free translation).

This change in perspective regarding the expectations of the course and what they learned could also be noted in their internship final papers:

As it is a course that sets a different goal from what we are used to, I stopped being a mere observer and became a protagonist in the classroom. The course challenges us to deal with the autonomy to teach classes and the professor is a tutor who supports us (Student Y, 12 free translation).

**GRAPH 2**
**DISTRIBUTION OF THE ANSWERS GIVEN BY THE STUDENTS TO QUESTION 3**

<table>
<thead>
<tr>
<th>Category</th>
<th>Students’ Distribution</th>
<th>Source: The authors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not specified</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>Educational theories</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Learning theories</td>
<td>8%</td>
<td></td>
</tr>
<tr>
<td>Problem solving</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>Teaching and pedagogical planning</td>
<td>17%</td>
<td></td>
</tr>
<tr>
<td>Ethnomathematics</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>Special Needs/Inclusive Education</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>Workshops and educational technology</td>
<td>42%</td>
<td></td>
</tr>
<tr>
<td>Students’ error analysis</td>
<td>6%</td>
<td></td>
</tr>
</tbody>
</table>

Graph 2 presents the aspects that students stated had contributed the most to their education as future teachers after attending both courses.

The most cited category was workshops and educational technology classes, with 42% of students. We combined these two topics into a single category because of the nature of the tasks developed during these classes. One of the aspects mentioned was the importance of motivating students to learn mathematics:

The variety of alternative methods, games, websites, videos and many other unusual ways of teaching will undoubtedly help me to show the beautiful side of mathematics to the students (Student 30, free translation).

The workshops called my attention so much because they’re different from the things seen in previous courses, and I noticed
that they could help me in the classroom. (Student 15, free translation).

Students also mentioned the importance of workshops for teachers to put themselves their pupils’ shoes, that is, to anticipate difficulties and essential prerequisites to develop tasks well. Such an attitude is a fundamental skill to be developed by good teachers, but it is only possible if future teachers are encouraged since early to reflect with their peers about materials, methods and lesson plans developed and applied in different contexts. Indeed, this skill is important because each classroom has a particular reality, but simulated practice, as we proposed in workshops, plays an important role in such reflection on questions that might arise during the application of the tasks in a real classroom.

For example, on their final paper, a couple of students mentioned that they had difficulty in finding material to perform tasks that they had planned previously to teach algebraic equations. However, they devised an alternative plan, which turned out to be more effective than expected:

During the planning, we thought about using a real scale to illustrate the concept of equity using objects with different weights. That option proved to be impracticable not only because the school did not have any material like that but also because finding and transporting the scale to the school would be difficult due to the distance, commuting and weight. Due to this challenge, we ended up developing an idea of geometrical shapes, cut out from pieces of paper, to work with equities, which proved to be a convenient choice with an educational reward greater than expected (Students F. and I., free translation).

The section above refers to an intervention that students would have to conduct during their mandatory internships. We noticed that the couple of students faced an ordinary situation in most public schools: lack of material. However, they decided to review the planning and prepared a new tool to apply in the classroom (cf. Picture 1). We consider decision-making and attitude extremely important for a more effective and fruitful performance of future teachers in the face of difficulties during the application of tasks.

The second topic considered relevant by the students (17%) concerns general guidance on how to develop pedagogical plans, lesson plans and didactic sequences. During the lectures, we emphasized the importance of the curriculum, the political and pedagogic project and other documents that orient teaching. Questions about teaching plans
were explored to enable future teachers to plan clear objectives for classes, as well as the content to teach, the methods used and different ways to evaluate the tasks proposed.

PICTURE 1
IMAGE OF THE MATERIAL TAILORED BY THE STUDENTS

Another crucial point was the difficulty in evaluating which material available is the most suitable for each teaching situation. A student mentioned that although there is a wide variety of available materials, not only on online resources but also in teaching centers, it is difficult to adapt the materials or even the lecture itself to address the content:

Even though we find many ready-made tasks on the Internet, it is challenging to select which of them to approach some content. Many times, the proposal of the ready-made tasks does not have the approach to the content that you want to teach, and frequently, they are not very flexible, which means that you have to change all your lecture content (Student 20, free translation).

We are aware that the processes of systematization, organization and planning of what one intends to execute, the choice of materials
and changes to them for a specific situation are not straightforward. Future teachers must comprehend planning and practice it during their early education. This is mentioned in the following reports:

The major difficulty was to change something for the students because they have much difficulty in mathematics (Student 28, free translation).
The major difficulty is to write what you thought in a systematic way (Student 13, free translation).

We noticed that many students understood that the preparation of a lesson plan requires not only knowledge of the content and creativity, but also assertive and concise planning, and in accordance with the objectives of the task. Many students were able to apply these proposals during their mandatory internships in the course and could experience the difficulty that might be faced in a classroom, as we can see through the following excerpts:

I believed before that explaining well was enough to reach a goal. However, after this course, I figured out that it depends much more on the student. We must analyze errors, work on pupils’ wishes, work to catch the student’s attention and not only to explain well (Student 30, free translation).
I imagined that teaching was only a matter of knowledge transmission, in which I must fulfill a goal set beforehand (Student 15, free translation).

Consequently, through those reports, it is possible to realize the relevance of approaching such content, for instance, workshops and guidance, to prepare didactical plans, for the education of future mathematics teachers. It is also vital to encourage teacher candidates to know and access the centers for further training in teaching and mathematics laboratories, when they are available in the university. Not infrequently students do not benefit from these spaces, even after graduation, because of lack of time or even not knowing they exist. We encourage the diffusion of the activities developed in such spaces through social media and websites, as well as offering them in distance education.
Finally, Graph 3 shows that 56% of the students considered that the tasks performed during both courses contributed to their reflection on how to act in the classroom. The reports of the students varied, but we noticed that not only the tasks but also the mandatory readings were outstanding in their education. Additionally, the contact with diverse theories and research on education and mathematics education was enlightening for the students. Some examples of their reports follow:

Readings give the theoretical basis for teaching plans. Tasks and workshops always inspire us or give a clue when we cannot think beyond the traditional lecture. And the discussions helped me to see new points of view. (Student 25, free translation).

Theoretical aspects were mentioned as an important factor for lesson planning. These words are significant because, as we showed before, one of the aims of the course is to lead students to realize the importance of critical reflection about theoretical aspects in mathematics and how they can interfere with pedagogical practices:

I believe that, in my case, as I attended both courses, I benefitted from the workshops and tasks of the first semester and applied them during my internship in the second semester. Thinking broadly, knowing different tasks increases the ways in which teachers can work on the same content, which is very useful since the same task will for sure not work for all students. (Student 26, free translation).

The way in which the same task can vary, according to the profile of a class, was considered an essential factor. In this way, a
student mentioned the importance of knowing a range of approaches to render his or her action more effective:

They contributed to broaden our knowledge concerning basic schooling in general (mathematics topics, interaction with students). Also, during our internships, we are tested in real teaching situations. (Student 10, free translation).

I believe that both courses contributed to improve my views as a teacher about my old practices in the classroom, which were based on assumptions opposed to the good development of classes, such as being too strict or not considering the opinions of students when constructing classes. Methods courses guaranteed the basis of many mathematics topics and helped me create a different view and assign students a more active role and watch them beyond the content of the course. (Student 17, free translation).

One of the attitudes that the course intended to change among the students, as future teachers, was the fact that many of them did not consider the opinions of their students or did not give voice to them when they planned classes. We believe that this is an extremely relevant point since the knowledge the teacher expects his or her students to construct should be worked on together with what the student expects, which means there must be communication and interaction between the characters involved in a classroom. Regarding the last report, we can see precisely the appreciation of this active attitude of the student and the opportunities that the teacher can provide his or her students with to foster more effective learning in mathematics classes.

**FINAL REMARKS**

We believe that the mathematics laboratory can play a relevant role in the education of mathematics teachers, first because it can provide teaching materials and space for practical tasks, seeking opportunities to lead students to reflect on their prior ideas, to foster their creativity (OLIVEIRA; ALENCAR, 2007) in order to achieve their goals of a class. Our study shows that the mathematics laboratory has potential to generate reflections by teacher candidates, leading them to rethink their conceptions of teaching practices (Graph 1) and encouraging them to comprehend the importance of creativity, which they evidenced by valuing the development of workshops and educational technology (Graph 2).

It is important to emphasize that, to exercise creativity, it is crucial to keep our minds free from worries that could interfere with...
finding new solutions (CSIKZENTMIHALY, 1996). For this reason, the mathematics laboratory becomes the primary place to exercise creativity for teacher candidates because there they can test, validate with their peers and find new strategies to teach specific content without being concerned about the results, schedules and evaluations, which are part of everyday work of the teachers who are already in the classroom. Consequently, if the creativity of teacher candidates is not exercised before going into the classroom, they will hardly have the opportunity to develop it when they are already working as teachers.

Furthermore, the mathematics laboratory offers a place to reflect. By discussing and reflecting on essential issues inherent in their future work as teachers in the classroom, students put themselves in an active position in the face of the necessities of a classroom, as we showed when we discussed the results presented in Graph 3. When we mentioned, for example, the lack of material and to the fact that a couple of students reviewed their planning, we are promoting, in the mathematics laboratory, the education of reflective, critic and active teachers, and corroborating the theoretical perspective we adopted (EWBANK, 1971; PASSOS, 2009; PASSOS; GAMA; COELHO, 2007).

Through students’ perceptions of the aforementioned aspects as relevant in terms of the theoretical basis, we infer that mathematics teacher education must offer opportunities for creating, testing and reflecting with their colleagues to develop over time the abilities that could prepare them to face classroom adversities.

Using the mathematics laboratory for teacher education, we reached five points considered extremely relevant, which our teacher candidates must recognize during their education: (i) to know different didactical materials recognizing their potentialities and limitations, but understanding above all that their mediation as teachers is crucial to the efficiency of such materials in the classroom; (ii) to learn the importance of creativity in the processes of planning, creation and development of teaching activities to meet the needs of their students; (iii) to know the importance of working in partnership with other teachers – in mathematics or other areas – insofar as the teaching and learning process occurs through interaction and sharing of knowledge with others; (iv) to comprehend the importance of planning for teaching and to learn its limitations, since the classroom environment involves the interaction and creativity of the students; and (v) last but not least, to recognize the complexity involved in the teaching and learning processes of mathematics, knowing how to act critically in the face of adversities encountered in mathematics classes.

Other aspects should be considered, but we believe these five mentioned above can consistently help future mathematics teaching practices in basic education given the current reality of mathematics
education in Brazil. By making teaching practice classes into a real mathematics laboratory, we are contributing not only to the training of future teachers but also to the quality of the learning of our students in basic education.

REFERENCES


