Compressive Deformation Behavior Modeling of AZ31 Magnesium Alloy at Elevated Temperature Considering the Strain Effect

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The true stress–strain data from isothermal hot compression tests, in the temperature range of 300-500 °C and strain rate of 0.001-0.1s⁻¹, were employed to study the flow behavior of AZ31 and to develop constitutive equation based on an Arrhenius-type equation. The flow stress increases with the decrease of deformation temperature and the increase of strain rate, which can be represented by Zener–Hollomon parameter in an exponential equation. The influence of strain was incorporated in the developed constitutive equation by considering the effect of strain on material constants. The results show that the proposed constitutive equations give a precise estimate for high temperature flow stress AZ31 alloy, which means it can be used for numerical simulation of hot deformation process and for choosing proper deformation parameter in engineering practice accurately.

Keywords: magnesium alloy, flow behavior, constitutive equations, numerical modeling

1. Introduction

To analyze the hot deformation processes (e.g. forging, rolling and etc.) it is necessary to describe the change in mechanical response under external loadings. This should be conducted in terms of constitutive equations which relate the stress and strain values to the related thermomechanical conditions of temperature and strain rate. In this regard, a number of previous researches have been devoted to assess such equations for conventionally hot worked magnesium alloys. Some have applied a power law equation to describe the relationship between stress and strain rate at low stresses1,5. Others have employed an exponential relationship at high stresses2,6. A phenomenological method was proposed by Sellars and McTegart7 where the flow stress is expressed by the hyperbolic laws in an Arrhenius-type equation, has also been applied8,9. Qin et al.10 developed a model to determine the flow stress of magnesium alloy during hot deformation. Quan et al.11 predicted a constitutive model for the dynamic recrystallization evolution of AZ80 magnesium alloy based on stress-strain data. However, there are always some limitations for the original constitutive model. So, in order to accurately describe and predict the flow behaviors for the different metals or alloys, considerable amount of works had been done to modify this equation by considering the special effects of the forming processing parameters. Slooff et al.12 have noted that a strain-dependent parameter should be incorporated to correct the constitutive behavior. Some investigations have established the constitutive equations for various metals and alloys such as 9Cr–1Mo (P91) steel13, aluminum alloys14-16, V150 grade oil casing steel17, H62 brass alloy18 incorporating the effect of the strain. However it has not well documented in case of magnesium alloys yet.

The present work deals with developing a proper constitutive based model using hyperbolic sine equations considering the effect of strain. The main objective is to describe the high temperature flow behavior of AZ31 alloy, as the most common wrought AZ magnesium alloy series.

2. Experimental Procedure

The experimental material was AZ31 magnesium alloy (Mg–2.9Al–0.85Zn–0.3Mn, wt. %) which was received as-hot rolled plates with 22 mm thickness. The cylindrical hot compression testing specimens were machined in the sizes of Φ 8 × H 12 mm. In all as-rolled specimens the deformation axis was selected to be parallel to the rolling direction. As was well-documented the rolled plates would have a texture with basal plane mainly parallel to normal direction. In the present work the deformation axis of the all specimens is normal to the basal planes. The isothermal hot compression tests were carried out at temperature range of 300-500 °C with initial strain rate of 0.001, 0.01 and 0.1 s⁻¹. The specimens were first heated up to the deformation temperature and held isothermally for 5 min, prior to straining. The specimens were then compressed to a true strain of 0.4 using an Instron-4208 universal testing machine, equipped with electrical resistance furnace, which can maintain temperature variation of ±5K. This was followed by quenching them in water right after straining.

3. Result and Discussion

3.1. Flow stress characteristics

True stress–true strain curves obtained from the hot compression tests of the rolled specimens are presented in Figure 1. As is observed, the peak flow stress is sensitive to
temperature and strain rate, with the peak flow stress being shifted to lower stresses and lower strains as the rate of deformation is reduced or the deformation temperature is increased. The related characteristics will be discussed in the following sections.

3.2. Constitutive equations

As is well established the correlation between the flow stress ($\sigma$), temperature (T) and strain rate ($\dot{\varepsilon}$), particularly at high temperatures, can be expressed by an Arrhenius type equation. Moreover, the effects of temperature and strain rate on deformation behavior may also be represented by the Zener-Holloman parameter ($Z$) in an exponent type equation. These are mathematically expressed as:

$$Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right)$$  

$$\dot{\varepsilon} = AF(\sigma)\exp \left( \frac{-Q}{RT} \right)$$  

where

$$F(\sigma) = \begin{cases} \sigma^{n_1} & \alpha\sigma < 0.8 \\ \exp(\beta\sigma) & \alpha\sigma < 1.2 \\ [\sinh(\alpha\sigma)]^n & \text{for all } \alpha \sigma \end{cases}$$

where R is the universal gas constant (8.314 J mol$^{-1}$ K$^{-1}$); T is the absolute temperature in K; Q is the activation energy (kJ mol$^{-1}$); A, $\beta$, $n_1$, $\alpha$ and $n$ are the materials constants, $\alpha = \beta/n_1$.

3.3. Determination of materials constants

True stress-true strain data from the compression tests at various processing conditions were employed to calculate the materials constants of the constitutive equations. The evaluation procedure of material constants at true strain of 0.2 as an example is as follows. For low and high stress levels, substituting the values of $F(\sigma)$ in Equation 2 gives the following relationships, respectively:

$$\dot{\varepsilon} = B \sigma^{n_1}$$  

$$\dot{\varepsilon} = C \exp(\beta\sigma)$$

where B and C are the material constants. Taking Logarithm of both sides of Equation 4 and 5 yields:

$$\ln(\sigma) = \frac{1}{n_1} \ln(\dot{\varepsilon}) - \frac{1}{n_1} \ln(B)$$

$$\sigma = \frac{1}{\beta} \ln(\dot{\varepsilon}) - \frac{1}{\beta} \ln(C)$$

The value of $n_1$ and $\beta$ is obtained from the mean slope values of $\ln(\sigma)$ vs. $\ln(\dot{\varepsilon})$ plot and $\sigma$- $\ln(\dot{\varepsilon})$ plot (Figure 2). For rolled microstructure, these values were found to be 6.66 and 0.2 MPa$^{-1}$. The value of $\alpha = \beta/n_1$ was calculated to be 0.03 MPa$^{-1}$. These values are in good agreement with

![Figure 1. The typical true stress-strain curves of the experimental alloy obtained by hot compression tests.](image)
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previous studies\textsuperscript{20-22}. For all the strain levels, Equation 2 is rewritten as:

\[ \dot{\varepsilon} = A \left[ \sinh (\alpha \sigma) \right]^n \exp \left( \frac{Q}{RT} \right) \]

Taking the logarithm of both sides of the above Equation gives:

\[ \ln \left[ \sinh (\alpha \sigma) \right] = \frac{\ln \dot{\varepsilon}}{n} + \frac{Q}{nRT} - \frac{\ln A}{n} \]

The slope of \( \ln \left[ \sinh (\alpha \sigma) \right] \) vs. \( \ln \dot{\varepsilon} \) yields \( 1/n \) (Figure 3) and for a particular strain rate, differentiating Equation 8 gives:

\[ Q = R n \frac{d \ln \left[ \sinh (\alpha \sigma) \right]}{d \left( \frac{1}{T} \right)} \]

Thus, the \( Q \) parameter is determined from the slopes of \( \ln \left[ \sinh (\alpha \sigma) \right] \) vs. \( 1/T \) (Figure 4), through averaging the values under different strain rates. \( \ln A \) is also easily found from the interception of \( \ln \left[ \sinh (\alpha \sigma) \right] \) vs. \( \ln \dot{\varepsilon} \), The activation energy is obtained to be in the range of 144-152 kJ/mol for different strain values in the temperature range of 300-500 °C. The calculated activation energy values are close to Mg self diffusion energy (136 kJ/mol\textsuperscript{23,24}). These obtained \( Q \) values fall in the range that reported by previous researchers for hot deformation of magnesium alloys\textsuperscript{4,25}. As is well known, some deviation in deformation activation energy is acceptable due to the nature of linear regression method used for acquiring the \( Q \)-value.

3.4. Compensation of strain

It has been recently shown that the deformation activation energy and material constants are strongly influenced by the strain\textsuperscript{14-18}. Therefore, compensation of strain may have a significant effect on the accuracy of the flow stress prediction and should be taken into account in order to derive the proper constitutive equations. The influence of strain in the constitutive equation is incorporated by assuming that the activation energy (\( Q \)) and material constants (i.e. \( n \), \( \beta \), \( \alpha \), \( n \) and \( \ln A \)) are polynomial function of strains. In the present work, the values of the material constants were evaluated at various strains (in the range of 0.1-0.4) at the intervals of 0.05, the corresponding curves of which are shown in Figure 5. These values were then employed to fit the polynomial function. A fifth order polynomial, as shown in Equation 11, was found to represent the influence of strain on the material constants with a very good correlation and generalization.

\[ Q = C_0 + C_1 \varepsilon + C_2 \varepsilon^2 + C_3 \varepsilon^3 + C_4 \varepsilon^4 + C_5 \varepsilon^5 \]
\[ \ln A = D_0 + D_1 \varepsilon + D_2 \varepsilon^2 + D_3 \varepsilon^3 + D_4 \varepsilon^4 + D_5 \varepsilon^5 \]
\[ \alpha = E_0 + E_1 \varepsilon + E_2 \varepsilon^2 + E_3 \varepsilon^3 + E_4 \varepsilon^4 + E_5 \varepsilon^5 \]
\[ n = F_0 + F_1 \varepsilon + F_2 \varepsilon^2 + F_3 \varepsilon^3 + F_4 \varepsilon^4 + F_5 \varepsilon^5 \]

Once the materials constants are evaluated, the flow stress at a particular strain can be predicted. Accordingly, the constitutive equation that relates flow stress and Zener-Holloman parameter can be written in the following form (considering the Equation 1 and Equation 8):

\[ \sigma = \frac{1}{\alpha} \left\{ \left[ \frac{Z}{A} \right] + \left[ \frac{Z}{A} \right]^2 \right\}^{1/2} \]

Figure 2. Relation between \( \ln (\sigma) \) and \( \ln (\dot{\varepsilon}) \) of rolled AZ31 experimental alloy.

Figure 3. Relationship between \( \ln [\sinh (\alpha \sigma)] \) and \( \ln (\dot{\varepsilon}) \) of rolled microstructure.

Figure 4. Relationship between \( \ln [\sinh (\alpha \sigma)] \) and 1000/T of rolled microstructure.
3.5. Verification of constitutive equation

The developed constitutive equation (considering the compensation of strain) has been verified through comparing the experimental and predicted data (Figure 6). As is observed a good agreement has been obtained between the experimental and predicted stress values. The predictability of the constitutive equation is also quantified employing standard statistical parameters such as correlation coefficient (R) and average absolute relative error (AARE), as is shown in Figure 7. These are expressed as:

![Figure 5](image1.png) ![Figure 6](image2.png)

**Figure 5.** Variation of (a) $\alpha$ (b) $n$ (c) $Q$ and (d) $\ln A$ with true strain for rolled microstructure.

**Figure 6.** Comparison between the experimental and predicted flow stress at different strain rate for Rolled initial microstructure.
The correlation between the experimental and predicted flow stress data from the proposed constitutive equation over the entire range of strain, strain rate and temperature.

\[ R = \frac{\sum_{i=1}^{N} (E_i - \bar{E})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{N} E_i^2 \sum_{i=1}^{N} (P_i - \bar{P})^2}} \]  

(13)

\[ \text{AARE (%) = } \frac{1}{N} \sum_{i=1}^{N} \left| \frac{E_i - P_i}{E_i} \right| \times 100 \]  

(14)

where E is the experimental finding and P is the predicted value obtained from the constitutive equation. The E and P are the mean values of E and P, respectively. The N is the number of data which were employed in the investigation. The correlation coefficient is a commonly used statistical parameter and provides information about the strength of linear relationship between the observed and the calculated values.

4. Conclusion

The high temperature deformation behavior of wrought AZ31 magnesium alloy has been investigated by performing isothermal hot compression tests. Based on the experimental stress–strain data, constitutive analysis of AZ31 was carried out. The following conclusions can be drawn:

- Within the range of experiment, the flow stress of AZ31 increases with the increase of strain rate and the decrease of deformation temperature. The influence of strain was incorporated in the constitutive equation by considering the effect of strain on material constants (i.e., n, Q and ln A). A 5th order polynomial was used to represent the influence of strain on these material constants with good correlation;

- The flow stress can be predicted precisely using the constitutive equation (considering the compensation of strain) under the tested deformation conditions. The average absolute relative error associated with the prediction for the whole temperature and strain rate range was 9.78% and the correlation coefficient was 0.989 for rolled structure.

References


