Multivariable H₂ and H∞ Control for a Wind Energy Conversion System – A Comparison

The Wind Energy Conversion System (WECS) is a nonlinear system, highly dependent on a stochastic variable characterized by sudden variations, and subjected to cyclical disturbances caused by operational phenomena. Thus, the quality of a WECS controller is measured by its capacity to deal with unmodeled dynamics, stochastic signals, and periodic, as well as non-periodic disturbances. Since the WECS’ objectives can be easily specified in terms of maximum allowable gain in the disturbance-to-output transfer functions, H₂ and H∞ methodologies can be good options for designing a WECS stabilizing controller, combining specifications such as: disturbance attenuation, asymptotic tracking, bandwidth limitation, robust stability, and trade-off between performance and control effort. Designs for WECS multivariable feedback controllers based on H₂ and H∞ methodologies are presented in this paper. The performances of both controllers are computationally simulated, analyzed and compared in order to identify the advantages and drawbacks of each controller design.

Keywords: Wind Energy Conversion System, Control Theory, H₂ and H∞ control

Introduction

The reproduction of the current energy scenery is impracticable due to problems to overlap the negative effects associated to progressive use of the conventional resources, such as the inevitable exhaustion of the fossil fuels and the environmental problems. Considering the growing energy demand, the importance of renewable and pollution-free technologies must increase in the future energy strategies. This fact has attracted great interest in the development of Wind Energy Conversion Systems (WECS).

The basic configuration of a WECS is a wind turbine (WT) coupled to an electric generator, either directly or by a gear-box. In spite of its simplicity, WECS represents an interesting control problem. Due to difficulty in physical phenomena characterizing by means of experimental investigation, WECS modeling becomes a complex problem. The aerodynamic characteristics of a WT are nonlinear and highly dependent on wind speed, which is characterized by sudden variations and behaves simultaneously as energy supply and disturbance signal. A ripple torque is introduced into WECS by operational phenomena such as tower shadow, wind shear, yaw misalignment and shaft tilt (Freris, 1990). The speed and/or power control of a WECS can be achieve by adjusting the generator torque (Novak et al., 1995) or varying the pitch angle of the blades in some WT configurations (Waszczuk et al., 1981). Thus, a WECS is a nonlinear multivariable system, in which the control system has to deal with several uncertainties, parameter variations, nonlinearities, noise, unmodeled dynamics, periodic and non-periodic disturbances.

In this context, the quality of a WECS control system is measured by its stochastic properties and its capacity to establish a trade-off between detrimental dynamic load reduction and energy conversion maximization, shaping the system dynamics in order to satisfy performance and stability specifications (Leithead et al., 1991). Another important control objective is to reduce the influence of wind fluctuation and ripple torque at any rotation speed, because these cause large power fluctuations and unavoidable vibrations with detrimental effects to WECS (Dessaint et al., 1986). The classic methods do not offer a completely satisfactory solution to WECS control design, resulting in controllers that do not offer the necessary robustness for both stability and performance (Dessaint et al., 1986; Lefebvre and Dubé, 1988; Leithead et al., 1991). Considering that the WECS control objectives can be easily specified in terms of maximum allowable gain in the disturbance-to-output transfer functions, H₂ and H∞ methodologies can be good options for designing a WECS stabilizing controller, since both approaches combine specifications, such as: disturbance attenuation, asymptotic tracking, bandwidth limitation, robust stability and trade-off between performance and control effort. The H₂ methodology is particularly appropriate in situations where disturbance rejection and noise suppression are important, while H∞ is usually preferred when the robustness to plant uncertainties is the dominant issue (Maciejowski, 1989; Skogestad and Postlethwaite, 2001).

The designs of multivariable feedback controllers based on H₂ and H∞ methodologies are presented in this paper. The structure shown in Fig. 1 was used for the rotation control for an upwind variable-pitch horizontal axis WT (HAWT) coupled to an induction generator, which is connected to the electric network via power electronic converters. The performances of both controllers are computationally simulated, analyzed and compared so as to identify advantages and drawbacks of each design.
WECS Model

A nonlinear model of a WECS with suitable complexity was developed for computational simulations. Five distinct WECS subsystems are considered in this modeling: wind, aerodynamics, drive train, pitch actuator and generator.

Wind

The WECS operation is highly dependent on wind speed, a stochastic variable characterized by sudden variations which simultaneously behaves as the energy supply and disturbance signal. Although the wind is a multidimensional stochastic process that depends on time and spatial coordinates, a two-dimensional model is generally enough to evaluate the dynamics of WECS (Waszynczuk et al., 1981). Due to a phenomenon known as “wind shear”, wind speed depends on height. Its value at the representative point 3/4 of the cord of i-th blade at instant t is given by (Golding, 1977; Waszynczuk et al., 1981; Freris, 1981):

\[ V_i = V_i^0 \left[ 1 + \frac{3}{4} \frac{R}{H} \sin(\theta) \right] \]

where \( a \) is a coefficient that depends on local topography, \( \theta \) is the spatial angle of the i-th blade, \( R \) is the WT radius, and \( H \) is the height of the tower that supports the WT. The wind speed \( V_i \) at height \( H \) can be described by four components (Rohatgi and Pereira, 1996; Leith and Leithhead, 1997):

\[ V_H = \Gamma + V_G + V_R + \Delta V \]

where \( \Gamma \) is the effective average wind speed at height \( H \). The discrete longitudinal wind gust \( V_G \) at instant \( t \) can be described as (Hwang and Gilbert, 1978; Anderson and Bose, 1983; Raina and Malik, 1985):

\[ V_G = \frac{3\Gamma}{2\ln\left(\frac{H}{h_o}\right)} \left[ 1 - \exp\left( -\frac{V_i\Delta T_G}{1.48H} \right) \right]^{-1.5} \left[ 1 - \cos\left( 2\pi \frac{t - T_G}{\Delta T_G} \right) \right] \]

where \( T_G \) is the instant when the gust begins, \( \Delta T_G \) is the gust duration, and \( h_o \) is a parameter known as roughness height. Another kind of sudden wind speed variation considered in this modeling is the ramp component \( V_R \), which is given by (Anderson and Bose, 1983):

\[ V_R = V_{R\text{MAX}} \left( \frac{t - T_R}{\Delta T_R} \right) \]

where \( V_{R\text{MAX}} \) is the peak of the ramp, \( T_R \) is the instant when the ramp begins and \( \Delta T_R \) is the ramp duration. For small \( \Delta T_R \), the ramp component can be used as an approach to the wind step. The wind speed stochastic component is the wind fluctuation \( \Delta V \), which can be estimated as (Waszynczuk et al., 1981):

\[ \Delta V = 2\sum_{j=1}^{N} S(\psi_j) \Delta \psi_j^2 \cos(\psi_j + \phi) \]

where \( \psi_j = (i-1/2)\Delta \psi, \phi \) is an independent random variable with uniform density in the interval of 0 to 2\( \pi \), and \( S(\psi) \) is a power spectral density given by:

\[ S(\psi_j) = \frac{2K_v F^2 |\psi_j|}{\pi \left[ 1 + \left( \frac{F \psi_j}{\mu \pi} \right)^2 \right]^2} \]

where \( K_v \) is the superficial drag coefficient, \( F \) is the turbulence scale, and \( \mu \) is the mean wind speed at the reference height. For good results, it is suggested that \( N = 50 \) and \( \Delta \psi \) be between 0.5 and 2.0 rad/s (Anderson and Bose, 1983).

WT Aerodynamics

The aerodynamics of a WT is normally described by dimensionless coefficients, which define the WT ability to convert kinetic energy of moving air into mechanical power \( C_p \) or torque \( C_q \) (Novak et al., 1995; Medeiros et al., 1996). Both coefficients \( C_p \) and \( C_q \) depend on the constructive aspects of the WT blades and are nonlinear functions of pitch angle \( \beta \), yaw angle \( \theta \) and a parameter known as tip-speed ratio \( \lambda \), which is defined for the i-th blade of WT as:

\[ \lambda_i = \frac{R \omega_i}{V_i} \]

where \( \omega_i \) is the rotation of the WT i-th blade. Admitting that the WT is always aligned with the wind direction (\( \theta = 0^\circ \)), the aerodynamic torque \( Q_{ai} \) of the WT’s i-th blade is given by (Waszynczuk et al., 1981; Freris, 1990; Novak et al., 1995):

\[ Q_{ai} = \frac{1}{2} \rho \pi R^2 \frac{C_i(\lambda_i, \beta)}{\lambda_i} V_i^2 = \frac{1}{2} \rho \pi R^2 \frac{C_i(\lambda_i, \beta)}{\lambda_i} V_i^2 \]

where \( \rho \) = air density.

Pitch Actuator

Some WTs have an electro-hydraulic device to adjust the pitch angle \( \beta \) of its blades, which can be modeled as (Johnson and Smith, 1976):

\[ \tau \frac{d^2 \beta}{dt^2} + 2\zeta \tau \frac{d\beta}{dt} + \beta = \beta_c \]

where \( \beta_c \) is the command signal, \( \tau \) is the time constant of the pitch actuator and \( \zeta \) is the damping factor.

Drive Train

The drive train of a WECS can be modeled as a set of masses connected by flexible shafts according to the block diagram shown in Fig. 2 (Novak et al., 1995; Hori et al., 1999). Admitting an ideal gear-box, the mechanical coupling system of a WT with \( n \) blades can be described by the classical rotational dynamics:

- i-th blade:

\[ J_i \dot{\omega}_i + D_i \omega_i = Q_{ai} - D_{ai}(\omega_i - \omega) - Q_{nih} \]

- hub:

\[ J_h \dot{\omega}_h + D_h \omega_h = \sum_{i=1}^{N} [D_{ai}(\omega_i - \omega) - Q_{nih}] - D_{ah}(\omega_h - \omega) - Q_{nhh} \]
generator:

\[ J_g \omega_g + D_g \omega_g = D_{mg} (\omega_m - \omega_g) + Q_{mg} - Q_g \]  

(12)

- shaft torques:

\[ Q_{mph} = K_{mh} (\omega_m - \omega_h) \]  

(13)

\[ Q_{mphg} = K_{hg} (\omega_m - \omega_g) \]  

(14)

where \( \omega_h \) = hub rotation, \( \omega_g \) = generator rotation, \( J_i \) = \( i \)-th blade inertia, \( J_h \) = hub inertia, \( J_g \) = generator inertia (including gear-box inertia), \( D_i \) = \( i \)-th blade damping, \( D_h \) = hub damping, \( D_g \) = generator damping, \( D_{mh} \) = \( i \)-th blade-hub connection damping, \( K_{mh} \) = shaft damping, \( K_h \) = \( i \)-th blade-hub connection stiffness, \( K_{hg} \) = shaft stiffness, \( Q_{mph} \) = \( i \)-th blade torque, \( Q_{mphg} \) = shaft torque and \( Q_g \) = generator torque.

Figure 2. WECS model for simulation.

Generator Torque

The electric generator converts the rotational mechanical energy of the WT into electric energy for customers. For variable speed WECSs, the electric generator must be connected to the grid using power electronic converters. In this case, the generator torque is independent of the WECS dynamics (Novak et al., 1995) and can be considered as a system input in the WECS model. Since the dynamics of electric systems are extremely fast if compared to drive train dynamics, a quasi-static model is assumed for the electric generator.

Controller Design for WECS

Nominal Linear Model

In order to design \( H_2 \) or \( H_\infty \) controllers, it is necessary to obtain a nominal linear model for the WECS. In the nonlinear \( C_{p,\lambda} \) and \( C_{q,\lambda} \) characteristics of a WT for \( \beta = 0^\circ \), which is shown in Fig. 3, it is possible to identify two distinct regions in the WT operation. The stall region (A) is characterized by a positive slope, resulting in an unstable operation with sudden and significant drops in the aerodynamic torque. The stable operational region (B) is characterized by a negative slope, corresponding to normal WT operation, where the aerodynamic torque \( Q_a \) can be linearized as (Novak et al., 1995; Rocha et al., 2001; Rocha and Martins Filho, 2003):

\[ \dot{Q}_a = aV + \gamma \dot{\omega}_g + \kappa \beta \]  

(15)

where \( a \) is a scaling factor for torque disturbance due to wind variations \( V \), \( \gamma \) denotes feedback speed coefficient from the drive train, and \( \kappa \) represents the pitch control gain. In steady state, \( V \) is the wind fluctuation \( \Delta V \), which can be assumed for design purposes as a white noise with zero mean (Waymouch et al., 1981). Since it is desirable to operate at maximum \( C_{p,\lambda} \), the aerodynamic torque linearization can be performed in the corresponding \( \lambda_{opt} \), which is always situated in the normal operation region. Considering \( F_{x_{mn}} \) as the nominal wind speed on the WECS location, the coefficients \( a, \gamma, \) and \( \kappa \) can be easily computed from WT data as:

\[ a = \frac{\partial Q}{\partial V} \bigg|_{\lambda_{opt}, \beta} = \frac{3}{2} \rho A R C_{p_{\lambda_{opt}}} F_{x_{mn}} \]  

(16)

\[ \gamma = \frac{\partial Q}{\partial \omega} \bigg|_{\lambda_{opt}, \beta} = -\frac{1}{2} \rho A R^2 C_{p_{\lambda_{opt}}} F_{x_{mn}} \]  

(17)

\[ \kappa = \frac{\partial Q}{\partial \beta} \bigg|_{\lambda_{opt}, \omega} = \frac{1}{2} \rho A R V_{x_{mn}} \frac{\partial C_{p_{\lambda_{opt}}}}{\partial \beta} \bigg|_{\lambda_{opt}, \omega} \]  

(18)

Figure 3. Aerodynamic characteristics of a WT.

Although a real mechanical drive train has rigid disks, flexible shaft elements with distributed mass and stiffness, an approximated two-mass model shown in Fig. 4 is enough to design a controller for WECS (Freris, 1990; Novak et al., 1995). Admitting an ideal gear-box and reducing all quantities to the primary side, the mechanical coupling can be described as (Leith and Leithead, 1997):
\[ J_i \omega_i + D_i \omega_i = Q_g - Q_{nbg} - D_{nbg} (\omega_i - \omega_x) \]  
\[ J_g \omega_g + D_g \omega_g = D_{nbg} (\omega_i - \omega_g) + Q_{nbg} - Q_g \]  
\[ Q_{nbg} = K_{bg} (\omega_i - \omega_g) \]

where \( J_i \) is the total WT inertia, and \( D_i \) = total WT damping.

The linearized equations 15, 19, 20 and 21 constitute the nominal linear state model of a pitch regulated WT. One of the control inputs is the generator torque \( Q_g \), which represents the electric load mechanically connected to generator. It is adjustable and virtually independent from WECS dynamics (Novak et al., 1995). The second control input is the generator rotation \( \beta \), which greatly impacts the control system due to its active influence on the WT’s aerodynamic efficiency. In this context, WECS configures a multivariable system.

Since the dynamics of the pitch actuator is very fast if compared to WT dynamics, it can be considered as unmodeled uncertainties in this approach.

Control Objectives

Aiming to exploit the trade-off between the control requirements, the nominal model has to be manipulated using weighting functions to obtain a generalized system \( G(s) \) shown in Fig. 5, given by:

\[
G(s) = \begin{bmatrix} x = Ax + Bu + w + Bu & \end{bmatrix}
\]

where \( x \) = state vector, \( u \) = control signals, \( w \) = exogenous inputs, \( z \) = control objectives outputs and \( y \) = measured outputs. The exogenous inputs \( w \) are signals determined by external processes or environments that influence the dynamics of the system, such as reference signals, commands, disturbances and noises.

\[
\text{Figure 5. Generalized system.}
\]

The main control requirement of WECS is to reduce detrimental dynamic loads on the shaft, which is obtained by minimizing the difference \( \Delta \omega = \omega_i - \omega_x \) with a fixed gain \( K_x \). Another important control requirement for fixed and variable speed WECS is the WT rotation control, which can be defined as the reduction of the rotation error \( e_i = \omega_i - \omega_x \). Thus, the second control objective output \( z_2 \) is generated by weighting \( e_i \), with a PI function:

\[
K_x(s) = \frac{z_2(s)}{e_i(s)} = K_x + \frac{K_i}{s}
\]

which implies in the augmentation of the original nominal model with an integrator. The control design has to minimize the effects of wind fluctuation and ripple torque over the energy delivered to electric load, generating the third control objective output \( z_3 \), which is obtained by weighting the generator torque \( Q_g \) with a fixed gain \( K_g \).

Finally, it is necessary to limit the bandwidth of pitch control input \( \beta \) by weighting it with a fixed gain \( K_\beta \). The exogenous inputs \( w \) on WECS are the rotation reference \( \omega_x \) and wind fluctuation \( \nu \).

Due to practical constraints relative to the assembly, cost and maintenance of the sensors, the generator rotation \( \omega_g \) is considered the only measured output \( y \). Considering \( u = [Q_g \; \beta] \), and \( x = [\omega_i \; \omega_g \; Q_{nbg} \; e_i] \), the nominal WECS model augmented with weighting functions is given by:

\[
s = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
y = [0 \; 0 \; 0 \; \omega_g] + [0 \; 0 \; 0 \; w] + [0 \; 0 \; 0 \; 0] u
\]

H₂ Methodology

The \( H_2 \) controller design can be formalized as an optimization problem, where the goal is to find a controller \( K_2 \) that internally stabilizes the system \( G(s) \), so that \( H_2 \) norm:

\[
\| H_{\text{in}} \|_2 = \frac{1}{\sqrt{2 \pi}} \int_0^\infty [H_{\text{in}}(j\omega)]H_{\text{in}}^*(-j\omega)d\omega
\]

is minimized, where \( H_{\text{in}} \) denotes the transfer function matrix from exogenous inputs \( w \) to objective outputs \( z \). This \( H_2 \) optimization problem is equivalent to the conventional LQG problem (Skogestad and Postlethwaite, 2001) involving a cost function:

\[
J = \int_0^\infty \begin{bmatrix} x^T C C x + C D y + y^T D^T y \end{bmatrix} dt
\]

with correlated white noises \( \xi \) (states) and \( \eta \) (measurements) entering in the system via \( w \) channel associated with the correlation function:

\[
E \begin{bmatrix} \xi(t) & \xi(t) \xi(t) \end{bmatrix} = \begin{bmatrix} B_B B_B' & B_B D_{\eta} \\ D_{\xi} B_B' & D_{\xi} D_{\eta} \end{bmatrix} \delta(t-t)
\]

This problem can be solved by the resolution of the following two Riccati equations:

\[
Y_2 A^* + AY_2 - Y_2 C_{\xi} C_{\xi}^* Y_2 + B_B B_B' = 0
\]

\[
A'X_1 + X_1 A - X_1 B_{\eta} C_{\eta}^* Y_2 + C_{\xi} C_{\xi}^* = 0
\]

resulting in an \( H_2 \) optimal controller \( K_2(s) \) given by:

\[
K_2(s) = \begin{bmatrix} X = (A - B_{\eta} B_{\eta}^* Y_2 C_{\xi} C_{\xi}^*)^{-1} \hat{x} + Y_2 C_{\eta}^* y \\
- B_{\eta} C_{\xi} \hat{x} \end{bmatrix}
\]
**H∞ Methodology**

Feedback control design can be also formalized in terms of $H_\infty$ norm optimization. The sub-optimal $H_\infty$ control problem is to find all admissible compensators $K_\infty(s)$ which internally stabilize the generalized system $G(s)$ and minimize the norm (Doyle et al., 1989):

$$\|H_\infty\|_\infty = \sup_\alpha \sigma(H_\infty)$$

(33)

such that $\|H_\infty\|_\infty < \varepsilon$. Considering $D_{11} = 0$ and $D_{22} = 0$, the solution of this problem can be given by:

$$K_\infty(s) = \left[ (A_x - B_2B'_x^2X_x - L_xC_x)\hat{x} + L_{x,y} \right]^{-1} B'_2^2X_x\hat{x}$$

(34)

where

$$A_x = A + \varepsilon^{-2}B_2B'_xX_x$$

(35)

$$L_x = \left( I - \varepsilon^{-2}Y_xX_x \right)^T Y_xC_x^2$$

(36)

and $X_x$ and $Y_x$ are the solutions for two Riccatti equations:

$$A'X_x + X_xA - X_xB_2B'_xX_x + C_xC_x = 0$$

(37)

$$Y_xA + A'Y_x - Y_xC_xC_xY_x + B_2B'_x = 0$$

(38)

where

$$B_2B'_x = \varepsilon^{-2}B_2B'_x - B_2B'_x$$

(39)

$$C_xC_x = \varepsilon^{-2}C_xC_x - C_xC_x$$

(40)

The existence of a solution for $H_\infty$ control problem is assured by the following conditions: $X_x \geq 0$, $Y_x \geq 0$ and the eigenvalues $\rho(X_x,Y_x) \leq \varepsilon^2$. The best solution for sub-optimal/optimal $H_\infty$ controller can be computed using the loop-shifting two-Riccati formulae (Chang and Safonov, 1996).

**Simulation Results**

**Plant Description**

The WECS considered in this paper consists of an upwind Horizontal Axis WT coupled to a 2.5MW four-pole electric generator by a gear-box (ratio 1:102.5) as shown in Fig. 6 (Wasynchuck et al., 1981). This WT has two blades (NACA230XX series airfoil), each one with a length of 45.72 m, where the outer 30% corresponds to the variable-pitch section controlled by a servohydraulic actuator. The generator and other support equipment are enclosed in a nacelle, which is mounted atop a tower with 60.96 m where wind measurements are performed. A yaw control allows the correct alignment of the WT rotor with the wind direction. The complex nonlinear and stochastic mathematical model presented in the section “WECS Model” is used to simulate this WECS, while the nominal model described in the subsection “Nominal Linear Model” is used to design the controllers. The scheme for dynamic simulation for this closed-loop WECS is described in Fig. 7. The main data of this WECS are presented in table 1, including an approach for the power coefficient obtained from the blade geometry. To compute the parameters of the nominal model, the effective average wind speed is considered as 7m/s.
The frequency response of the open-loop WECS is shown in Fig. 8. Although high frequency wind fluctuations are well rejected, WECS is very affected by low frequency wind disturbances. It is noted that the control input $\beta$ is more effective on WT regulation than $Q_g$, although it reduces energy conversion efficiency. Torsional modes can be excited by sudden wind variations and/or operational disturbances since this WECS presents a resonance peak on:

$$\omega_{\alpha} = \sqrt{\frac{K_{bf}}{J_{t} + \frac{1}{J_{g}}}} = 1.9752 \text{ rad/s}$$

**H₂ Controller Performance**

Considering the WECS presented in the subsection “Plant Description”, the use of the H₂ design procedure results in the following controller:

$$K_{s}(s) = \frac{0.07592s^4 - 0.6462s^3 - 3.923s^2 - 2.761s - 0.7238}{s^8 + 4.504s^7 + 11.09s^6 + 13.27s^5 + 7.454s + 7.547 \times 10^5 - 0.2345s^6 - 0.6949s^5 - 1.269s^4 - 0.5969s - 0.07599}$$

The frequency response of the closed-loop WECS with an H₂ controller is shown in Fig. 9. High frequency wind fluctuations are submitted to strong attenuation. For frequencies below 0.7 rad/s, the sensitivity function ($\epsilon / \omega_{\alpha}$) decays rapidly when the frequency tends to zero, as shown in its Bode plots, satisfying the requirements related to disturbance rejections. Bode plots of the complementary sensitivity function ($\omega_{\alpha} / \omega_{\beta}$) show that the H₂ controller attenuates measurements noise above 0.7 rad/s, assuring good robustness against uncertainties above this frequency. In regards to the rotation difference $\Delta \omega$, the excitation of torsional modes is difficult due to an adequate attenuation of reference variations and/or operational disturbances. Although power fluctuations on the electric load are attenuated, the system's response to variations of the electric torque $Q_e$ will be slow.

The simulation results presented in Fig. 10 show the dynamic behavior of the fixed-speed closed-loop WECS when submitted to a wind gust with duration of 90 s. After this event, both control inputs are simultaneously used in the rotation regulation, and $\omega_\alpha$ returns to its reference value $\omega_{\alpha}^{ref}$ after approximately 5 minutes. Considering a variable speed operation, the rotation reference must be adjusted to:

$$\omega_{\alpha} = \frac{\omega_{\alpha}^{ref} V}{R}$$

The simulation results presented in Fig. 11 verify the effect of a wind step variation of 7.5 m/s to 9.5 m/s in the dynamic behavior of the variable-speed closed-loop WECS. In this case, $\omega_\alpha$ follows speed reference $\omega_{\alpha}^{ref}$, reaching zero error after 10 minutes. Aiming to adjust $\omega_\alpha$, the generator torque $Q_g$ is practically duplicated, increasing the energy delivered to the electric load. Considering that $\beta$ has a detrimental effect on energy conversion efficiency, the relatively small contribution of this control input on rotation regulation is positive for variable speed WECS. The system's operation does not excite any torsional modes and the noises introduced by wind fluctuation are filtered by the H₂ controller.

### Table 1. WECS data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
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<td>Cut-in wind speed</td>
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<tr>
<td>Cut-off wind speed</td>
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<tr>
<td>Linearized Aerodynamic Parameters</td>
<td></td>
</tr>
<tr>
<td>$\omega_{sp}$</td>
<td>17.0 exp^6.5002</td>
</tr>
<tr>
<td>$\omega_{st}$</td>
<td>12.5</td>
</tr>
<tr>
<td>$\omega_{tsp}$</td>
<td>17.0</td>
</tr>
<tr>
<td>$\omega_{tst}$</td>
<td>12.5</td>
</tr>
<tr>
<td>$\omega_{opt}$</td>
<td>-2345</td>
</tr>
<tr>
<td>$\omega_{opt}^{ref}$</td>
<td>-2345</td>
</tr>
</tbody>
</table>

The simulation results presented in Fig. 11 verify the effect of a wind step variation of 7.5 m/s to 9.5 m/s in the dynamic behavior of the variable-speed closed-loop WECS. In this case, $\omega_\alpha$ follows speed reference $\omega_{\alpha}^{ref}$, reaching zero error after 10 minutes. Aiming to adjust $\omega_\alpha$, the generator torque $Q_g$ is practically duplicated, increasing the energy delivered to the electric load. Considering that $\beta$ has a detrimental effect on energy conversion efficiency, the relatively small contribution of this control input on rotation regulation is positive for variable speed WECS. The system's operation does not excite any torsional modes and the noises introduced by wind fluctuation are filtered by the H₂ controller.
Controller Performance

Considering the WECS presented in the subsection “Plant Description”, the optimal $H_\infty$ controller is obtained with $\varepsilon = 0.0674$:

$$K_{\infty}(s) = \begin{bmatrix}
2.297s^4 + 2.253s^3 - 8.93s^2 - 2.685s - 0.3474
\
s^6 + 16.25s^5 + 52.89s^4 + 92.42s^3 + 51.75s^2 + 5.18\times10^{-7}
\end{bmatrix}$$

$$-1.138s^4 - 3.18s^3 - 6.059s^2 - 2.58s - 0.22$$

$$s^6 + 16.25s^5 + 52.89s^4 + 92.42s^3 + 51.75s^2 + 5.18\times10^{-7}$$

The frequency response of closed-loop WECS with the $H_\infty$ controller is shown in Fig. 12. Notice that high frequency wind fluctuations are strongly attenuated. The sensitivity function decays rapidly for frequencies below 0.7 rad/s and the complementary sensitivity function is attenuated above 0.7 rad/s, assuring the requirements related to disturbance rejections and robustness against uncertainties. The $H_\infty$ controller provides adequate attenuation of the reference variations or operational disturbances and, if compared to the $H_2$ controller, it provides a greatest attenuation for power fluctuations in the grid, resulting in an extremely slow response for variations on electric torque $Q_e$.

In relation to a fixed-speed operation, the dynamic behavior of the $H_\infty$ closed-loop WECS when submitted to a wind gust with...
duration of 180 s is shown in Fig. 13. If compared with the $H_2$ controller, this controller presents better robustness, without unstable behavior when submitted to greatest disturbances. The control system is able to reject the effects of this wind disturbance using simultaneously $\beta$ and $Q_g$. The turbine speed $\omega_t$ returns to its reference value $\omega_{t0}$ approximately at the end of the wind gust. The simulation results for variable speed closed-loop WECS when submitted to wind step variation from 7.5 m/s to 9.5 m/s are shown in Fig. 14. Notice that the $\beta$ performance on the rotation adjustment is improved. Although $\omega_t$ will eventually reach the reference $\omega_{t0}$, this adjustment is extremely slow.

Conclusions

$H_2$ or $H_\infty$ optimal feedback control problem involves finding a controller $K$ for a generalized system $G(s)$, using optimization techniques for the respective norms. Although both approaches present several similarities, the $H_\infty$ methodology results in a more conservative controller than the $H_2$ methodology, since the disturbance signal dependency is considered in the $H_2$ controller design. Thus, the $H_\infty$ controller presents a better robustness than a similar $H_2$ controller, but its dynamic response is extremely slow. Although the $H_\infty$ solution can be relatively flexible, admitting sub-optimal controllers, the tendency is to use the $H_\infty$ controller in applications involving regulation problems, such as fixed-speed closed-loop WECS, where the output has to stay at determined value despite the presence of great disturbances. In counterpart, the fast response of the $H_2$ controller is more adequate for applications involving tracking problems, such as variable-speed closed-loop WECS, since it is necessary to follow a reference imposed by the wind speed to obtain maximum energy conversion. In this context, an interesting option for WECS controller designs can be the multi-objective $H_2/H_\infty$ optimal control approach, where several channels associated with different norms are established, aiming to simultaneously attend several performance criteria.
References


