# On the Helly Defect of a Graph

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## Abstract

The *Helly defect* of a graph $G$ is the smallest integer $i$ such that the iterated clique graph $K^i(G)$ is clique-Helly. We prove that it is NP-hard to decide whether the Helly defect of $G$ is at most 1.

**Keywords:** Clique graphs, clique-Helly graphs, Helly defect

## 1 Introduction

In this work, we consider the following question, on iterated clique graphs. Given a graph $G$ and an integer $i \geq 0$, is the $i$-iterated clique graph of $G$ a clique-Helly graph? Since clique-Helly graphs can be recognized in polynomial time [11], for $i = 0$ the answer of this question can be given in polynomial time. In this work, we prove that the above problem is NP-hard for $i = 1$. In fact, the NP-hardness holds for a more general problem stated in Theorem 6.

In general, write that a set $S$ is a $k$-set when $|S| = k$, a $k^+$-set when $|S| \geq k$, and a $k^-$-set when $|S| \leq k$. This same notation will also apply for families of sets.

Let $\mathcal{F}$ be a family of subsets $F_i$ of some set. A *core* of $\mathcal{F}$ is any subset of $\bigcap_{F_i \in \mathcal{F}} F_i$. Let $p, q$ be integers, $p \geq 1$ and $q \geq 0$. Say that $\mathcal{F}$ is *(p,q)-intersecting* when every $p^-$-subfamily of it has a $q^+$-core. In addition, $\mathcal{F}$ is $(p,q)$*-Helly* when every $(p,q)$-intersecting subfamily of it has a $q^+$-core. Consequently, the classical *p-Helly* families of sets [2, 3] correspond to the case $(p, 1)$-Helly, while the Helly families correspond to the notation $(2, 1)$-Helly.

Let $G$ be a graph. A *complete* of $G$ is a subset of pairwise adjacent vertices of it. A *clique* is a maximal complete. A *clique-transversal* of $G$ is a subset of vertices intersecting all cliques. The problem of deciding whether a given subset is a clique-transversal of $G$ has been proved to be Co-NP-Complete by Erdös, Gallai and Tuza [8]. Say that $G$ is *(p,q)-clique-Helly* when the family of the cliques of $G$ is $(p, q)$-Helly. When $G$ is (2,1)-clique-Helly, write simply *clique-Helly*.

The *clique graph* $K(G)$ of $G$ is the intersection graph of the cliques of $G$. The *i-th iterated clique graph* of $G$, denoted $K^i(G)$, is defined as follows: $K^0(G) = G$, while $K^i(G) = K(K^{i-1}(G))$, $i > 0$.

The *Helly defect* of a graph $G$ is the smallest $i$ such that $K^i(G)$ is clique-Helly. If $K^i(G)$ is not clique-Helly, for any finite $i$, say that its Helly defect is infinite. Trivially, the Helly defect of a clique-Helly graph is 0.

A graph $G$ is *periodic* when $K^i(G) = G$, for some $i$. In addition, the smallest $i$ satisfying $K^i(G) = G$ is the *period* of a periodic graph $G$.

Escalante [9] proved that if $K^i(G)$ is clique-Helly, then $K^j(G)$ is clique-Helly for any $j \geq i$, and that if $G$ is clique-Helly, then $K^2(G)$ is an induced subgraph of $G$.

Let $G$ be a periodic clique-Helly graph, and let $p \geq 1$ be its period. Clearly, $G = K^{2p}(G)$. By Escalante's result, $K^{2p}(G)$ is an induced subgraph of $K^2(G)$, therefore $G = K^2(G)$. This implies that $p \leq 2$. This argument shows that if $H$ is a periodic graph with period strictly greater than 2, then $K^i(H)$ cannot be clique-Helly for any $i \geq 0$, and thus its Helly defect is infinite.

Bandelt and Prisner [1] proved that the Helly defect of a chordal graph is at most 1. In [4] there are examples of graphs with Helly defect $i$, for any finite $i$.

In this paper, we prove that it is NP-hard to decide whether the clique graph of $G$ is $(2, q)$-Helly, for any fixed $q \geq 1$. Consequently, it is NP-hard to decide whether $G$ has Helly defect at most 1.

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## 2 The proof

In this section, we prove that it is NP-hard to recognize if the Helly defect of a graph is at most 1. We need first some definitions and results.

Let $G$ be a graph and $T$ a triangle of it. The *extended triangle* of $G$, relative to $T$, is the subgraph of $G$ induced by subset of vertices of $G$ which form a triangle with (at least) two vertices of $T$.

**Theorem 1** [6, 11] *$G$ is a clique-Helly graph if and only if every of its extended triangles contains a universal vertex.*

Extended triangles can be generalized as follows: let $C$ be a $p$-complete of a graph $G$, $p \geq 3$. The *$p$-expansion of $G$ relative to $C$* is the subgraph of $G$ induced by the vertices forming a $p$-complete with $p-1$ vertices of $C$.

Let $\mathscr{F}$ be a subfamily of cliques of $G$. The *clique subgraph induced by $\mathscr{F}$ in $G$*, denoted by $G[\mathscr{F}]_c$, is the subgraph of $G$ formed exactly by the vertices and edges belonging to the cliques of $\mathscr{F}$.

**Lemma 2** [5] *Let $G$ be a graph, $C$ a $p$-complete of it, $H$ the $p$-expansion of $G$ relative to $C$, and $\mathscr{C}$ the subfamily of cliques of $G$ that contain at least $p-1$ vertices of $C$. Then $G[\mathscr{C}]_c$ is a spanning subgraph of $H$.*

**Proof.** We have to show that $V(G[\mathscr{C}]_c) = V(H)$. Let $v \in V(H)$. Then $v$ is adjacent to at least $p-1$ vertices of $C$. Hence, $v$ together with those $p-1$ vertices form a $p$-complete, which is contained in a clique that contains at least $p-1$ vertices of $C$. Therefore, $v \in V(G[\mathscr{C}]_c)$. Now, consider $v \in V(G[\mathscr{C}]_c)$. Then $v$ belongs to some clique containing $p-1$ vertices of $C$. That is, $v$ is adjacent to at least $p-1$ vertices of $C$, and hence $v \in V(H)$. Consequently, $V(G[\mathscr{C}]_c) = V(H)$. Furthermore, both $H$ and $G[\mathscr{C}]_c$ are subgraphs of $G$, but $H$ is induced. Thus $E(G[\mathscr{C}]_c) \subseteq E(H)$. □

Let $q$ be a positive integer. The graph $\Phi_q(G)$ is defined in the following way: the vertices of $\Phi_q(G)$ correspond to the $q$-completes of $G$, two vertices being adjacent in $\Phi_q(G)$ if the corresponding $q$-completes in $G$ are contained in a common clique. We remark that $\Phi_q$ is precisely the operator $\Phi_{q,2q}$, studied in [10]. An interesting property of $\Phi_q$ is that there exists a bijection between the subfamily of $q^+$-cliques of $G$ and the family of cliques of $\Phi_q(G)$ [5]. The following definitions are possible due to this property: If $C$ is a $q^+$-clique of $G$, denote by $\Phi_q(C)$ the clique that corresponds to $C$ in $\Phi_q(G)$. If $C'$ is a clique of $\Phi_q(G)$, denote by $\Phi_q^{-1}(C')$ the $q^+$-clique that corresponds to $C'$ in $G$. If $\mathscr{F}$ is a subfamily of $q^+$-cliques of $G$, define $\Phi_q(\mathscr{F}) = \{\Phi_q(C) \mid C \in \mathscr{F}\}$. If $\mathscr{C}$ is a subfamily of cliques of $\Phi_q(G)$, define $\Phi_q^{-1}(\mathscr{C}) = \{\Phi_q^{-1}(C) \mid C \in \mathscr{C}\}$.

**Lemma 3** [5] *Let $G$ be a graph, $\mathscr{F}$ a subfamily of $q^+$-cliques of it, $\mathscr{C} = \Phi_q(\mathscr{F})$, and $H = \Phi_q(G)$. Then $H[\mathscr{C}]_c$ contains a universal vertex if and only if $G[\mathscr{F}]_c$ contains $q$ universal vertices.*

**Proof.** If $H[\mathscr{C}]_c$ contains a universal vertex $x$, then every clique of $\mathscr{F}$ contains the $q$-complete of $G$ that corresponds to $x$, that is, $G[\mathscr{F}]_c$ contains $q$ universal vertices. Conversely, if $G[\mathscr{F}]_c$ contains $q$ universal vertices forming a $q$-complete $Q$ of $G$, then every clique of $\mathscr{C}$ contains the vertex of $H$ that corresponds to $Q$, that is, $H[\mathscr{C}]_c$ contains a universal vertex. □

The proof of the next lemma is easy, and thus we will omit it:

**Lemma 4** [5] *Let $C$ be a $(p+1)$-complete of a graph $G$, and let $\mathscr{C}$ be a $p^-$-subfamily of cliques of $G$ such that every clique of $\mathscr{C}$ contains at least $p$ vertices of $C$. Then $\mathscr{C}$ has a $1^+$-core.* □

The following theorem is a characterization of $(p,q)$-clique-Helly graphs:

**Theorem 5** [5] *Let $p > 1$ be an integer. A graph $G$ is $(p,q)$-clique-Helly if and only if every $(p+1)$-expansion of $\Phi_q(G)$ contains a universal vertex.*

**Proof.** Suppose that $G$ is a $(p,q)$-clique-Helly graph and there exists a $(p+1)$-expansion $T$, relative to a $(p+1)$-complete $C$ of $\Phi_q(G)$, such that $T$ contains no universal vertex.

Let $\mathscr{C}$ be the subfamily of cliques of $H = \Phi_q(G)$ that contain at least $p$ vertices of $C$. Let $\mathscr{F} = \Phi_q^{-1}(\mathscr{C})$. Consider a $p^-$-subfamily $\mathscr{F}' \subseteq \mathscr{F}$. Let $\mathscr{C}' = \Phi_q(\mathscr{F}')$. By Lemma 4, $\mathscr{C}'$ has a $1^+$-core. That is, $H[\mathscr{C}']_c$ contains a universal vertex. This implies, by Lemma 3, that $G[\mathscr{F}']_c$ contains $q$ universal vertices. Thus, $\mathscr{F}'$ has a $q^+$-core, that is, $\mathscr{F}$ is $(p,q)$-intersecting. Since $G$ is $(p,q)$-clique-Helly, we conclude that $\mathscr{F}$ has a $q^+$-core and $G[\mathscr{F}]_c$ contains $q$ universal vertices. By using Lemma 3 again, $H[\mathscr{C}]_c$ contains a universal vertex. Moreover, by Lemma 2, $H[\mathscr{C}]_c$ is a spanning subgraph of $T$. However, $T$ contains no universal vertex. This is a contradiction. Therefore, every $(p+1)$-expansion of $H = \Phi_q(G)$ contains a universal vertex.

Conversely, assume by contradiction that $G$ is not $(p,q)$-clique-Helly. Let $\mathscr{F} = \{C_1, \ldots, C_k\}$ be a minimal $(p,q)$-intersecting subfamily of cliques of $G$ which does not have a $q$-core. Clearly, $k > p$.

By the minimality of $\mathscr{F}$, the subfamily $\mathscr{F}\backslash C_i$ has a $q^+$-core $Q_i$, for $i = 1, \ldots, k$. It is clear that $Q_i \not\subseteq C_i$. Moreover, every two distinct $Q_i, Q_j$ are contained in a same clique, since $k \geq 3$. Hence the sets $Q_1, Q_2, \ldots, Q_{p+1}$ correspond to a $(p+1)$-complete $C$ in $\Phi_q(G)$.

Let $\mathscr{C}$ be the subfamily of cliques of $H = \Phi_q(G)$ that contain at least $p$ vertices of $C$. Let $\mathscr{C}' = \Phi_q(\mathscr{F}) = \{\Phi_q(C_1), \ldots, \Phi_q(C_k)\}$. Since every $C_i \in \mathscr{F}$ contains at least $p$ sets from $Q_1, Q_2, \ldots, Q_{p+1}$, it is clear that the clique $\Phi_q(C_i)$ of $H$ contains at least $p$ vertices of $C$. Therefore, $\Phi_q(C_i) \in \mathscr{C}$, for $i = 1, \ldots, k$.

Let $T$ be the $(p+1)$-expansion of $H$ relative to $C$. By Lemma 2, $H[\mathscr{C}]_c$ is a spanning subgraph of $T$. Therefore, $V(Q) \subseteq V(T)$, for every $Q \in \mathscr{C}$. In particular, $V(\Phi_q(C_i)) \subseteq V(T)$, for $i = 1, \ldots, k$. By hypothesis, $T$ contains a universal vertex $x$. Then $x$ is adjacent to all the vertices of $\Phi_q(C_i)\backslash\{x\}$, for $i = 1, \ldots, k$. This implies that $\Phi_q(C_i)$ contains $x$, otherwise $\Phi_q(C_i)$ would not be maximal. Thus, $\mathscr{C}'$ has a $1^+$-core and $H[\mathscr{C}']_c$ contains a universal vertex. By Lemma 3, $G[\mathscr{F}]_c$ contains $q$ universal vertices, that is, $\mathscr{F}$ has a $q^+$-core. This contradicts the assumption for $\mathscr{F}$. Hence, $G$ is a $(p, q)$-clique-Helly graph. □

Next, we state the main result of this paper.

**Theorem 6** *Let $q \geq 1$ be a fixed integer. Given a graph $G$, it is NP-hard to decide whether $K(G)$ is $(2, q)$-clique-Helly.*

**Proof.**

The proof is a transformation from the following problem. Given a graph $G$ and a clique $Q$ of it, does $Q$ intersect all the cliques of $G$? (In other words, is $Q$ a *clique-transversal* of $G$?) This problem was shown to be Co-NP-complete in [7].

Given a graph $G$ and a clique $Q$ of it, we have to construct a graph $H$ such that $Q$ is a clique-transversal of $G$ if and only if $K(H)$ is $(2, q)$-clique-Helly. The construction of $H$ is as follows: consider first the graph formed by three disjoint copies of $G$, denoted by $G_a, G_b, G_c$. Add six vertices forming the set $V_1 = \{a'_1, a'_2, b'_1, b'_2, c'_1, c'_2\}$, and add the following edges:

$a'_j v$ for all $v \in V(G_a)$ and $j = 1, 2$;
$b'_j v$ for all $v \in V(G_b)$ and $j = 1, 2$;
$c'_j v$ for all $v \in V(G_c)$ and $j = 1, 2$;
$a'_1 a'_2$; $b'_1 b'_2$; $c'_1 c'_2$; $a'_2 b'_1$; $b'_2 c'_1$; $c'_2 a'_1$.

Add now $q$ vertices for each copy of $G$, forming the set $V_2 = \{a_1, a_2, ..., a_q,\ b_1, b_2, ..., b_q, c_1, c_2, ..., c_q\}$, and add the edges:

$a_i v$ for all $v \in V(G_a)$ and $1 \leq i \leq q$;
$b_i v$ for all $v \in V(G_b)$ and $1 \leq i \leq q$;
$c_i v$ for all $v \in V(G_c)$ and $1 \leq i \leq q$;
$a_i a'_j$ for $1 \leq i \leq q$ and $j = 1, 2$;
$b_i b'_j$ for $1 \leq i \leq q$ and $j = 1, 2$;
$c_i c'_j$ for $1 \leq i \leq q$ and $j = 1, 2$.

For each vertex $w \in Q$, consider its three copies in $G_a, G_b, G_c$, and collapse them into a single vertex, preserving all its adjacencies. For simplicity, the vertex of $H$ corresponding to a vertex $w \in Q$ will also be denoted by $w$, and we will refer to $Q$ as a complete of $H$.

Finally, add more $3q$ vertices forming the set $V_3 = \{d_1, d_2, ..., d_q, d'_1, d'_2, ..., d'_q,$
$d''_1, d''_2, ..., d''_q\}$ and the edges:

$dw$ for $d \in V_3$, $w \in Q$;
$d_i c'_2$ and $d_i a'_1$ for $1 \leq i \leq q$;
$d'_i a'_2$ and $d'_i b'_1$ for $1 \leq i \leq q$;
$d''_i b'_2$ and $d''_i c'_1$ for $1 \leq i \leq q$.

The construction of $H$ is completed (see Figure 1). Clearly, the clique $Q$ of $G$ corresponds to the complete $Q$ of $H$, while any other clique $Q' \neq Q$ of $G$ corresponds to three complete sets $Q'_a, Q'_b, Q'_c$, located at $G_a, G_b, G_c$, respectively.

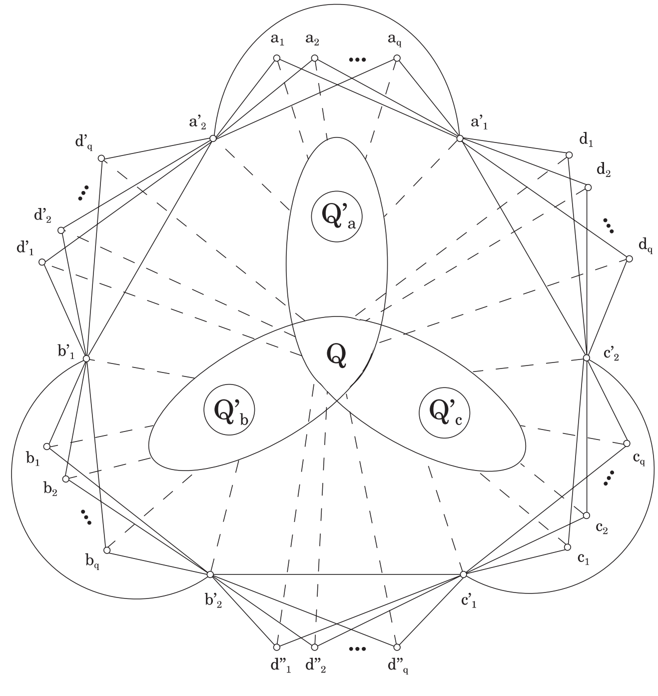


Figure 1: Construction of $H$.

By construction, observe that every clique of $H$ is formed by a copy of a clique of $G$ together with three new vertices: two from $V_1$ and one from $V_2 \cup V_3$.

We proceed with the proof of the theorem.

Suppose that $Q$ is a clique-transversal of $G$. This implies that $Q$ intersects any clique $Q'$ of $G$. Since each clique of $H$ contains a copy of a clique of $G$ by construction, $Q$ intersects any clique of $H$. Therefore any clique of $H$ that contains $Q$ is a clique-transversal of $H$. Consider the following cliques of $H$:

$$Q \cup \{a'_2, b'_1, d'_i\} \text{ for } 1 \le i \le q.$$

Since these $q$ cliques contain $Q$, they are clique-transversals of $H$. Clearly, a clique which is a clique-transversal of a graph $G'$ is a universal vertex of $K(G')$. So the clique graph of $H$ contains $q$ universal vertices, hence $K(H)$ is $(2, q)$-clique-Helly.

Conversely, suppose that $Q$ is not a clique-transversal of $G$. Consider the following three families of cliques of $H$:

$$C_1 = \{Q \cup \{a'_2, b'_1, d'_i\}, \text{ for } 1 \le i \le q\};$$
$$C_2 = \{Q \cup \{b'_2, c'_1, d''_i\}, \text{ for } 1 \le i \le q\};$$
$$C_3 = \{Q \cup \{c'_2, a'_1, d_i\}, \text{ for } 1 \le i \le q\}.$$

It is clear that every clique in $C_i, i = 1, 2, 3$, is a vertex of $K(H)$. Let $C_1^K, C_2^K, C_3^K$ be the subsets of vertices of $K(H)$ corresponding to the cliques in the families $C_1$, $C_2$, and $C_3$, respectively. Since all these cliques contain $Q$, all the vertices of $C_1^K \cup C_2^K \cup C_3^K$ are pairwise adjacent. Therefore $C_1^K, C_2^K$ and $C_3^K$ are completes of $K(H)$. Moreover, it implies the existence of a clique containing these 3 completes. Since each one of these completes contains $q$ vertices, the graph $\Phi_q(K(H))$ contains three vertices $v_1, v_2, v_3$ corresponding to them. Since there is a clique in $K(H)$ containing $C_1^K, C_2^K, C_3^K$, it follows that $\{v_1, v_2, v_3\}$ is a triangle $T$ of $\Phi_q(K(H))$. Let $T'$ be the extended triangle relative to $T$. Consider the following additional families of cliques of $H$:

$$C_4 = \{Q'_a \cup \{a'_1, a'_2, a_i\}, \text{ for } 1 \le i \le q\};$$
$$C_5 = \{Q'_b \cup \{b'_1, b'_2, b_i\}, \text{ for } 1 \le i \le q\};$$
$$C_6 = \{Q'_c \cup \{c'_1, c'_2, c_i\}, \text{ for } 1 \le i \le q\};$$

where $Q'_a, Q'_b, Q'_c$, are the three copies of a clique $Q'$ of $G$ that does not intersects $Q$. Clearly, $Q'$ exists because $Q$ is not a clique-transversal of $G$. Clearly, every clique in $C_i, i = 4, 5, 6$, is a vertex of $K(H)$. Let $C_4^K, C_5^K, C_6^K$ be the subsets of vertices of $K(H)$ corresponding to the cliques in the families $C_4, C_5$, and $C_6$, respectively. Since any clique in $C_4$ contains $Q'_a$, it follows that $C_4^K$ is a complete of $K(H)$. Analogously, $C_5^K$ and $C_6^K$ are completes of $K(H)$. Consider the cliques $Q'_a \cup \{a'_1, a'_2, a_1\}$, $Q'_b \cup \{b'_1, b'_2, b_1\}$, and $Q'_c \cup \{c'_1, c'_2, c_1\}$ that belong to the families $C_4$, $C_5$, and $C_6$, respectively. Since they are disjoint, their corresponding vertices in $K(H)$ are not adjacent. Consequently, there exists no clique in $H$ containing a pair of completes from $C_4^K, C_5^K, C_6^K$. Therefore the vertices $v_4, v_5, v_6$ of $\Phi_q(K(H))$, corresponding to $C_4^K, C_5^K$, and $C_6^K$, respectively, form an independent set.

The following argument shows that $v_4, v_5, v_6$ belong to $T'$. All the cliques of the families $C_1$ and $C_4$ contain the vertex $a'_2$. Hence the corresponding vertices in $K(H)$ are pairwise adjacent. Therefore there is a clique in $K(H)$ containing the completes $C_1^K$ and $C_4^K$. Consequently $v_1v_4$ is an edge of $\Phi_q(K(H))$. All the cliques of the families $C_3$ and $C_4$ contain the vertex $a'_1$, meaning that $v_3v_4$ is also an edge of $\Phi_q(K(H))$. Select a clique of the family $C_4$ and another of the family $C_2$. Since they do not intersect, the corresponding vertices in $K(H)$ are not adjacent. Hence there is no clique in $H$ containing the completes $C_2^K$ and $C_4^K$. Consequently $v_2v_4$ is not an edge of $\Phi_q(K(H))$. By the same argument, we conclude that $v_5$ is adjacent to the vertices $v_1$ and $v_2$, but not to $v_3$; and $v_6$ is adjacent to $v_2$ and $v_3$, but not to $v_1$. Therefore, no vertex $v_i$, for $1 \le i \le 6$, is universal in $T'$.

Denote $A_4 = Q'_a \cup \{a'_1, a'_2\} \cup \{a_i : 1 \le i \le q\}$ and $A_5 = Q'_b \cup \{b'_1, b'_2\} \cup \{b_i : 1 \le i \le q\}$. Note that all the cliques of $C_4$ and $C_5$ are subsets of $A_4$ and $A_5$, respectively. Note also that $A_4 \cap A_5 = \emptyset$, and the only edge joining vertices of $A_4$ and $A_5$ is $a'_2b'_1$. Therefore, any clique sharing vertices simultaneoulsy with the cliques of $C_4$ and $C_5$ must contain this edge. The vertices that are adjacent simultaneously to $a'_2$ and $b'_1$ form the set $\{d'_i : 1 \le i \le q\} \cup Q$. Consequently, each clique that contains $a'_2$ and $b'_1$ contains solely vertices of this latter set. Since $Q$ is a complete and $\{d'_i : 1 \le i \le q\}$ is an independent set, there are $q$ cliques of $H$ that contain the edge $a'_2b'_1$, and they correspond exactly to the cliques of $C_1$. Therefore the only cliques of $H$ that share vertices with all the cliques of $C_4$ and $C_5$ are the cliques of $C_1$. Since their cardinalities are all equal to $q$, $C_1^K$ is the only $q$-complete of $K(H)$ which is contained in a clique that contains $C_4^K$ or $C_5^K$. Hence $v_1$ is the only one vertex simultaneously adjacent to $v_4$ and $v_5$ in $\Phi_q(K(H))$. Consequently, $T'$ does not have a universal vertex. By Theorem 5, it follows that $K(H)$ is not $(2, q)$-clique-Helly. □

**Corollary 7** *It is NP-hard to verify whether the Helly defect of a graph is at most one.*

## 3 Conclusions

We have proved that the problem of recognizing whether a given graph has Helly defect at most one is NP-hard.

A related problem is to recognize whether the Helly defect of a given graph is finite. It is not known whether this problem is decidable.

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