Introduction

Power plants have particular control systems to ensure stable operation. The satisfactory operation of a power system requires a frequency control that keeps it within acceptable limits when the system is submitted to significant load variation. As the electric network frequency is common to all the system, a change on the active power at one point will be reflected on the net as a frequency variation. The design of proper control systems for hydraulic turbines remains a challenging and important problem due to the nonlinear plant characteristics, increasing number of interconnections, development of large generating units and big load changes and disturbances (Eker, 2004).

The primary control system is composed of the speed sensor, the controller, the actuator and the hydraulic supply system. Its main functions are to maintain the angular speed constant and equal to its nominal value and to change the distributor position when the load varies or the operation conditions (as head) changes in both isolated and grid-connected modes. The isolated operation mode occurs for distributed energy systems or when there is some failure in the tie-line connection to the grid. In this case, the performance requirements for the primary control system are higher because of more severe load oscillations. The report by Tripathy and Bhardwaj (1996) on the control of a small hydro-turbine driven generator concludes that the frequency and load can not be satisfactorily controlled in the isolated mode but only in the grid connected mode, by the combined effects of the speed governor and the integral tie-line bias control.

The development of numerical models for hydropower plants has proved to be useful in understanding plant characteristics and in predicting the system dynamic response for new control systems designs. These models can be used during commissioning of new hydropower plants or for tuning and implementation of new control systems, among other applications (Mansoor et al., 2000).

The present work deals with the operation and control of hydropower plants and its basic equipments. An analysis of the dynamic behavior of a hydropower plant is presented. The main objectives are twofold. The first is to model an actual plant using a nonlinear model based on differential equations with parameters that can be easily estimated or obtained from field tests. The second objective is to study the primary control system for the plant in isolated mode in order to define useful sets of parameters for the chosen controllers. Each operation condition has its requirements so the controller parameters that are adequate to one condition may not be adequate to another one. The use of adaptive control is an option to satisfy different operating conditions. Otherwise, the usual procedure to define the controller parameters is to consider the isolated mode that imposes the most severe operation requirements and guarantees that the stability will be sustained in this case. This procedure is adopted in the stability study of this work.

Nomenclature

\[ A = \text{conduit’s transversal section, m}^2 \]
\[ D = \text{derivative gain} \]
\[ D_{pf} = \text{constant which depends on the load} \]
\[ f = \text{loss coefficient of the conduit, s}^2/\text{m}^5 \]
\[ g = \text{gravitational acceleration, m/s}^2 \]
\[ G_T = \text{distributor transfer function} \]
\[ G_C = \text{controller transfer function} \]
\[ h_p = \text{pressure at the bifurcation, m} \]
\[ h_f = \text{friction pressure loss, m} \]
\[ h_b = \text{friction loss on the common conduit, m} \]
\[ h_i = \text{friction loss on the individual conduit, m} \]
\[ h_1 = \text{pressure at the turbine’s admission, m} \]
\[ h_2 = \text{net head (pressure at turbine’s admission), m} \]
\[ h_3 = \text{head water level, m} \]
\[ h_4 = \text{tail water level, m} \]
\[ h_5 = \text{static pressure (defined by the gross head) of the water column, m} \]
\[ h_{so} = \text{static pressure of the water column at the bifurcation, m} \]
\[ h_{so} = \text{static pressure of the water column at the turbine’s admission, m} \]
\[ H = \text{generator inertia constant} \]
\[ I = \text{integral gain} \]
\[ J = \text{performance index} \]
\[ K = \text{controller’s gain} \]
\[ l = \text{conduit’s length, m} \]
\[ P = \text{proportional gain} \]
\[ P_e = \text{electrical power, W} \]
\[ P_g = \text{generated power, W} \]
\[ P_m = \text{mechanical power, W} \]
\[ q = \text{flow rate, m}^3/\text{s} \]
\[ q_c = \text{flow rate in the common conduit, m}^3/\text{s} \]
\[ q_i = \text{flow rate in the individual penstock, m}^3/\text{s} \]
\[ r = \text{transient droop} \]
\[ S_n = \text{nominal apparent power, V.A} \]


\[ T = \text{time, s} \]
\[ T_i = \text{actuator’s time constant, s} \]
\[ T_{D} = \text{derivative time constant, s} \]
\[ T_{g} = \text{gate time constant, s} \]
\[ T_{I} = \text{integral time constant, s} \]
\[ T_{w} = \text{controller zero parameter, s} \]
\[ T_{w} = \text{water starting time constant of the conduit, s} \]
\[ T_{w} = \text{water starting time constant of the common conduit, s} \]
\[ T_{w} = \text{time constant of each individual penstock, s} \]
\[ y = \text{gate position} \]

**Greek Symbols**

\[ \omega = \text{angular speed, rad/s} \]
\[ \omega_o = \text{nominal angular speed, rad/s} \]
\[ \rho = \text{water specific mass, kg/m}^3 \]

**Subscripts**

base relative to nominal values

c relative to common conduit

f relative to friction

i relative to individual conduit

o relative to static

w relative to water

---

**Model Equations**

### Hydraulic Circuit

The model is based on the assumption that water is an incompressible fluid and that the penstocks are rigid. Although it is well known that effects of fluid compressibility and duct elasticity are important especially for long conduits (Souza Jr. et al., 1999), a simpler model was adopted in this work because its main objective is the analysis of the controller. Two types of plants are presented: single penstock and multiple penstocks with a common conduit.

**Single Penstock.** A single penstock plant has one conduit supplying each turbine. From the momentum balance, the rate of change of flow in the penstock is (IEEE, 1992):

\[ (h_{w} - h_{l} - h_{f}) \frac{gA}{l} = \frac{dq}{dt} \]

(1)

\[ h_{f} = f q^2 \]

(2)

In per unit (pu) it becomes

\[ (\bar{h}_{w} - \bar{h}_{l} - \bar{h}_{f}) \frac{1}{T_{w}} = \frac{dq}{dt} \]

(3)

The water starting time is defined as:

\[ T_{w} = \frac{1}{h_{base}} \frac{gA}{q_{base}} \]

(4)

where \( q_{base} \) and \( h_{base} \) are the nominal values for \( q \) and \( h \). If the penstock presents different sections, then,

\[ T_{w} = \sum \frac{1}{h_{base}} \frac{gA}{q_{base}} \]

(5)

**Multiple Penstocks with a Common Conduit.** A plant with multiple penstocks with a common conduit is represented in the Fig. 1.

From the equations presented, the rate of flow change in the penstocks is (IEEE, 1992)

\[ (h_{w} - h_{l} - h_{f}) \frac{gA}{l} = \left( \frac{dq}{dt} + \cdots + \frac{dq}{dt} \right) \]

(6)

\[ (h_{w} - h_{l} - h_{f}) = T_{w} \frac{dq_{c}}{dt} \]

(7)

\[ q_{c} = \sum q_{i} \]

(8)

For the individual penstock

\[ (h_{w} - h_{l} - h_{f}) - (h_{l} - h_{f} - h_{g}) = T_{w} \frac{dq_{i}}{dt} \]

(9)

The equation can then be written as

\[ (h_{w} - h_{l}) - T_{w} \frac{dq_{c}}{dt} - h_{g} = T_{w} \frac{dq_{i}}{dt} \]

(10)

\[ \frac{dq_{c}}{dt} = \sum \frac{dq_{i}}{dt} \]

(11)

**Equipments**

The basic equipments on a stability study of a hydropower plant are the turbine and the generator. The following models represent their operation and the distributor action.

**Distributor.** The relation between the gate position and the prime control system signal depends on the distributor actuator dynamic behavior and, for a single-stage actuator, can be described by a first order system,

\[ G_{y}(s) = \frac{K}{(T_{i} s + 1)} \]

(12)

**Turbine.** The turbine can be modeled by its valve characteristic as (De Jaeger et al., 1994)

\[ q = G \sqrt{h_{l}} \]

(13)

\[ G \text{ is defined as } \]

\[ G(s) = \frac{G_{y}}{1 + T_{s} s} \]

(14)

\[ G_{y}(y) = d_{o} + d_{1} y + d_{2} y^2 \]

(15)

where \( y = 1 \) for nominal position and \( y = 0 \) for closed position.
The mechanical power, generated by the turbine, can be written as (IEEE, 1992)
\[ P_m = \rho g h_l q - K_f \omega^3 \] (16)

The turbine’s loss coefficient \((K_f)\) is defined as a second order equation,
\[ K_f = a_f q^2 + b_f q + c_f \] (17)

Generator. The difference between the values of the mechanical power \((P_m)\) and the electrical power, which is the power put in the electrical network, \((P_e)\) causes a variation on the axis torque that generates the angular speed variation. Defining the generator inertia constant \(H\) as
\[ H = \frac{\text{kinetic energy at nom. speed}}{\text{nominal apparent power}} = \frac{1}{2} \frac{J \omega_0^2}{S_N} \] (18)

Then, the following equation can be written:
\[ \frac{2HS_N}{\omega_0} \frac{d\omega}{dt} = P_m - P_e \] (19)

or, in per unit,
\[ 2H \frac{d\bar{\omega}}{dt} = \bar{P}_m - \bar{P}_e \] (20)

It can be also defined that
\[ \Delta P_m - \Delta P_e = 2H s \Delta \bar{\omega} \] (21)

The electrical power can be written as function of the load power \((P_G)\) as (Kundur, 1994)
\[ \Delta P_e = \Delta P_G + P_G D_{pf} \omega_0 \Delta \bar{\omega} \] (22)

As result, it can be written:
\[ \Delta P_m - \Delta P_G = 2H s \Delta \bar{\omega} + P_G D_{pf} \omega_0 \Delta \bar{\omega} \] (23)
\[ \Delta \bar{\omega} = \frac{\bar{P}_m - \bar{P}_G}{2H s + P_G D_{pf} \omega_0} \] (24)

Controllers

The simple schematic view of the primary control system is shown in Fig. 2. The feedback constant \((R_c)\) is the frequency droop. Typical values are between 0 and 5%. The following controllers were studied in this work.

**Traditional Controller.** The traditional controller has the following transfer function:
\[ G_c(s) = \frac{1 + sT_f}{s T_f} \] (25)

**Proportional Integral.** The transfer function of this controller is
\[ G_c(s) = K_c \left(1 + \frac{1}{T_f s}\right) \] (26)

**Proportional Integral and Derivative.** This controller has the following transfer function:
\[ G_c(s) = K_c \left(1 + \frac{1}{T_f s} + T_D s\right) \] (27)

**Proportional Integral and Proportional Derivative.** This controller is the combination of a proportional integrative controller and a proportional derivative controller. It has a different schematic view from the others as it is presented in Fig. 3.

**Plant Modeling and Simulation**

Field Tests Results of an Actual Hydropower Plant

Field tests results of an actual hydropower station with three Francis turbines built at the Paranapanema River in Brazil are used to evaluate the model and to calculate its parameters. Figure 4 shows the plant circuit and Tab. 1 presents its main geometric characteristics.
Table 1. Hydraulic circuit.

<table>
<thead>
<tr>
<th>Circuit</th>
<th>L [m]</th>
<th>d [m]</th>
<th>A [m²]</th>
<th>τw turbine [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conduit 1</td>
<td>1728.0</td>
<td>6.16</td>
<td>29.84</td>
<td>0.74</td>
</tr>
<tr>
<td>Conduit 2</td>
<td>355.0</td>
<td>4.00</td>
<td>12.57</td>
<td>0.36</td>
</tr>
<tr>
<td>Individual 1.2.3</td>
<td>40.0</td>
<td>2.20</td>
<td>3.80</td>
<td>0.13</td>
</tr>
<tr>
<td>Spiral Case 1.2.3</td>
<td>16.6</td>
<td>1.81</td>
<td>2.57</td>
<td>0.08</td>
</tr>
<tr>
<td>Draft tube 1.2.3</td>
<td>10.9</td>
<td>2.36</td>
<td>4.37</td>
<td>0.03</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>1.34</td>
</tr>
</tbody>
</table>

The head water level ($h_M$) is 479.2 meters and the tail water level ($h_J$) varies from 303.7 to 311.8 meters. So the net head varies approximately from 165.4 to 173.5 meters. The inertia of the generator ($J$) is 600 ton.m².

The optimal operation conditions are:
- speed: 450 rpm;
- power: 32.37 MW;
- flow rate: 21.0 m³/s;
- net head: 168 m.

The net head is calculated by

$$h_i = h_M - h_J - 0.000964 \sum q^3$$  \hspace{1cm} (28)

The stability study is conducted based on the range of operation close to the optimal conditions. This range comprises net head varying from 167 to 169 meters (operating conditions shown in Tabs. 2, 3 and 4). The loss coefficient $K_f$ is determined by the results on the graphic showed in Fig. 5. The gate opening time, that is the total time to open the gate (from $y=0.0$ to $y=1.0$), is 9.4 seconds.

Table 2. Operation points – 167 m net head.

<table>
<thead>
<tr>
<th>Y [pu]</th>
<th>q [m³/s]</th>
<th>η [%]</th>
<th>P [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>15.3</td>
<td>91.5</td>
<td>22.6</td>
</tr>
<tr>
<td>0.6</td>
<td>18.0</td>
<td>93.5</td>
<td>27.4</td>
</tr>
<tr>
<td>0.7</td>
<td>20.9</td>
<td>94.4</td>
<td>32.0</td>
</tr>
<tr>
<td>0.8</td>
<td>23.3</td>
<td>94.0</td>
<td>35.6</td>
</tr>
<tr>
<td>0.9</td>
<td>25.3</td>
<td>93.5</td>
<td>38.5</td>
</tr>
<tr>
<td>1.0</td>
<td>27.4</td>
<td>92.7</td>
<td>41.4</td>
</tr>
</tbody>
</table>

Table 3. Operation points – 168 m net head.

<table>
<thead>
<tr>
<th>Y [pu]</th>
<th>q [m³/s]</th>
<th>η [%]</th>
<th>P [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>15.3</td>
<td>91.5</td>
<td>23.0</td>
</tr>
<tr>
<td>0.6</td>
<td>18.0</td>
<td>93.6</td>
<td>28.0</td>
</tr>
<tr>
<td>0.7</td>
<td>21.0</td>
<td>94.4</td>
<td>32.4</td>
</tr>
<tr>
<td>0.8</td>
<td>23.4</td>
<td>94.1</td>
<td>36.0</td>
</tr>
<tr>
<td>0.9</td>
<td>25.4</td>
<td>93.5</td>
<td>39.0</td>
</tr>
<tr>
<td>1.0</td>
<td>27.5</td>
<td>92.8</td>
<td>42.0</td>
</tr>
</tbody>
</table>

Parameters Calculation

The base values used to calculate the model parameters are presented in Tab. 5. The parameters of the model are listed in Tab. 6.

Table 5. Base values used to calculate the parameters in per unit.

<table>
<thead>
<tr>
<th>qbase [m³/s]</th>
<th>hbase [m]</th>
<th>Pbase [kW]</th>
<th>ωbase [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate at the optimal point</td>
<td>Gross head at the nominal point</td>
<td>Nominal Power</td>
<td>Nominal Speed</td>
</tr>
<tr>
<td>21.0</td>
<td>168.4</td>
<td>35966.7</td>
<td>7.5</td>
</tr>
</tbody>
</table>

The parameters $G_i$ and $T_i$ of Table 6 are defined as follows:

$$G_i = \frac{1}{P_n D'_\varphi \omega_0}$$ \hspace{1cm} (29)

$$T_i = \frac{2H}{P_n D'_\varphi \omega_0}$$ \hspace{1cm} (30)

The gate time constant is assumed 1.0 second. For the actuator, $T_i$ represents the time necessary to open the gate from $y=0.0$ to $y=0.632$, so $T_i = 5.94$ seconds. This yields the following first order system:

$$G_s (s) = \frac{1}{(5.94 s + 1)}$$ \hspace{1cm} (31)
Table 6. Model parameters in per unit.

<table>
<thead>
<tr>
<th>$T_a$ [s]</th>
<th>$r$</th>
<th>$\tilde{a}_i + \tilde{d}_i y + \tilde{d}_j y^2$</th>
<th>$H$</th>
<th>$G_i$</th>
<th>$T_i$</th>
<th>$\tilde{a}_i q^2 + \tilde{b}_j q + \tilde{c}_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.34</td>
<td>0.002524</td>
<td>$-0.570 + 2.023y - 0.142y^2$</td>
<td>0.469</td>
<td>7.407</td>
<td>6.951</td>
<td>$0.197q^2 - 349q + 204$</td>
</tr>
</tbody>
</table>

Model Results

The model results for one turbine operating with different gate positions are compared to the field results. The deviations were very low, varying from -0.54% to 0.76%, as shown in Tab. 7. These results were considered satisfactory to validate the model of this hydropower plant.

Table 7. Model deviations compared to actual results.

<table>
<thead>
<tr>
<th>$Y$ [pu]</th>
<th>$q$ [%]</th>
<th>$h_r$ [%]</th>
<th>$P_m$ [%]</th>
<th>$P_G$ [%]</th>
<th>$\Delta \omega$ [pu]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.60%</td>
<td>-0.01%</td>
<td>0.76%</td>
<td>0.76%</td>
<td>0.0006</td>
</tr>
<tr>
<td>0.8</td>
<td>0.38%</td>
<td>-0.03%</td>
<td>0.29%</td>
<td>0.29%</td>
<td>0.0004</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.54%</td>
<td>0.01%</td>
<td>-0.15%</td>
<td>-0.15%</td>
<td>0.0010</td>
</tr>
<tr>
<td>1.0</td>
<td>0.04%</td>
<td>0.06%</td>
<td>0.49%</td>
<td>0.49%</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

Primary Control System Analysis

The controllers studied are the traditional, PI, PID and PI-PD, and the frequency. In order to compare the controllers’ performances and to determine the optimal parameters, a performance index is introduced. It evaluates the speed deviation when the system is submitted to a load variation represented by a ramp from 0.779 to 1.009 pu in 10 seconds.

The chosen performance index ($I_p$) is the Integral of the Absolute Error (IAE), where the absolute error (actual frequency, $\omega$, minus nominal frequency, $\omega_0$) is integrated over the analysis time span (Duarte-Mermoud and Prieto, 2004). It is written as

$$I_p = \int |\omega - \omega_0| \, dt$$  \hspace{1cm} (32)

Traditional Controller

The range of values is $T_r$ varying from 0.5 to 2.5, and $r$ varying from 1.0 to 20. Values outside this range did not improve the stability of the system. The performance index calculated in this range is presented in Fig. 6. It shows that the lower both parameters are, the higher is the index. The lowest value of the performance index is $I_p = 0.664$, for $T_r = 0.5$ and $r = 1.0$ m.

Proportional Integral (PI) Controller

The proportional ($P = K_p$) and integral ($I = K_i / T_r$) gains are evaluated for $P$ varying from 0 to 20, and $I$ varying from 0 to 40, that is the range of values proposed by IEEE (1988).

The lower value of the performance index for the PI controller is $I_p = 0.456$, for $P = 4.0$ and $I = 40.0$. In Fig. 7, the surface shows that for higher values for the integral parameter there is a significant reduction of the index when these values vary from 0 to 25. However, for values higher than 25 the reduction is not significant and it is verified that, for values higher than 40, the system performance is no more influenced by this parameter. Besides, for low values of the integral parameter, higher values of the proportional parameter reduce the performance index. On the other hand, for high values of the integral parameter, the proportional gain does not influence significantly the system’s performance.

Figure 6. Performance Index as function of $T_r$ and $r$ – Traditional Controller.

Figure 7. Performance Index as function of $P$ and $I$ – PI Controller.
Proportional Integral Derivative (PID) Controller

For the PID controller, the range is also the recommended by IEEE (1988). The proportional gain \( P = K_c \) varies from 0 to 20, the integral gain \( I = K_c / T_I \) from 0 to 40 and the derivative gain \( D = K_c / T_D \) from 0 to 20. The best performance index is \( I_p = 0.457 \), obtained for \( P = 2.0, I = 40 \) e \( D = 1.0 \), as shown in Fig. 8.

Figure 8. Performance Index as function of D and I – PID Controller – \( P=4.0 \).

Proportional Integral - Proportional Derivative (PI-PD) Controller

In this work, \( P_1 \) is considered 1.0 so there is no proportional gain in the first stage of the controller. The adjustable parameters are \( P_2, I \) and \( D \), and \( P_2 \) varies from 0 to 20, \( I \) from 0 to 40 and \( D \), from 0 to 15. The results in Fig. 9 show that for higher values of \( D \), the system has a lower performance index. For values over \( D = 15 \), the response became too oscillatory and stability is not achieved. The minimum value of the index is \( I_p = 0.657 \), for \( P_1 = 1.0, D = 15, P_2 = 5.0 \) and \( I = 40 \).

Figure 9. Performance Index as function of \( P_2 \) e I – PI-PD Controller – \( P_1=1.0 \) e \( D=9.45 \).

After analyzing each controller submitted to different values of its adjustable parameters, the set that yields the best system performance is presented on Tab. 8. The lower performance index is for the PI controller and the higher is for the Traditional Controller.

Figure 10 shows that for all controllers the system becomes stable in about 50 seconds. The traditional controller response is the most oscillatory, and presents the higher initial peak of -0.07 pu. The responses of the PI and PID controllers are very close to each other. They present an initial peak of -0.68 pu and are damped and not oscillatory. The PI-PD controller has also a not very oscillatory response but the system stabilizes slowly and has an initial peak of 0.60 pu.

Table 8. Optimal values of the performance index.

<table>
<thead>
<tr>
<th></th>
<th>Traditional</th>
<th>PI</th>
<th>PID</th>
<th>PI-PD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal value</td>
<td>0.664</td>
<td>0.456</td>
<td>0.457</td>
<td>0.658</td>
</tr>
</tbody>
</table>

Figure 10. Comparison of the controllers when the system is submitted to a ramp \( \Delta P_G = +0.23 \) pu.

Controller Behavior for a Demand Curve

In order to verify the dynamic behavior of the PI controller with the best performance, the demand curve for a typical day of operation in the hydropower plant mentioned before is presented in Fig. 11. The curve’s critical stability regions are those with higher demand variation. Figure 12 presents the speed variation when the system is submitted to the demand curve. The speed is kept to stability limits and the system has a good dynamic behavior with the use of the PI controller.
Conclusions

This work combines the nonlinear model analysis with a primary control system optimization. The main objectives, the model validation and the definition of the controller’s parameters, are achieved. The model’s results, with its parameters calculated based on an actual hydropower plant operating with one turbine are satisfactory. They present deviations of flow, power and speed lower than 1.0%. These deviations are due to approximations on the model parameters. The analysis shows that the PI controller presents the best performance index ($I_P=0.456$) and the traditional controller has the worst performance ($I_P=0.664$). The response of the PI controller plant model for a real demand curve presents a behavior within the stability criteria for this type of power plants and the mechanical power follows the demand power.

References


