Scaling of An Impacted Reticulated Dome Using Partial Similitude Method

Demin Wei

Chenxi Hu*

a School of Civil Engineering and Transportation, South China University of Technology, China. Email: dmwei@scut.edu.cn, maradonan2@sina.com

*Corresponding author

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Abstract

Strain-rate effects can distort model testing with geometrically-similar models. In impact modelling this problem is usually addressed by revising the impact conditions, but such kind of method is inadequate for modelling impact on a reticulated dome. A new technique was proposed and tested. Apart from adjusting the impact conditions, the technique adds additional mass to components of the model to balance the strain-rate effects. That allows studying in model scale more complex structures in which the strain rate varies over the structure’s components. Model scale tests of impact on a reticulated dome showed good agreement with full scale in terms of displacements and axial forces on the structure’s rods. Those results verify the effectiveness of the new technique.

Keywords:
Reticulated Domes; Scaling Laws; Strain-rate Effects; Impact; Model Testing.

1 INTRODUCTION

Accidents and explosions causing damage are a matter of growing concern in civil engineering after the 9/11 terrorist attacks. The dynamic response of reticulated domes subjected to impact loading has been investigated theoretically and experimentally in recent years (Wang X. et al, 2016; Lin L. et al, 2015; Wei D. and Hu C., 2015; Wu C. et al, 2014; Wang X. and Lei J., 2012; Zheng L., 2012; Zheng L. and Chen Z., 2011; Fan F. et al, 2010). In order to validate either theoretical or numerical results, experiments are indispensable. They are, however, always carried out on scaled-down models so as to be efficient and cost-effective. Indeed, for large-span structures such as the reticulated dome to be discussed here, a full-size model test would be impossibly large. Generally the dynamic response of prototypes can be predicted adequately by scaled-down model tests and through applying scaling laws (Ding B. et al, 2015; Wang D. et al, 2011; Li H. et al, 2006).

Scaling laws for impacted structures have been investigated continuously. Johnson (1972) has used a dimensionless damage number $D_n$ to identify various impact regions in metals. Zhao (1998) improved this number by taking into account the geometrical effects and applied it to the dynamic response of simple impacted structures. The derivation of these numbers are based on a set of governing equations. However, it is always difficult to deduce the dimensionless numbers through theoretical equations. A more general method for the scaling of impacted structure is dimensional analysis (Buckingham E., 1914; Westine P. S. et al, 1973), by which Nevill (1963) and Duffey (1970) achieved scaling laws for structures subjected to explosion, impact and thermal loading.

Suppose that the small-scale model and the full-scale prototype are made of the same material and that the scaling factor of the model is $\gamma_e$. Simple dimensional analysis shows that the ratio of model strain to prototype strain $\gamma_e$ is one to one, which leads to a unit ratio of the stress $\gamma_e$. Jones (2011) has detailed the analysis procedure and he
explains that these ratios constitute the geometrically similar scaling law. It is widely used in static and dynamic experiments. That classic law can, however, be distorted due to gravity, material strain-rate sensitivity and fracture (Xu Z. et al, 2016; Yang F. J. et al, 2013; Jackson K. E., 1994). Complete and perfect solutions for those kind of distortions are always impossible so that researchers resort to partial similar method or sequential method such as in references (Soedel W., 1971; Shokrieh M. M. and Askari A., 2013).

This study aimed at correcting the scaling laws to account for strain-rate effects. At the outset, express the dynamic stress $\sigma_d$ on the prototype in the generic form of a material constitutive law as

$$\sigma_d = \sigma_s g(\dot{\varepsilon})$$

(1)

where $\sigma_s$ and $\dot{\varepsilon}$ denote the static stress and the stain rate. $g(\dot{\varepsilon})$ is a generic function of $\dot{\varepsilon}$ representing strain rate effects. If the prototype of the structure is geometrically scaled by $\gamma_l$, then its dynamic stress is written as

$$\sigma_d = \sigma_s g(\dot{\varepsilon} \gamma_l)$$

(2)

Apparently the dynamic stress ratio is not 1 contrary to the geometrically similar scaling law. In general, the model used to represent the prototype is smaller in the size for structural engineering experiments. However, the equations and method in this paper can also be applied to models larger than the prototype since $\gamma_l$ can either be smaller or larger than 1, but the smaller case is considered in the paper. Non-scaling phenomena become even more obvious as the model’s scale gets smaller or larger. This will result in underestimates or overestimates about the behaviour of the prototype, as can be seen in Norman Jones’ (1987) experiments and Booth’s et al. (1983) tests. Size effect in composite material behaviour for inelastic analysis are also highlighted in the works of Viot et al. (2008) and Wisnom (1999).

Drazetic et al. (1994) provided an inspiring way of dimensional analysis to correct non-scaling phenomenons. Alves & Oshiro (2006) further exploited this idea, and properly scaled strain rate sensitive models by correcting either initial speed or the mass of the impactor. Based on the idea of changing the impact velocity, Oshiro & Alves (2009) dealt with the scaling of structures subjected to impact loads when using a power law constitutive equation. Mazzariol & Alves (2014) developed a method to represent impacted structures using scaled models made of different materials with a combination of modifying velocity and mass of the impactor. However, as found in the research of our concerns, None of these methods are precise enough to describe complex structures, particularly for the reticulated shell concerned in this paper. Therefore, a new technique based on partial similarity and dimensional analysis is proposed here. The core manipulation of the technique is adding additional mass to components of the dome according to the developed scaling law to balance the strain-rate induced size effects. The amount of mass must be scaled such that the dimensionless number describing the dome’s dynamic response is unchanged. At last, the technique is demonstrated by numerical analysis of the prototype and the models of an impacted reticulated dome.

**2 SCALING METHOD**

Since geometrically similar scaling (a law denoted as MI) is not suitable for strain rate-sensitive structures, dimensional analysis is needed to find a new dimensionless number on which to base the scaling law. Buckingham’s (1914) $\pi$ theorem is a useful dimensional analysis tool.

Seven variables must be considered to acquire the scaled input variables of the structure: the impacting mass $M$, the impact velocity $V$, a characteristic dimension of the structure $l$, material density $\rho$, response time $t$, dynamic stress $\sigma_d$ and the strain $\varepsilon$. Input variables include the impact velocity, the mass of the impactor and the mass of components of the structure. $[l]$, $[\sigma_d]$ and $[t]$ are a set of fundamental dimensions, hence the dimensionless $\pi$ terms calculated through Buckingham $\pi$ theorem are $\pi_1 = M^{-1}l^1 \rho$, $\pi_2 = l^{-1}t^1 V$, $\pi_3 = \varepsilon$, $\pi_4 = l^{-2} M^{-1} t^2 \sigma_d$

If the impact response of the model and the prototype are to be similar, their dimensionless numbers should be equal, specifically $\pi_{iM} = \pi_{im}$ $(i=1,2,3,4)$ where $iM$ and $im$ denote the model and the prototype respectively. It can be deduced from $\pi_{2M} = \pi_{2m}$ and $\pi_{4p} = \pi_{4m}$ that

$$\gamma_M = \gamma_1^1 \gamma_2^2 \gamma_{\sigma_d}$$

(3)
where $\gamma$ is the scaling factor of the variable denoted by the subscript. For example, $\gamma_M = M_m / M_p$. It is reasonable to configure the material constitutive law in the general form of equation (1)

$$\sigma_d = \sigma_s g(\dot{e}),$$

(1)

where $\dot{e}$ is the strain rate. Scaling the variables of Equation (1) and substituting $\gamma_M$ into Equation (2) leads to

$$\gamma_M = \gamma_l \gamma_v \gamma_s g(\dot{e}_m) / g(\dot{e}_p)$$

(4)

Strain rate for the prototype is related with that for the model according to the scaling of the model and the impact velocity as $\dot{e}_p = \dot{e}_m \gamma_l / \gamma_v$. Substituting this equation into Equation (4) results in

$$\gamma_M = \gamma_l \gamma_v \gamma_s g(\dot{e}_m) / g(\dot{e}_m \gamma_l / \gamma_v)$$

(5)

from which it is easy to see the non-scaling of dynamic stress induced by strain rate effects. Practically, $\gamma_l$ is chosen on purpose by researchers in the set of experiments. As a consequence, either the mass or the initial velocity of the impactor should be revised to balance Equation (5).

The general method of altering the impact conditions (denoted as MII) works well for scaling structures that deform with a constant strain rate. It is even applicable for situations in which the strain rate changes over the time provided that the average strain rate is properly selected. However, a more general situation is that the strain rate varies over different parts of an impacted structure at the same time, a situation impossible to deal with by only revising the impact conditions.

A solution for this involves taking a closer look at the equation $\pi_{1p} = \pi_{1m}$, which can be expressed as

$$\gamma_M = \gamma_l \gamma_v \gamma_s g(\dot{e}_m) / g(\dot{e}_m \gamma_l / \gamma_v)$$

(6)

Reorganizing Equation (6) yields

$$\gamma_s = \gamma_v \gamma_s g(\dot{e}_m) / g(\dot{e}_m \gamma_l / \gamma_v)$$

(7)

which suggests that the density of the model’s material should also be revised according to the strain rate.

From a structural mechanics point of view, the strain rate in Equation (5) is different from that in Equation (7). The scaling of the impactor’s mass or its initial impact speed should be combined with an evaluation of the strain rate near the impact point. The material density of certain parts of the structure must be corrected based on the strain rate at that point. Equations (5) and (7) thus provide a more comprehensive understanding of the scale revision procedure. It will involve adjusting the impactor’s mass or velocity to balance the strain rates in the impact region (whose displacements during structural responses are quite small thus can be neglected) and adjusting the material density of the model to balance the strain rates in the unimpacted region (whose displacements cannot be neglected). This new method is denoted as MIII. Apparently MII can be regarded as a reduced method of MIII.

Adjusting the impactor’s mass is much more convenient than adjusting its impact velocity, since the latter needs iteration. As a result, mass adjustments were used in the following numerical analysis. In that case the scaling factor of impact velocity $\gamma_v = 1$. In practice it is also difficult to alter the material’s density while keeping other material properties unchanged. An alternative approach is to add additional mass to components of the model. For example, if components of the prototype are rod-shaped, then the scaling factor of components’ mass per metre is $\gamma_m = y_0 A_p / y_0 A_p (A_p$ denotes section area of the rod). As a result, the added mass per metre $\Delta m$ on rods of the model can be calculated as $\Delta m = \rho A_p (y_0 - 1)$, which by Equation (7) lead to

$$\Delta m = \rho A_p \left[ \gamma_v \gamma_s g(\dot{e}_m) / g(\dot{e}_m \gamma_l / \gamma_v) - 1 \right]$$

(8)
where \( \rho \) specifically refers to the material density of components. It is important to note that the added mass should not provide any added strength to the model, so the material strength of any masses attached to the rods must be much lower than that of the model’s material. This requirement ensures that the act of adding the mass is completely equivalent to altering the density of the material.

In summary, then, the complete scaling technique MIII for a structure subjected to impact is listed as follow:

I. Determine representative strain rate \( \dot{\epsilon}_m \) for different parts of the model.

II. Calculate the actual mass of the impactor in model scale according to Equation (6). Then calculate the mass that should be added to components according to Equation (8).

III. Estimate the dynamic response of the prototype according to the results with the model using revised scaling factors.

Step I is based on geometrically similar-model (MI) and \( \dot{\epsilon}_m \) is calculated as half of the maximum strain rate. This procedure was adopted according to Perrone and Bhadra (1979,1984).

The scaling factors summarized in Table 1. It should be kept in mind that \( \gamma_\rho \) for MIII models is realized through adding additional mass in real tests, though with finite element modelling \( \gamma_\rho \) can be adjusted by simply revising the density of the material.

### 3 FINITE ELEMENT MODELLING

Consider the single-layer, reticulated dome shown in Figure 1 with a span \( L=40m \) and height \( H=8m \). The dome’s radius is \( R=29m \). All 36 joints on the outer ring are hinged to the foundation. The dome is composed of hollow rods with a circular cross-section fabricated from Chinese steel Q235 (Ministry of Construction, 2014). The rods whose impact responses are to be discussed are labeled in Figure 1(c). They are rigidly connected by welded joints and can be divided into 3 categories depending on their direction. The outer radius of the radial rods is 73.0mm and their wall thickness is 4.5mm (\( \Phi 73.0 \times 4.5 \)). Using the same nomenclature the diagonal rods are \( \Phi 83.0 \times 4.0 \) and the latitudinal rods are \( \Phi 63.5 \times 5.5 \). The density of the steel is \( \rho=7850kg/m^3 \) and its elastic modulus \( E=206GPa \). The Poisson’s ratio \( \mu=0.3 \). In simple tension, the yield limit \( \sigma_y \) and the ultimate strength \( \sigma_b \) of the material are 235MPa and 450MPa respectively. The mass impactor is assumed to strike the dome vertically at its apex with an impact speed \( V=30m/s \). The impactor is a 370mm cube weighing \( M=400kg \). The striking position and the striking direction is chosen to represent a typical condition of the impact. Such kind of impact is accepted as a benchmark of tests referring to (Wang X. et al, 2016; Lin L. et al, 2015; Wei D. and Hu C., 2015; Wu C. et al, 2014; Wang X. and Lei J., 2012; Zheng L., 2012; Zheng L. and Chen Z., 2011; Fan F. et al, 2010).
The dynamic response of the prototype and its scaled-down model are solved numerically using commercial FE package ANSYS/LS-DYNA. That allows easy comparison of geometrical scaling method (MII) with reduced scaling method (MIII) and the proposed new method (MIII). The results with models designed according to these methods can be scaled up to predict the prototype’s behaviour and the accuracy of their results can be compared. Models at 1/10, 1/20, 1/50 and 1/100 scale were prepared for this purpose, so the spans of the models were 4m, 2m, 0.8m and 0.4m respectively.

3.1 Material model

Piecewise linear plasticity model is chosen as the material constitutive law which is based on the well-known Cowper-Symonds constitutive equation (Cowper G. R. and Symonds P. S., 1957)

\[
\sigma_d(\varepsilon, \dot{\varepsilon}) = \sigma_1 \left(1 + \left(\frac{\dot{\varepsilon}}{C}\right)^p\right)
\]

where C and P are material constants. Typically C=40 and P=5 for the Q235 steel to be used in the prototype and the model. Strain rate here actually refers to plastic strain rate which causes the strain rate effects with the Cowper-Symonds constitutive law.

The adopted constitutive curve is presented in Figure 2. 7 interpolation points signaled in the figure are input into the programme: (0.000, 235), (0.003, 290), (0.007, 330), (0.018, 375), (0.040, 400), (0.100, 425), (0.200, 450).
3.2 Element meshing

The rods are modeled as Beam 161, and each rod is divided into 5 equi-length elements. Joints connecting those rods are modeled as rigid points, while its mass is represented by Mass 166. The impactor is treated as the rigid body and it is simulated with Solid 164.

3.3 Verification of the FE method

A reticulated shell of 0.9m span is taken as an example from Wang’s (Wang D. et al, 2011) experimental study. The shell is subjected to an impact of a 6.7kg cylinder free fall from the height of 0.03m at the apex. The FE method used in this paper is applied to the experiment model and the results are compared with that from the original reference, which is listed in Table 2.

<table>
<thead>
<tr>
<th>Rod number</th>
<th>Maximum axial stress</th>
<th>Maximum apex displacement</th>
<th>Minimum apex displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FE</td>
<td>M&lt;sub&gt;1&lt;/sub&gt;&lt;sup&gt;a&lt;/sup&gt;</td>
<td>M&lt;sub&gt;1&lt;/sub&gt;&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>Test</td>
<td>-52.3</td>
<td>-56.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-50.9</td>
<td>-55.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.93</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.94</td>
<td>-</td>
</tr>
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<td></td>
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<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.33</td>
<td>-</td>
</tr>
</tbody>
</table>

It can be seen from Table 2 that the FE method used in this paper is quite satisfactory. As a result, it is used to simulated the response of the reticulated shell (scaled-down models as well as the prototype) as follows.

4 NUMERICAL RESULTS

Determining representative strain rates for different parts of the model is essential for MIII. It can be inferred from the continuity of the material that strain rate varies continuously through the rods. For simplicity, each rod is divided into 5 (the same as the element meshing number for rods) equal parts and representative strain rate is acquired from the center of these parts.

Strain rate history in the part adjacent to the apex (point A) of the prototype is depicted in Figure 3 as an example. The maximum strain rate is 1.4820 from the figure, so the representative strain rate is 0.7410. Dividing 0.7410 by γ leads to a representative strain rate for the model. If γ = 0.1, then \( \dot{\varepsilon}_m = 7.410 \). As a result, the scaling factor of density for the part is 1.1645 calculated from the combination of Equation (7) and (9). In a real test, the added mass should be 1.417kg according to Equation (8).
Representative strain rates in rod 1 through rod 4 of the prototype are reported in Figure 4.

Scaled displacements and axial forces are compared between the prototype and the models below:

4.1 Scaling of displacement

Under impact, the dome deforms with a local axisymmetric dimple inside the second ring of latitudinal rods counting from the apex as shown in Figure 5. It is signaled as zone A and can be defined as impact region. The vertical displacements of the outside parts of zone A are less than 14mm for the prototype, thus it can be defined as unimpacted region. The ultimate deformation is best exhibited by depicting the longitudinal rods.

Since the deformation of longitudinal rods in plane DAB is axially symmetric and it mainly happens inside the second ring, the displacements of rods 1 and 4 are enough to reflect the structural deformation. It is easy to see that the scaling of displacement equals $\gamma_l$. So deformation results for models with MI, MII or MIII scaling should be divided by $\gamma_l$ to predict deformation in the prototype.
Figure 6 shows the deformation responses of rods 1 and 4. The relative error of the values predicted using MI and MII clearly increases as dimensional scaling factor becomes smaller. The results for MIII are, however, stable and accurate for γ<sub>l</sub>=0.1 to 0.01. The predicted deformation profiles for the MIII models almost coincide with those of the prototype. The reduced method (MII) is better than geometrically similar scaling (MI) however. The results using MII are accurate for γ<sub>l</sub>=0.1. The relative errors in the apex displacement predicted by MI are beyond 5% for all dimensions, so using it is not suggested.

![Deformation responses of rods 1 and 4](image)

**Figure 6:** Ultimate vertical displacements of rods 1 and 4. e<sub>1</sub>, e<sub>2</sub> and e<sub>3</sub> are the relative errors of methods MI, MII and MIII respectively for the apex displacement compared with the prototype.

### 4.2 Scaling of axial force

An important mechanical behavior for the rods is their axial force history. Rod 1 was taken as an example to evaluate the scaling of axial force history since it is heavily influenced by the impact. Indeed, the maximum axial force appears in rod 1.

Combining Equations (3) and (4) yields

\[
\gamma_{\sigma_d} = \gamma_\sigma \frac{g(\dot{\varepsilon}_m)}{g(\dot{\varepsilon}_m \gamma_f / \gamma_V)}
\]  

(10)

According to dimensional analysis, the scaling factor for axial force can be calculated as

\[
\gamma_F = \gamma_f^2 \frac{g(\dot{\varepsilon}_m)}{g(\dot{\varepsilon}_m \gamma_f / \gamma_V)}
\]  

(11)
Since the material is the same for the model and the prototype, $\gamma_\sigma = 1$ and that can be neglected in the equation. The predicted axial force in prototype scale is computed by dividing the model results by $\gamma_F$.

Figure 7 depicts axial force histories for rod 1 for the models and the prototype. Positive and negative values refer to pull and push respectively. The response period selected goes from the initial impact time (0s) to 0.2s during which the response is most significant. The maximum axial force appears at the peak of the first wave. For the prototype, the maximum axial force on rod 1 is 424kN. The first wave of the profile predicted by MIII fits well with the prototype for all dimensions, with good agreement for the peak and the trough. The first peak is much larger in the MI model and with MII the first trough is much deeper than in the prototype. Both MI and MII failed to monitor the first and the most important wave. As for the first peak, which is also the maximum during the period, the model by MIII gives an accurate prediction of axial force for all dimensions with relative errors less than 0.20%. Although the relative error of the first peak predicted by MII is no more than 5% for $\gamma_l=0.1$, 0.05 and 0.02, the trough value is far lower than in prototype scale, which is unacceptable.

The detailed peak and trough values are listed in Table 3. These extreme values appear in the early stage of the impact history for each rod. Unlike rod 1, however, the trough is absolutely larger than the peak for rods 2, 3 and 4. The results with MIII are generally more stable and accurate than those using the other two methods in terms of both the peak and the trough.

Figure 7: Axial force history for rod 1. $f_1$, $f_2$ and $f_3$ refer to relative errors in the predicted maximum axial force using MI, MII and MIII modelling respectively.
5 Discussion

A key principle of dimensional analysis is that variables sharing the same dimension should be scaled equally. However, for the present scaling method (MIII) it is not necessary to alter the elastic modulus $E$ of the material or the static stress $\sigma_0$ with the dynamic stress $\sigma_d$. That is based on the assumption that the structure’s deformation is so large that the elastic part can be ignored, so the scaling method can be conceived as partial similitude method.

The tests here were performed using finite element method calculations, but in reality the proposed scaling method should be applied in laboratory tests. The main obstacle is a lack of information about the strain rate in the model. As a result, FEM simulation should be conducted to gather strain rate estimates for each part before a real test.

Since the strain rate changes with time for each part of the structure, the strain rate used in equations (6)–(8) can be chosen as half of the maximum strain rate. This has been proposed by Perrone and Bhadra (1979, 1984) and it is the only uncertain factor for the MIII scaling procedure. However, we can see through the results that the approximate procedure is good enough with ultimate vertical displacement relative error no more than 0.6% for all scaling cases.

This new scaling method can also be applied by changing the initial impact speed according to Equation (5), as presented in the works of Oshiro & Alves (2009) and Mazzariol & Alves (2014). Several iterations may be need to determine the value of $\gamma_l$. Therefore, Equation (8) is rewritten as

$$\Delta m = \rho A_p [g(\hat{\varepsilon}_m) / g(\hat{\varepsilon}_m \gamma_l / \gamma_V) - 1]$$

which is the mass that should be added to each component.

Equation (11) relates $\gamma_l$ with the square of $\gamma_p$, indicating a larger error for the force than for the displacement. That explains why the results in Section 4.1 are much better than those in Section 4.2.

6 CONCLUSIONS

A new technique based on partial similitude and dimensional analysis has been proposed. The innovative manipulation is adding mass to components of a structure in model scale to balance varied strain rate effects.

The new technique was tested using the case of an impact on a reticulated dome through using finite element modelling, and the results were compared with those of reduced scaling method (MII) and geometrically similar scaling method (MI). MII only provide an accurate prediction of the rods’ displacements for $\gamma=0.1$ scaling, whereas the new method predicts the rods’ displacements well for all of the scale factors tested. MI failed to predict the first wave of axial force for all dimensions, whereas the profile of the new method coincided with that of the prototype.
Geometrically-similar scaling failed to predict either the displacements or the axial force since it doesn’t take strain rate effects into consideration. In that point, the new technique significantly broadens the applicability of the scaling method to more complex, strain rate-sensitive impact problems.

References


