Abstract
In this paper, an optimal controller for integrated longitudinal and lateral closed loop vehicle/driver dynamics proposed to follow desired path in various driving maneuvers, which also improved maneuverability and stability of vehicle over desired path. Designed controller imposed corrected steering angle and torque on the wheels to keep the vehicle on the desired trajectory whilst modified its handling properties. In the next stage, performance of proposed optimal linear quadratic regulator (LQR) controller compared with Proportional-integrated-derivative (PID) one. The proposed controllers has been implemented on vehicle eight degree of freedom model in MATLAB/Simulink. Then the effects of adaptive controller on vehicle path following has been examined for various maneuvers, by driving on the lane change, J-turn, double lane-change and desired tracks. Finally, longitudinal dynamic performance of vehicle has been investigated during severe braking conditions. Simulation results indicated the dominate efficiency of controller on the vehicle stabilization and path following. Also, it improved longitudinal dynamics performance by preventing wheel lock and reducing stopping distance.

Keywords
Vehicle path following; handling; optimal controller; integrated lateral & longitudinal dynamic.

1 INTRODUCTION
In the last decade, important researches have been undertaken to improve safety driving and reducing accidents. Consequently considerable attention has been given to the development of the stability and steerability control systems such as electronic stability program (ESP), different active steering, and active braking control systems. An important issue for the chassis control systems is to control the lateral vehicle motion variables such as the yaw rate and side-slip angle by controlling the vehicle yaw moment. Active steering systems in front (AFS) or both in front
and rear (4WS) can effectively improve the steerability performance in the linear region of the
tire (Hwang et al., 2008; Jinlai et al., 2011). Kang et al. (2008) investigated the steering controller
for path-tracking and a speed controller for improving the safety of lateral vehicle behavior. In
critical high lateral acceleration situations that the tire is in nonlinear region, however, a direct
yaw moment control (DYC) system can make the vehicle stable (Shuai et al., 2014; Mashadi et
al., 2014; Yamakado, 2012).

Tchamna and Youn (2013) presented a new approach for controlling the yaw rate and side-slip
of a vehicle without neglecting its longitudinal dynamics and without making simplifying assump-
tions about its motion. A sliding-mode controller is used to develop a differential braking control-
er for tracking a desired vehicle yaw rate for a given steering wheel angle, while keeping the ve-
hicle’s side-slip angle as small as possible. Kazemi et al. (2000) presented a new method for finding
slip control law of Anti-lock Braking System (ABS), based on sliding mode control method. A
four wheel vehicle model with seven degrees of freedom were considered. Slip of each wheel con-
trolled separately so that it remained in the desired range for every kind of road condition, and
by tuning desired slip undesired yaw on miu-split surfaces has been prevented. One of the adverse
effects of sliding mode approaches are high-frequency fluctuations. An optimization-based braking
pressure control laws for the front and rear wheels were analytically designed based on a non-
linear two-axle vehicle model (Mirzaeinejad and Mirzaei, 2010b). The integral feedback technique
was also appended to the design method to increase the robustness of the controller in the pres-
ence of modelling uncertainties. Mashadi et al. (2013) developed the optimal path following con-
troller based on genetic algorithm for lateral dynamics of vehicle. Simulation results demonstrated
that the proposed controller was able to effectively keep the vehicle path appropriately close to
the desired path even in the presence of the driver commands. The main objective of this paper is
designing optimal controller of integrated lateral/longitudinal vehicle dynamics based on control-
ling combined slips to improve vehicle handling properties and tracking desired path.

The vehicle path control and handling improvement is treated by Yang et al. (2009); Horiuchi
et al. (1999) respectively. They used integrated AFS and DYC systems by application of non-
linear predictive control. The AFS was used in low lateral accelerations and the DYC for high-g
maneuvers wherein the tires were saturated and couldn’t produce enough lateral forces to control
the vehicle on the path. This combination showed that the vehicle maneuverability and stability
can be remarkably improved. Mashadi et al. (2014) proposed a GA-PID controller with optimized
gains for the control of integrated driver/DYC system. Doumiati et al. (2013) investigated the
coordination of active front steering and rear braking in a driver-assist system for vehicle yaw
control by using robust $H_{\infty}$ approach. A fuzzy method to control ESP and AFS proposed in (Chu
et al., 2012). They applied genetic algorithm to optimize the control rule to ensure the correctness
and accuracy of the control rule.

It has been shown that DYC is the most effective method on vehicle motion control compared
with the other conventional control systems such as four wheel steering (4WS) (Abe, 1999; Selby
et al., 2001). The 4WS control, which depends on the relation between tire lateral force and the
steer angle as a control command, is efficient in a range where the lateral acceleration is low. But,
in high lateral accelerations, the steer input loses its direct effectiveness on tire lateral force and
thus on the yaw moment. Therefore, the lateral dynamics parameters, yaw rate and side-slip an-
gle, can no longer be controlled by the steer command.
In previous works, regardless of simultaneous lateral and longitudinal vehicle dynamic modeling system, controller with fewer degrees of freedom has been used to path following or improving handling conditions (Mahmoodi et al., 2013; Mokhiamar and Abe, 2002; Mirzaeinejad and Mirzaei, 2010a; Mashadi and Majidi, 2014). Various control methods such as sliding mode, PID, optimal LQR, and fuzzy approaches for vehicle dynamics control has been used in references frequently (Zhang and et al., 2009; Silva and Sousa, 2011; Guo et al., 2013). Wu et al. (2010) proposed a new integrated robust model based on $H_\infty$ controller matching chassis controller to improve vehicle handling performance and lane keep ability. In order to evaluate the effectiveness of controller vehicle lane keeping and stability is tested by a closed loop driver/vehicle model under a transverse forces.

In this paper for a suitable desired model for vehicle handling, combined lateral and longitudinal vehicle dynamics are developed to be tracked by integrated AFS/DYC control system. Firstly, an optimal LQR and PID controller are designed for improving stability, maneuverability and path following of comprehensive vehicle model. Then, considering some admissible tracking errors, an optimal yaw moment control law is developed to reduce the external yaw moment as much as possible by adequately following the desired path and improving its handling properties. An optimal problem is formulated to track the proposed comprehensive vehicle model for yaw rate, side-slip angle and longitudinal slip. Longitudinal dynamic of the vehicle is controlled with the throttle/brake pedal and longitudinal slip. Also, the lateral dynamic, side slip angle and yaw rates are controlled through integrated AFS/DYC system. Therefore, according to the road profile and vehicle location, vehicle tracks the desired path with minimal lateral deviation and heading angle error.

2 VEHICLE MODELLING

In this article, to simulate the control of a vehicle during the path following at various maneuvers, a non-linear 8-DOF vehicle model that includes both lateral and longitudinal dynamics is used. A schematic of typical front wheel steering passenger car is illustrated in Figure 1. The DOFs associated with this model are the longitudinal velocity $u$, the lateral velocity $v_y$, the yaw rate $r$, the roll rate $\Phi$, and four wheel rotational speeds, $w_{fl}$, $w_{fr}$, $w_{rl}$ and $w_{rr}$. Sources of nonlinearity includes nonlinear behavior of tires, nonlinearities exist in the longitudinal and the lateral tire normal load transfers, the roll steer effects, and the camber angle changes due to the vehicle roll.

![Figure 1: Vehicle 8 dof model (Mahmoodi et al., 2013).](image-url)
The governing equation of longitudinal dynamic, the lateral motion (lateral velocity, yaw rate and roll angle) are given as follows respectively:

\[
M(\dot{u} - rv) = \sum F_x
\]

\[
M(\dot{v} + ru) + m_s h_s \dot{\Phi} = \sum F_y
\]

\[
I_{xz} \ddot{\Phi} - I_{xx} \dot{r} = \sum M_z
\]

\[
I_{zx} \dot{\Phi} - I_{zz} \ddot{r} = \sum M_x
\]

where \( M \) and \( m_s \) are the total mass and the rolling mass respectively, \( I_{zz} \) and \( I_{xx} \) are mass moment of inertia about z-axis and x-axis, \( I_{xz} \) is the product of inertia with respect to x and z axes. \( h_s \) is the height of sprung mass CG to roll axis. The terms \( \sum F_x \) and \( \sum F_y \) are external forces along the x and y directions and \( \sum M_x \) and \( \sum M_z \) are the sums of the moments acting around the roll and yaw axes of the vehicle-fixed coordinate system and can be evaluated from:

\[
\sum F_x = F_{xfl} + F_{xfr} + F_{xrl} + F_{xrr}
\]

\[
\sum F_y = F_{yfl} + F_{yfr} + F_{yrl} + F_{yrr}
\]

\[
\sum M_z = a(F_{ylf} + F_{ylr}) - b(F_{ylf} + F_{ylr}) + T/2[(F_{zfl} + F_{zrl}) - (F_{zfr} + F_{zrr})] + \sum M_{zi}
\]

\[
\sum M_x = -m_s h_s (\ddot{v} + ru) + (m_s h_s - k_\phi) \dot{\Phi} - C_\phi \Phi
\]

In the above equations, \( a \) and \( b \) are respectively the distances measured from the CG to the front and the rear axles. \( T \) is the track width, and \( K_\phi \) and \( C_\phi \) are the overall roll stiffness and the damping coefficient respectively.

External forces \( \sum F_x \) and \( \sum F_y \), can be related to the tractive and the lateral tire forces through below equations:

\[
F_{xi} = F_{ui} \cos \delta_i - F_{ui} \sin \delta_i \quad \text{with} \quad i = fl, fr, rl, rr
\]

\[
F_{yi} = F_{ui} \cos \delta_i + F_{ui} \sin \delta_i \quad \text{with} \quad i = fl, fr, rl, rr
\]

Given that the Vehicle is the front wheel steering, steering angle (\( \delta_i \)) can be considered as:

\[
\delta_{fl} = \delta_{fr} = \delta_f + \Delta \delta_f + K_{rf} \Phi
\]

\[
\delta_{rl} = \delta_{rr} = K_{rr} \Phi
\]

In the equations (11) and (12), \( K_{rf} \) and \( K_{rr} \) are steer by roll coefficient for the front and rear wheels which depend on suspension geometry.
2.1 Tyre model

In this paper the mathematical model proposed by Fiala (1954), is used to the analysis of lateral force due to side slip of the tire. It is commonly called Fiala’s theory and is related to the tire cornering characteristics which takes into account the interactions between longitudinal and side forces as:

\[ F_y = \begin{cases} \left| u \right| F_z \text{sgn}(\alpha) & \alpha \geq \alpha_{cr} \\ \left| u \right| F_z \text{sgn}(\alpha)(1 - H^3) & \alpha \leq \alpha_{cr} \end{cases} \]  \hspace{1cm} (13)

\[ u = u_{\text{max}} - (u_{\text{max}} - u_{\text{min}})S_{so} \]  \hspace{1cm} (14)

\[ S_{so} = \sqrt{S_s^2 + \tan^2 \alpha} \]  \hspace{1cm} (15)

\[ \alpha_{cr} = \tan^{-1} \left( \frac{3u |F_z|}{C_{\alpha}} \right) \]  \hspace{1cm} (16)

\[ H = 1 - \frac{C_{\alpha} \tan \alpha}{3u |F_z|} \]  \hspace{1cm} (17)

where, \( \alpha \), \( S_s \), \( S_{so} \) are side slip angle, Longitudinal slip ratio the comprehensive slip ratio respectively. Lateral side slip angle for front (\( \alpha_f \)) and rear tires (\( \alpha_r \)) are given as:

\[ \alpha_f = \delta - \tan^{-1} \left( \frac{\nu + ar}{u} \right) \]  \hspace{1cm} (18)

\[ \alpha_r = - \left( \frac{\nu - br}{u} \right) \]  \hspace{1cm} (19)

According to integrated lateral/longitudinal dynamic of vehicle, vertical tire loads on wheels can be expressed as:

\[ F_{z1} = \frac{mg}{2} \left\{ \frac{b}{l} - \frac{a_z}{g} \left( \frac{h}{l} \right) \right\} + K_R \left\{ \frac{a_y}{g} \left( \frac{h}{T} \right) - \left( \frac{m_a}{m} \right) \left( \frac{h_a}{T} \right) \sin \Phi \right\} \]  \hspace{1cm} (20)

\[ F_{z2} = \frac{mg}{2} \left\{ \frac{a}{l} + \frac{a_z}{g} \left( \frac{h}{l} \right) + (1 - K_R) \left\{ \frac{a_y}{g} \left( \frac{h}{T} \right) - \left( \frac{m_a}{m} \right) \left( \frac{h_a}{T} \right) \sin \Phi \right\} \right\} \]  \hspace{1cm} (21)

\[ F_{z3} = \frac{mg}{2} \left\{ \frac{b}{l} - \frac{a_z}{g} \left( \frac{h}{l} \right) - K_R \left\{ \frac{a_y}{g} \left( \frac{h}{T} \right) - \left( \frac{m_a}{m} \right) \left( \frac{h_a}{T} \right) \sin \Phi \right\} \right\} \]  \hspace{1cm} (22)

\[ F_{z4} = \frac{mg}{2} \left\{ \frac{a}{l} + \frac{a_z}{g} \left( \frac{h}{l} \right) - (1 - K_R) \left\{ \frac{a_y}{g} \left( \frac{h}{T} \right) - \left( \frac{m_a}{m} \right) \left( \frac{h_a}{T} \right) \sin \Phi \right\} \right\} \]  \hspace{1cm} (23)

where \( a_z \) and \( a_y \) are longitudinal and lateral acceleration respectively. \( K_R \) is the ratio of the front roll stiffness to the total roll stiffness which determines the front/rear distribution of total lateral load transfer, and \( h \) denotes the height of CG relative to the ground.

2.2 Closed loop vehicle model

In order to realize the simulation results, a driver model utilized in closed loop vehicle model, for the purpose of developing and testing of vehicle stability.

There is a general consensus that in driver model control occurs at two levels (Gordon et al., 2002; Moon and Choi, 2011): preview control (open loop feedforward), in which the driver anticipates the path ahead and makes an appropriate steering action based on knowledge of the vehicle dynamics; and compensatory control (closed-loop feedback), in which the driver compensates for errors in the preview control and for disturbances. The compensatory task involves the human operator controlling a system to minimize an error.

It has been shown that the proposed integrated human drive model can control a vehicle in the same way human manual driving does in various road curvature situations. It can also represent a normal driver's driving motion. Consequently, the integrated human driver model presented in this study can be used into a closed-loop simulation and for the development of a vehicle's intelligent safety system.

One popular method is treating the human control behavior as a linear continuous feedback control, and expressing it as a transfer function. Various transfer functions have then been proposed to suit different conditions. Here, the following transfer function is used (Abe, 2004):

\[
H(s) = h \left( \tau_p s + 1 + \frac{1}{\tau_i s} \right) e^{-\tau_e s}
\]  

Firstly, a driver will have a time delay to decide and make an action. This is represented by \(\tau_e\); and the time lag is expressed by \(\tau_L\); Certain time delay between the driver's perception and reaction exits which ranging from 0.1-0.5s. In the proposed driver model ion this paper time delay has been selected 0.3 seconds.

The control action that the human operator can do includes proportional (\(h\)), derivative (\(\tau_p\)) and integrated (\(\tau_i\)) control actions to regulate the driver commands based on inputs. In other words, the vehicle driver, in general is considered as both the compensator and the estimator, with certain time delay, in a common closed-loop control system.

3 CONTROLLER DESIGN

The purpose of control system proposed in this paper is controlling the vehicle to follow a desired path, whereas maintains the vehicle actual motions, yaw rate and slip angles, close to their desired responses with a minimum external yaw moment, for improving vehicle stability and handling condition. To achieve this aim, an optimal LQR and PID approaches should be applied for development of the yaw moment and steering angle control law. The control system is under consideration here is shown in Fig. 2.

At the first step in the design of driver/vehicle controller, one should develop an integrated lateral/longitudinal vehicle dynamic model, which is a good representation of the comprehensive vehicle dynamics, for steady state vehicle handling property analysis. The standard form of this model can be found throughout the literature, such as (Mashadi et al., 2014; Vahedi et al., 2011).
Despite to control the vehicle in desired path, relationship between the vehicle and the intended path identified by expressing in terms of a lateral position error, $y_e$ (the lateral distance between the vehicle and the intended path), and an orientation angle error ($\psi - \psi_r$). The lateral deviation ($y_e$) and heading error ($\psi$) of the vehicle are computed by augmenting this model with vehicle variables as:

$$\dot{y_e} = u \sin(\psi) + v \cos(\psi)$$

$$\dot{\psi} = \dot{\psi}_v - \dot{\psi}_r$$

where indexes $v$ and $r$ are representative of vehicle and road respectively and $\dot{\psi}_r$ is defined as road curvature rate which equals longitudinal velocity divided by road curvature, $(u/R)$.

Combining of vehicle model equations with external yaw moment and equations (25) and (26) can be described by the following state space equations based on small heading angle error and constant longitudinal vehicle speed assumptions:

$$\dot{X} = AX + E\delta + BM_z + K\left(1/R\right)$$

where,

$$a_{11} = -2\frac{C_{af} + C_{ar}}{Mu}, \quad a_{12} = 2\frac{bC_{ar} - aC_{af}}{Mu} - U, \quad a_{21} = 2\frac{bC_{ar} - aC_{af}}{I_z u}$$

$$a_{22} = -2\frac{a^2 C_{af} + b^2 C_{ar}}{I_z u}, \quad c_1 = 2\frac{C_{af}}{M}, \quad c_2 = -2\frac{aC_{af}}{I_z}, \quad b = \frac{1}{I_z}$$

For the vehicle model, the lateral velocity $v$ and the yaw rate $r$ are considered as the two state variables while the yaw moment $M_z$ is the control input, which must be calculated through the control law. Furthermore, the vehicle steering angle $\delta$ is considered as the external disturbance which controlled by driver model that should be added to vehicle model states through driver model.
In order to develop the yaw moment control law for improving vehicle handling properties, the linear quadratic regulator (LQR) method is considered here as a suitable tool. In order to compare controller law effects, PID controller applied to achieve compare with LQR method.

3.1 Optimal LQR controller

To control a vehicle to track a driver intended path with constant longitudinal velocity thereby LQR theory, the performance index that penalizes the tracking errors and control expenditure is formulated as:

\[ J = \frac{1}{2} \int_{t_i}^{t_f} \left[ w_1 (y - y_d)^2 + w_2 (\psi - \psi_d)^2 + w_3 (\nu - \nu_d)^2 + w_4 (r - r_d) + w_5 (\delta - \delta_d)^2 + w_6 M_z^2 \right] dt \]  

(29)

where, \( M_z \) denotes external yaw moment. The subscript \( d \) denotes the desired response of each variable. First and second terms in the performance index are lateral deviation and heading error, which are representations of vehicle path following. Third and fourth term in performance index denote handling and stability property of vehicle and fifth term is vehicle steering angle which regulated by driver. \( w_1 - w_6 \) are weighting factors which indicate the relative importance of the corresponding terms.

The typically defined optimal control consists of the state variable feedback signal and the disturbance feedforward signal that is related to the road specification, are expressed as:

\[ M_z = K_v v + K_r r + K_\psi \psi + K_\delta \delta + K_\psi \psi + K_{\psi \psi} \psi + K_{R R} R \]  

(30)

\( K_v, K_r, K_\psi, K_\delta, K_{\psi \psi}, K_{R R} \) are known as the state gains of lateral velocity, yaw rate, heading angle, steering angle, steering angle rate, and lateral displacement respectively, which act on the vehicle states and \( K_{R R} \) is the preview gain, acting on the previewed path information.

Consequently, equation (29) can be rewritten in the standard form of the optimal control as:

\[ J(u) = \frac{1}{2} \int_{0}^{\infty} \left[ U^T R U + (X_d - X)^T Q (X_d - X)^2 \right] dt \]  

(31)

where \( X_d \) is the desired values that the vehicle states should track, \( Q \) is a positive semi-definite state weighting matrix, and \( R \) the positive semi-definite control weighting matrix. In order to solve the LQR problem the Hamiltonian function can have the following form:

\[ H = \frac{1}{2} \left[ (X - X_d)^T Q (X - X_d) + U^T R U + P^T (AX + BU + EW) \right] \]  

(32)

where \( P \) is the Lagrange multipliers vector can be written as:

\[ P(t) = [P_1, P_2, P_3, P_4, P_5, P_6]^T \]  

(33)
Following the general approach of a typical optimal control problem, the following equations, as the necessary conditions, must be satisfied:

\[
\begin{align*}
\dot{X} &= \frac{\partial H}{\partial P} = AX + BU + EW \\
\dot{P} &= -\frac{\partial H}{\partial X} = -Q(X - X_d) - A^T P \\
0 &= \frac{\partial H}{\partial U} = RU + B^T P
\end{align*}
\] (34)

Now suppose that the Lagrange multipliers can be written in the following form:

\[
P = KX + S
\] (35)

where \( K \) is symmetric matrix of feedback gains and \( S \) represents feedforward gains matrix. Hence controller input can be written as:

\[
U = -R^{-1}B^T(KX + S)
\] (36)

Combining equations (32), (34) and (35) lead to

\[
\dot{K}X + K\dot{X} + \dot{S} = -Q(X - X_d) - A^T P
\] (37)

Time varying gains cause complication and the divergence in solutions. Also, these values converge rapidly to constant values. Therefore, this variable is ignored \((\dot{K} = \dot{S} = 0)\). This leads to

\[
\begin{align*}
KA + A^T K + Q - KBR^{-1}B^T K &= 0 \\
A^T - KBR^{-1}B^T &S - QX_d + KFW = 0
\end{align*}
\] (38)

The first equation in equation (19) is known as the Riccatti equation. The solution for this equation will determine the elements of \( K \) and can be solved numerically (Kirk, 2004; Goodarzi et al., 2006) by using MATLAB software. Substituting \( K \) into the second equation of equation (19) and solving for \( S \)

\[
S = -\left[ A^T - KBR^{-1}B^T \right]^{-1} KFW
\] (39)

With the matrices \( S \) and \( K \) and substituting in equation (36), and then the controller input can be determined as:

\[
M_z = \frac{1}{I_z w_i} \left[ K_n y_{de} + K_n y_{de} + K_n \psi + K_n \dot{\psi} + K_n \delta + K_n \dot{\delta} + K_{load} \left( \frac{1}{R} \right) \right]
\] (40)

Values of the state gains are set up based on time variation, in such a way that the optimal controller is able to control the vehicle’s track and stability. It results in minimum lateral deviation and obtaining vehicle stability and maneuverability over various paths. Their variation in different range of vehicle longitudinal speed are illustrated in Figs. 3.
Figure 3: Feedforward and feedback state gains (lateral speed, lateral deviation, yaw rate, heading angle, steering angle, steering angle derivative, and road radius) versus longitudinal speed (m/s).

3.2 PID controller

The PID controller is based on simple ideas. As illustrated in Fig. 4, the idealized formula the transfer function of a PID controller is described as equation (41), which, the controller output is a combination of three terms:

- The proportional term acts to current errors.
- Past errors are accounted for by the integral term.
- The derivative term anticipates future errors by linear extrapolation of the error.

\[
G_c(s) = k_p + \frac{k_i}{s} + k_ds
\]  

(41)

A proportional controller \((K_p)\) will have the effect of reducing the rise time and will reduce but never eliminate the steady-state error. An integral control \((K_i)\) will have the effect of eliminating the steady-state error for a constant or step input, but it may make the transient response slower. A derivative control \((K_d)\) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response. Note that these correlations may not be exactly accurate, because \(K_p\), \(K_i\), and \(K_d\) are dependent on each other. In fact, changing one of these variables can change the effect of the other two. For this reason, tradeoff between control gains has been performed. In this study the method used to tune the PID controller was the poles allocation (Alonso, 2013; Cominos and Munro, 2002). In the design of the PID controller, the study is based on the performance index of these:

1. Settling time less than 2s, within 1% of final value;
2. Overshoot of step responsive less than 10%;
3. Steady-state error of step responsive is 0.

4 SIMULATION RESULTS

Simulation has been performed for both vehicle longitudinal and lateral dynamics with PID, Optimal LQR and without controllers. Simulations were run in Matlab/Simulink software. In the first step, vehicle longitudinal dynamic performance during braking has been compared for three cases. Then in the next stage, vehicle path following over various maneuvers has been investigated with and without controllers. Then vehicle handling properties analyzed for lane change maneuver. Finally, vehicle obstacle avoidance by previewing path has been compared for PID and LQR controllers.
4.1 Longitudinal dynamic results

In order to investigate the effects of controller on vehicle longitudinal dynamic, stopping distance and braking force on tire are compared for uncontrolled, PID and optimal LQR controllers in Figs. 5 respectively.

![Graphs showing stopping distance and tire torque for different controllers.](image)

**Figure 5:** Stopping distance and tire lateral force comparison for (a) optimal LQR controller, (b) PID controller and (c) without controller.

Results reveal that forces act on tire in uncontrolled mode, proportional to the longitudinal tire slip are less. Therefore, road holding and steerability of vehicle reduce. Whereas, in controlled modes, tire slip limited in sufficient range which result in improving maneuverability and reducing stooping distance (time). Also, Figure 5 depicts that optimal controller has the minimum time to stop and best performance.

In the next stage for a closer look at the vehicle's steerability, wheel and vehicle speeds variation during braking are compared for three above mentioned conditions in Fig. 6. It can be observed without controller, after 10 seconds of braking, wheel speed becomes zero while the vehicle stops after 17 seconds. In other words, in uncontrolled mode, the wheel locked after 10 seconds...
and steerability drastically reduced. However with the addition of controllers, vehicle and wheel speeds simultaneously reached zero, and wheel locking mode didn’t occur. Also, results in table 6 indicate that stopping time and distance are less for optimal LQR controller in comparison to PID one.

![Vehicle speed and wheel speed](image)

**Figure 6:** Speed variation during braking for uncontrolled, PID and optimal LQR controllers.

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>LQR</th>
<th>PID</th>
<th>Without control</th>
</tr>
</thead>
<tbody>
<tr>
<td>tire locking time</td>
<td>-</td>
<td>1 sec</td>
<td>7 sec</td>
</tr>
<tr>
<td>stopping</td>
<td>13 sec</td>
<td>15 sec</td>
<td>17 sec</td>
</tr>
<tr>
<td>Longitudinal slip</td>
<td>0.2</td>
<td>0.2</td>
<td>7</td>
</tr>
</tbody>
</table>

*Table 1: Comparison of vehicle longitudinal performance.*

The simulation results indicate that, the wheel slip tracking error is remarkably decreased by the proposed controller. Moreover, the achieved control input is entirely smooth and suitable for implementation, which prevents from wheel locking and steerability losing.

### 4.2 Lateral dynamics simulation results

Vehicle simulation results has been performed over different paths: quadratic, lanechange, double lanechange, and j-turn maneuvers for uncontrolled, PID, and optimal LQR controllers in Figs 7.

As shown in Figs. 7, vehicle without controller cannot follow desired path properly and in some maneuvers losses stability and critical handling situation acurse. Whilst, additional of controller improves stability and vehicle can track desired path with minimal deviation. Also, results depict that designed controllers are adaptive to various maneuvers, whereas optimal controller performance is better than PID one. In the next stage vehicle state variation during standard lanechange maneuver has been illustrated in Figs. 8, for side slip angle, lateral velocity, acceleration, and deviation respectively.
Compared results indicate that, optimal LQR controller improves vehicle handling conditions. Lateral velocity and acceleration always remains below the critical margins (1 m/s and 0.8 m/s²) [29]. So, vehicle can track desired path with minimum deviation with maintain stability and steerability. In order to analyze the controller performance control efforts (external yaw moment and corrective steer angle) are compared for PID and optimal LQR controller in Figs. 9 and 10, respectively. It is obvious that PID controller needs more control efforts, which results in extra work load on driver/vehicle model. Whereas, optimal LQR controller minimizes external yaw moment and corrective steering angle.

### 4.3 Obstacle avoidance

In order to enhance the effects of the controls under more severe maneuvering conditions, the single lane change test with obstacle in path with LQR and PID controllers is executed. This can be utilized to make a more intelligent choice with consideration of driver model preview distance. Simulation results for lane change maneuver with obstacle in straight part of lane change (5th second), at a constant speed of 15 m/s are shown in Figs. 8.
As shown in Figs. 11 LQR controller could follow desired path with preview of upcoming road profile. This can be utilized to make a more intelligent choice with consideration of driver model look-ahead distance obstacle avoidance and stability maintaining, even though lateral acceleration and velocity has undulation because of instantaneous steering. Optimal controller stabilize the vehicle states properly after disturbances, which makes vehicle steerable and stable after severe steering conditions.
5 CONCLUSION

This paper presented the comprehensive dynamic model for vehicle path following and improving its handling properties. An integrated vehicle safety control strategy for vehicle longitudinal and lateral stability was designed to optimally maintain steerable and stability of vehicle under different scenarios with minimizing control efforts. So, the effectiveness of integrated direct yaw moment control and corrective steering angle with optimal LQR and PID approaches evaluated in the closed-loop driver/vehicle system, for path following. The proposed control law was developed based on tracking vehicle parameters (yaw and lateral velocities) in related to previewed path by driver (lateral deviation). Controller applied corrective steering angle and direct yaw moment to maintain vehicle stability and improving maneuverability. For this purpose, a number of simulations were conducted on an 8-DOF nonlinear driver/vehicle closed-loop model for a various maneuvers. Simulation results clarified that the closed-loop driver vehicle response was stable even under severe maneuvers, in which an uncontrolled vehicle is unstable.
References


