Transverse load distribution of skew cast-in-place concrete multicell box-girder bridges subjected to traffic condition

Abstract
Concrete multicell box-girder bridges are a common choice among the designers for various ranges of bridges. In order to provide safer and greater speed of traffic, the roadway is built as straight as possible. The use of skewed bridges has increased considerably in the recent years for roadway. The skewed bridges have quite different mechanical behavior from the straight bridges, although for skew angles less than 20 degrees, it is reasonably safe to ignore the effect of skew angles and analyze that at the straight bridge. In this study, in developing an analytical solution, an extensive parametric study was carried out to determine the maximum positive and negative stress distribution factors and to calculate the maximum distribution factor of deflection along the mid-span of skewed multicell box-girder bridges. A total of 240 representative bridges numerical models were selected and analyzed using SAP2000 finite element software. It was found that the span length, number of boxes, number of lanes and skew angles significantly affected the distribution factors of stress and deflection. Finally, several equations were proposed for stress and deflection distribution factors of multicell box-girder bridges for the application of American Association of State Highway and Transportation officials load and resistance factor design live loads.

Keywords
Skewed bridges, Vehicle, Load distribution factor, Finite element analysis

1 INTRODUCTION

Concrete multicell box-girder bridges (MCB) are the most common type of highway bridges (Song et al., 2003). These bridges have excellent torsional and stiffness, equipped with elegance. The MCB bridges can be built as skew bridges in large urban areas to meet several requirements, including natural or man-made obstacles, complex intersections and space limitations. Although advanced computer techniques can determine the effect of vehicle loads (Lin and Weng, 2004) and distribution of live loads in concrete bridges, until recently, the original “s-over” equations in the American Association of State Highway and Transportation officials (AASHTO, 2002) are used for the calculation of live load distribution factor of skewed bridges. The “s-over” equations are developed for straight bridges and the effects of skew angle and continuity are not included in this code. Such simplification leads to very conservative results for long span bridges, and to unsafe results for short span bridges (Huang et al., 2004; Huo and Zhang, 2008; Sotelino et al., 2004). Several investigations have been carried out to find the effect of skew angle on the live load distribution factor. Ebeido and Kennedy (Ebeido and Kennedy, 1996) observed that as skew angle was less than 30 degrees, neglecting the effect of skew angle was considered...
safe and bridge could be design as right bridge. Some researchers suggest new equations of live load distribution factor for moment and shear based on the data generated from the parametric study on skew continuous slab on girder bridges (Bishara et al., 1993; Khaleel and Itani, 1990). Recent investigation indicated that, however, the effect of secondary components were not taken into account in current bridge design standards, the presence of intermediate diaphragms (ID) highly influence shear and moment distribution factor of skewed bridges (Barr et al., 2001; Cai et al., 2009; Khaloo and Mirzabozorg, 2003; Li and Ma, 2010).

In addition, American Association of State Highway and Transportation officials load and resistance factor design (AASHTO, 2008) takes into account more bridge parameters than the AASHTO standard (AASHTO, 2002) and includes several extensions to basic distribution factor, such as continuity and the skew effect. For instance, the AASHTO LRFD specification presented several skew correction factor (SCF) expressions for shear and moment distribution factors of skewed bridges, however, the accuracy of those is still questionable (Huo et al., 2003; Zhang, 2008). To develop the preciseness of LRFD formulas for distribution factor, Zhang (Zhang, 2008) proposed new skew correction factor expressions for various types of bridge cross sections.

In addition, concrete bridges are expected to crack in the tensile and extreme deflection regions, under heavy truck load conditions and, therefore, the proper reinforcement with high tensile strength material must be provide. To this purpose, the stress and deflection distribution of bridges on transverse and longitudinal direction should be determined. Although many investigations were performed to predict the live load distribution factor of skewed bridges, only limited numbers concentrated on determining the maximum distribution of tensile and compressive stress, and deflection of skewed bridges. In many bridge design procedure, the maximum positive and negative stress of bridges are obtain using the corresponding moment distribution factor formulas in corresponding cross sections. It should be noted that maximum tensile and compressive stress on the cross section are indeed localized, while the moment distribution factors formulas were obtained based on uniformly distribution of stress on bridge cross section. Since, in the most cases, specification's formulas provide highly conservative or unconservative results for stress distribution factor (Zoghi et al., 2008).

The main aim of this study is to investigate the maximum deflection, tensile and compressive stress distribution factor of concrete continuous skewed MCB bridges. A parametric study is performed on 240 prototype bridges to determine effective parameters on live load distribution factor of bridges. The parameters investigated included: skew angle, span length, number of boxes and number of lane. Using a statistical approach several empirical equations are deduced to determine maximum distribution factor of stress and deflection of skewed MCB bridges subjected to the AASHTO LRFD truck loads.

2 GEOMETRY AND DETAILS OF BRIDGE MODELS

In order to develop the live load distribution factor (LDF), a parametric study was carried out on 240 prototype skewed multicell box-girder bridges with various parameters. This parametric study covered a broad range of bridge properties found in practice. Nevertheless, only the parameters of superstructure were used in this study and the variation in substructure was not included in any of the finite element models. The span length of skewed MCB bridges measured along an unsupported edge of the bridges in plan is called the skew span, while the perpendicular distance between the supported lines is called the right span (Gupta and Misra, 2006). Fig. 1 shows the typical cross-sectional symbols for W, B, d, and Lc in Table 1. The bridge properties used for this scope were: (1) the skew span length, L; (2) number of boxes, NB; (3) number of loaded lanes, NL; and (4) skew angle, . The practical ranges of these parameters were selected using a span-to-depth ratio of 24, which has been observed to be the most economical (Hall et al., 1999; Heins, 1978). A preliminary investigation in this study showed that the changing the cross section of intermediate diaphragm and slab thickness had an insignificant effect.
on the live load distribution factor of the MCB bridges. For all bridges used in this study, the modules of elasticity $E$ of concrete and Poisson’s ratio $\nu$ were 22.80 GPa and 0.2, respectively.

### 3 BRIDGE NUMERICAL MODELING

The commercial finite element analysis program, SAP2000 version 12, was used in this study. A four node three dimensional shell element with six degree of freedom at each node were used to model the prototype multicell box-girder bridges. Top and bottom shell element of web are integrated with the top and bottom slab at connection points to ensure compatibility of deformation. The transvers solid diaphragms at supports were modeled using the same element with the size and properties of designated diaphragms (Huo et al., 2005). Fig. 3 illustrates a typical finite element mesh used in analyses of a four-cell MCB bridge.

The effect of bearing and piers on the live load distribution of skewed bridges have been investigated by many researchers (Chun, 2010; Dicleli and Erhan, 2009; Eom and Nowak, 2001; Suksawang and Nassif, 2007). It was observed that piers and bearing would not be affected by the live load in the finite element analysis, and that simulating boundary condition by hinge-roller supports can predict the bridge behavior properly. Therefore, in the presented study, only the superstructure of bridges was modeled and the effect of bearing and piers were neglected. The first abutment was treated as a hinge, at the bottom of each web, which resist both vertical and lateral displacement and all other supports were treated as roller, at the bottom of each web, with prevents only vertical translation.

Validations of the bridge modeling used in this study were performed. The criterion used for this verification was compared to Bridge No.14 of NCHR Project 12-18 (Huo et al., 2003). The comparison of the live load moment and shear distribution factor as shown in Fig. 4 indicated good agreement between the numerical modeling and the mentioned project (Table 2). Based on this validation, the same finite element modeling was used to analyze more of the MCB bridge models.

### Table 1 Parameters Considered in the Parametric Study (in Si Unit)

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Table 2 Comparison of Distribution Factor obtained from numerical modeling and those from NCHRP project 12-28 for bridge No.14

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</table>
4 LOADING CONDITION

The vehicular live loads, designated as HL 93, used in this study were based on the load specified by the American Association of State Highway and Transportation Officials (AASHTO, 2008). The designated HL 93, which consists of a design truck plus design lane load or the design tandem plus lane load, whichever governs, was used in this study to calculate the maximum positive stress and the negative stress at the pier of bridges. A case with 90% of two trucks spaced a minimum distance of 15.20 meter apart in the longitudinal direction plus 90% lane load was used to determine the maximum compressive stress at piers. Fortunately, the SAP2000 has certain feature of AASHTO LRFD, HL vehicular live loads applies only to certain types of bridge response, such as negative and positive stresses or deflection along the span. According to AASHTO LRFD, multiple present factors of 1.00, 0.85 and 0.65 for two, three, and four lane loadings, respectively, were also applied. Using the FEA for the three-dimensional bridges, the maximum stress and deflection were obtained by positioning the wheel loads at a distance of 0.61 meter from the curb edge of the bridge and then moving all live loads foot by foot in transverse direction. The live loads were applied according to the number of lanes as shown in Table 1. The location of live loads in the transverse direction of bridges is shown in Fig. 5. The adjacent wheel lines of the two trucks were placed 1.20 meter apart from each other.

5 STRESS DISTRIBUTION ON MCB BRIDGES

To determine the tensile and compressive distribution factor of skewed MCB bridges, it was necessary to find the location of maximum stress at both longitudinal and transvers directions of bridge superstructures. Fig 6 shows the variations in the normal stress distribution of a four-box prototype MCB bridges with and without skew angle. The vertical and horizontal axes represented normal stress and longitudinal direction, respectively. In addition, the positive values represent tensile stress, while the negative stress values in the figured are compressive stress. It can be observed that the maximum tensile stress (positive stress) for non-skewed bridge is obtained on the bottom slab at the mid-span and the maximum compressive stress (negative stress) is occurred over the intermediate supports of superstructure.
For skewed bridges, the maximum compressive stress takes place at the intermediate support line, in the same way as straight bridges, but the maximum tensile stress occurs at the section which passes through the center of each lane and is parallel to skewed abutment. Moreover, the maximum tensile stress of skewed MCB bridges is obviously higher than right ones because of intensifying effect of torsion in high skewed bridges and also changing the load path in skewed bridges (refer to Fig. 6).

Fig. 7 shows the variations of maximum tensile and compressive stresses in the bottom slab (lowest fiber) of the mid-span and intermediate support line of selected bridge. It can be seen that the largest stress are all located in the intersection of webs, bottom slab and diaphragm. The same trend is observed in skewed MCB bridges.

6 DISTRIBUTION FACTOR

Lateral distribution of the live loads is a major component of bridge design and control. The live load distribution factor (LDF) is commonly obtained as follows (Barker and Puckett, 1997):
Where $F_{\text{refined}}$ corresponds to the largest live loads in the girder from the refined methods; while $F_{\text{beamline}}$ corresponds to the maximum live loads from a simple beam-line model subjected to one lane of traffic. To determine the LDF of multicell box-girder bridges, the cross section was idealized by an equivalent I-beam, including the same size and properties of the skewed MCB bridges, as shown in Fig. 2. Each idealized beam includes one web, as well as bottom and top flanges. According to Eq. (1), the distribution factor of positive ($D\sigma_{po}$) and negative ($D\sigma_{ne}$) stresses and maximum deflection distribution ($D\delta$) were obtained by dividing the maximum response of the finite element models with the largest response from one of the idealized girders with a single lane of traffic.

\[
LDF = \frac{F_{\text{refined}}}{F_{\text{beamline}}}
\]  

(1)

7 DISCUSSION OF THE PARAMETRIC STUDY

A parametric study was performed to examine the effect of main parameters on the maximum distribution factors of deflection and tensile stress at the mid-span and compressive stress at the intermediate piers of prototype skewed MCB bridges.

Fig. 8 presents the influence of changing in the number of lanes on positive and negative stress distribution factor of skewed MCB bridges. It can be seen that number of lanes has an increasing influence on maximum negative and positive stress distribution factors. For instance, for a prototype bridge with span length of 30 meter, by growing the number lane loading from two to four, the positive and negative stress distribution factor increased by around 25% and 20%, respectively.
It also can be observed that the stress distribution factor of shorter spans is higher than those which have longer decks. Moreover, the influence of span length on distribution factor of maximum negative stress is more important than positive one.

Fig. 9 indicates the relationship between the number of boxes and stress distribution factor of positive and negative stress. The figure shows that stress distribution factor decreases as the number of boxes increase. This reduction is too drastic for shorter span bridges. For example, the maximum positive and negative stress of a 30 meter span length bridge, decreased by about 53% and 42%, respectively when the number of bridge changed from 2 to 4.

The effect on the presence of skew angle in the supports of skewed bridges was shown in fig. 10. It is presented in the form of the ratio of maximum distribution factor of the skewed bridges with those of the corresponding right bridges. Rs and Rd stand for the ratio of stress and deflection, respectively. The advantage of this method is that the results would be independent of the LRFD designated truck and therefore would be applied to other bridge specifications.
From fig. 10(a) it could be concluded that skew angle has an insignificant effect on positive stress distribution factor. Therefore, its effect would be neglected in developing new equations for positive stress distribution factor of MCB bridges.

In contrast, the skew angle has an insignificant influence on the negative stress distribution factor of MCB bridges. For example, the ratio Rs is ranging from 1.08 to 1.40 for 30 meter bridges and changing from 1.0 to 1.19 for those with 90 meter span lengths, as skew angle changes from 30 to 45 degrees. As a result, it could be noted that the effect of skew angle is more notable to short bridges, strikingly.

In the same manner as described above, the relationship between skew angle and maximum deflection distribution was drawn in fig. 10(b). It is obvious that there is a converse relationship between deflection and skew angle, so that the maximum deflection distribution factor increases with growing skew angle. However, the effect of skewed angle on maximum deflection is somewhat more significant for the short span bridges.

Fig. 11 shows the effect of changing in the number of lanes and boxes on the deflection distribution factor of bridges. It can be seen that an inverse relationship between maximum deflection distribution factor and number of boxes (refer to fig. 11a). Meanwhile, the effect of span length on deflection distribution changes in similar way as stress distribution factor in which shorter spans have more significant impact on live load distribution factor of skewed bridges. In contrast, a direct relationship between maximum deflection distribution factor and number of boxes can be found. However, the long bridge remains lesser influence on this relevance.

8 LIVE LOAD DISTRIBUTION FACTOR EQUATIONS OF THE SKEWED MCB BRIDGES

As described earlier, the skew angle changes the load path. The live load tends by nature to take a short cut to obtuse corners of the skewed bridge, so the lateral load distributions factor of straight bridges cannot be used for skewed ones, any longer. To consider the skew effect on stress and deflection of MCB bridges, it is a necessity to obtain new simplified equations for those factors. For this purpose, a statistical method based on least square regression (Dicleli and Erhan, 2009) on obtained data from the parametric analysis was applied. Several new equations were proposed to determine the maximum
distribution factor for positive stress and deflection at mid-span, and negative stress over intermediate support line of continuous skewed MCB bridges. One advantage of the new equations is that, unlike most bridge standards, the effect of continuity and skew were taken into account in proposed equations directly.

8.1 Distribution Factor of Positive Stress

As mentioned earlier, the effect of skew on positive distribution factors can be underestimated, so in developing new equation for positive stress distribution factor, only obtained data from straight bridges were used. The following equation was proposed for positive stress distribution factor of MCB bridges:

\[
D_{\sigma_{po}} = \frac{1.86 \times N_l^{0.47}}{L^{0.037} \times N_B^{1.13}}
\]  

(2)

8.2 Distribution Factor of Negative Stress

Fig. 5 shows that the skew angle greatly influenced distribution of negative stress \(D_{\sigma_{ne}}\), since in providing the new equation for negative stress distribution factor over the intermediate supports, the effect of skew abutment was taken into account as function of cosine (Eq. 3).

\[
D_{\sigma_{po}} = \frac{4.27 \times N_l^{0.58}}{L^{0.24} \times N_B^{1.08} \times (Cos\theta)^{0.40}}
\]  

(3)

The great advantage of eq. (3) is that, unlike to LRFD specification, do not need any separate skew correction factor.

8.3 Distribution Factor of Maximum Deflection

In the similar way, the minimum least square fit method was applied to deduce the following equation for maximum deflection distribution factors \(D_{\delta_S}\) at the mid-span of the skewed MCB bridges:

\[
D_{\sigma_{po}} = \frac{3.12 \times N_l^{0.55}}{L^{0.095} \times N_B^{1.35} \times (1 - 0.042 \times Tan\theta - 0.108 \times Tan^2\theta)}
\]  

(4)

the effect of skew angle on maximum deflection distribution factor was expressed as a function of tangent. This equation also can be used to obtain the maximum deflection of straight bridges, it this situation, the second phrase of equation will be equal to zero.

It should be noted that, the proposed equations (Eqs. 2-4) was obtained for the case of continuous multicell box-girder bridges with two equal spans. They were also can be used to simple supported...
bridges and even MCB bridges with two unequal continuous spans by taking the longest span length in equations.

9 VERIFICATION OF THE NEW DISTRIBUTION FACTOR EQUATIONS

To evaluate the accuracy of the proposed equations (Eqs. 2-4), the proposed stress and deflection distribution factors of several prototype MCB bridges were verified against distribution factor from the current methods of analysis. Accordingly, the distribution factors of stress and deflection were calculated using: (1) Finite element method, (2) non-orthogonal Grillage method, (3) Canadian Highway Bridge Design Code (CHBDC, 2000), (4) NCHRP Project 12-26 (Zokaie et al., 1993), and (5) AASHTO standard.

The comparison of results was presented in Table 3. It can be observed that there is a good agreement between adopted herein finite element analysis and proposed equations. The AASHTO specification and the Nchrp Project 12-26 live load distribution factor formulas which are basis of AASHTO LRFD formulas, generally predicted very conservative values for positive and negative stress distribution factors. It is due to the fact that these empirical equations were obtained using experimental and field testing on a limited number of simply support existing bridges. Both methods cannot calculate maximum deflection distribution factor of MCB bridges.

The discrepancies between proposed equations and grillage analysis is less than nine percentages due to assigning a lesser torsional and flexural stiffness to longitudinal and vertical members of grid plan model and also because of assuming uniformly distribution of stress on the cross section. Nonetheless, it can be concluded the non-orthogonal grillage analysis is a reliable and simple method to evaluate distribution of live loads subjected traffic load conditions.

To further verify the applicability of the proposed equations (Eq. 2-4), the average (AVG) and standard deviations (STD) of the ratios of the stress distribution factor from the proposed formulas to FEA results are presented in Table 4. The slightly greater than unity average indicates that the proposed formulas can be used reasonably in the prediction of stress and deflection distribution factors. The low variance of the proposed equations to rigorous analysis data for stress and deflection means an acceptable data with a low variety form of FEA results.

Fig. 5 shows that the skew angle greatly influenced distribution of negative stress ($D\sigma_{ne}$), since in providing the new equation for negative stress distribution factor over the intermediate supports, the effect of skew abutment was taken into account as function of cosine (Eq. 3).

### Table 3 Comparison of different Distribution Factor’s methods

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### Table 4  Average, Standard Deviation and Variance of the ratio proposed equations to FEA

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<tr>
<td>Negative stress</td>
<td>1.016</td>
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<td>Deflection</td>
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**10 SUMMARY AND CONCLUSIONS**

Based on the results of an extensive parametric study on continuous skewed multicell box-girder bridges the following conclusions and recommendations were drawn:

1. The three–dimensional finite element modeling by SAP 2000 is appropriate for evaluating the behavior of skewed bridges.
2. For straight bridges, the maximum tensile stress occurs in the mid-span of longitudinal direction, however, it is provided at the cross section along a line passing among the mid-span of each lane in skewed multicell box-girder bridges.
3. The bridge span length, skew angle, number of boxes and number of lane loadings are the most crucial parameters that affect stress and deflection distributions factor of these types of bridges.
4. The simplified empirical equations were deduced for distribution factor of tensile stress, negative stress and deflection of the skewed multicell box-girder bridges.
5. The effect of skew angle on positive stress distribution factor was negligible.
6. There is a good agreement between finite element analysis, non-orthogonal grillage method and proposed equations. It was discovered that grillage analysis can be used to determine bridge responses.
7. The AASHTO specification and AASHTO LRFD overestimate positive and negative stress distribution factor of skewed multicell box-girder bridges.
8. Proposed equations can be used to determine maximum distribution factors of simply supported bridges and those with two unequal spans, by setting the longest span length into equations.

**References**


