# A Precise Algorithm for Computing Sun Position on a Satellite 

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#### Abstract

To meet the high precision sun tracking needs of a space deployable membrane solar concentrator and other equipment, an existing algorithm for accurately computing the sun position is improved. Firstly, compared with other theories, the VSOP (variation seculaires des orbits planetaires) 87 theory is selected and adopted to obtain the sun position in the second equatorial coordinate system. Comparing the results with data of the astronomical almanac from 2015 , it is found that the deviation of the apparent right ascension does not exceed 0.17 arc seconds, while that of the apparent declination does not exceed 1.2 arc seconds. Then, to eliminate the difference in the direction of the sun position with respect to the satellite caused by the size of the satellite's orbit, a translation transform is introduced in the proposed algorithm. Finally, the proposed algorithm is applied to the orbit of the satellite designated by SJ-4 (shijian-4). Under the condition that both of the existing and improved algorithms adopt the VSOP87 theory to compute sun position in the second equatorial coordinate system, the maximum deviation of the azimuth angle on the $\mathrm{SJ}-4$ is 35.19 arc seconds and the one of pitch angle is 19.93 arc seconds, when the deviation is computed by subtracting the results given by both algorithms. In summary, the proposed algorithm is more accurate than the existing one.


KEYWORDS: Space solar concentrator, Sun tracking, VSOP87, Translation transform, SJ-4 satellite.

## INTRODUCTION

Space solar power is the main energy source for a satellite, which is provided by solar arrays at present (Ma 2001; Osipov et al. 2017). The amount of energy provided per unit area of a solar array depends on the angle between the normal direction of the solar array and the sun vector (Lin 2010). Therefore, the higher the tracking accuracy of the solar array with respect to the sun, the greater the power available to the satellite. Meanwhile, as the related technology of space deployable antenna is becoming more and more mature (Miura and Miyazaki 1990; Guest and Pellegrino 1996; Huang 2001; He and Zheng 2018), a space deployable convergent membrane solar concentrator, whose structure and principle of operation are schematically depicted in Fig. 1, is expected to be used as an energy supply equipment for satellites in the future (Zhang et al. 2009; 2016). Compared with a flat solar array, a space deployable convergent membrane solar concentrator concentrates the sunlight received by the reflector onto a small plane of focus, requiring greater solar tracking accuracy. From another perspective, the improvement of the accuracy of sun tracking will reduce the weight of the corresponding power supply equipment and the cost for transportation will be cut down accordingly (Duan 2017). In summary, solar arrays, space deployable convergent membrane solar concentrators and other optical energy supply equipment demand greater sun tracking accuracy, which essentially depends on the accuracy of the algorithms used for computing sun position on the satellite.

[^0]At present, the algorithm for computing the sun position with respect to a satellite is divided in four steps: firstly, the current time and the orbital elements of the satellite are obtained; secondly, the sun position in the second equatorial coordinate system is computed; thirdly, the computed sun position is transformed into the orbit coordinate system by a rotation transformation; finally, the sun position in the coordinate system of the satellite body is obtained. In the process of transforming the sun position from the second equatorial coordinate system to the orbit coordinate system, the existing algorithm ignores the error in direction caused by the different position of the satellite in the orbit and its distance from the Earth. That is to say, the line from the satellite to the Sun and the one from the Earth to the Sun are considered to be ideally parallel (Zhai et al. 2009; Lin 2010; Zhang et al. 2011; Zhu 2011).

In the above algorithm, the computation accuracy of the second step plays a major role on the whole accuracy. For this step, several high precision methods have been proposed in solar engineering, with both numerical (Pitjeva 2005; Hilton and Hohenkerk 2010) and analytical methods (Simon et al. 2013). Due to their dependence on mass historical observations, numerical methods may include large amounts of data and exhibit instability. In comparison, analytical methods are simpler and more reliable. From the aspect of tracking control, analytical methods are more suitable. In analytical methods, VSOP theories are the most popular. In 1982, Bretagnon published his first planetary theory VSOP (variation seculaires des orbits planetaires) 82, which consists of long series of periodic terms for each of the major planets, from Mercury to Neptune (Bretagnon 1982). The inconvenience of the VSOP82 solution is that one does not know where the different series should be truncated when no full accuracy is required. Fortunately VSOP87, which was an extension of VSOP82, solved the problem well (Bretagnon and Francou 1988). When use is made of the complete VSOP87 theory, a high accuracy, better than 0.01 arc second, is obtained (Meeus 1998). Then, VSOP theories continued to develop further, such as VSOP2000 and VSOP2013, which are more accurate than VSOP87 (Simon et al. 2013). However, these methods require the computation of additional series of periodic terms while VSOP87 is accurate enough for space engineering. So, VSOP87 is the best choice here.

Due to the already discussed weakness of the existing algorithm, based on VSOP87 theory, an improved algorithm is proposed, which gives additional accuracy. In the following, firstly, the whole computation process of the improved algorithm is described in four steps. Secondly, a simulation of the improved algorithm for a large elliptic orbit is carried out to validate the proposed algorithm and assess its accuracy. Finally, the work of this paper is summarized.


Figure 1. The space large deployable convergent membrane solar concentrator.

## COMPUTATION PROCESS OF THE IMPROVED ALGORITHM

The improved algorithm is divided into four steps, as shown in Fig. 2. The first step is to obtain the current UTC time and instantaneous orbital elements of the satellite. The second step is to compute the apparent right ascension, declination and the earth-sun distance in the second equatorial coordinate system by VSOP87 theory. The third step is to transfer the sun vector to the orbit coordinate system by rotation and translation transforms. Finally, the sun vector is transferred to the coordinate system of the satellite body, according to the attitude information of the satellite.


Figure 2. Procedure for the improved algorithm.

## CORRELATIVE COORDINATE SYSTEMS

## Second Equatorial Coordinate System (Inertial Coordinate System) $\mathrm{O}_{i} X_{i} Y_{i} Z_{i}$

The origin $\mathrm{O}_{\mathrm{i}}$ is taken as the center of the Earth. The direction $\mathrm{O}_{\mathrm{i}} \mathrm{Z}_{\mathrm{i}}$ points to the north along the Earth's axis, $\mathrm{O}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}}$ points to the vernal equinox, $\mathrm{OiY}_{\mathrm{i}}$ is formed as the cross-product of the two previous vectors (right-hand rule) and $\mathrm{O}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}} \mathrm{Y}_{\mathrm{i}}$ is the equatorial plane, as shown in Fig. 3a.


Figure 3. Correlative coordinate systems: (a) second equatorial coordinate system and orbit coordinate system; (b) orbit coordinate system and coordinate system of satellite body.

## Orbit Coordinate System $\mathrm{O}_{0} \mathrm{X}_{\mathrm{o}} \mathrm{Y} \mathrm{Z}_{\mathrm{o}}$

The orbit coordinate system is determined by the orbit plane and by the center of the Earth. The origin $\mathrm{O}_{\mathrm{o}}$ coincides with the centroid of the satellite. $\mathrm{O}_{0} \mathrm{Z}_{\mathrm{o}}$ points to the center of the Earth. $\mathrm{O}_{0} \mathrm{Y}_{0}$ points to the normal direction of the orbit. $\mathrm{O}_{0} \mathrm{X}_{\mathrm{o}}$ is formed as the right-hand rule. When the shape of the orbit is circular, $\mathrm{O}_{0} \mathrm{X}_{\mathrm{o}}$ points to the motion direction of the satellite. The orbit coordinate system is shown in Figs. 3a and 3b.

## Coordinate System of Satellite Body $\mathrm{O}_{\mathrm{b}} \mathrm{X}_{\mathrm{b}} \mathrm{Y}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}$

The origin $\mathrm{O}_{\mathrm{b}}$ is the centroid of the satellite and the three orthogonal axes of $\mathrm{X}_{\mathrm{b}}, \mathrm{Y}_{\mathrm{b}}$ and $\mathrm{Z}_{\mathrm{b}}$ are respectively parallel to the inertial axes of the satellite, which are fixedly connected with the satellite body. When the attitude of the satellite with respect to the Earth is not changed, the coordinate system of the satellite body coincides with the orbit coordinate system. The coordinate system of the satellite body is shown in Fig. 3b.

## COMPUTATION OF THE SUN VECTOR IN SECOND EQUATORIAL COORDINATE SYSTEM

## VS0P87 Theory

As aforementioned, Bretagnon and Francou created VSOP87 planet theory in 1987, which gives the periodic terms to compute planetary heliocentric coordinate including heliocentric longitude, latitude and radius vector. When use is made of the complete VSOP87 theory, a high accuracy, better than 0.01 arc second, is obtained. For the Earth it contains 2425 terms, namely 1080 terms for the Earth's longitude, 348 for the latitude, and 997 for the radius vector. However, this big amount of numerical data is unfavorable for onboard tracking control. Instead, by selecting important terms from the VSOP87, an error not exceeding 1 arc second between the years -2000 and +6000 can be obtained (Meeus 1998), which is accurate enough to tracking control and not complicated to implement.

## COMPUTATION OF APPARENT LONGITUDE AND LATITUDE OF THE SUN

As aforementioned, the heliocentric longitude $L$, heliocentric latitude $B$, and radius vector $R$ of the Earth contain many periodic terms according to the VSOP87 theory. Appendix II of the literature (Meeus 1998) gives the most important periodic terms from the VSOP87 theory. In Chapter 31 of Meeus (1998), the series labelled $L 0, L 1, L 2, L 3, L 4, L 5, B 0, B 1, R 0, R 1, R 2, R 3, R 4$ for the Earth are provided. Each horizontal line in the list of Appendix II (Meeus 1998) represents one periodic term and contains four numbers: the first number is the label of the term in the series and the following three numbers are referred to as $\mathrm{A}, \mathrm{B}, \mathrm{C}$, respectively. The value of each term is given by $\operatorname{Acos}(B+C \tau)$, where $\tau$ is the time measured in Julian millennia from the epoch 2000.0. The required longitude $L$, latitude $B$ and radius vector $R$ (distance to the Sun in astronomical units) are obtained from Eq. 1:

$$
\left\{\begin{array}{l}
L=\frac{\left(L 0+L 1 \tau+L 2 \tau^{2}+L 3 \tau^{3}+L 4 \tau^{4}+L 5 \tau^{5}\right) \cdot 180^{\circ}}{10^{8} \cdot \pi}  \tag{1}\\
B=\frac{(B 0+B 1 \tau) \cdot 180^{\circ}}{10^{8} \cdot \pi} \\
R=\frac{\left(R 0+R 1 \tau+R 2 \tau^{2}+R 3 \tau^{3}+R 4 \tau^{4}\right)}{10^{8}}
\end{array}\right.
$$

Then, geocentric longitude $\Theta$ and latitude $\beta$ of the Sun are computed by Eq. 2:

$$
\begin{equation*}
\Theta=L+180^{\circ}, \quad \beta=-B \tag{2}
\end{equation*}
$$

## Conversion to the FK5 System

The Sun's longitude $\Theta$ and latitude $\beta$ are referred to the mean dynamical ecliptic and equinox of the date defined by the VSOP87 (Bretagnon et al. 1986). This reference frame differs very slightly from the standard FK5 (Fifth Fundamental Catalogue) system (Meeus 1998), which is based on absolute and quasi-absolute catalogues with mean epochs later than 1900 (Fricke et al. 1988). These catalogues consist of about 85 catalogues giving observations from 1900 to about 1980. The observations presented in these catalogues were made with meridian circles, vertical circles, transit instruments, and astrolabes. The conversion of $\Theta$ and $\beta$ to the FK5 system can be performed by making use of the following equations (Eq. 3) (Meeus 1998):

$$
\left\{\begin{array}{l}
\lambda^{\prime}=\Theta-1^{\circ} .397 T-0^{\circ} .00031 T^{2}  \tag{3}\\
\Delta \Theta=-\left(\frac{0.09033}{3600}\right)^{\circ} \\
\Delta \beta=\left(\frac{0.03916}{3600}\right)^{\circ}\left(\cos \lambda^{\prime}-\sin \lambda^{\prime}\right) \\
\Theta=\Theta+\Delta \Theta \\
\beta=\beta+\Delta \beta
\end{array}\right.
$$

where $T$ is the time in centuries from epoch 2000.0.

## Apparent Place of the Sun

The Sun's longitude $\Theta$ obtained thus far is the geometric longitude of the Sun. To obtain the apparent longitude $\lambda$, the effects of nutation and aberration should be taken into account (Meeus 1998).

For nutation, add the nutation in longitude $\Delta_{\Psi}$ to $\Theta$ according to Eq. 4 (Seidelmann 1980):

$$
\left\{\begin{array}{l}
M=280^{\circ} .4665+36000^{\circ} .7698 T  \tag{4}\\
M^{\prime}=218^{\circ} .3165+481267^{\circ} .8813 T \\
\Omega=\frac{\left(125.04452-1934.136261 T+0.0020708 T^{2}+T^{3} / 450000\right) \cdot 180^{\circ}}{\pi} \\
\Delta \psi=-\left(\frac{17.20}{3600}\right)^{\circ} \sin \Omega-\left(\frac{1.32}{3600}\right)^{\circ} \sin 2 M-\left(\frac{0.23}{3600}\right)^{\circ} \sin 2 M^{\prime}+\left(\frac{0.21}{3600}\right)^{\circ} \sin 2 \Omega \\
\Theta=\Theta+\Delta \psi
\end{array}\right.
$$

Then, to the aberration, apply the correction to $\Theta$ obtained by Eq. 5 (Meeus 1998), where $R$ is the earth-sun distance in astronomical units. Following this procedure, one obtains the Sun's apparent longitude $\lambda$.

$$
\begin{equation*}
\lambda=\Theta-\frac{\left(\frac{20.4898}{3600}\right)^{\circ}}{R} \tag{5}
\end{equation*}
$$

Finally, the apparent right ascension $\alpha$ and declination $\delta$ are computed from the apparent longitude $\lambda$ and latitude $\beta$ by means of Eq. 6, which represent a coordinate transformation from the geocentric ecliptical coordinate system to the second equatorial coordinate system (Meeus 1998):

$$
\left\{\begin{array}{l}
\tan \alpha=\frac{\sin \lambda \cos \varepsilon-\tan \beta \sin \varepsilon}{\cos \lambda}  \tag{6}\\
\sin \delta=\sin \beta \cos \varepsilon+\cos \beta \sin \varepsilon \sin \lambda
\end{array}\right.
$$

where $\varepsilon$ stands for the true obliquity of the ecliptic and $\varepsilon$ contains nutation in obliquity $\Delta \varepsilon$, which can be obtained by Eq. 7 (Seidelmann 1980; Laskar 1986):

$$
\left\{\begin{array}{rl}
U= & T / 100  \tag{7}\\
\Delta \varepsilon= & \left(\frac{9.20}{3600}\right)^{\circ} \cos \Omega+\left(\frac{0.57}{3600}\right)^{\circ} \cos 2 M+\left(\frac{0.10}{3600}\right)^{\circ} \cos 2 M^{\prime}-\left(\frac{0.09}{3600}\right)^{\circ} \cos 2 \Omega \\
\varepsilon 0= & 23^{\circ} .4392911-\left(4680^{\circ} .93 / 3600\right) U-\left(1.55^{\circ} / 3600\right) U^{2} \\
& \left(1999^{\circ} .25 / 3600\right) U^{3}-\left(51^{\circ} .38 / 3600\right) U^{4}-249^{\circ} .67 / 3600 U^{5} \\
& -\left(39^{\circ} .05 / 3600\right) U^{6}+\left(7^{\circ} .12 / 3600\right) U^{7}+\left(27^{\circ} .87 / 3600\right) U^{8} \\
& +\left(5^{\circ} .79 / 3600\right) U^{9}+\left(2^{\circ} .45 / 3600\right) U^{10} \\
\varepsilon= & \varepsilon 0+\Delta \varepsilon
\end{array} .\right.
$$

## COMPARISON WITH THE CHINESE ASTRONOMICAL ALMANAC

To check the accuracy of VSOP87 algorithm, the values of apparent right ascension $\alpha$ and declination $\delta$, which are computed according to VSOP87 algorithm, are compared with data given by the Chinese astronomical almanac (Purple Mountain Observatory Chinese Academy of Science 2015) at zero hour of the first day each month in 2015. The results of the comparison are shown in Table 1. Deviation results between them are plotted in Fig. 4. Figure 4 shows that the maximum deviation of the apparent longitude $\alpha$ is less than 0.17 arc seconds and that of the apparent declination $\delta$ is less than 1.2 arc seconds. Compared with corresponding data of the literature (Zhang et al. 2011), the accuracy has been improved, especially for the apparent right ascension $\alpha$.


Figure 4. Comparison between VSOP87 algorithm and the Chinese Astronomical Almanac data for the year of 2015.
Table 1. Values of $\alpha$ and $\delta$ given by the VSOP87 algorithm and the Chinese Astronomical Almanac.

| Month | VSOP87 algorithm | Chinese astronomical almanac | Deviation |
| :---: | :---: | :---: | :---: |
| 1 | $18 \mathrm{~h} 44 \mathrm{~m} 30.41 \mathrm{~s} ;-23^{\circ} 02^{\prime} 27.12^{\prime \prime}$ | $18 \mathrm{~h} 44 \mathrm{~m} 30.55 \mathrm{~s} ;-23^{\circ} 02^{\prime} 26.3^{\prime \prime}$ | $-0.14^{\prime \prime} ;-0.82^{\prime \prime}$ |
| 2 | $20 \mathrm{~h} 56 \mathrm{~m} 59.27 \mathrm{~s} ;-17^{\circ} 14^{\prime} 59.75^{\prime \prime}$ | $20 \mathrm{~h} 56 \mathrm{~m} 59.44 \mathrm{~s} ;-17^{\circ} 14^{\prime} 59.3^{\prime \prime}$ | $-0.17^{\prime \prime} ;-0.45^{\prime \prime}$ |
| 3 | $22 \mathrm{~h} 46 \mathrm{~m} 24.27 \mathrm{~s} ;-7^{\circ} 47^{\prime} 26.06^{\prime \prime}$ | $22 \mathrm{~h} 46 \mathrm{~m} 24.35 \mathrm{~s} ;-7^{\circ} 47 \mathrm{r} 26.0^{\prime \prime}$ | $-0.08^{\prime \prime} ;-0.06^{\prime \prime}$ |
| 4 | $0 \mathrm{~h} 40 \mathrm{~m} 15.26 \mathrm{~s} ; 4^{\circ} 19^{\prime} 53.0^{\prime \prime}$ | $0 \mathrm{~h} 40 \mathrm{~m} 15.19 \mathrm{~s} ; 4^{\circ} 19.52 .7^{\prime \prime}$ | $0.07^{\prime \prime} ; 0.3^{\prime \prime}$ |
| 5 | $2 \mathrm{~h} 31 \mathrm{~m} 37.21 \mathrm{~s} ; 14^{\circ} 54^{\prime} 42.22^{\prime \prime}$ | $2 \mathrm{~h} 31 \mathrm{~m} 37.05 \mathrm{~s} ; 14^{\circ} 54^{\prime} 41.5^{\prime \prime}$ | $0.16^{\prime \prime} ; 0.72^{\prime \prime}$ |
| 6 | $4 \mathrm{~h} 34 \mathrm{~m} 21.44 \mathrm{~s} ; 21^{\circ} 58^{\prime} 35.65^{\prime \prime}$ | $4 \mathrm{~h} 34 \mathrm{~m} 21.32 \mathrm{~s} ; 21^{\circ} 58^{\prime} 34.6^{\prime \prime}$ | $0.12^{\prime \prime} ; 1.05$ |
| 7 | $6 \mathrm{~h} 38 \mathrm{~m} 38.10 \mathrm{~s} ; 23^{\circ} 08^{\prime} 17.10^{\prime \prime}$ | $6 \mathrm{~h} 38 \mathrm{~m} 38.09 \mathrm{~s} ; 23^{\circ} 08^{\prime} 15.9^{\prime \prime}$ | $0.01^{\prime \prime} ; 1.2^{\prime \prime}$ |

Table 1. Continuation...

| Month | VSOP87 algorithm | Chinese astronomical almanac | Deviation |
| :---: | :---: | :---: | :---: |
| 8 | $8 \mathrm{~h} 43 \mathrm{~m} 31.31 \mathrm{~s} ; 18^{\circ} 08^{\prime} 44.70^{\prime \prime}$ | $8 \mathrm{~h} 43 \mathrm{~m} 31.42 \mathrm{~s} ; 18^{\circ} 08^{\prime} 43.6^{\prime \prime}$ | $-0.11^{\prime \prime} ; 1.1^{\prime \prime}$ |
| 9 | $10 \mathrm{~h} 39 \mathrm{~m} 36.49 \mathrm{~s} ; 8^{\circ} 28^{\prime} 18.86^{\prime \prime}$ | $10 \mathrm{~h} 39 \mathrm{~m} 36.61 \mathrm{~s} ; 8^{\circ} 28>18.1^{\prime \prime}$ | $-0.12^{\prime \prime} ; 0.76^{\prime \prime}$ |
| 10 | $12 \mathrm{~h} 27 \mathrm{~m} 33.16 \mathrm{~s} ;-2^{\circ} 58^{\prime} 33.22^{\prime \prime}$ | $12 \mathrm{~h} 27 \mathrm{~m} 33.22 \mathrm{~s} ;-2^{\circ} 58^{\prime} 33.6^{\prime \prime}$ | $-0.06^{\prime \prime} ; 0.38^{\prime \prime}$ |
| 11 | $14 \mathrm{~h} 23 \mathrm{~m} 28.05 \mathrm{~s} ;-14^{\circ} 14^{\prime} 59.39^{\prime \prime}$ | $14 \mathrm{~h} 23 \mathrm{~m} 28.00 \mathrm{~s} ;-14^{\circ} 14^{\prime} 59.4^{\prime \prime}$ | $0.05^{\prime \prime} ; 0.01^{\prime \prime}$ |
| 12 | $16 \mathrm{~h} 26 \mathrm{~m} 51.63 \mathrm{~s} ;-21^{\circ} 42^{\prime} 36.18^{\prime \prime}$ | $16 \mathrm{~h} 26 \mathrm{~m} 51.53 \mathrm{~s} ;-21^{\circ} 42^{\prime} 35.9^{\prime \prime}$ | $0.1^{\prime \prime} ;-0.28^{\prime \prime}$ |

## TRANSFORM OF THE SUN POSITION FROM THE SECOND EQUATORIAL COORDINATE SYSTEM TO the Coordinate system of the sateluite body

Since the deviation of the sun position in the second equatorial coordinate system obtained by VSOP87 algorithm is of about 1 arc second compared with the Chinese astronomical almanac data, it is not acceptable to ignore the error caused by the size of the orbit. So the mathematical model of the existing algorithm is improved, as shown in Fig. 5.


Figure 5. Deviation of sun position on the satellite caused by the size of the orbit.

## THE PROCESS OF THE IMPROVED ALGORITHM

Firstly, the position vector of the $\operatorname{Sun} \mathrm{L}_{\mathrm{i}}$ in $\mathrm{O}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}} \mathrm{Y}_{\mathrm{i}} \mathrm{Z}_{\mathrm{i}}$ is given by Eq. 8:

$$
\boldsymbol{L}_{i}=\left(1.495978707 \cdot 10^{8} \cdot R\right)\left[\begin{array}{c}
\cos \delta \cos \alpha  \tag{8}\\
\cos \delta \sin \alpha \\
\sin \delta
\end{array}\right]
$$

Then, $L_{i}$ is transferred from $O_{i} X_{i} Y_{i} Z_{i}$ to $O_{o} X_{o} Y_{o} Z_{o}$ by the transform matrix $T$ including rotation matrix $R$ and translation matrix $P$. The conversion is described by Eqs. 9 and 10:

$$
\begin{gather*}
R=R_{y}\left(-90^{\circ}-\omega\right) R_{x}\left(i-90^{\circ}\right) R_{z}(\Omega)  \tag{9}\\
\boldsymbol{L}_{o}=R \boldsymbol{L}_{i}+\boldsymbol{P} \tag{10}
\end{gather*}
$$

where $\omega$ is the argument of perigee, $i$ is the orbital inclination and $\Omega$ is the right ascension of the ascending node; $R_{x}, R_{y}, R_{z}$ denote the unit rotation matrix around the axis $x, y, z$, respectively, and $P$ is the position vector of the satellite in orbit.

The position vector of the Sun in $\mathrm{O}_{\mathrm{b}} \mathrm{X}_{\mathrm{b}} \mathrm{Y}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}, \mathrm{L}_{\mathrm{b}}$, is obtained by Eq. 11:

$$
\begin{equation*}
\boldsymbol{L}_{\boldsymbol{b}}=R_{x}(\theta) R_{y}(\varphi) R_{z}(\psi) \boldsymbol{L}_{o} \tag{11}
\end{equation*}
$$

where $\varphi, \theta, \Psi$ denote the rolling angle, pitch angle and yaw angle of the satellite, respectively.
The unit position vector of Sun in $\mathrm{O}_{\mathrm{b}} \mathrm{X}_{\mathrm{b}} \mathrm{Y}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}, L_{b}^{u}$ is given by the modular operation to $L_{b}$ (Eq. 12):

$$
\begin{equation*}
\boldsymbol{L}_{b}^{u}=\frac{\boldsymbol{L}_{b}}{\left\|\boldsymbol{L}_{b}\right\|} \tag{12}
\end{equation*}
$$

The azimuth angle $\phi$ is defined as the angle between the projection of $L_{b}^{u}$ on the plane of $\mathrm{O}_{\mathrm{b}} \mathrm{X}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}$ and the negative direction of the axis $\mathrm{O}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}$ in $\mathrm{O}_{\mathrm{b}} \mathrm{X}_{\mathrm{b}} \mathrm{Y}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}$, and is given by Eq. 13:

$$
\begin{equation*}
\phi=\arccos \frac{-\boldsymbol{L}_{b}^{u}(z)}{\sqrt{\boldsymbol{L}_{b}^{u}(x)^{2}+\boldsymbol{L}_{b}^{u}(z)^{2}}} \tag{13}
\end{equation*}
$$

where $L_{b}^{u}(\mathrm{x})$ and $L_{b}^{u}(\mathrm{x})$ denote the components of $L_{b}^{u}$ in the direction of axis $x$ and $z$, respectively.
The pitch angle $\gamma$ is defined as the angle between $L_{b}^{u}$ and the plane of $\mathrm{O}_{\mathrm{b}} \mathrm{X}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}$ in $\mathrm{O}_{\mathrm{b}} \mathrm{X}_{\mathrm{b}} \mathrm{Y}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}$, which is given by Eq. 14 :

$$
\begin{equation*}
\gamma=\arccos \sqrt{\boldsymbol{L}_{b}^{u}(x)^{2}+\boldsymbol{L}_{b}^{u}(z)^{2}} \tag{14}
\end{equation*}
$$

## SIMULATION OF THE IMPROVED ALGORITHM FOR A LARGE ELLIPTICAL ORBIT AND ANALYSIS OF THE ACCURACY

The satellite named SJ-4 has been launched from China with the mission of studying the environmental effect of charged particles in space (Hu and Chen 1994). Here, the orbit of SJ-4 is used as a simulation example. The orbital elements are given in Table 2 (Heavens Above 2016). It is assumed that the satellite adopts the three-axis stabilization attitude and $\mathrm{O}_{\mathrm{b}} \mathrm{X}_{\mathrm{b}} \mathrm{Y}_{\mathrm{b}} \mathrm{Z}_{\mathrm{b}}$ coincides with $\mathrm{O}_{0} \mathrm{X}_{\mathrm{o}} \mathrm{Y}_{\mathrm{o}} \mathrm{Z}_{\mathrm{o}}$, that is, the satellite has no change of flight pose with respect to the orbit coordinate system.

Table 2. The orbital elements of SJ-4.

| Elements | Values |
| :---: | :---: |
| Eccentricity | 0.5675908 |
| Inclination $\left({ }^{\circ}\right)$ | 28.7578 |
| Perigee Altitude $(\mathrm{km})$ | 232 |
| Apogee Altitude $(\mathrm{km})$ | 17585 |
| RAAN $\left({ }^{\circ}\right)$ | 126.1640 |
| Argument of perigee $\left({ }^{\circ}\right)$ | 288.1275 |
| Initial Mean Anomaly $\left({ }^{\circ}\right)$ | 20.0596 |

The orbital elements are substituted in the improved algorithm. Then the curves of $\phi$ and $\gamma$ in a flight cycle of the satellite are computed and plotted, as shown in Figs. 6 and 7. When the satellite enters the Earth's shadow area, the value of $\phi$ or $\gamma$ are zero, as shown in Figs. 6 and 7.


Figure 6. Curve of the azimuth angle $\phi$ in a flight period.


Figure 7. Curve of the pitch angle $\gamma$ in a flight period.

It is assumed that both the existing and improved algorithms adopt the VSOP87 theory, proposed in this paper, to compute the apparent right ascension $\alpha$ and declination $\delta$ in $\mathrm{O}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}} \mathrm{Y}_{\mathrm{i}} \mathrm{Z}_{\mathrm{i}}$ in order to assess the accuracy of the improved algorithm. Only the error caused by the size of orbit when SJ-4 is at different positions in its orbit is considered. Then, the azimuth angle $\phi$ and the pitch angle $\gamma$ are computed making use of the two algorithms. The difference in azimuth angle given by the two algorithms, $\sigma$, is plotted in Fig. 8, and the one corresponding to the pitch angle, $\tau$, is plotted in Fig. 9. Table 3 shows the values of azimuth angle $\phi$ and their deviation $\sigma$ at six key points of Fig. 8. At the same time, Table 4 shows the values of pitch angle $\gamma$ and their deviation $\tau$ at six key points of Fig. 9. Figure 8 shows that the maximum absolute value of $\sigma$ is 35.19 arc seconds at flight time equal to 171 min . Figure 9 shows that the maximum absolute value of $\tau$ is 19.93 arc seconds at flight time equal to 231 min .

Table 3. Values of azimuth angle at six key moments.

| Time (min) | Azimuth angle (") |  |  |
| :---: | :---: | :---: | :---: |
|  | Improved algorithm | Existing algorithm | Deviation |
| 0 | 2.656197 e 5 | 2.656160 e 5 | 3.67 |
| 33 | 7.461378 e 4 | 7.460010 e 4 | 13.68 |
| 63 | 2.97365 e 3 | 2.97876 e 3 | -5.11 |
| 171 | 1.6798979 e 5 | 1.6795460 e 5 | 35.19 |
| 264 | 4.0102255 e 5 | 4.0101763 e 5 | 4.92 |
| 314 | 2.6132750 e 5 | 2.613238 e 5 | 3.67 |

Table 4. Values of pitch angle at six key moments.

| Time (min) | Pitch angle (") |  |  |
| :---: | :---: | :---: | :---: |
|  | Improved algorithm | Existing algorithm | Deviation |
| 0 | 1.3523874 e 5 | 1.3524983 e 5 | -11.09 |
| 7 | 1.3524505 e 5 | 1.3525832 e 5 | -13.27 |
| 94 | 1.3538215 e 5 | 1.3536379 e 5 | 18.36 |
| 231 | 1.3550967 e 5 | 1.3552960 e 5 | -19.93 |
| 280 | 1.3559584 e 5 | 1.3558883 e 5 | 7.01 |
| 314 | 1.3561850 e 5 | 1.3562990 e 5 | -11.40 |



Figure 8. Curve of the deviation of the azimuth angle $\sigma$ in a flight period.


Figure 9. Curve of the deviation of the pitch angle $\tau$ in a flight period.

## CONCLUSION

In this paper, a precise method for computing the sun position on a satellite is proposed, improving the accuracy given by other algorithms, as shown by simulation results. The results of this research provide a theoretical basis for spaceborne equipment that need to track the Sun accurately. The improvement in accuracy includes two aspects, which are summarized below:

- Compared with the data of 2015 astronomical almanac, the deviation of the apparent right ascension $\alpha$ is not greater than 0.17 arc second and the one of apparent declination $\delta$ is not greater than 1.2 arc seconds, which is better than the results from other literature.
- The maximum value of the deviation in azimuth angle $\sigma$ on the orbit of SJ-4 is 35.19 arc seconds and the one in pitch angle $\tau$ is 19.93 arc seconds compared with existing algorithms. The results indicate that the error of the sun position caused by the size of the orbit does exist and reaches the order of the arc second.


## AUTHORS' CONTRIBUTION

Conceptualization, Zheng T and Zheng F; Methodology, Zheng T and Zheng F; Investigation, Zheng T, Rui X and Ji X; Writing Original Draft, Zheng T and Rui Xi; Writing - Review and Editing, Zheng T and Ji X.

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