Band Structure in Carbon Nanostructure Phononic Crystals

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1. Introduction

Recently, artificial periodic composites known as phononic crystals (PCs) have been quite studied. They are created by arranging periodically two or more materials with different vibrational properties. Elastic/acoustic mismatch between the constituent materials can be considered to arise either from difference of material properties/ geometric parameters (e.g., density, elastic modulus, cross-sectional area) - continuum-scale theory, or from interatomic force constants and masses - atomic-scale theory.

PCs have received renewed attention because they exhibit band gaps where there are only mechanical (elastic or acoustic) evanescent waves and no mechanical propagating waves. Novel physical properties of PCs arise from the possibility of creating phononic band gaps and negative refraction (phonon branches with negative group velocity). Phononic band gaps are similar to electronic and photonic band gaps in semiconductors and photonic crystals, respectively. The physical origin of phononic and photonic band gaps can also be understood at micro-scale using the classical wave theory to describe Bragg and Mie resonances, respectively, based on the scattering of mechanical and electromagnetic waves propagating within the crystal. PCs have many applications, such as vibration isolation technology, acoustic barriers/filters, noise suppression devices, surface acoustic devices, architectural design, sound shields, acoustic diodes, elastic/acoustic metamaterials (EM/AM), also known as locally resonant phononic crystals (LRPC), and thermal metamaterials (TM), also known as phononic thermocrystals or locally resonant phononic thermocrystals.

Phononic thermocrystals can reduce the thermal conductivity in a nanostructured semiconducting material without affecting other important factors, especially electrical conductivity. LRPCs, differently from traditional PCs which create Bragg-type band gaps, present locally resonant (LR) band gaps. LR band gaps can be obtained in a frequency range of orders of magnitude lower than that given by Bragg limit. LR band gaps arise in the vicinity of the local resonator natural frequency while Bragg-type band gaps typically occur at wavelengths of the order of unit cell size. The concept of EM or LRPC generally involves local resonators, and the periodicity is advantageous (creates Bragg-type band gaps), but it is not necessary in an EM.

Researchers have been studied PCs on mm scales, with frequency band gaps ranging from GHz and kHz to MHz, respectively. However, they present poor electronic and optical applications. More recently, with the advance of nanomaterial fabrication, phonononic crystals have been studied and it is possible to control wave propagation in a frequency range from hypersonic to thermal. Only a few studies have focused on carbon microstructure and carbon nanostructure PCs in a hypersonic frequency range for elastic wave propagation control. Anjos & Arantes studied the influence of carbon microtubes in an epoxy matrix achieving band gaps ranging from GHz scale. These researchers only studied hollow circular cross section geometry in a square lattice.
2. 2D Carbon Nanostructure Phononic Crystal Model by PWE Method

This section presents PWE formulation for a 2D PC. PWE method, also known as ω(k) method, where k is the Bloch wave vector, is a semi-analytical method used to predict the band structure. PWE is one of the most used methods to calculate the elastic band structure of PCs and it has been applied in micro and nanophononic crystals (NPCs). PWE method is used to solve the constitutive equations based on classical elasticity theory, ignoring nanoscopic size effects, similar to other studies. Quantum effects should be investigated in a near future.

We also consider two-dimensional periodicity, i.e. 2D PC, isotropic elastic solid and wave propagation in the xy plane. Complete band gaps between XY (longitudinal-transverse vibration) and Z (transverse vibration) modes are observed for all types of carbon nanostructures - circular, hollow circular, square and rotated square with a 45° angle of rotation with respect to the x and y axes. We apply the plane wave expansion (PWE) method to obtain the elastic band structure.

To the best of our knowledge, it is the first time that the elastic band structure of a carbon nanostructure PC is analyzed varying the inclusion geometries and lattice types for mechanical vibration control.

2.1 2D Phononic Crystal Model

Figure 1 (a-e) sketches the cross section of carbon nanostructure/epoxy PC unit cells taking into account square, rectangular, triangular, honeycomb and Kagomé lattices.

We consider 1/4 of the FBZ as the actual first irreducible Brillouin zone (FIBZ) for square, rectangular, triangular, honeycomb and Kagomé lattices.

Note that Eqs. (4-6) are written using Voigt notation. Equations (4-6) can also be rewritten in a vectorial notation:

\[ \rho \ddot{u}_i = \nabla \cdot (c_{ijkl} \nabla u_i) + \kappa (\nabla \cdot u_i) + \rho \ddot{u}_i, \]  

\[ \rho \ddot{u}_j = \nabla \cdot (c_{ijkl} \nabla u_j) + \kappa (\nabla \cdot u_j) + \rho \ddot{u}_j, \]  

\[ \rho \ddot{u}_3 = (c_{11} u_{3,1} + c_{12} u_{2,2})_1 + [c_{66} (u_{1,2} + u_{2,1})]_x. \]

The main purpose of this study is to investigate the elastic band structure, also known as dispersion relation, of carbon nanostructure (inclusion)/epoxy (matrix) PCs with wave propagation in the xy plane and two-dimensional periodicity in a square, rectangular, triangular, honeycomb and Kagomé lattices. We consider the following carbon nanostructure geometries - circular, hollow circular, square and rotated square with a 45° angle of rotation with respect to the x and y axes. We apply the plane wave expansion (PWE) method to obtain the elastic band structure.
where \( c_{16} = \frac{1}{2} (c_{11} - c_{12}) = c_{44} = G, \frac{E}{2(1+\nu)} \) is the shear modulus, \( E \) is the Young’s modulus, \( \nu \) is the Poisson’s ratio, \( c_{11} = \lambda + 2\mu, c_{12} = \lambda, \frac{\lambda}{(1+\mu)(1-2\mu)} \) and \( \mu = G \) are known as Lamé coefficients. \( \nabla_i = (\partial/\partial x_j)e_i + (\partial/\partial x_j)e_i, u_i = u_1, u_2, u_3 \) and \( e_i \) \( (i = 1,2) \) are the basis vectors in the real space.

In addition, considering a PC, we can note that \( c_{11} = c_{11}(r), c_{12} = c_{12}(r), c_{66} = c_{66}(r), c_{44} = c_{44}(r) \), \( \rho = \rho(r) \) because we consider two different materials - carbon and epoxy, and \( u_i = u_i(r,t) \). For a two-dimensional periodicity (the system has translational symmetry in z direction and the material parameters depend only on the x and y coordinates), then \( r = x e_i + y e_i (x,y \in \mathbb{R}) \) is the two-dimensional spatial vector.

In order to eliminate the factor time in Eqs. (4-6), we apply the temporal Fourier transform. Applying the Floquet-Bloch’s theorem\(^{56,57} \), expanding Bloch wave amplitude as Fourier series in reciprocal space and considering wave propagation in the xy plane \( (k_z = 0) \), we can write:

\[
\begin{align*}
U_i(k, r) &= e^{i k x} U_{k}(r) = e^{i k x} \\
&= \sum_{g=-\infty}^{\infty} U_{k}(g) e^{i k x} = \sum_{g=-\infty}^{\infty} U_{k}(g) e^{i (k + g) x},
\end{align*}
\]

(9)

where \( U_{k}(r) \) is the Bloch wave amplitude. Note that \( U_{k}(r) = U_{k}(r + \mathbf{F}) \) and \( U_i(r + \mathbf{F}) = U_i(r) e^{i k_x x} \), where \( e^{i k_x x} \) is called the Bloch periodic boundary condition, \( k = k e_1 + k e_1, k_1, k_2 \in \mathbb{R} \) are the point coordinates in the reciprocal space. The basis vectors in reciprocal space \( \mathbf{b}_i \) \( (i = 1,2) \) are defined as \( \mathbf{a}_1 = 2\pi \hat{x}, \mathbf{a}_2 = 2\pi \hat{y}, \) \( \mathbf{b}_1 = 2\pi \hat{x}, \mathbf{b}_2 = 2\pi \hat{y} \), \( \mathbf{a}_i \) \( (i = 1,2) \) are the components of the lattice vector \( \mathbf{F} = (\mathbf{a}_1 + q \mathbf{a}_2), \hat{p}, \hat{q} \in \mathbb{Z} \).

The lattice vector components for square lattice are \( a_1 = a e_i \) \( (i = 1,2) \), for rectangular lattice \( a_1 = a e_1, a_2 = a e_2 \), for triangular lattice \( a_1 = a e_1, a_2 = \frac{\sqrt{3}}{2} e_1 + \frac{\sqrt{3}}{2} e_2 \), for honeycomb lattice \( a_1 = a e_1 + \frac{\sqrt{3}}{2} e_1, a_2 = \frac{2\sqrt{3}}{3} e_1 + \frac{2\sqrt{3}}{3} e_2 \), and for Kagomé lattice \( a_1 = a e_1 + a \sqrt{3} e_2, a_2 = -ae_1 + a \sqrt{3} e_2 \).

The reciprocal lattice vector is defined as \( g = \frac{2\pi}{a} (m \hat{a} + n \hat{b}) \) \( (m, n \in \mathbb{Z}) \) for square lattice, \( g = \frac{2\pi}{a \sqrt{2}} \hat{a} + \frac{2\pi}{a \sqrt{2}} \hat{b} \) for rectangular lattice, \( g = \frac{2\pi}{a \sqrt{3}} \hat{a} + \frac{2\pi}{a \sqrt{3}} \hat{b} \) for triangular lattice, \( g = \frac{2\pi}{a \sqrt{3}} \hat{a} + \frac{2\pi}{a \sqrt{3}} \hat{b} \) for honeycomb lattice and \( g = \frac{2\pi}{a} (m \hat{a} + n \hat{b}) \) for Kagomé lattice. Note that \( g \) is a two-dimensional vector because we consider two-dimensional periodicity.

Furthermore, we may expand \( c_{11}, c_{12}, c_{66}, c_{44} \) in Fourier series on the reciprocal space as:

\[
P(r) = \sum_{g=-\infty}^{\infty} P(g) e^{i \mathbf{g} \cdot \mathbf{r}},
\]

(10)

where \( P \) is one of the \( c_{11}, c_{12}, c_{66}, c_{44} \) \( \rho \) and \( g \) has the same expressions of \( g \) with \( \mathbf{m}, \mathbf{n} \in \mathbb{Z} \). We use \( g \) instead of \( g \) to highlight the difference between the Fourier series expansion of material properties and displacements.

Substituting Eqs. (9) and (10) in Eqs. (4-6), with \( \hat{g} = g \) + \( g \), multiplying by \( e^{i \mathbf{g} \cdot \mathbf{r}} \) and integrating over the unit cell, we may write:

\[
(K - \omega^2 M)q = 0,
\]

(11)

where

\[
\mathbf{K} = \begin{bmatrix}
K_{11} & K_{12} & 0 \\
K_{21} & K_{22} & 0 \\
0 & 0 & K_{33}
\end{bmatrix}
\]

(12)
The sub-matrices in Eq. (12) are given by:

\[
\mathbf{K}_{11} = c_{11} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2} + c_{66} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2},
\]

\[
\mathbf{K}_{12} = c_{12} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2} + c_{66} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2},
\]

\[
\mathbf{K}_{21} = c_{12} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2} + c_{66} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2},
\]

\[
\mathbf{K}_{22} = c_{11} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2} + c_{66} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2},
\]

\[
\mathbf{K}_{33} = c_{33} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2} + c_{44} \left( \mathbf{g}^2 - \mathbf{g} \right)(k + g)_{1}(k + g)_{2},
\]

The matrix \( \mathbf{M} \) in Eq. (11) is expressed by:

\[
\mathbf{M} = \begin{bmatrix}
\rho(\mathbf{g}^2 - \mathbf{g}) & 0 & 0 \\
0 & \rho(\mathbf{g}^2 - \mathbf{g}) & 0 \\
0 & 0 & \rho(\mathbf{g}^2 - \mathbf{g})
\end{bmatrix},
\]

The vector \( \mathbf{q} \) in Eq. 11 is given by:

\[
\mathbf{q} = \begin{bmatrix}
u_{ik}(\mathbf{g}) \\
u_{ik}(\mathbf{g}) \\
u_{ik}(\mathbf{g})
\end{bmatrix}.
\]

Equation (11) is an infinite system of equations, thus we truncate the Fourier series, i.e. \( m, m, n, \bar{n} = \{-M, \ldots, M\} \) and the total number of plane waves is \((2M + 1)^2 \). Equation (11) represents a generalized eigenvalue problem of \( \omega^2(k) \) and should be solved for each \( k \) into the FIBZ for square, rectangular, triangular, honeycomb and Kagomé lattices (Figure 1 (f-h)).

The Fourier coefficients are defined as:

\[
P(\mathbf{g}) = \begin{cases} fP_{A} + (1 - f)P_{B} & \text{if } \mathbf{g} = 0 \\
(P_{A} - P_{B})F(\mathbf{g}) & \text{if } \mathbf{g} \neq 0,
\end{cases}
\]

where indexes \( A \) and \( B \) in Eq. (20) are related to inclusion (carbon nanostructure) and matrix (epoxy), respectively, \( F(\mathbf{g}) \) is the structure function and \( f \) is the filling fraction of each type of carbon nanostructure, considering circular section of radius \( \bar{r} \), square section of width \( 2l \), rotated square section of width \( 2l \) with a \( 45^\circ \) angle of rotation with respect to \( x \) and \( y \) axes and hollow circular section with external radius \( \bar{R} \) and internal radius \( \bar{R}, \bar{R} > \bar{r} \). Hollow cylinder inclusion has an internal radius \( \bar{r} \) of epoxy and a thickness \( \bar{R} - \bar{r} \) of carbon nanostructure, similar to Anjos & Arantes (10).

The filling fraction is defined as \( f = S_{j}/S_{c} \) for square, rectangular and triangular lattices, whereas for honeycomb and Kagomé lattices it is defined as \( f = 2S_{j}/S_{c} \) and \( f = 3S_{j}/S_{c} \), respectively, where \( S_{j} \) is the cross section area of the carbon nanostructures and \( S_{c} = \lambda_{a} \times \lambda_{j} \) is the cross section area of the unit cell. Note that differently from the other lattices honeycomb and Kagomé lattices have two and three inclusions per unit cell, respectively. The filling fractions for square, rectangular, triangular, honeycomb and Kagomé lattices are:

\[
f = \begin{cases}
\pi \bar{r}^2/a^2 & \text{for circular section} \\
4f^2/a^2 & \text{for square section} \\
4f^2/a^2 & \text{for rotated square section} \\
\pi(\bar{R}^2 - \bar{r}^2)/a^2 & \text{for hollow section}
\end{cases}
\]

\[
f = \begin{cases}
\pi \bar{r}^2/(a_{1}a_{2}) & \text{for circular section} \\
4f^2/(a_{1}a_{2}) & \text{for square section} \\
4f^2/(a_{1}a_{2}) & \text{for rotated square section} \\
\pi(\bar{R}^2 - \bar{r}^2)/(a_{1}a_{2}) & \text{for hollow section}
\end{cases}
\]

\[
f = \begin{cases}
2\pi \bar{r}^2/\sqrt{3}a^2 & \text{for circular section} \\
8f^2/\sqrt{3}a^2 & \text{for square section} \\
8f^2/\sqrt{3}a^2 & \text{for rotated square section} \\
2\pi(\bar{R}^2 - \bar{r}^2)/\sqrt{3}a^2 & \text{for hollow section}
\end{cases}
\]

\[
f = \begin{cases}
4\pi \bar{r}^2/3\sqrt{3}a^2 & \text{for circular section} \\
16f^2/3\sqrt{3}a^2 & \text{for square section} \\
16f^2/3\sqrt{3}a^2 & \text{for rotated square section} \\
4\pi(\bar{R}^2 - \bar{r}^2)/3\sqrt{3}a^2 & \text{for hollow section}
\end{cases}
\]

\[
f = \begin{cases}
\pi \sqrt{3} \bar{r}^2/2a^2 & \text{for circular section} \\
2\sqrt{3}f^2/a^2 & \text{for square section} \\
2\sqrt{3}f^2/a^2 & \text{for rotated square section} \\
\pi \sqrt{3}(\bar{R}^2 - \bar{r}^2)/2a^2 & \text{for hollow section}
\end{cases}
\]

respectively.

The structure function for square, rectangular and triangular lattices \( F(\mathbf{g}) \) is defined as:

\[
F(\mathbf{g}) = \frac{1}{S_{c}} \int_{\Omega} e^{-\mathbf{g} \cdot \mathbf{r}} d^{2}r.
\]
The integral in Eq. (26) is performed over the cross section of the unit cell. The structure functions for square, rectangular, and triangular lattices are:

\[
P_f(\mathbf{g}) = \begin{cases} 
\frac{2f(\mathbf{g})}{\pi^2} & \text{for circular section} \\
\frac{2f(\mathbf{g})}{\pi^2} & \text{for square section} \\
\left[ \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_i \right) \right) \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_f \right) \right) \right] & \text{for rectangular section} \\
\left[ \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_i \right) \right) \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_f \right) \right) \right] & \text{for triangle section} \\
\left[ \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_i \right) \right) \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_f \right) \right) \right] & \text{for hollow section} \
\end{cases}
\]

for square rotated section

\[
P_f(\mathbf{g}) = \frac{2f(\mathbf{g})}{\pi^2} \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_i \right) \right) \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_f \right) \right) \]

for triangular section

\[
P_f(\mathbf{g}) = \frac{2f(\mathbf{g})}{\pi^2} \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_i \right) \right) \left( \sin \left( \frac{\nu}{2}\mathbf{g} \cdot \mathbf{u}_f \right) \right) \]

for hollow section

where \( \mathbf{g} = [\mathbf{g}] \) and \( \mathbf{g}_{1,2} = [\mathbf{g}_{1,2}] \).

The structure function for honeycomb lattice \( F_{h}(\mathbf{g}) \) is defined as:

\[
F_{h}(\mathbf{g}) = \cos(\mathbf{g} \cdot \mathbf{u}) - \frac{1}{S_c} \int e^{-\mathbf{g} \cdot \mathbf{r}} d^2 r = \frac{1}{2} \left[ 3 \sum_{i=1}^{3} e^{-\mathbf{g} \cdot \mathbf{r} \cdot \mathbf{u}_i} F(\mathbf{g}) \right],
\]

where \( \mathbf{u}_i = -\alpha(0, 1/2) \) are the vectors that define the central position of the two scatterers (carbon nanostructures) into the honeycomb unit cell. We chose these vectors similar to Gao et al.93, Thus, the structure functions of carbon nanostructures for honeycomb lattice are the same of Eq. (26) multiplied by \( \cos(\mathbf{g} \cdot \mathbf{u}_i) \), considering \( f \) from Eq. (24). Note that Eq. (28) is not multiplied by \( 2\cos(\mathbf{g} \cdot \mathbf{u}_i) \) as other authors83,93,95, because we define the filling fraction as \( f = 2S_c/S_a \) and not as \( f = S_c/S_a \).

The structure function for Kagomé lattice \( F_{k}(\mathbf{g}) \) is defined as:

\[
F_{k}(\mathbf{g}) = \frac{1}{3} \sum_{i=1}^{3} e^{-\mathbf{g} \cdot \mathbf{r} \cdot \mathbf{u}_i} F(\mathbf{g})
\]

where \( \mathbf{u}_i = -\alpha(0, 1/2) \), \( \mathbf{u}_i = -\alpha(0, 1/2) \), \( \mathbf{u}_i = -\alpha(0, 1/2) \), \( \mathbf{u}_i = -\alpha(0, 1/2) \), are the vectors that define the positions of the three carbon nanostructures into the Kagomé unit cell. Note that the Eq. (29) is multiplied by 1/3, because we define the filling fraction as \( f = 3S_c/S_a \) and not as \( f = S_c/S_a \). It is possible to rewrite Eq. (29) separating its real and imaginary parts:

\[
F_{k}(\mathbf{g}) = \Re\left[F_{k}(\mathbf{g})\right] + j\Im\left[F_{k}(\mathbf{g})\right],
\]

\[
\Re\left[F_{k}(\mathbf{g})\right] = \frac{1}{3} \sum_{i=1}^{3} \cos(\mathbf{g} \cdot \mathbf{r} \cdot \mathbf{u}_i) F(\mathbf{g}),
\]

\[
\Im\left[F_{k}(\mathbf{g})\right] = \frac{1}{3} \sum_{i=1}^{3} \sin(\mathbf{g} \cdot \mathbf{r} \cdot \mathbf{u}_i) F(\mathbf{g}),
\]

where \( \mathbf{u}_{1,2} = [\mathbf{u}_{1,2}] \).

3. Results and Discussion

The physical parameters of carbon nanostructure (A) and epoxy (B) are listed in Table 1. We use some physical parameters of Anjos & Arantes10. We calculate the elastic band structure considering initially a fixed filling fraction, 0.335, for the four carbon nanostructure cross section geometries - circular, hollow circular, square and rotated square with a 45° angle of rotation with respect to the x, y axes, considering square, rectangular, triangular, honeycomb and Kagomé lattices. In the course of the numerical calculations, the integers \( m, n, \tilde{n} \) are limited to the interval [-10, 10] for all results, i.e. 441 plane waves. This resulted in a good convergence. We restrict the band structure plots until a maximum reduced frequency, i.e. \( \Omega = \omega a/2\pi c_{44}, \) of 2, where \( c_{44} = f c_{44a} + (1-f) c_{44b} \). The reduced frequency for rectangular lattice is calculated considering \( a = \sqrt{a_i^2 + a_j^2} \).

Figure 2 (a-d) compares the band structure of a square lattice illustrated in Figure 1 (a) and (f) for the four types of carbon nanostructures, considering XY (red) and Z (blue) modes. We plot the band structure in the three principal symmetry directions of the FIBZ (see Figure 1 (f-h)). Plots are given in terms of reduced frequency versus reduced Bloch wave vector \( \mathbf{K} = k a/2\pi \). The reduced Bloch wave vector for rectangular lattice is calculated considering \( a = \sqrt{a_i^2 + a_j^2} \) . In Figure 2 (a), two complete band gaps are obtained for circular carbon nanostructure.

The relation between parameters \( \tilde{R} \) and \( \tilde{R} \) for hollow circular carbon nanostructure is fixed, i.e. \( \tilde{R} - \tilde{R} \), on the band structure. The influence of carbon microstructure thickness for square lattice was investigated by Anjos & Arantes10. Figure 2 (b) presents one complete band gap and we can observe that the first branches occur in higher frequencies compared to the other carbon nanostructures.

Figure 2 (c) shows one complete band gap for square carbon nanostructure and it is the broadest one for square lattice, with a bandwidth of 0.3478, approximately. When these square carbon nanostructures are rotated 45° with respect to x and y axes, another three narrow complete gaps are created in higher frequencies as illustrated in Figure 2 (d). Even though square carbon nanostructure presents the broadest complete band gap for square lattice, rotated square carbon nanostructure presents more complete band gaps.

| Table 1. Physical parameters of carbon nanostructure (A) and epoxy (B). |
|-----------------|------------------|
| Geometry/Property | Value            |
| Lattice parameter (\(a_i\)) | 30 x 10^9m      |
| Lattice parameter (\(a_j\)) | 25 x 10^9m      |
| Filling fraction (\(f\)) | 0.335            |
| Mass density (\(\rho_A\)) | 1800 kg/m³, 120 kg/m³ |
| Young’s modulus (\(E_A, E_B\)) | 130 x 10^9Pa, 6 x 10^9Pa |
| Poisson’s ratio (\(\nu_A, \nu_B\)) | 0.4, 0.35       |
Figure 2. Elastic band structures of XY (red) and Z (blue) modes of carbon nanostructures in an epoxy matrix for square lattice. The following types of inclusions are considered - circular (a), hollow circular (b), square (c) and rotated square with a 45° angle of rotation with respect to the x, y axes.
Figure 3. Elastic band structures of XY (red) and Z (blue) modes of carbon nanostructures in an epoxy matrix for rectangular lattice. The following types of inclusions are considered - circular (a), hollow circular (b), square (c) and rotated square with a 45° angle of rotation with respect to the $x, y$ axes.
Figure 4. Elastic band structures of XY (red) and Z (blue) modes of carbon nanostructures in an epoxy matrix for triangular lattice. The following types of inclusions are considered - circular (a), hollow circular (b), square (c) and rotated square with a 45° angle of rotation with respect to the $x, y$ axes.
Figure 5. Elastic band structures of XY (red) and Z (blue) modes of carbon nanostructures in an epoxy matrix for honeycomb lattice. The following types of inclusions are considered - circular (a), hollow circular (b), square (c) and rotated square with a 45° angle of rotation with respect to the x, y axes.
Figure 6. Elastic band structures of XY (red) and Z (blue) modes of carbon nanostructures in an epoxy matrix for Kagomé lattice. The following types of inclusions are considered - circular (a), hollow circular (b), square (c) and rotated square with a 45° angle of rotation with respect to the x, y axes.
Figure 7. Elastic band gap widths of carbon nanostructures in an epoxy matrix as a function of filling fraction for square lattice. The following types of inclusions are considered - circular (a), hollow circular (b), square (c) and rotated square with a 45° angle of rotation with respect to the $x, y$ axes.

Figure 3 (a-d) shows the band structures for rectangular lattice. For circular, hollow circular and square carbon nanostructures, the number of complete band gaps of rectangular lattice is similar to square lattice. However, for rotated square carbon nanostructure, only two complete band gaps are created. Square carbon nanostructure presents the broadest complete band gap, as well as in a square lattice, with 0.4309 of bandwidth for rectangular lattice.

Figure 4 (a-d) shows the band structures for triangular lattice. For square and rotated square carbon nanostructures, we observe a wide complete band gap with close bandwidths, i.e. 0.3801 and 0.3865, respectively. The broadest complete band gap is observed for circular carbon nanostructure with a complete band gap width of 0.4141 (Figure 4 (a)). For hollow inclusion, two complete band gaps are opened up. Comparing the band structures of triangular lattice to the square and rectangular lattices, we observe that triangular lattice presents broader complete band gaps for all carbon nanostructures, except for square carbon nanostructure, where rectangular lattice presents the broadest complete band gap.

Figures 5 and 6 illustrate the band structures for honeycomb and Kagomé lattices, respectively. Square and rotated square carbon nanostructures present close behavior for both lattices. The band structure for circular carbon nanostructure shows...
Figure 8. Elastic band gap widths of carbon nanostructures in an epoxy matrix as a function of filling fraction for rectangular lattice. The following types of inclusions are considered - circular (a), hollow circular (b), square (c) and rotated square with a 45° angle of rotation with respect to the x, y axes.

We also observe that Kagomé lattice presents better performance (broader band gaps) than honeycomb lattice for square and rotated square carbon nanostructures, whereas honeycomb lattice shows better performance than Kagomé lattice for circular and hollow circular carbon nanostructures.

In Figures 7-11, we analyze the elastic band gap widths ($\Delta\Omega$) as a function of filling fraction for all types of carbon nanostructures and lattices considered. For square lattice, Figure 7, the geometries that provide the best behavior are circular and square carbon nanostructures with high band gap widths in a wide range of filling fraction. Among circular and square geometries, square carbon nanostructure presents the best performance because it presents complete band gaps opened up over a wide range of filling fraction defined by $0.06 \leq f \leq 0.86$. Hollow circular carbon nanostructure presents
only three small complete band gaps, whereas rotated square carbon nanostructure does not improve the square carbon nanostructure performance.

For rectangular lattice, Figure 8, carbon nanostructures present less complete band gaps than square lattice for circular, square and rotated square geometries. All carbon nanostructures in square lattice present broader band gap widths than rectangular lattice in a large range of filling fraction. However, band gap widths are higher for rectangular lattice, considering circular, square and rotated square carbon nanostructures.

We can observe from Figure 9 that circular carbon nanostructure presents the broadest band gap widths for triangular lattice and they are opened up over a large range of filling fraction, i.e. $0.01 \leq f \leq 0.81$.

Comparing triangular lattice, Figure 9, to square and rectangular lattices, Figures 7 and 8, respectively, it is not simple to identify which carbon nanostructure presents the
Figure 10. Elastic band gap widths of carbon nanostructures in an epoxy matrix as a function of filling fraction for honeycomb lattice. The following types of inclusions are considered - circular (a), hollow circular (b), square (c) and rotated square with a 45° angle of rotation with respect to the \( x, y \) axes.

The best performance among the lattice types, because there are two important variables, that is to say the band gap width (the higher, the better) and the filling fraction range in which the band gaps are opened (the broader, the better).

From triangular, square and rectangular lattices, we observe that square carbon nanostructure presents the best performance for square lattice because many complete band gaps are opened up in a large range of filling fraction. However, in a specific range of filling fraction, square carbon nanostructure in rectangular lattice presents higher band gap widths. Circular, hollow circular and rotated square carbon nanostructures present the best performances for triangular lattice. Comparing the best performances until now, that is to say square and circular carbon nanostructures for square and triangular lattices, Figures 7 (c) and 9 (a), respectively, the best performance is found for circular carbon nanostructure in triangular lattice, because it presents high band gap width in a large range of filling fraction.

From Figures 10 and 11, we can observe that Kagomé lattice shows better performance than honeycomb lattice considering circular, square and rotated square carbon nanostructures. However, the difference among these lattices is subtle. Hollow circular carbon nanostructure for honeycomb lattice shows better performance than for Kagomé lattice.
Comparing honeycomb and Kagomé lattices to other lattices analyzed, we can observe that for all carbon nanostructures, the best behavior is found for square and triangular lattices.

4. Conclusion

We obtain broad complete band gaps between XY and Z modes where there are no elastic propagating waves, only evanescent waves. We firstly analyze the elastic band structure of carbon nanostructures in an epoxy matrix for a fixed filling fraction of 0.335. For square lattice, the broadest complete band gap is found for square carbon nanostructure with 0.3478 of bandwidth. This carbon nanostructure geometry is also the best one for rectangular lattice with 0.4309 of bandwidth. The broadest band gap for triangular lattice is observed for circular carbon nanostructure with a complete band gap width of 0.4141. The band structures of honeycomb and Kagomé lattices also opens up complete band gaps, however, all of them narrower than the complete band gaps of square, rectangular and triangular lattices.

We also analyze the elastic band gap widths as a function of filling fraction for all types of carbon nanostructures and lattices. Considering triangular, square and rectangular lattices, we observe that square carbon nanostructure presents the best performance for square lattice, whereas circular, hollow circular and rotated square carbon nanostructures present best performances for triangular lattice. For honeycomb and Kagomé lattices, Kagomé lattice presents the best performance.
for circular, square and rotated square carbon nanostructures. However, the difference among these lattices is subtle.

In a general way, the best performance observed for the nanophononic crystal studied is found for circular carbon nanostructures in a triangular lattice. Finally, we consider square, rectangular, triangular, honeycomb and Kagomé lattices with carbon nanostructures perfectly embedded in an elastic background. This means that we neglect the effects due to decohesion of carbon nanostructures from epoxy matrix and due to roughness at the interface between carbon nanostructures and epoxy matrix. These defects can modify elastic wave propagation in composite materials, altering their band structure. The elastic complete band gaps in carbon nanostructure PCs enlarge the potential applications for vibration management in GHz.

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6. References


55. Band Structure in Carbon Nanostructure Phononic Crystals


