Abstract

A numerical method is developed for the buckling analysis of moderately thick plate with different boundary conditions. The procedure use the finite strip method in conjunction with the refined plate theory (RPT). Various refined shear displacement models are employed and compared with each other. These models account for parabolic, hyperbolic, exponential, and sinusoidal distributions of transverse shear stress, and they satisfy the condition of no transverse shear stress at the top and bottom surfaces of the plates without using a shear correction factor. The number of independent unknown functions involved here is only four, as compared to five functions in the shear deformation theories of Mindlin and Reissner. The numerical results of present theory are compared with the results of the first-order and the other higher-order theories reported in the literature. From the obtained results, it can be concluded that the present study predicts the behavior of rectangular plates with good accuracy.

Keywords

Refined plate theory; buckling analysis; finite strip method; shear deformation plate theories.

1 INTRODUCTION

The buckling behavior of orthotropic and laminated composite plates has been extensively studied, and various plate theories have been developed on the basis of transverse shear deformation effect.

The classical plate theory (CPT), which totally disregards the transverse shear deformation effect, provides reasonable results for thin plates (Das, 1963; Harik and Ekambaram, 1988; Bao et al., 1997; Leissa and Kang, 2002, 2005; Eisenberger and Alexandrov, 2003; Hwang and Lee, 2006; Ovesy et al., 2012); however, for moderately thick plates, it underestimates the deflections and
overestimates the buckling loads and natural frequencies. To overcome this shortcoming of the
CPT, many shear deformation plate theories, which account for the transverse shear deformation
effects, have been developed including the first-order shear deformation theory (FSDT) developed
by Reissner (1945); Mindlin (1951). The FSDT accounts for the transverse shear deformation
effect, but requires a shear correction factor to satisfy the stress-free conditions at the top and
bottom surfaces of the plate (Dawe and Roufaeil, 1978; Wang et al., 2001; Bui and Rondal, 2008).
Although the FSDT provides a sufficiently accurate description of response for thin to moderate-
ly-thick plates, it is not convenient to use due to the difficulty of determining an accurate shear
correction factor. Thus, to avoid the use of a shear correction factor, many higher-order shear
deformation plate theories (HSDTs) were proposed, including the theories of Reddy (1984); Amb-
barsumian (1958); Levinson (1980); Murthy (1981); Kaczkowski (1968); Panc (1975); Karama et
al. (2009, 2003); Mantari et al. (2012); Zenkour (2005); Mechab et al. (2012); Touratier (1991);
Benyoucef et al. (2010); Atmane et al. (2010); Soldatos (1992). Although the HSDTs with five
unknowns provided sufficiently accurate results for thin to thick plate, their equations of motion
were more complicated than those of the FSDT and CPT. Therefore, Shimpi (2002) developed a
two-variable refined plate theory (RPT) which is simple to use. The Shimpi’s theory is based on
the assumption that the in-plane and transverse displacements consist of bending and shear com-
ponents, and that the bending components do not contribute to shear forces and, likewise, the
shear components do not contribute to bending moments. The most significant feature of this
theory is that it applies transverse shear strains across the thickness as a quadratic function and
satisfies the zero stress boundary conditions at the top and bottom surfaces of the plate without
using a shear correction factor. Also, by having fewer unknowns in the equations, this theory en-
joya simpler form which is close to that of the classical plate theory. Some of the most impor-
tant papers written based on this theory are:

Shimpi and Patel (2006a) extended the RPT to the vibration of isotropic plates. The RPT was
applied to orthotropic plates by Shimpi and Patel (2006b) in the bending and vibration problems.
Thai and Kim (2012, 2011) derived the Levy solution of the RPT for the bending, buckling, and
vibration of orthotropic plates. Kim et al. (2009) derived the Navier solution of the RPT for the
buckling of orthotropic plates. Vo and Thai (2012) adopted the RPT for the buckling and vibra-
tion analyses of laminated beams. Recently, the RPT has been extend to nanobeams (2012), na-
noplates (2013, 2011), functionally graded sandwich plates (2011), and functionally graded plates
(2012). Most of the studies based on the refined plate theory has been confined to the use of a
particular function for the prediction of transverse shear deformation and have been conducted by
using the Navier and Levy solutions.

In this paper, various simple higher-order shear deformation plate theories for the buckling of
orthotropic and laminated composite plates are developed. These theories account for parabolic,
hyperbolic, exponential, and sinusoidal distributions of transverse shear stress, and they satisfy
the condition of no transverse shear stress at the top and bottom surfaces of the plates without
using a shear correction factor. The number of unknown functions involved here is only four,
compared to five functions in the case of shear deformation theories of Mindlin and Reissner
which by removing this one unknown, we can save in the volume, time and cost of extra compu-
tations. The analysis employs the finite strip method. This method is applied to study the local
instability of thick plates under compression with different boundary conditions. The numerical
results of present theory are compared with the results of the first-order and the other higher-order theories reported in the literature. This paper is organized into the following sections. In section 2, the different shear strain shape functions are presented and its application in the finite strip procedure is overviewed. Numerical results and discussions are presented in section 3. In section 4, some concluding remarks are highlighted.

2 THEORETICAL FORMULATION

2.1 Refined plate theory (Basic assumptions)

Consider the plate and a cartesian coordinate system as shown in Figure 1.

![Figure 1: Illustrations of displacements and plate meshing arrangement.](image)

The assumptions of the present theory are as follows:

i. The displacements are small in comparison with the plate thickness and, therefore, the resulting strains are infinitesimal.

ii. The transverse normal stress $\sigma_z$ is negligible in comparison with the in-plane $\sigma_x$ and $\sigma_y$.

iii. The transverse displacement $w$ includes two components of bending $w_b$ and shear $w_s$. These components are functions of coordinates $x, y$.

$$w(x, y, z) = w_b(x, y) + w_s(x, y)$$  \hspace{1cm} (1)

iv. The in-plane displacements $u$ and $v$ consist of extension, bending, and shear components.

$$u = u_e + u_b + u_s \text{ and } v = v_e + v_b + v_s$$  \hspace{1cm} (2)
The bending components $u_b$ and $v_b$ are assumed to be similar to the displacements given by the CPT. Therefore, the expressions for $u_b$ and $v_b$ are

$$u_b = -z \frac{\partial w_b}{\partial x} \quad \text{and} \quad v_b = -z \frac{\partial w_b}{\partial y} \quad (3.\text{a})$$

The shear components $u_s$ and $v_s$, in conjunction with $w_s$, give rise to the $f_i(z)$ variations of shear strains $\gamma_{xz}, \gamma_{yz}$ and hence to shear stresses $\sigma_{xz}, \sigma_{yz}$ along the plate thickness $h$ in such a way that shear stresses $\sigma_{xz}, \sigma_{yz}$ are zero at the top and bottom surfaces of the plate. Consequently, $u_s$ and $v_s$ can be expressed as

$$u_s = f_i(z) \frac{\partial w_s}{\partial x} \quad \text{and} \quad v_s = f_i(z) \frac{\partial w_s}{\partial y} \quad (3.\text{b})$$

The objective of this paper is to develop various models to employ the new functions $f_i(z)$ for the buckling analysis of orthotropic and laminated composite plates under compression loading. These functions are shown in Table 1 and are depicted in Figure 2.

<table>
<thead>
<tr>
<th>$f_i(z)$ function</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(z) = \frac{z}{2} \left( \frac{h^2}{4} - \frac{z^2}{3} \right) - z$</td>
<td>Parabolic shear deformation theory (PSDT)</td>
</tr>
<tr>
<td>$f_2(z) = h \sinh \left( \frac{z}{h} \right) - z \cosh \left( \frac{1}{2} \right) - z$</td>
<td>Hyperbolic shear deformation theory (HSDT)</td>
</tr>
<tr>
<td>$f_3(z) = ze^{-\frac{z^2}{h^2}} - z$</td>
<td>Exponential shear deformation theory (ESDT)</td>
</tr>
<tr>
<td>$f_4(z) = \frac{h}{\pi} \sin \left( \frac{\pi z}{h} \right) - z$</td>
<td>Sinusoidal shear deformation theory (SSDT)</td>
</tr>
</tbody>
</table>

Table 1: Different shear strain shape functions.

Figure 2: Variation of functions $f_i(z)$ along the plate thickness.
Functions \( f_i(z) \) must be chosen to satisfy the following constraints:

\[
\int_{-h/2}^{h/2} f_i(z) \, dz = 0, \quad \left. \frac{\partial f_i(z)}{\partial z} \right|_{z=\pm h/2} = -1
\]  

(4)

2.2 Kinematics

Based on the assumptions made in the preceding section and using equations (1) through (3b), the displacement field can be obtained as

\[
u(x, y, z) = u_0(x, y) - z \frac{\partial w_b}{\partial x} + f_i(z) \frac{\partial w_s}{\partial x}
\]

\[
v(x, y, z) = v_0(x, y) - z \frac{\partial w_b}{\partial y} + f_i(z) \frac{\partial w_s}{\partial y}
\]

\[
w(x, y, z) = w_b(x, y) + w_s(x, y)
\]  

(5)

where \( u \) and \( v \) are the in-plane displacements at any point \((x, y, z)\) in direction of \(x\) and \(y\) respectively; and \( u_0 \) and \( v_0 \) denote the in-plane displacements of point \((x, y, 0)\) on the mid-plane in \(x\) and \(y\) direction respectively, and \( f_i(z) \) is placed from Table 1.

The kinematic relations can be obtained as follows:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial y} & \frac{\partial v}{\partial y}
\end{bmatrix}
+ z \begin{bmatrix}
-\frac{\partial^2 w_b}{\partial x^2} & -\frac{\partial^2 w_b}{\partial x \partial y} \\
-\frac{\partial^2 w_b}{\partial y^2} & -\frac{\partial^2 w_b}{\partial x \partial y} \\
-\frac{\partial^2 w_b}{\partial x \partial y} & -\frac{\partial^2 w_b}{\partial y^2}
\end{bmatrix}
+ f(z) \begin{bmatrix}
\frac{\partial^2 w_s}{\partial x^2} & \frac{\partial^2 w_s}{\partial x \partial y} \\
\frac{\partial^2 w_s}{\partial y^2} & \frac{\partial^2 w_s}{\partial x \partial y} \\
2 \frac{\partial^2 w_s}{\partial x \partial y}
\end{bmatrix}
, \begin{bmatrix}
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix} = g(z) \begin{bmatrix}
\frac{\partial w_s}{\partial y} \\
\frac{\partial w_s}{\partial x}
\end{bmatrix}
\]  

(6.a)

where

\[ g(z) = \frac{df(z)}{dz} + 1 \]  

(6.b)

2.3 Constitutive equations

It is assumed that the laminate is manufactured from orthotropic layers of pre-impregnated unidirectional fibrous composite materials (see Figure 3). Neglecting \( \sigma_z \), the stress-strain relations for each layer in the \((x, y, z)\) coordinate system may be written as

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy} \\
\sigma_{yz} \\
\sigma_{zz}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{12} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{66} & 0 & 0 \\
0 & 0 & 0 & Q_{44} & 0 \\
0 & 0 & 0 & 0 & Q_{55}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix}
\]  

(7)
where $Q_{ij}$ are the plane stress-reduced stiffness values, which are known in terms of the engineering constants in the material axes of the layers:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{16} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \quad (8)$$

where $E_1$ and $E_2$ are the Young’s moduli; $\nu_{12}$ and $\nu_{21}$ are the Poisson’s ratios, and $G_{12}$, $G_{23}$ and $G_{13}$ are the shear moduli.

By performing a coordinate transformation, the stress-strain relations in the global coordinate system can be obtained as

$$
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy} \\
\sigma_{yz} \\
\sigma_{xz}
\end{bmatrix} =
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\
\bar{Q}_{16} & \bar{Q}_{26} & Q_{66} & 0 & 0 \\
0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\
0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix} \quad (9)
$$

and the compact form of Eq. (9) will be

$$\{\sigma\} = \bar{Q}\{\varepsilon\} \quad (10)$$

The components of $\bar{Q}$ for each laminated plate has been discussed by Reddy (2004).
Figure 4: Pre-buckling stresses in a strip.  

Figure 5: Pre-buckling system of displacements in a strip.

2.4 Finite strip method

In this section, the rectangular plate is modeled by a number of finite strips, each of which has three equally spaced nodal lines (see Figure 5) (Cheung, 1976). For the $m^{th}$ harmonic, the displacement parameters of nodal line $i$ are

$$\{ \delta \}_{im} = \begin{cases} [u_{im}, v_{im}, w_{im}^b, \theta_{im}^b, w_{im}^s, \theta_{im}^s]^T, & \text{for } i = 1, 3, \\ [u_{im}, v_{im}]^T, & \text{for } i = 2 \end{cases}$$

(11.a)

where

$$w^b = w_a, \quad w^s = w, \quad \theta^b = \frac{\partial w}{\partial x}, \quad \text{and} \quad \theta^s = \frac{\partial w}{\partial x}$$

(11.b)

The unknown displacement field functions (Eq. (5)) are assumed as follows:

$$u_0 = \sum_{m=1}^{r} \sum_{i=1}^{3} X S_m \{ \delta \}_{im}$$

(12.a)

$$v_0 = \sum_{m=1}^{r} \sum_{i=1}^{3} Y S'_m \{ \delta \}_{im}$$

(12.b)

$$w_b = \sum_{m=1}^{r} \sum_{i=1}^{3} R^b S_m \{ \delta \}_{im}$$

(12.c)

$$w_s = \sum_{m=1}^{r} \sum_{i=1}^{3} R^s S_m \{ \delta \}_{im}$$

(12.d)

in which

$$S'_m = \frac{d(S_m)}{dy}$$

(13)
where \( r \) is the number of harmonics and \( S_m \) is the \( m^{th} \) term of the basic function series (see Appendix) corresponding to particular end conditions, and \( X, Y, R^b, R^s \) are the interpolation matrices defined by Eq. (14).

\[
X = \begin{bmatrix} \frac{1}{2} \eta(\eta - 1) & 0 & 0 & 0 & 0 & 0 & 1 - \eta^2 & 0 & \frac{1}{2} \eta(\eta + 1) & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14a)
\]

\[
Y = \begin{bmatrix} 0 & \frac{1}{2} \eta(\eta - 1) & 0 & 0 & 0 & 0 & 0 & 1 - \eta^2 & 0 & \frac{1}{2} \eta(\eta + 1) & 0 & 0 & 0 \end{bmatrix} \quad (14b)
\]

\[
R^b = \begin{bmatrix} 0 & 0 & \frac{1}{4}(1 - \eta)^2(2 + \eta) & b_x(1 + \eta)(1 - \eta)^2 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}(1 + \eta)^2(2 - \eta) & -\frac{b_x}{8}(1 - \eta)(1 + \eta)^2 & 0 & 0 \end{bmatrix} \quad (14c)
\]

\[
R^s = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{4}(1 - \eta)^2(2 + \eta) & b_x(1 + \eta)(1 - \eta)^2 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}(1 + \eta)^2(2 - \eta) & -\frac{b_x}{8}(1 - \eta)(1 + \eta)^2 \end{bmatrix} \quad (14d)
\]

In the above equations, \( \eta = \frac{2r}{b} \) and \( b_x \) is the strip width.

It should be noted that the Hermitian cubic polynomials used in the interpolation functions of \( w_b \) and \( w_s \) in the \( x \) direction, guarantee the inter-element continuity of the transverse displacement \( w \) and of its first derivatives \( \partial w_b / \partial x \) and \( \partial w_s / \partial x \). The linear and nonlinear buckling strain vectors \( \{ \varepsilon_L \} \) and \( \{ \varepsilon_{NL} \} \) are given by

\[
\{ \varepsilon_L \} = \left[ \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial z}, \frac{\partial w}{\partial x} \right]^T \quad (15)
\]

\[
\{ \varepsilon_{NL} \} = \left[ \frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 \right]
\]

\[
\left[ \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}, \frac{\partial w}{\partial y} \right] \quad \left[ \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial w}{\partial x} \right] \quad \left[ \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial w}{\partial x} \right]
\]

\[
\{ \varepsilon_L \} = \sum_{m=1}^{r} \sum_{i=1}^{3} \left[ B \right]_{im} \{ \delta \}_{im} \quad \{ \varepsilon_{NL} \} = \sum_{m=1}^{r} \sum_{i=1}^{3} \left[ B \right]_{im} \{ \delta \}_{im} \quad (17)
\]

where \( \left[ B \right]_{im} \) is the strain matrix.

The total strain energy \( U \) stored during buckling may be written as

\[
U = \frac{1}{2} \int_V \{ \varepsilon_L \}^T \{ \sigma \} dV \quad (18)
\]

where \( V \) is the volume of the strip. Hence, by substituting Eqs. (10) and (17) into Eq. (18) the stiffness matrix is obtained from

\[ U = \frac{1}{2} \{ \delta \}^T \{ K \}_{ijmn} \{ \delta \}_{jn} \]  

(19)

in which \( \{ K \}_{ijmn} \) is the stiffness matrix corresponding to nodal lines \( i \) and \( j \), and it can be expressed as

\[ \{ K \}_{ijmn} = \int_{a}^{b} \int_{0}^{1} [B]_{im}^T \{ \mathcal{Q} \} [B]_{jn} \, dx dy dz \]  

(20)

where \( m \) and \( n \) denote the related series terms.

The strip is subjected to in-plane stresses \( \sigma_x \) and \( \sigma_y \) shown in Figure 4. The potential energy reduction of these stresses \( (V_p) \) during buckling is given by

\[ V_p = \frac{1}{2} \int_V \{ \varepsilon_{NL} \}^T \{ \sigma \} dV \]  

(21)

By appropriate substitution, the stability matrix \( \{ K_G \} \) can be obtained from

\[ V = \frac{1}{2} \{ \delta \}^T \{ K_G \}_{ijmn} \{ \delta \}_{jn} \]  

(22)

in which

\[ \{ K_G \}_{ijmn} = \frac{1}{2} \int_V \left\{ [G_u]_{im}^T \{ \sigma \}^0 [G_u]_{jn} + [G_v]_{im}^T \{ \sigma \}^0 [G_v]_{jn} + [G_w]_{im}^T \{ \sigma \}^0 [G_w]_{jn} \right\} dV \]  

(23)

Where

\[ \left\{ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right\} = \sum_{m=1}^{r} \sum_{i=1}^{3} [G_u]_{im} \{ \delta \}_{im} \]  

(24.a)

\[ \left\{ \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right\} = \sum_{m=1}^{r} \sum_{i=1}^{3} [G_v]_{im} \{ \delta \}_{im} \]  

(24.b)

\[ \left\{ \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} \right\} = \sum_{m=1}^{r} \sum_{i=1}^{3} [G_w]_{im} \{ \delta \}_{im} \]  

(24.c)

and

\[ \{ \sigma \}^0 = \begin{bmatrix} \sigma_x^0 \\ \sigma_y^0 \\ \sigma_y^0 \end{bmatrix} \]  

(25)

In the equations (24a-c), \( [G_u]_{im} \), \( [G_v]_{jn} \) and \( [G_w]_{im} \) are the stability matrices. Once the stiffness matrix \( \{ K \}_{ijmn} \) and stability matrix \( \{ K_G \}_{ijmn} \) have been derived, and combined for each com-
posite strip, they can be assembled into the respective global matrices $[K]$ and $[K_G]$ using standard procedures. The buckling problem can then be solved by eigenvalue equations

$$([K] - \lambda [K_G]) \{\Delta\} = 0$$

where $\lambda$ is a scaling factor related to the critical load and $\{\Delta\}$ is the eigenvector.

### 3 NUMERICAL RESULTS

The numerical program has been written in the MATLAB environment which can model various boundary conditions and three types of isotropic, orthotropic and laminated composite plates.

In this section, to verify the accuracy of the RPT in predicting the buckling behavior of orthotropic and asymmetric cross-ply laminates under different boundary conditions, various numerical examples are presented for laminates with the following properties, and the results of the RPT are compared with those of the classical plate theory (CPT), first-order shear deformation theory (FSDT) and higher-order shear deformation plate theory (HSDT). The explanations of various displacement models are given in Table 2.

<table>
<thead>
<tr>
<th>Material type (1) Reddy (2004)</th>
<th>$E_1/E_2$ varied, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $v_{12} = 0.25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material type (2) Reddy (2004)</td>
<td>$E_1/E_2$ varied, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $v_{12} = 0.25$</td>
</tr>
</tbody>
</table>

To more conveniently present the numerical results in graphical and tabular forms, they are dimensionless using the following relation:

$$\bar{N} = N_{cr} \left( \frac{a^2}{E_2 h^2} \right)$$

In obtaining the results, plate strips with 14 degrees of freedom have been used. Also in all the results, except the mentioned cases, one harmonic and 10 strips have been used.

In all the tables and figures, $a$, $b$ and $h$ are the plate width, length and thickness, respectively; and $k$ is shear correction factor for the first-order shear deformation theory (FSDT).

<table>
<thead>
<tr>
<th>Model</th>
<th>Theory</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPT</td>
<td>Classical plate theory</td>
<td>3</td>
</tr>
<tr>
<td>FSDT</td>
<td>First-order shear deformation theory</td>
<td>5</td>
</tr>
<tr>
<td>TSDT</td>
<td>Third-order deformation theory</td>
<td>5</td>
</tr>
<tr>
<td>Present PSDT</td>
<td>Parabolic shear deformation theory</td>
<td>4</td>
</tr>
<tr>
<td>Present HSDT</td>
<td>Hyperbolic shear deformation theory</td>
<td>4</td>
</tr>
<tr>
<td>Present ESDT</td>
<td>Exponential shear deformation theory</td>
<td>4</td>
</tr>
<tr>
<td>Present SSDT</td>
<td>Sinusoidal shear deformation theory</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2: Description of various displacement models.
3.1 Buckling analysis of simply-supported square orthotropic plate

The dimensionless buckling loads of the simply-supported square orthotropic plate \( (a = b) \) have been presented in Tables 3 and 4 as well as Figures 6, 8 and 9. Material type (1), 10 strips and the first harmonic are used. The results obtained from the RPT numerical solution agree well with the Kim’s Navier solutions and the FSDT results. Also, the difference between the results of the present theory, FSDT \( (k = 5/6) \), and CPT have been illustrated in Figures 6 and 7 as an increase in the \( \frac{a}{h} \) ratio and in Figures 8 and 9 as an increase in the elasticity modulus. As shown in Table 3, the differences between the results of the present study and FSDT \( (k = 5/6) \), and between the results of the present study and FSDT \( (k = 1) \) are 15.42\% and 1.6\%, respectively, for the same case of square orthotropic plate \( (a = b = 5h \text{ and } E_1/E_2 = 40) \). The buckling load of a square orthotropic plate subjected to in-plane biaxial pressure was presented in Table 4 and Figure 7, which for converging the results, we used the first two harmonics \( (m = 1 \text{ and } m = 2) \) and 10 strips. The first two buckling mode shapes of a simply supported square orthotropic plate boundary conditions and \( a/h = 5 \) and subjected to in-plane uniaxial compressive load is depicted in Figure 10.

<table>
<thead>
<tr>
<th>( b/h )</th>
<th>Theories</th>
<th>Orthotropic ( E_1/E_2 = 10 )</th>
<th>Orthotropic ( E_1/E_2 = 25 )</th>
<th>Orthotropic ( E_1/E_2 = 40 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td><strong>Present</strong> PSDT</td>
<td>6.2126</td>
<td>9.0109</td>
<td>10.5133</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>6.3478</td>
<td>9.1039</td>
<td>10.5785</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 2/3 )</td>
<td>5.5679</td>
<td>7.1122</td>
<td>7.7411</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 5/6 )</td>
<td>6.1804</td>
<td>8.2199</td>
<td>9.1085</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 1 )</td>
<td>6.6715</td>
<td>9.1841</td>
<td>10.3463</td>
</tr>
<tr>
<td>10</td>
<td><strong>Present</strong> PSDT</td>
<td>9.2655</td>
<td>16.6319</td>
<td>22.1168</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>9.3732</td>
<td>16.7719</td>
<td>22.2581</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 2/3 )</td>
<td>8.8988</td>
<td>14.7011</td>
<td>18.3575</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 5/6 )</td>
<td>9.2733</td>
<td>15.8736</td>
<td>20.3044</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 1 )</td>
<td>95415</td>
<td>16.7699</td>
<td>21.8602</td>
</tr>
<tr>
<td>20</td>
<td><strong>Present</strong> PSDT</td>
<td>10.6138</td>
<td>21.2759</td>
<td>30.9730</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>10.6534</td>
<td>21.3479</td>
<td>31.0685</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 2/3 )</td>
<td>10.4926</td>
<td>20.4034</td>
<td>28.8500</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 5/6 )</td>
<td>10.6199</td>
<td>20.9528</td>
<td>30.0139</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 1 )</td>
<td>10.7066</td>
<td>21.3363</td>
<td>30.8451</td>
</tr>
<tr>
<td>50</td>
<td><strong>Present</strong> PSDT</td>
<td>11.0709</td>
<td>23.1080</td>
<td>34.9503</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>11.0780</td>
<td>23.1225</td>
<td>34.9717</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 2/3 )</td>
<td>11.0497</td>
<td>22.9366</td>
<td>34.4886</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 5/6 )</td>
<td>11.0721</td>
<td>23.0461</td>
<td>34.7487</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 1 )</td>
<td>11.0871</td>
<td>21.3363</td>
<td>34.9244</td>
</tr>
<tr>
<td>100</td>
<td><strong>Present</strong> PSDT</td>
<td>11.1398</td>
<td>23.3971</td>
<td>35.6067</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>11.1415</td>
<td>23.4007</td>
<td>35.6120</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 2/3 )</td>
<td>11.1343</td>
<td>23.3527</td>
<td>35.4852</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 5/6 )</td>
<td>11.1400</td>
<td>23.3810</td>
<td>35.5538</td>
</tr>
<tr>
<td></td>
<td>FSDT ( k = 1 )</td>
<td>11.1438</td>
<td>23.3999</td>
<td>35.5996</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>11.1628</td>
<td>23.4949</td>
<td>35.8307</td>
</tr>
</tbody>
</table>

Table 3: Nondimensional critical buckling loads of simply-supported (SSSS) square plates subjected to uniaxial compression.
<table>
<thead>
<tr>
<th>$b/h$</th>
<th>Theories</th>
<th>Orthotropic $E_1/E_2 = 10$</th>
<th>$E_1/E_2 = 25$</th>
<th>$E_1/E_2 = 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Present PSDT</td>
<td>2.7453$^a$</td>
<td>3.2417$^a$</td>
<td>3.5995$^a$</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>2.8549$^a$</td>
<td>3.3309$^a$</td>
<td>3.4800$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 2/3$</td>
<td>2.5042$^a$</td>
<td>2.7332$^a$</td>
<td>2.8303$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 5/6$</td>
<td>2.8319$^a$</td>
<td>3.1422$^a$</td>
<td>3.2822$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 1$</td>
<td>3.1027$^a$</td>
<td>3.4933$^a$</td>
<td>3.6793$^a$</td>
</tr>
<tr>
<td>10</td>
<td>Present PSDT</td>
<td>4.5555</td>
<td>5.9363$^a$</td>
<td>7.1217$^a$</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>4.6718</td>
<td>6.0646$^a$</td>
<td>7.2536$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 2/3$</td>
<td>4.4259</td>
<td>5.4351$^a$</td>
<td>6.0717$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 5/6$</td>
<td>4.6367</td>
<td>5.8370$^a$</td>
<td>6.6325$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 1$</td>
<td>4.7708</td>
<td>6.1425$^a$</td>
<td>7.0909$^a$</td>
</tr>
<tr>
<td>20</td>
<td>Present PSDT</td>
<td>5.3069</td>
<td>7.5993$^a$</td>
<td>9.5835$^a$</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>5.3267</td>
<td>7.6643$^a$</td>
<td>9.6614$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 2/3$</td>
<td>5.2463</td>
<td>7.3701$^a$</td>
<td>8.9895$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 5/6$</td>
<td>5.3100</td>
<td>7.5546$^a$</td>
<td>9.3049$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 1$</td>
<td>5.3533</td>
<td>7.6834$^a$</td>
<td>9.5297$^a$</td>
</tr>
<tr>
<td>50</td>
<td>Present PSDT</td>
<td>5.5355</td>
<td>8.2653$^a$</td>
<td>10.6409$^a$</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>5.5390</td>
<td>8.2784$^a$</td>
<td>10.6576$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 2/3$</td>
<td>5.5249</td>
<td>8.2199$^a$</td>
<td>10.5111$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 5/6$</td>
<td>5.5361</td>
<td>8.2566$^a$</td>
<td>10.5810$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 1$</td>
<td>5.5436</td>
<td>8.2812$^a$</td>
<td>10.6282$^a$</td>
</tr>
<tr>
<td>100</td>
<td>Present PSDT</td>
<td>5.5699</td>
<td>8.3710$^a$</td>
<td>10.8129$^a$</td>
</tr>
<tr>
<td></td>
<td>Kim et al. (2009a)</td>
<td>5.5707</td>
<td>8.3744$^a$</td>
<td>10.8172$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 2/3$</td>
<td>5.5672</td>
<td>8.3593$^a$</td>
<td>10.7788$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 5/6$</td>
<td>5.5700</td>
<td>8.3687$^a$</td>
<td>10.7972$^a$</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 1$</td>
<td>5.5719</td>
<td>8.3751$^a$</td>
<td>10.8095$^a$</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>5.5814</td>
<td>8.4069$^a$</td>
<td>10.8715$^a$</td>
</tr>
</tbody>
</table>

$^a$ (10 strips and first two harmonics)

Table 4: Nondimensional critical buckling loads of simply-supported (SSSS) square plates ($a = b$) subjected to biaxial compression.

Figure 6: The effect of side-to-thickness ratio on the critical buckling load of square plates subjected to uniaxial compression; $E_1/E_2 = 25$. 

Figure 7: The effect of side-to-thickness ratio on the critical buckling load of square plates subjected to biaxial compression; $E_1/E_2 = 25$.

Figure 8: The effect of modulus ratio on the critical buckling load of square plates subjected to uniaxial compression; $a/h = 10$.

Figure 9: The effect of modulus ratio on the critical buckling load of square plates subjected to uniaxial compression; $a/h = 20$.
3.2 Buckling analysis of simply-supported square orthotropic plate with various shear deformation theories

Table 5 has listed the critical buckling loads obtained from various shear deformation theories for simply-supported orthotropic square plates subjected to uniaxial compression. Material type (1), 10 strips and the first harmonic are used to solve the problem. As shown in Table 5, the non-dimensional buckling loads obtained by sinusoidal and exponential functions are greater than those obtained by hyperbolic and parabolic functions.

<table>
<thead>
<tr>
<th>b/h</th>
<th>present $f_i(z)$</th>
<th>Orthotropic $E_1/E_2 = 10$</th>
<th>Orthotropic $E_1/E_2 = 25$</th>
<th>Orthotropic $E_1/E_2 = 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$f_1(z)$ (PSDT)</td>
<td>6.2126</td>
<td>9.0109</td>
<td>10.5133</td>
</tr>
<tr>
<td></td>
<td>$f_2(z)$ (HSDT)</td>
<td>6.2122</td>
<td>9.0090</td>
<td>10.5094</td>
</tr>
<tr>
<td></td>
<td>$f_3(z)$ (ESDT)</td>
<td>6.2390</td>
<td>9.0921</td>
<td>10.6577</td>
</tr>
<tr>
<td></td>
<td>$f_4(z)$ (SSDT)</td>
<td>6.2637</td>
<td>9.0897</td>
<td>9.9596</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 1$</td>
<td>6.6715</td>
<td>9.1841</td>
<td>10.3463</td>
</tr>
<tr>
<td>20</td>
<td>$f_1(z)$ (PSDT)</td>
<td>10.6138</td>
<td>21.2759</td>
<td>30.9730</td>
</tr>
<tr>
<td></td>
<td>$f_2(z)$ (HSDT)</td>
<td>10.6138</td>
<td>21.2759</td>
<td>30.9729</td>
</tr>
<tr>
<td></td>
<td>$f_3(z)$ (ESDT)</td>
<td>10.6168</td>
<td>21.2885</td>
<td>31.0011</td>
</tr>
<tr>
<td></td>
<td>$f_4(z)$ (SSDT)</td>
<td>10.6253</td>
<td>21.3218</td>
<td>31.0692</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 1$</td>
<td>10.7066</td>
<td>21.3363</td>
<td>30.8451</td>
</tr>
<tr>
<td>100</td>
<td>$f_1(z)$ (PSDT)</td>
<td>11.1398</td>
<td>23.3971</td>
<td>35.6067</td>
</tr>
<tr>
<td></td>
<td>$f_2(z)$ (HSDT)</td>
<td>11.1398</td>
<td>23.3971</td>
<td>35.6067</td>
</tr>
<tr>
<td></td>
<td>$f_3(z)$ (ESDT)</td>
<td>11.1399</td>
<td>23.3977</td>
<td>35.6080</td>
</tr>
<tr>
<td></td>
<td>$f_4(z)$ (SSDT)</td>
<td>11.1402</td>
<td>23.3992</td>
<td>35.6116</td>
</tr>
<tr>
<td></td>
<td>FSDT $k = 1$</td>
<td>11.1628</td>
<td>23.3999</td>
<td>35.5996</td>
</tr>
</tbody>
</table>

Table 5: Nondimensional critical buckling loads obtained by various $f_i(z)$ for simply-supported square plates subjected to uniaxial compression.

3.3 Buckling analysis of square orthotropic plate with different boundary conditions

The non-dimensional buckling loads of square orthotropic plates ($a = b$) with different boundary conditions have been shown in Table 6 and Figure 11. In this section, the boundary conditions of two loaded ends are simply supported and side edges boundary conditions are considered as sim-
ply supported, clamped and free. Material type (1), 10 strips and the first harmonic term is used to solve the problem. In Table 6, a comparison has been made between the critical buckling loads of thin plates \( a/h = 100 \) achieved by the present RPT numerical solution, the Levy-Thai solution (2011) and the CPT solution. The changes of the critical buckling load with thickness ratio and PSDT model are shown in Figure 11. In Table 6, \( \beta_1 \) and \( \beta_2 \) are the load parameters that indicate the loading conditions. Positive values for \( \beta_1 \) and \( \beta_2 \) indicate that the plate is subjected to biaxial compressive loads. Also, a zero value for \( \beta_1 \) or \( \beta_2 \) shows uniaxial loading in the \( x \) or \( y \) direction, respectively. The buckling mode shapes of a square orthotropic plate with various boundary conditions and \( a/h = 5 \), subjected to in-plane uniaxial pressure are shown in Figure 12.

<table>
<thead>
<tr>
<th>((\beta_1, \beta_2))</th>
<th>(E_1/E_2)</th>
<th>Method</th>
<th>Boundary condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 ((0,1))</td>
<td>Present</td>
<td>11.1398</td>
<td>45.5671</td>
</tr>
<tr>
<td></td>
<td>PSDT</td>
<td>11.1415</td>
<td>45.5714</td>
</tr>
<tr>
<td></td>
<td>Thai et al. (2011)</td>
<td>11.1628</td>
<td>45.9207</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>11.1628</td>
<td>45.9207</td>
</tr>
<tr>
<td>25 ((0,1))</td>
<td>Present</td>
<td>23.3971</td>
<td>107.3591</td>
</tr>
<tr>
<td></td>
<td>PSDT</td>
<td>23.4007</td>
<td>107.3597</td>
</tr>
<tr>
<td></td>
<td>Thai et al. (2011)</td>
<td>23.4007</td>
<td>107.3597</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>23.4949</td>
<td>109.3141</td>
</tr>
<tr>
<td>20 ((1,1))</td>
<td>Present</td>
<td>35.6067</td>
<td>167.8998</td>
</tr>
<tr>
<td></td>
<td>PSDT</td>
<td>35.6120</td>
<td>167.8887</td>
</tr>
<tr>
<td></td>
<td>Thai et al. (2011)</td>
<td>35.6120</td>
<td>167.8887</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>35.8307</td>
<td>172.7103</td>
</tr>
<tr>
<td>40 ((1,1))</td>
<td>Present</td>
<td>5.5698</td>
<td>20.1498</td>
</tr>
<tr>
<td></td>
<td>PSDT</td>
<td>5.5707</td>
<td>20.1558</td>
</tr>
<tr>
<td></td>
<td>Thai et al. (2011)</td>
<td>5.5707</td>
<td>20.1558</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>5.5814</td>
<td>20.2904</td>
</tr>
<tr>
<td>25 ((1,1))</td>
<td>Present</td>
<td>11.6984</td>
<td>47.4986</td>
</tr>
<tr>
<td></td>
<td>PSDT</td>
<td>11.7003</td>
<td>47.5122</td>
</tr>
<tr>
<td></td>
<td>Thai et al. (2011)</td>
<td>11.7003</td>
<td>47.5122</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>11.7475</td>
<td>48.2668</td>
</tr>
<tr>
<td>10 ((1,1))</td>
<td>Present</td>
<td>17.8031</td>
<td>74.3589</td>
</tr>
<tr>
<td></td>
<td>PSDT</td>
<td>17.8060</td>
<td>74.3794</td>
</tr>
<tr>
<td></td>
<td>Thai et al. (2011)</td>
<td>17.8060</td>
<td>74.3794</td>
</tr>
<tr>
<td></td>
<td>CPT</td>
<td>17.9154</td>
<td>76.2450</td>
</tr>
</tbody>
</table>

**Table 6:** Comparison between nondimensional critical buckling loads of square orthotropic plates with different boundary conditions \( a/h = 100 \).

### 3.4 Buckling analysis of simply-supported square asymmetric cross-ply laminated plate

The critical buckling loads of two-layer asymmetric cross-ply laminated plates under uniaxial and biaxial loadings are presented in Table 7 for modulus ratios \( E_1/E_2 = 10, 25, 40 \) and material type (1). In Tables 7 and 8 as well as Figure 13, 10 strips and the first harmonic term are used to solve the problem. In Table 8, a simply-supported asymmetric cross-ply \((0/90)_n\) \( (n = 2,3,5) \) square laminate subjected to uniaxial compressive load on sides \((x = 0,a)\) and with modulus ratios \( E_1/E_2 = 40 \) is considered. Material type (2) is used. Table 8 shows a comparison between the results obtained by using various models and the 3-D elasticity solutions given by Noor (1975). The results clearly indicate that the present theories predict the buckling loads more accurately than the identical HSDTs.
Figure 11: The effect of side-to-thickness ratio on the critical buckling load of square plates with different boundary conditions subjected to uniaxial compression along the y-axis; $E_1/E_2 = 10$ and PSDT model.

Figure 12: The buckling mode shapes of a square orthotropic plate with various boundary conditions; A: SSSS, B: SSSC, C: SSBC, D: SSFF.

The effect of side-to-thickness ratio on the buckling load of simply-supported four-layer (0/90/0/90) square laminates has been presented in Figure 13 with modulus ratios $E_1/E_2 = 40$. Buckling mode shape of a square laminated composite plate (0/90/0/90) with simply supported boundary conditions and $a/h = 5$, subjected to in-plane uniaxial pressure have been illustrated in Figure 14.
<table>
<thead>
<tr>
<th>$b/h$</th>
<th>Theories</th>
<th>$E_1/E_2 = 10$</th>
<th>$E_1/E_2 = 25$</th>
<th>$E_1/E_2 = 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Present (PSDT)</td>
<td>Present (HSDT)</td>
<td>Present (SSDT)</td>
</tr>
<tr>
<td>10</td>
<td>Present (PSDT)</td>
<td>5.6641</td>
<td>8.1636</td>
<td>10.4671</td>
</tr>
<tr>
<td></td>
<td>Present (HSDT)</td>
<td>5.6636</td>
<td>8.1620</td>
<td>10.4639</td>
</tr>
<tr>
<td></td>
<td>Present (SSDT)</td>
<td>5.6645</td>
<td>8.1652</td>
<td>10.4705</td>
</tr>
<tr>
<td>20</td>
<td>Present (PSDT)</td>
<td>6.1777</td>
<td>9.1378</td>
<td>11.9961</td>
</tr>
<tr>
<td></td>
<td>Present (HSDT)</td>
<td>6.1775</td>
<td>9.1373</td>
<td>11.9951</td>
</tr>
<tr>
<td></td>
<td>Present (SSDT)</td>
<td>6.1778</td>
<td>9.1383</td>
<td>11.9972</td>
</tr>
<tr>
<td>100</td>
<td>Present (PSDT)</td>
<td>6.3662</td>
<td>9.5102</td>
<td>12.6016</td>
</tr>
<tr>
<td></td>
<td>Present (HSDT)</td>
<td>6.3662</td>
<td>9.5101</td>
<td>12.6015</td>
</tr>
<tr>
<td></td>
<td>Present (SSDT)</td>
<td>6.3662</td>
<td>9.5102</td>
<td>12.6016</td>
</tr>
<tr>
<td></td>
<td>CLPT</td>
<td>6.374</td>
<td>9.526</td>
<td>12.628</td>
</tr>
</tbody>
</table>

Table 7: Nondimensional critical buckling load of simply-supported asymmetric cross-ply square plates ($a = b$).

Figure 13: The effect of side-to-thickness ratio on nondimensionlized uniaxial buckling load of simply-supported four-layer $(0/90/0/90)$ square laminates subjected to uniaxial buckling; $E_1/E_2 = 40$.
<table>
<thead>
<tr>
<th>Number of layers</th>
<th>Source</th>
<th>$\bar{N}$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Exact (Noor, 1975)</td>
<td>21.2796</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>TSDT (Reddy, 2004)</td>
<td>22.5790</td>
<td>6.11</td>
</tr>
<tr>
<td></td>
<td>FSDT (Reddy, 2004)</td>
<td>22.8060</td>
<td>7.17</td>
</tr>
<tr>
<td></td>
<td>RPT (Kim et al., 2009b)</td>
<td>22.5700</td>
<td>6.06</td>
</tr>
<tr>
<td></td>
<td>Present (PSDT)</td>
<td>22.3306</td>
<td>4.93</td>
</tr>
<tr>
<td></td>
<td>Present (HSDT)</td>
<td>22.3370</td>
<td>4.95</td>
</tr>
<tr>
<td></td>
<td>Present (SSDT)</td>
<td>22.3044</td>
<td>4.81</td>
</tr>
<tr>
<td></td>
<td>CLPT</td>
<td>30.3591</td>
<td>42.67</td>
</tr>
<tr>
<td>6</td>
<td>Exact (Noor, 1975)</td>
<td>23.6689</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>TSDT (Reddy, 2004)</td>
<td>24.4596</td>
<td>3.34</td>
</tr>
<tr>
<td></td>
<td>FSDT (Reddy, 2004)</td>
<td>24.5777</td>
<td>3.84</td>
</tr>
<tr>
<td></td>
<td>RPT (Kim et al., 2009b)</td>
<td>24.4581</td>
<td>3.33</td>
</tr>
<tr>
<td></td>
<td>Present (PSDT)</td>
<td>24.2258</td>
<td>2.352</td>
</tr>
<tr>
<td></td>
<td>Present (HSDT)</td>
<td>24.2267</td>
<td>2.356</td>
</tr>
<tr>
<td></td>
<td>Present (SSDT)</td>
<td>24.2264</td>
<td>2.355</td>
</tr>
<tr>
<td></td>
<td>CLPT</td>
<td>33.5817</td>
<td>41.88</td>
</tr>
<tr>
<td>10</td>
<td>Exact (Noor, 1975)</td>
<td>24.9636</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>TSDT (Reddy, 2004)</td>
<td>25.4225</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>FSDT (Reddy, 2004)</td>
<td>25.4500</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>RPT (Kim et al., 2009b)</td>
<td>25.4225</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>Present (PSDT)</td>
<td>25.1976</td>
<td>0.937</td>
</tr>
<tr>
<td></td>
<td>Present (HSDT)</td>
<td>25.1975</td>
<td>0.936</td>
</tr>
<tr>
<td></td>
<td>Present (SSDT)</td>
<td>25.2100</td>
<td>0.986</td>
</tr>
<tr>
<td></td>
<td>CLPT</td>
<td>35.2316</td>
<td>41.13</td>
</tr>
</tbody>
</table>

Table 8: Nondimensionalized uniaxial buckling load of simply-supported asymmetric cross-ply (0/90/...) square laminates with \((a/h = 10)\) and \(E_1/E_2 = 40\).

![Buckling mode shape](image)

**Figure 14:** Buckling mode shape of a square laminated composite plate (0/90/0/90) with simply supported boundary conditions and \(a/h = 5\).

## 4 CONCLUSIONS

The finite strip numerical solution and the use of the refined plate theory for orthotropic and laminated composite plates at different boundary conditions have been investigated. Also in this...
solution, the results of various transverse shear functions have been compared. The important findings of this analysis can be expressed as follows:

1- In this paper, we employed the four transverse shear functions of PSDT, HSDT, ESDT and SSDT (Table 1). In section 3.2 (Table 5), all four functions are used for the analysis of different plate samples and demonstrated that the non-dimensional buckling loads of the PSDT and HSDT functions are less than those obtained the ESDT and SSDT functions. Therefore in Section 3.1, we only used the PSDT function for analysis.

2- In Section 3.4 (Table 8), it is shown that the results obtained by the PSDT function are closer to the exact solution.

3- The present theory yields more accurate buckling load values than the first-order shear deformation theory.

4- The buckling loads of the hyperbolic transverse shear function has a good accuracy compared with those of the first-order shear deformation theory.

5- The buckling loads of the exponential transverse shear function is usually higher than those of the first-order shear deformation theory.

6- This paper provided many examples for the the analysis of orthotropic plates with different boundary conditions and subjected to uniaxial and biaxial loading situations. Examples of laminated composite plats with different layers and sizes are presented in section 3.4, in all cases good accuracy is observed.

7- The most significant feature of this theory is that it may apply the transverse shear strains across the thickness as parabolic, sinusoidal, hyperbolic and exponential functions. Also, by having fewer unknowns in the equations, this theory enjoys a simpler form which is close to that of the CPT.

References


Appendix

Basic function \((S_m)\):

(1) Both ends simply-supported

\[ S_m = \sin \frac{\mu_m y}{b} \quad \text{where} \quad \mu_m = m\pi \]

(2) Both ends clamped

\[ S_m = \sin \frac{\mu_m y}{b} - \sinh \frac{\mu_m y}{b} - a_m \left[ \cos \frac{\mu_m y}{b} - \cosh \frac{\mu_m y}{b} \right] \]

in which \( a_m = \frac{\sin \mu_m - \sinh \mu_m}{\cos \mu_m - \cosh \mu_m} \) and \( \mu_m = \frac{2m + 1}{2} \pi \)

(3) One end simply-supported and the other end clamped

\[ S_m = \sin \frac{\mu_m y}{b} - a_m \sinh \frac{\mu_m y}{b} \]

in which \( a_m = \frac{\sin \mu_m}{\sinh \mu_m} \) and \( \mu_m = \frac{4m + 1}{4} \pi \)

(4) One end clamped and the other end free

\[ S_m = \sin \frac{\mu_m y}{b} - \sinh \frac{\mu_m y}{b} - a_m \left[ \cos \frac{\mu_m y}{b} - \cosh \frac{\mu_m y}{b} \right] \]

in which \( a_m = \frac{\sin \mu_m + \sinh \mu_m}{\cos \mu_m + \cosh \mu_m} \) and \( \mu_m = \frac{2m - 1}{2} \pi \)