Dynamic response of functionally graded skew shell panel

Abstract

The dynamic response of functionally graded skew shell is investigated using a $C_0$ finite element formulation. Reddy’s higher order theory has been employed to perform the analysis and the volume fractions of the ceramic and metallic components are assumed to follow simple linear distribution law. The present study attempts to focus mainly on the influence of skew angle on frequency parameter and displacement of shell panel with various geometries. Comprehensive numerical results are demonstrated for cylindrical, spherical and hypar shells for different boundary conditions and skew angles. The findings obtained for functionally graded skew shell panels are new and can be used as benchmark for researchers in this field.

Keywords

Skew shell, functionally graded material, finite element formulation, higher order shear deformation theory.

1 INTRODUCTION

Due to commodious applications of functionally graded material (FGM) in various fields of engineering, it enthralled the attention of many researchers worldwide. Moreover, the smooth and continuous change of mechanical properties across the preferred direction made them to occupy forefront in the material research. Understanding the vibration characteristics and dynamic behavior of members made of such materials is of prime importance from structural design point of view.

Owing to the above reasons, a large number of works have been devoted to conceive the vibration characteristics and dynamic response of functionally graded plates and shells exposed to thermo-mechanical loads. Consequently, many theories were developed to model the structure that accurately predicts its response under different loading environment. Some of the most widely adopted theories available in the scientific literature include first order shear deformation theory [26] and higher order shear deformation theory [24, 25]. In the past years, first order shear deformation theory (FSDT) which neglects the effects of transverse shear strain is used to accomplish the linear as well as non linear response of shells. For example, Zhao et al.[6] carried out static and vibration analysis of functionally graded cylindrical shell using element-free kp-Ritz method and found that the volume fraction exponent plays significant role in predicting the response of the shell; Kim et al. [4] presented the
nonlinear analysis of FGM plates and shells using analytical solution and assumes the properties in terms of volume fraction exponent that follows sigmoid function; Arciniega and Reddy [3] presented a tensor based finite element formulation for large deformation analysis of FGM shells; and Reddy and Chin [10] examined the dynamic response of functionally graded plates and shells under thermomechanical environment. But the use of FSDT depends on the shear correction factor which is the cumbersome one to decide. Moreover, the theory may not be accurate in case of thick shells.

To explicate the shortcomings of the first order shear deformation theory many higher order shear deformation theories (HSDT) were developed in due course of time. It is noteworthy to mention that, Reddy’s higher order shear deformation theory [24] is the most widely implemented by many researchers, where the realistic parabolic variation of transverse shear strain has been taken into account to eliminate the use of shear correction factor. Here, we cite the papers where the higher order theory [24] is successfully implemented with some analytical tools. Yang and Shen [18] analyzed the effect of thermal field on free and forced vibration analysis of functionally graded plates that combines the Reddy’s higher order shear deformation plate theory with Galerkin technique. The plates with properties in between ceramic and metal components do not show the intermediate response, when the properties are considered as temperature dependent. Static and dynamic response of functionally graded plates using meshless local petrov-Galerkin approach in conjunction with higher order theory has been done by Qian et al.[19]. Mori-Tanaka method that includes interactions between various elastic constants is used to estimate the properties of the functionally graded plate. Neves et al. [14] extended the Carrera’s unified formulation to perform vibration analysis of cylindrical shells. Two cases of transverse displacement (constant transverse displacement and quadratic variation with thickness coordinate) are considered to determine the frequency parameter of the cylindrical shell panels. Isvandzibaei and Moarrefzadeh[5] performed the free vibration analysis of FGM shells and influence of different parameters on frequency characteristics of shell are discussed briefly. Yang and Shen[15] examined the free vibration and stability analysis of FGM cylindrical shell panels under thermal and mechanical loads. Reddy’s higher order theory, Galerkin technique and Blotin’s method are applied to study the response of the shell panels under static and periodic loads. Setareh and Isvandzibaei[8] studied the vibration characteristics of functionally graded cylindrical shell using Reddy’s higher order shear deformation theory. Influence of constituent volume fraction on frequency parameter was studied using Nickel and stainless steel shell panels. Pradyumna and Bandyopadhyay[13] located the unstable regions in functionally graded shell panels with different geometry (cylindrical, spherical, hypar and conical) using finite element formulation.

Other studies include the dynamic response and stability analysis of functionally graded shells by various numerical techniques. Ng et al. [9] carried out the stability analysis of FGM cylindrical shells using Bolotin’s method. It is mentioned that control over the response of the plate can be achieved by proper variation of volume fraction exponent. Dynamic response of FGM shell under point load was investigated by Han et al.[12]. Nezhadi and Ayob [1] studied the dynamic response of the functionally graded cylindrical shell using Rayleigh-Ritz technique. Moreover, studies pertaining to special cases like shells embedded with piezoelectric layers are also considerable in number. Among them are: Wu and Syu[11], who studied the static response of functionally graded piezoelectric shells; and Alibeigloo et al.[7], who analyzed the free vibration of functionally graded piezo shells. On the whole, it can be interpreted that functionally graded materials are widely used in diverse fields of engineering, where situations like structural elements subjected to ultra high temperatures and sudden change in tempera-
ture within a fraction of seconds are encountered. Despite of high cost of this material which is consid-
ered as a drawback, proper design and tailoring of such material to suit different requirement made
them to stand in the row of advanced materials.

To date, vibration and dynamic solution of functionally graded shell panels are limited to rectangu-
lar plan form only. Hence, an attempt is made to fill the apparent void exists in the literature by pre-
senting the finite element solution to non rectangular plan form such as skew shells which have wide
range of applications in modern construction industry. Reddy’s higher order shear deformation theory
[24] which satisfies the condition of zero transverse shear stress at top and bottom of the shell is im-
plemented. The formulation also incorporates the term for twist curvature \(1/R_{xy}\) which plays a vital
role to analyze the special forms like hypar shell, which is not yet done in any other formulation that
incorporates Reddy’s higher order theory. The present study is divided into two parts. The first part
gives deep insight about the vibration characteristics of various forms of functionally graded skew
shells (cylindrical, spherical and hypar) by incorporating different parameters such as skew angle \(\alpha\),
thickness ratio \((a/h)\), curvature ratio \((R/a)\) and boundary conditions (simply supported and clamped).
In the second part, dynamic response of skew shell is performed using Newmark integration scheme
[16]. It is anticipated that the present results paves the way for researchers who are involved in the
area of functionally graded skew shells.

2 MODELING AND FORMULATION

2.1 Shell geometry

A shell element having skew boundary with Cartesian coordinate system is depicted in Fig. 1. The mid
surface of the shell is assumed as origin for the material coordinate system. The top surface of the shell
\((z=+h/2)\) is rich in ceramic content, whereas the bottom surface of the shell \((z=-h/2)\) is rich in metal
content. A nine noded isoparametric Lagrangian shell element (Fig. 2) having seven nodal unknowns is
employed to model the present shell element. For analysis of skew shells, the edges of the boundary
elements are not parallel to the global axes \((x, y)\) of the shell. Hence it is required to carry out the
necessary transformation from global axes to local axes by using nodal transformation matrix \([T]\).

For the shell finite element used in the present study the following transformation matrix \([T]\) is utilized.

\[
[T] = \begin{pmatrix}
\cos \alpha & -\sin \alpha & 0 & 0 & 0 & 0 & 0 \\
\sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \alpha & -\sin \alpha & 0 & 0 \\
0 & 0 & 0 & \sin \alpha & \cos \alpha & 0 & 0 \\
0 & 0 & 0 & 0 & \cos \alpha & -\sin \alpha & 0 \\
0 & 0 & 0 & 0 & \sin \alpha & \cos \alpha & 0 \\
\end{pmatrix}
\]

(1)

where \(\alpha\) is the skew angle of the shell. For the elements which does not lies on skew edges no transfor-
mation will be required. For hypar shells, the surface equation can be expressed in the following man-
ner. It should be noted that the ratio \(c/a\) implies the twist curvature for hypar shell.
\[ z = 4 \frac{c}{ab} xy + \frac{cx}{a} + \frac{cy}{b} \]  

(2)

Figure 1  Plan view of FGM skew shell.

Figure 2  IsoparametricLagrangian element in natural co-ordinate system.

2.2 Effective properties of shell

Due to the dissimilarity of material properties along certain direction, it is necessary to evaluate the properties accurately using suitable method. Different schemes were proposed in the literature and some of them are: three phase model of Frohlich and Sack [20]; self consistent scheme [21]; Mori-Tanaka technique [22]; mean field approach [23]; Voigt method; and the representative volume element. Most widely adopted methods in the literature are Mori-Tanaka technique and Voigt method. In the present study Voigt method is employed to estimate the effective properties, such as, Young’s modulus \((E)\), Poisson’s ratio \((\gamma)\) and mass density \((\rho)\) of the shell panel as a function of position.

Based on the linear distribution law, effective properties of the shell constituents \((E, \gamma \text{ and } \rho)\) are expressed in terms of volume fraction of the ceramic and metal content as mentioned below.
\[ E(z) = \left[ E_t - E_b \right] \left( \frac{z}{h} + \frac{1}{2} \right)^n + E_b \]

\[ \gamma(z) = \left\{ \gamma_t - \gamma_b \right\} \left( \frac{z}{h} + \frac{1}{2} \right)^n + \gamma_b \]

\[ \rho(z) = \left( \rho_t - \rho_b \right) \left( \frac{z}{h} + \frac{1}{2} \right)^n + \rho_b \]

where the subscripts “\( t \)" and “\( b \)" refers to the top and bottom of the surface of the shell respectively, \( n \) is the non-negative key parameter that describes the optimum distribution of constituents along the thickness direction of the shell. It takes the value between zero and infinity (i.e., zero corresponds to ceramic portion and infinity corresponds to metal portion). Since the variation of Poisson’s ratio is negligible, it is assumed as constant in the present analysis.

The constitutive relationship of functionally graded shell may be written as,

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy} \\
\sigma_{yz} \\
\sigma_{xz}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{21} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{33} & 0 & 0 \\
0 & 0 & 0 & Q_{44} & 0 \\
0 & 0 & 0 & 0 & Q_{55}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix}
\]  

(4)

where \( Q_{ij} \) contains the terms elastic moduli \((E)\) and Poisson’s ratio \((\gamma)\), in which \( E \) alone is the function of depth as given below.

\[ Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, \quad Q_{12} = Q_{21} = -\frac{\gamma E(z)}{1-\nu^2}, \quad Q_{44} = Q_{55} = \frac{E(z)}{2(1+\gamma)} \]

Here, the Young’s modulus \((E)\) and Poisson’s ratio \((\gamma)\) of the panel at any height \((z)\) of the shell can be easily estimated by using Equation (3). It should be noted that, the term \( \left( \frac{z}{h} + \frac{1}{2} \right)^n \) involving in Equation (3) implies the volume fraction of the ceramic content \((V_c)\) present in the panel considered. Further, the correlation between the volume fraction of ceramic \((V_c)\) and metal \((V_m)\) components is given by the relation \(V_c + V_m = 1.0\).

### 2.3 Displacement field

To describe the deformation profile of the shell panel, a special form of displacement field proposed by Reddy [24] is chosen, where the in-plane displacement fields \((u\) and \(v)\) are expanded as cubic functions of the thickness coordinate \((z)\), while the transverse displacement \((w)\) variable has been assumed to be constant through the thickness. Any other choice of displacement field would either not satisfy the stress-free boundary conditions or lead to a theory that would involve more dependent unknowns than...
those in the first-order shear deformation theory [24]. Also, the theory leads to the parabolic distribution of transverse shear stresses and therefore the need of shear correction co-efficient could be avoided. According to Reddy’s higher order shear deformation theory [24], the in-plane displacements \((u\) and \(v)\) and transverse displacement \((w)\) are expressed in terms of corresponding displacements at the mid surface \((u_0, v_0\) and \(w_0)\) by the following expression.

\[
\begin{align*}
\textbf{u}(x, y, z) &= u_0(x, y) + z\theta_x(x, y, z) + z^2\xi_x(x, y, z) + z^3\zeta_x(x, y, z) \\
\textbf{v}(x, y, z) &= v_0(x, y, z) + z\theta_y(x, y, z) + z^2\xi_y(x, y, z) + z^3\zeta_y(x, y, z) \\
\textbf{w}(x, y) &= w_0(x, y)
\end{align*}
\]

where \(u, v\) and \(w\) are the displacements of any general point in the shell. The parameters \(u_0, v_0\) and \(w_0\) are the displacements of points which are in the mid-surface (i.e., reference surface) of the shell and \(\theta_x, \theta_y\) are the bending rotations defined at the mid-surface about the \(y\) and \(x\) axes respectively. \(\xi_x, \xi_y, \zeta_x\) and \(\zeta_y\) are higher order terms appears in Taylor’s series expansion and solved by the condition of zero transverse shear strains \((\gamma_x(x, y, \pm h/2) = \gamma_y(x, y, \pm h/2) = 0)\) at the top and bottom of the shell surface. Thus, incorporation of the above condition in Equation (5) leads to the expression for unknown higher order terms \((\xi_x, \xi_y, \zeta_x\) and \(\zeta_y)\). Finally, by substituting the values of unknown higher order terms \((\xi_x, \xi_y, \zeta_x\) and \(\zeta_y)\) in Equation (5) and rearranging all the terms that appears in the displacement field \((u\) and \(v)\), the following final expression may be obtained.

\[
\begin{align*}
\textbf{u}(x, y, z) &= u_0(x, y, z) + z\theta_x(x, y, z) - \frac{4z^3}{3h^2} \left( \theta_x + \frac{\partial w}{\partial x} \right) \\
\textbf{v}(x, y, z) &= v_0(x, y, z) + z\theta_y(x, y, z) - \frac{4z^3}{3h^2} \left( \theta_y + \frac{\partial w}{\partial y} \right) \\
\textbf{w}(x, y) &= w_0(x, y)
\end{align*}
\]

In Equation (6), it is to be noted that the in-plane displacement field \((u\) and \(v)\) invites the problem of \(C_1\) continuity by the presence of second order derivatives in the strain part. The problem of choosing \(C_1\) elements are well known due to its practical applications. In order to overcome the problem of \(C_1\) continuity requirement at the time of finite element implementation the terms involving derivatives of transverse displacement are treated as separate field variables, i.e., \(\psi_x = \left( \theta_x + \frac{\partial w}{\partial x} \right)\) and \(\psi_y = \left( \theta_y + \frac{\partial w}{\partial y} \right)\).

Hence by above substitution, the final in-plane displacement fields \((u\) and \(v)\) for skew shell with the coordinate axes \((x', y', z')\) can be modified as

\[
\begin{align*}
\textbf{u}(x', y', z') &= u_0(x', y', z') + z\theta_x \left( 1 - \frac{4z^2}{3h^2} \right) - \frac{4z^3}{3h^2} \psi_x \\
\textbf{v}(x', y', z') &= v_0(x', y', z') + z\theta_y \left( 1 - \frac{4z^2}{3h^2} \right) - \frac{4z^3}{3h^2} \psi_y
\end{align*}
\]
Hence, the basic field variables interpreted in the present study are $u_0$, $v_0$, $w_0$, $\theta_x$, $\theta_y$, $\phi_{x*}$ and $\phi_{y*}$ for each node thus forming a total of 63 nodal unknowns for the element.

### 2.4 Mathematical formulation

#### 2.4.1 Strain displacement relation

All the formulation in the present study is confined to linear elastic behavior with small displacements and hence small strains. The linear strain-displacement relations according to Sander’s shell theory are

$$
\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{R_x} $$
$$
\varepsilon_y = \frac{\partial v}{\partial y} + \frac{w}{R_y} $$
$$
\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_y} $$
$$
\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - C_1 \frac{u}{R_x} - C_1 \frac{v}{R_y} $$
$$
\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - C_1 \frac{v}{R_x} - C_1 \frac{u}{R_y} $$

Equations (8)

Where $R_x$, $R_y$ represents the radii of curvature in the $x$ and $y$ directions respectively and $R_{xy}$ is the twist radii of curvature. $C_1$ is the tracer that helps to reduce the approximation in to Love’s shell theory and it is taken as unity in the present formulation. To combine equation (6), (7) and (8), the strain terms may be re-written as

$$
\varepsilon_x = \varepsilon_{x0} + z \left( 1 - \frac{4z^2}{3h^2} \right) k_x - \frac{4z^3}{3h^2} k_x^* $$
$$
\varepsilon_y = \varepsilon_{y0} + z \left( 1 - \frac{4z^2}{3h^2} \right) k_y - \frac{4z^3}{3h^2} k_y^* $$
$$
\gamma_{xy} = \gamma_{xy0} + z \left( 1 - \frac{4z^2}{3h^2} \right) k_{xy} - \frac{4z^3}{3h^2} k_{xy}^* $$
$$
\gamma_{xz} = \phi_{x0} + z \left( 1 - \frac{4z^2}{3h^2} \right) k_{xz} - \frac{4z^3}{3h^2} k_{xz}^* - \frac{4z^2}{h^2} k_{xz}^{**} $$
$$
\gamma_{yz} = \phi_{y0} + z \left( 1 - \frac{4z^2}{3h^2} \right) k_{yz} - \frac{4z^3}{3h^2} k_{yz}^* - \frac{4z^2}{h^2} k_{yz}^{**} $$

Equations (9)
where the different terms involved in the above equation are defined in the following fashion.

\[
\begin{align*}
\{ e_{xw}, e_{yw}, \gamma_{yw} \} &= \left\{ \frac{\partial u}{\partial x} + w_x \frac{\partial v}{\partial y} + w_y \frac{\partial u}{\partial y} + \frac{2w_x}{R_x}, \frac{\partial v}{\partial y} + \frac{2w_y}{R_y} \right\} \\
\{ \phi_x, \phi_y \} &= \left\{ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial x} - C_i \frac{\partial u}{R_x} - C_i \frac{\partial v}{R_y}, \frac{\partial v}{\partial y}, \frac{\partial \psi}{\partial y} + \theta, \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial x} \right\} \\
\{ k_{xw}, k_{yw}, \sigma_{xw}, \sigma_{yw}, \kappa_{xw}, \kappa_{yw} \} &= \left\{ -C_i \frac{\theta}{R_x} - C_i \frac{\theta}{R_y} - C_i \frac{\theta}{R_x} - C_i \frac{\theta}{R_y}, -C_i \frac{\psi}{R_x} - C_i \frac{\psi}{R_y}, -C_i \frac{\psi}{R_x} - C_i \frac{\psi}{R_y}, \frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x} + \psi \right\} \\
\{ f \} &= \frac{\partial^2}{\partial t^2} \left\{ f \right\} = -\omega^2 \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} = -\omega^2 \begin{bmatrix} F \end{bmatrix} \{ f \} \\
\end{align*}
\]

\[ \text{2.4.2 Free vibration analysis} \]

The acceleration at any point within the element may be expressed in terms of the mid-surface parameters \( (u_0, v_0, \text{and } w_0) \) as

\[
\{ f \} = \frac{\partial^2}{\partial t^2} \left\{ f \right\} = -\omega^2 \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} = -\omega^2 \begin{bmatrix} F \end{bmatrix} \{ f \} \\
\]

Where \( \{ f \} = \begin{bmatrix} u_0, v_0, w_0, \theta_x, \theta_y, \psi_x, \psi_y \end{bmatrix}^T \) and the matrix \( [F] \) contains the terms involving \( z \) and \( h \) as expressed below.

\[
[F] = \begin{bmatrix} 1 & 0 & 0 & z & 0 & -4z^3 \\ 0 & 1 & 0 & 0 & z & -4z^3 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \\
\]

Again the matrix \( \{ f \} \) is decoupled into matrix \( [C] \) that contains the interpolation functions \( (N_i) \) and global displacement vector \( \{ X \} \).

\[
\{ f \} = [C] \{ X \} \\
\]
where the global displacement vector \{X\} contains the nodal unknowns for all the nine nodes and thus forming the matrix of order 63x1. i.e., \{X\} = [u_1 \ v_1 \ w_1 \ \theta_1 \ \varphi_1 \ w_2 \ \theta_2 \ \varphi_2 \ \ldots \ \theta_9 \ \varphi_9], where i=1-9.

The shape functions in \{C\} associated with the present nine noded Lagrangian element are as given below.

\[ N_i = \frac{1}{4}(\xi-1)(\eta-1)\xi\eta, N_2 = \frac{1}{4}(\xi+1)(\eta-1)\xi\eta, N_3 = \frac{1}{4}(\xi+1)(\eta+1)\xi\eta, \]
\[ N_4 = \frac{1}{4}(\xi-1)(\eta+1)\xi\eta, N_5 = -\frac{1}{2}(1-\xi^2)(1-\eta)\eta, N_6 = -\frac{1}{2}(1+\xi)(\eta^2-1)\xi, \]
\[ N_7 = -\frac{1}{2}(\xi^2-1)(1+\eta)\eta, N_8 = -\frac{1}{2}(\xi-1)(\eta^2-1)\xi, N_9 = (1-\xi^2)(1-\eta^2). \]

Finally utilizing the Equations (10) and (11), the mass matrix of an element may be expressed as,

\[ [m] = \int_A [C]^T [L] [C] dA \] (12)

where matrix \{L\} can be written as

\[ [L] = \int z [F]^T [F] dz \] (13)

Where \(\varrho\) is the density of the material estimated from Equation (1). Hence the governing equation for free vibration analysis becomes,

\[ ([K] - \omega^2 [M]) \{X\} = \{0\} \] (14)

where \([M]\), \([K]\) and \(\omega\) are global mass matrix, global stiffness matrix and frequency parameter. The right hand side of the above equation zero represents the problem of free vibration analysis. Eigen value algorithm is utilized to extract the mode shapes of the shell panel.

\[ \text{2.5 Dynamic Response} \]

For the problem of forced vibration Equation (14) is modified to incorporate damping matrix \{C\} and the force vector \{q\} at the right hand side. Hence the governing equation for forced vibration analysis becomes,

\[ [M] \ddot{U} + [C] \dot{U} + [K] U = \{q\} \] (15)
Where, \([M]\) and \([K]\) represent the global mass matrix and stiffness matrix respectively. \([C]\) is the Rayleigh damping matrix and it is considered as below.

\[
[C] = \alpha[M] + \beta[K]
\]  

(16)

In the above form, \(\alpha\) and \(\beta\) are constants to be determined from two given damping ratios corresponding to two unequal frequencies of vibration. \(\{q\}\) appearing in Equation (12) is the dynamic pressure applied on the top of the shell. It is given by,

\[
q(x, y, t) = q_0 F(t)
\]  

(17)

Here, \(q_0\) is the maximum amplitude and \(F(t)\) is a dynamic load shape function of time domain. In the present analysis \(F(t)\) is taken as unity for the case of suddenly applied load. The extension of the linear acceleration method known as Newmark integration method [16] is used to obtain the transient response of the system. A step-by-step procedure for the problem of dynamic response is summarized below.

1. For the problem under consideration the stiffness matrix \([K]\), mass matrix \([M]\) and damping matrix \([C]\) are formed as an initial step.
2. The magnitude of displacement \((U)\), velocity \((\dot{U})\) and acceleration \((\ddot{U})\) at time \(t=0\) are initialized.
3. The time step \(\Delta t\) is chosen, and parameters \(\alpha\) and \(\beta\) are to be determined from damping ratios that corresponds to two unequal natural frequencies obtained from free vibration analysis.
4. The co-efficients are determined from the expression given below.

\[
a_0 = \frac{1}{\alpha \Delta t^2}; a_1 = \frac{\beta}{\alpha \Delta t}; a_2 = \frac{1}{\alpha \Delta t} - 1; a_3 = \frac{\beta}{\alpha} - 1; a_4 = \frac{\Delta t^2}{2} \left( \frac{\beta}{\alpha} - 2 \right); a_5 = \Delta t(1 - \beta); a_6 = \delta \Delta t
\]  

(18)

5. Effective stiffness matrix \([K']\) is formed as

\[
[K'] = [K] + a_0[M] + a_1[C]
\]  

(19)

6. The above formed effective stiffness matrix \([K']\) is triangularized. Then, effective loads \([R']\) are calculated at time \(t+\Delta t\).
7. From the effective stiffness matrix \([K']\) and load matrix \([R']\) generated from step 5, the displacement is solved for time \(t+\Delta t\) and subsequently the velocity and acceleration can be estimated at time interval \(t+\Delta t\).
3 DISCUSSION ON NUMERICAL PROBLEMS

This section is broken down into three parts: (1) The accuracy and efficiency of the present finite element formulation are validated with the existing literature data for free and forced vibration analysis; (2) Vibration analysis is done for skew shells with cylindrical \((R_x=R, R_y=R_{xy}=\infty)\) and hyperbolic \((R_x=R_{xy}=-R, R_y=\infty)\) geometry; and (3) Transient response of the cylindrical skew shell is studied under suddenly applied dynamic pressure. Properties of the ceramic and metal constituents adopted to perform these analyses are mentioned below.

FGM I: Silicon Nitride (Si₃N₄)/ Stainless steel (SUS304):
\[ E_c = 322.27\text{GPa}, E_m = 207.78\text{GPa}, \gamma = 0.3, \rho_c = 2370, \rho_m = 8166. \]

FGM II: Silicon carbide (SiC)/ Aluminium (Al):
\[ E_c = 427\text{GPa}, E_m = 70\text{GPa}, \gamma_c = 0.17, \gamma = 0.3, \rho_c = 3210, \rho_m = 2707. \]

All the results presented herein are in non-dimensional forms and following are the different non-dimensional parameters implemented in the study.

\[ \Omega = \Omega a^2 \sqrt{12\rho_m(1-\gamma^2)/E_mh^2} \]

\[ w(a/2,b/2) = \frac{wE_mh}{qa^2} \]

\[ t = \frac{t}{\sqrt{E_m/qa^2\rho_m}} \]

\[ \sigma_x(a/2,b/2) = \frac{\sigma_x h^2}{qa^2} \]

3.1 Free vibration analysis– validation study

The free vibration of FGM I cylindrical shell with simply supported boundary condition is demonstrated in Table 1. The mode shapes of first four frequencies for different power law exponent \(n = 0.0, 0.2, 2.0, 10.0\) and 1000 (very high value), with geometric properties \(a/R = 0.1\) and \(a/h = 10\) are investigated. The source papers considered for comparison purposes are: Neves et al.[14], who adopted higher order shear deformation theory in conjunction with Carrera’s unified formulation [28-30] and collocation radial basis techniques [31-34]; Pradyumna and Bandyobadyay [17], who presented free vibration solution using higher order shear deformation theory [25] combined with finite element formulation; and Yang and Shen [15], who carried out the vibration analysis using higher order shear deformation theory [27] and semi analytical approach. It may be concluded that the present results exhibit close range with the above cited reference data for the maximum number of cases.
Table 1  Vibration modes of square cylindrical shell (FGM I) with clamped boundary condition.

<table>
<thead>
<tr>
<th>Power law exponent ((n))</th>
<th>References</th>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>Present(12X12)(^a)</td>
<td>74.503</td>
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\(^a\) indicates mesh size

Maximum exception cases in first frequency mode are observed with Pradyumna and Bandyopadhyay [17] for the values of power law exponent \(n = 0.0, 2.0\) and 10.0. The probable reason may be due to the different model of higher order theory involved in the reference paper by Pradyumna and Bandyopadhyay [17]. In case of Reddy’s higher order deformation theory [24] used by the authors, the unknowns present in the in-plane displacement fields are determined by satisfying the condition of zero transverse shear stress at the top and bottom surface to be zero, which is not in the case of displacement field proposed by Kant and Khare [25]. Also, for higher modes, the results obtained by the present study shows slight deviation from the reference data. The different methods proposed to extract the frequencies may be the influence factor for this deviation amongst the results.
3.2. Dynamic response - validation study

The above formulation is extended to study the transient response of shell panel having different skew angles (α) and power law exponent (n). Validation part considers the square simply supported FGM (Al/Zr0₂) plate with geometric properties a = b = 0.2m and h = 0.01m. The corresponding material properties are: \(E_c = 151\text{GPa}, \gamma_c = 0.3, \rho_c = 3000\ \text{kg/m}^3\) for Zirconia (Zr0₂) and \(E_m = 70\text{GPa}, \gamma_m = 0.3, \rho_m = 2707\ \text{kg/m}^3\) for Aluminium (Al). The plate is subjected to a uniformly distributed load of \(10^6\ \text{N/m}^2\) in upward direction and time step of 0.00001s is considered. Fig.3 reveals the comparison of present results with those of Praveen and Reddy [2] which is based on first order shear deformation theory [35, 36]. The results are compared for selected values of power law exponent \(n = 0.0, 1.0\ and 1000\), again a good agreement between the results is observed for all the values of n considered.

![Figure 3](image_url)

Figure 3  Transient response of plate (FGM I, simply supported) – Validation study.

3.3 New results

After examining the effectiveness of the present formulation, the study is extended to perform the vibration and dynamic analysis of FGM skew shells. The different shell forms such as cylindrical, spherical and hypar are considered to generate new results.

3.3.1 Skew cylindrical shell with various thickness ratios (a/h)

Table 2 present the non dimensional frequency of square clamped cylindrical skew shell with \(R/a = 5.0\) for several skew angles (α). Power law exponent (n) is varied from ceramic phase to metal phase according to Equation (1) to show its influence on frequency parameter. It is seen that, as the power law index rises, the frequency of the shell tends to reduce, which is also the most common observation in case of shells with no skew boundary. The low stiffness offered by the metal portion may be the contributing phenomenon for the above statement. Next, the increasing trend of the frequency with fall in thickness of the shell, due to dominance of mass effect is observed. Also, with the increase of skew angle of the shell (i.e., beyond skew angle 30°), the frequency parameter tends to increase at faster rate.
(nearly about 1.5-1.7 times). For thick shells ($a/h = 5.0$ and 10.0) with clamped boundary and skew angle 30°, the deviation in results is found from other cases ($a/h = 20.0, 50.0$ and 100). Therefore, it can be inferred that the present model will not accurately predict the frequency in case of thick clamped cylindrical skew shell predominantly for skew angle 30°. Fundamental frequency mode of simply supported cylindrical skew shell with $R/a = 5.0$ and different values of power law exponent ($n$) are established in Table 3. As estimated, the clamped skew shell establish higher frequency compared to simply supported shell, due to high stiffness.

Table 2 Non dimensional frequencies of square cylindrical shell (FGM II) with clamped boundary condition ($R/a=5.0$).

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Table 3  Non dimensional frequencies of square cylindrical shell (FGM II) with simply supported boundary condition (R/a=5.0).

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Table 4  Non dimensional frequencies of square spherical shell (FGM II) with clamped boundary condition (R/a=5.0).

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Table 5 Non-dimensional frequencies of square spherical shell (FGM II) with simply supported boundary condition ($R/a=5.0$).

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3.3.3 Skew cylindrical and spherical shell with several curvature ratios ($R/a$)

This example refers to the square cylindrical and spherical shell with $a/h=10$, power law exponent $n=1.0$, having simply supported and clamped boundary condition. Various values of $R/a$ ratio (0.2, 0.5, 5.0, 10.0, and 50.0) are selected to perform the study. Influence of $R/a$ ratio on frequency parameter for cylindrical, spherical skew shell with simply supported and clamped boundary condition are investigated in Fig.4 and Fig.5, respectively. It should be noted that, $R=1/radius$ of curvature is adopted in this case. Up to a certain value of $R/a$ (i.e., $(R/a)=2.0$), it endures decline tendency in hasty manner, after which it converges to a constant for all the values of $R/a$ considered. The shell with clamped boundary confirms elevated frequency compared to shell with simply supported boundary. Further-
more, spherical skew shell authenticates its superiority over cylindrical skew shell irrespective of the value of R/a considered.

3.3.4 Skew hypar shell with various c/a ratios

In this example, the term c/a is used as an indicator of the twist curvature of hypar shell. Effect of c/a ratio on frequency parameter for simply supported and clamped boundary conditions having geometric properties a/h=10.0 and power law exponent (n)=1.0 is illustrated in Table 6. When the c/a ratio improves from 0.0 to 0.3, the frequency of the shell increases for all the skew angles (α) considered. As anticipated, shell with clamped boundary shows more frequency than shell with simply supported boundary.
3.3.5 Cylindrical skew shell subjected to dynamic pressure

In order to generate new results for dynamic response of cylindrical (FGM I) skew shell the effects of different parameters such as skew angle \((\alpha)\), volume fraction index \((n)\), shell geometry (cylindrical and spherical) and aspect ratio \((b/a)\) are considered and the results are presented in the form of Figures (Figs. 6-9). Simply supported boundary condition is adopted to perform all the problems related to dynamic response of the panel and the displacement at the center of the shell is shown in all the figures. As a first illustration, in order to study the consequence of change of skew angle on the central displacement, cylindrical shell with \(a/h=10.0\) is used and depicted in Fig. 6. In this example, the value of the skew angle ranges from 15° to 60° and a linear variation of \(n\) \((n=1.0)\) is considered. Cylindrical shell with skew angle 30° endures the maximum displacement; and the minimum displacement is observed for skew angle 60°. Hence it is concluded that, an increase in skew angle contributes more stiffness to the shell under consideration thus recording minimum displacement at the center of the shell.

Fig. 7 reveals the consequence of aspect ratio \((b/a)\) on central displacement component for cylindrical shell with skew angle \((\alpha) =15°\). Four different cases of aspect ratio \((b/a=0.5, 1.0, 2.0\) and \(5.0)\) and skew angle 15° are chosen to perform the study. Smaller aspect ratio \((b/a=0.5)\) ensures maximum central displacement while the minimum value is observed for the value of \(b/a=5.0\). Shell with aspect ratio \(b/a=5.0\) exhibits negligible displacement is also observed in Fig. 7. In Fig. 8, skew shell \((a =15°)\) with two different geometry namely, cylindrical and spherical shells are considered for the study. As expected, the spherical shell report less deflection compared to cylindrical shell, thus ensuring its high stiffness. Next, the power law exponent \((n)\) is varied from ceramic \((n=0)\) to metal segment \((n=very\ high\ value)\), to study its influence on transient response of cylindrical skew shell as demonstrated in Fig. 9. Shell with pure metal \((n=very\ high\ value)\) gives maximum displacement, followed by composite shell and pure ceramic shell \((n=0.0)\). Dominance of stiffness effect offered by pure ceramic shell may be the possible cause for the above observation. At the end, variation of axial stresses over a period of time for cylindrical shell having skew angle 0° to 60° is also studied. The shell with skew angle \((\alpha) 30°\) gives maximum axial stress compared to other skew shells.
Figure 6  Influence of skew angle (α) on the transient response of cylindrical shell (FGM I, n =1.0, a/h=10.0)

Figure 7  Effect of aspect ratio (b/a) on dynamic response of cylindrical skew shell (FGM I, α=15°, n=1.0, a/h=10.0)
Figure 8  Influence of shell geometry on the dynamic response of cylindrical skew shell (FGM II, $\alpha = 15^\circ$, $n = 1.0$, $a/h = 10.0$)

Figure 9  Influence of power law exponent ($n$) on the dynamic response of cylindrical skew shell (FGM II, $\alpha = 15^\circ$, $a/h = 10.0$)
In the present paper the dynamic response of functionally graded skew shell has been studied by using a \( C_0 \) finite element formulation which is developed to overcome the issue of \( C_1 \) continuity associated with the present higher order shear deformation theory (HSDT). Different types of skew shell geometries are considered and various conclusions regarding the analysis are highlighted in the discussion section. The term for twist curvature is also included in the formulation to analyze special shell forms such as hypar shells. Based on the detailed study, the following observations are drawn regarding the free and forced vibration response of different types of functionally graded shells by varying different geometric and material parameters.

i. **Skew angle:** Increase in skew angle \( (a) \) exhibit higher frequency irrespective of the value of powerlaw exponent \( (n) \) considered and hence ensures minimum displacement. Also, shell with skew angle 30° gives the maximum axial stress.

ii. **Shell geometry:** Spherical skew shell establish better performance in vibration and transient response compared to cylindrical skew shell when boundary condition and other parameters (i.e., geometric properties and power law exponent) are kept constant.

iii. **Boundary conditions:** Skew shell with clamped boundary shows higher frequency than shell with simply supported boundary, due to the high rigidity in the first case.

iv. **Other parameters:** Due to preponderance effect of either mass or stiffness, fundamental frequency tends to decrease with the (a) raise in curvature ratio \( (R/a) \) and (b) fall-off in thickness ratio \( (a/h) \) for all the skew angles assumed.
The above conclusions may be helpful for the researchers affianced in analysis and design of skew shell panels, as they are reported for the first time.

References