Mathematical modeling for the management of the carrying capacity of aquaculture enterprises in lakes and reservoirs

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Abstract – Flooded areas of reservoirs and artificial lakes have been increasingly used for fish production; however, the waste generated by aquaculture has become a concern for the sustainable development of this activity. One of the main strategies adopted by management and regulatory agencies is the use of hydrodynamic models that calculate the carrying or nutrient load capacity of a particular water body and the effect of fish farming. These models are precise in the development of optimal strategies for feeding and waste calculation. This review paper addresses this topic and describes the methodology developed for the analysis and simulation of the carrying capacity for fish production, based on the integration of the Fish-PrFEQ nutritional bioenergetic model and the hydrodynamic model of Dillon & Rigler. This methodology allows evaluating the real contribution of aquaculture waste and assists in the planning and management of aquaculture in these aquatic environments, besides enabling and encouraging producers and the aquaculture industry to use fish food with better nutritional quality and lower environmental impact.

Index terms: hydrodynamic modeling, mass balance, nutritional bioenergetics, solid waste, sustainable aquaculture.

Modelagem matemática para gestão da capacidade de suporte de empreendimentos aquícolas em lagos e reservatórios

Resumo – Áreas inundadas de reservatórios e lagos artificiais estão sendo cada vez mais utilizadas para a produção de peixes; contudo, os resíduos lançados pela aquicultura tornaram-se uma preocupação para o desenvolvimento sustentável desta atividade. Uma das principais estratéias adotadas pelos órgãos gestores e fiscalizadores consiste no uso de modelos hidrodinâmicos que calculam a capacidade de suporte ou de carga de nutrientes de um determinado corpo hídrico e a influência dos cultivos de peixes. Esses modelos são precisos no desenvolvimento de estratégias ideais de alimentação e cálculo de resíduos. Este trabalho de revisão aborda esta temática e traz uma descrição da metodologia desenvolvida para análise e simulação da capacidade de suporte para produção de pescados, baseada na integração do modelo bioenergético nutricional “Fish-PrFEQ” com o modelo hidrodinâmico de Dillon & Rigler. Esta metodologia permite avaliar a real contribuição de resíduos aquícolas e auxilia no planejamento e na gestão da aquicultura nestes ambientes aquáticos, além de possibilitar e incentivar que os produtores e a indústria aquícola utilizem rações de melhor qualidade nutricional e menor impacto ambiental.

Termos para indexação: modelagem hidrodinâmica, balanço de massa, bioenergética nutricional, resíduos sólidos, aquicultura sustentável.

Introduction

Aquaculture is considered a viable and cheap source of high-quality protein, especially in developing countries, where there is a need to increase food production to guarantee food security (Béné et al., 2016). For this reason, the flooded areas of artificial lakes and ponds have being increasingly used for the aquaculture industry (Ayer & Tyedmers, 2009; Barton & Fløysand, 2010). In addition to food

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production, the expansion of this activity brings benefits to the regional economies, in the form of employment and income generation throughout the aquaculture production chain (Ross et al., 2011), and is an important alternative productive activity for populations affected by dams, for example (Béné et al., 2016).

In this scenario, according to Montanthini Neto & Ostrensky (2015), in order to produce a ton of tilapia, approximately 1,040 kg organic matter (OM), 45 kg N, and 14 kg P are released into the environment. Alves & Baccarin (2005) reported that 66% of the P obtained by intensive feeding in fish farms is absorbed by the sediment, 11% is dissolved in water, and 23% is incorporated by the farmed fish.

Therefore, the solid wastes generated by aquaculture become a concern for the sustainable development of the activity. Several researchers have shown that the residual products from different types of fish farming can be estimated by factorial mathematical models (Cho & Bureau, 1998; Lupatsch & Kissil, 1998; Yi, 1998; Bureau & Hua, 2010; Azevedo et al., 2011).

In this context, this paper presents a review of the topic and a description of the methodology developed to analyze and simulate the carrying capacity for fish production based on the integration of the “Fish-PrFEQ” nutritional bioenergetics model of Cho & Bureau (1998) with the hydrodynamic model of Dillon & Rigler (1974).

The adoption of this approach will allow evaluating each process or farm in a compartmentalized way, aiming to determine the real contribution of wastes in the aquatic environment, besides aiding in defining the carrying capacity of the reservoir for fish production. These measures will assist in monitoring zootechnical efficiency and in improving the regulations for public water concessions for aquaculture purposes, besides enabling and encouraging producers and the aquaculture industry to use feeds with better nutritional quality and lower environmental impact.

Environmental impact of aquaculture in lakes and reservoirs

The main impacts related to aquaculture in rivers, lakes, and reservoirs are associated to the increased flow of particles and dissolved nutrients in the environment (Sugiura et al., 2006; Azevedo et al., 2011; Gondwe et al., 2011; Canale et al., 2016); the mortality and loss of biodiversity of fishes (Sang, 2006); the contamination by chemical compounds (through the use of antibiotics, antiparasitics, anesthetics, and disinfectants) (Burridge et al., 2010); the lower dissolved oxygen concentrations (Hamblin & Gale, 2002); the occurrence of harmful algal blooms (Sowles, 2009); and the increase in the contents of organic matter and metals in the sediment (Xia et al., 2016). In addition to these factors, the following were observed: changes in the biodiversity of the microflora and benthic sediments (Buschmann et al., 2009); changes in the trophic structure and biological attributes of the diet of wild fishes due to the introduction of exotic species from aquaculture (Arthur et al., 2010; Carvalho et al., 2012; Ramos et al., 2014); dissemination of diseases that may affect wild populations of aquatic organisms (Israel, 2007); and, in some cases, direct conflicts with other users of water resources, which can cause adverse social effects (Béné & Obirih-Opahre, 2009).

Aquaculture effluents

Most of the effluents from aquaculture come from diets and from excess feed not consumed during feeding, resulting in solid and dissolved wastes (Bureau & Hua, 2010). The releases of P in continental water bodies (freshwater) are more alarming, because this nutrient is usually a limiting factor for plant (algae and macrophyte) growth. In marine environments, the outputs of nitrogenous wastes cause the greatest concern regarding environmental impact (Rabassó & Hernández, 2015); however, in some reservoirs, the high entry of N (coming mainly from the protein in the feed) can generate toxic ammonia in the water and endanger fish survival.

Hua & Bureau (2006) emphasize that solid waste (fecal material and food lost when feeding) can settle and impact the benthic ecosystem of inland and marine waters; therefore, estimating solid and dissolved wastes is the main strategy for monitoring and planning mitigation actions in environments where aquaculture farms are installed.

Nitrogenous wastes

The biological value of a given protein in the diet depends both on its digestibility and its balance of
amino acids in relation to the nutritional requirements of fish (NRC, 2011). The imbalance of amino acids can lead to decreased protein deposition and to increased nitrogen excretion (Cai et al., 2016).

Since the diets for fish contain high levels of protein (28 to 50%), a great amount of energy is supplied as nitrogen compounds, which leads to increased production of catabolites, as nitrite, nitrate, and ammonia, for example. In this way, saving protein content with the inclusion of digestible energy in the diet can significantly reduce pollution, due to the decrease in nitrogen end products.

Another strategy used to reduce pollution is using the ideal protein for the species. Botaro et al. (2007) showed that it is possible to reduce the content of digestible crude protein (CP) from 27 to 24.3% in diets for Nile tilapia (Oreochromis niloticus) reared in cages, and that this reduction can be achieved by a supplementation with amino acids.

In this scenario, Bureau & Hua (2010) highlighted the importance of digestible protein (DP) and digestible energy (DE) in the diet to increase N retention. Hargreaves (1998) conducted a literature review to identify the percentage of N retained by fish and its release into the environment in several aquaculture production systems. The author found differences from 19 to 21% retention with 73 to 86% of excreted nitrogenous compounds for species such as Ictalurus pagrus, O. niloticus, and Clarias macrocephalus, and that the use of diets with reduced levels of protein and increased levels of DE resulted in a decrease in the effect of feed pollution in the aquatic environment.

**Phosphate waste**

The way in which P is excreted by fish can have a direct effect on the enrichment of the aquatic environment and on algae growth. Usually P is excreted in soluble forms and particles: the soluble forms consist of organic P and PO$_4^{3-}$, which directly affect water quality, while the form of particles settles at the bottom of lakes and reservoirs or accumulates in the sediment (Tundisi & Tundisi, 2008; Canale et al., 2016).

Soluble P is readily available as a nutrient for plant growth, and a significant amount of the free fraction contained in total P is in the form of inorganic orthophosphate. The form of P consumed by fish will affect the amount of soluble P and particulate excreta, as well as the amount of P that could later be biologically degraded in the sediment (Canale et al., 2016). Therefore, the definition of nutrient inputs via aquaculture feed is of extreme importance for the sustainable development of this activity.

Wang et al. (2012) reported that wastes from salmon (Salmo salar) farms in Norway, in 2009, were released into the environment – equivalent to a discharge of about 404,000, 50,600, and 9,400 Mg C, N, and P, respectively, based on the total production of 1,02x10$^6$ Mg salmon. These results confirm those obtained by Chowdhury et al. (2013), who assessed O. niloticus fed different levels of protein in the diet (40, 38, and 35%) and found an increase of 4.2 to 5.0 kg in the excretion of P and a decrease of 46.2 to 40.9 kg in that of N per ton of tilapia produced. Montanhini Neto & Ostrensky (2015) also analyzed the potential waste load of the commercial production of O. niloticus. According to these authors, the total nutrient content in the waste generated per ton of biomass of produced tilapia was of 1,040.63 kg OM, 44.95 kg N, and 14.26 kg P, which represent 78% OM, 65% protein, and 72% P provided by feed.

Penczak et al. (1982) state that only 32% of P is used for the metabolism of fish, and the remaining 68% are transferred to the environment. Alves & Baccarin (2005) also reported that 66% of the P obtained by intensive feeding is deposited in the sediment, 11% is dissolved in water, and 23% is incorporated by the farmed fish; this emphasizes the need for programs for the management and control of aquaculture waste.

Tacon (2005) points out that, in 1985, the diets used in salmon farming in Chile contained 60% CP and only 6 to 8% lipids; however, in 2005, the average percentage of each of these nutrients was 35%, causing a decrease in the rates of excretion of metabolites by fish. For this author, these practical results minimized the polluting potential of the fish farms evaluated.

A similar situation occurred in the Norwegian and Canadian salmon industry, where a series of measures were adopted to reduce the release of nutrients from fish farms. These actions involved the optimization of feed composition, and improvements...
in feed digestibility and in processing technologies (Technical..., 2005; Bureau & Hua, 2010).

The average feed conversion of the diets used in Norway’s salmon industry reduced from 2.08, in 1974, to 1.00 in 2005 (Technical..., 2005). In Canada, the average feed conversion of the diets for salmon, in the 1980s, was 1.50, and, 20 years later, 1.10. Consequently, there was a decrease in excretion of 14 kg solid P per ton of fish produced (Bureau & Hua, 2010).

**Mathematical models applied to aquaculture**

Factorial modeling has been successfully used to estimate amino acid requirements; to improve the energy protein balance of diets for several animals, such as laying hens (Gous & Nonis, 2010), poultry, and swine (Sakomura et al., 2015); and also to determine the dietary needs of horses (Cordero et al., 2013) and cattle (Albertini et al., 2012).

In general, a deductive factorial model can be used to examine the relationship between the net requirement of an essential element for animals (the requirement for growth and the replacement of endogenous loss, for example) and the concentration in the diet needed to meet this requirement, with reduction in losses and excretions (NRC, 2011; Montanhini Neto & Ostrensky, 2015). It should be noted that each model considers the characteristics of the species, environment, and diet, among other factors that affect the final response to be evaluated.

Fish growth is a complex process that represents the results of a series of physiological and behavioral processes, involving food intake, the deposition of animal tissue, and the excretion of metabolites (Jobling, 2011). In each situation in commercial fish production, the knowledge of the growth rates in a given period, in relation to feed consumption, is essential for the analysis of the future viability of the venture.

Mathematical models that predict fish growth rates and feed requirements can be used to maximize efficiency and improve animal growth. These models can be a useful tool both for planning and managing production, as well as describing future scenarios; however, they must be used properly (Iwama & Tautz, 1981; Cho & Bureau, 1998; Dumas et al., 2010).

Despite the many attempts to develop mathematical expressions to describe fish growth, there is a wide range of approaches and concepts (Iwama & Tautz, 1981). It is common to find growth expressed in centimeters per month, instant growth rates, percentage in length change or percentage in weight change, often without any references to temperature, feed, or farming conditions (NRC, 2011).

Therefore, the adoption of an appropriate growth model allows estimating the (feed) requirements for the energy needs and growth rates of fish. This information allows the producer to solve several problems related to growth and feeding rates that arise in the routine of fish farming (Dumas et al., 2010).

Furthermore, it is possible, for example, to predict the average final weight of fish after a certain time of farming; to estimate the time required for the fish to reach a given commercial size, at a set temperature; or to decide on the necessary average temperature to produce a given size of fish in an exact period of time. In addition, a good mathematical model can also provide information about the biomass stock and daily feed, energy, and amino acid requirements (Iwama & Tautz, 1981; Bureau et al., 2002).

**Prediction models of body growth applied in fish farming**

To measure fish growth, the ratio of length or weight is usually used (Ricker, 1979; Bureau et al., 2002; Jobling, 2011). The simplest method of reporting growth is by evaluating the absolute increase in weight or growth. This implies that the relationship between time and weight is linear, and that the rate of absolute growth is the same, regardless of the size of the fish. However, the growth rate varies with the size of the fish, and the relative growth rate \((G_{rr})\) will allow comparing between treatments with fishes of different sizes (Hopkins, 1992). Relative growth \((G_{r})\) and the \(G_{rr}\) are mathematically expressed according to the following equations: \(G_{r} = (W_t - W_i)/W_i\) and \(G_{rr} = (W_t - W_i)/W_i \times \Delta t\), in which \(W_t\) is the weight at time \(t\); \(W_i\) is the initial weight; and \(\Delta t\) is the duration of the experiment (Ricker, 1979; Hopkins, 1992).

The relative growth rates are typically used in studies on fish nutrition and are presented as the percentage of weight gain per unit of time. However,
the GRR is restricted to the period of time calculated and cannot be easily converted to another time period (Hopkins, 1992). Therefore, other models and equations for growth can be used to obtain better growth simulations and values.

To eliminate the problem related with relative growth rates over time, another model of exponential growth rate recommended is the specific growth rate (SGR) coefficient (Ricker, 1979; Hopkins, 1992). It is usually reduced to an instantaneous growth rate or to a specific, intrinsic, exponential, logarithmic, or compound interest rate (Ricker, 1979). The logarithm of final (lnPf) and initial (lnPi) weights at a given time in days (d) is used, as shown in the following equation:

\[ \text{SGR} = \left\{ \frac{\ln(P_f) - \ln(P_i)}{d} \right\} \times 100. \]

Another equation that is very used in aquaculture is the one to calculate daily growth coefficient (DGC), given by:

\[ \text{DGC} = \left\{ \frac{(P_{f1/3} - P_{i1/3})}{d} \right\} \times 100. \]

Only the mean values of the weight at the beginning (P_i) and at the end (P_f) of animal growth are considered, being divided by the time in days (d) at a given exponential (1/3), which represents a ratio of exponential growth of 0.3333 and is used to adjust the growth curve that is not considered in the equation for SGR.

The equation for linear growth coefficient (LGC), as the other ones, does not reflect the actual trajectory of the animal during farming, since it considers only the final weight subtracted by the initial weight and divided by the number of farming days. It was believed that this would be a representation of what actually happened during farming; however, the used equations disregard oscillations and differences in growth related to water temperature and metabolic conditions during the period, as exemplified in the following equation:

\[ \text{LGC} = \frac{P_f - P_i}{d}. \]

Integration between growth models and fish energy requirements

The prediction models were developed to determine animal growth and feed consumption, as shown by the studies of Pfeffer & Pieper (1979) and Ricker (1979). These models are also used to determine metabolic excretion and nutrient bioavailability for several species and production systems, as observed in the works by Cho & Bureau (1998), Booth et al. (2010), Bureau & Hua (2010), Chowdhury et al. (2013), Bouwman et al. (2013), Bueno (2015), and Canale et al. (2016).

In general, the first models, of varying complexity, partition the energy ingested through the use of energy balance equations (Dumas et al., 2010). A simple model would be:

\[ C = ME + GR + E, \]

in which \( C \) is the energy ingested, \( ME \) is the metabolizable energy, \( GR \) is the growth retention, and \( E \) is the endogenous excretion (Jobling, 2011).

From this balance equation, an energy balance can be built using any period of time from the entire life cycle in a snapshot in time. Pfeffer & Pieper (1979) suggested a deductive model, containing empirical components, which was used to determine the dietary needs of essential elements for fish. The model included factors for dietary requirement (Edt), GR, E, and the availability of the element in the diet (A). These factors were empirically determined, but their relationship was deductively built as Edt = (GR + E) / A.

The factorial models evolved and, currently, are constructed by connecting a group of parameters based on scientific studies and empirical observations during farming, related to metabolic energy requirements for maintenance, fish growth potential, efficient use of energy and ingredients available in the feeds, and animal body composition. Therefore, growth scenarios, energy demand, and releases of wastes from aquaculture systems became more precise and applicable.
Nutritional bioenergetics

Bioenergetics describes the energy flow of nutrients within a biological system, for example, in a fish or a shrimp. This approach shows the biological process of using and transforming absorbed nutrients for energy, for the synthesis of the own body (NRC, 2011). The feed that is consumed is transformed in the body; complex chemical compounds are divided into more simple components – proteins into amino acids, carbohydrates into glucose, and lipids into fatty acids –; and the energy released from the metabolic processes, is used for maintenance, production, and reproduction (Strand, 2005).

The metabolic expenditure by an animal is often measured as the amount of heat produced, which is often called respiration (R). By analyzing the difference between ME and R, the retained energy is obtained, usually referred to as production (Pd): Pd = ME - R.

A part of this energy is lost in feces, but there are also losses by urinary excretion and by the gills through diffusion on body surface. Two forms of energy can be defined: DE and ME; DE is able to transform itself into ME.

According to Jobling (2011), when an animal is under starvation (C = 0), the body tissues are catabolized to support respiration, the production (retained energy) is negative, and the animal loses body mass. However, if an animal ingests some food, but the energy retained is null over time (i.e., Pd = 0), there is a balance, and the animal meets the requirements for maintenance.

Therefore, determining and providing diets that allow meeting the ideal energy requirements will make it possible to maintain the metabolic functions, increase the production (growth, fat, and reproduction), and minimize losses and wastes generated by the metabolism of fish.

Bioenergetic factorial model applied in fish farming

According to Cho et al. (1982), the principles of the bioenergetic factorial model were applied to fish, in 1914, by Ege & Krogh and, in 1939, by Ivlev. Many studies regarding the use and waste of energy have been carried out since then for various species of fish (Kaushik & Médale, 1994; Booth et al., 2010; Canale et al., 2016).

Strand (2005) reports that models based on similar principles had already been previously proposed by other researchers (Kerr, 1971). However, the model developed by Kitchell et al. (1974), used to simulate the growth of bluegill (Lepomis macrochirus), was the most effective and was later used as a standard in research on poikilotherms, representing the bioenergetics model approach (Cui & Xie, 2000).

This model has been applied to several different species, such as, for example: Phoxinus phoxinus (Cui & Xie, 2000), O. niloticus (Yi, 1998; Chowdhury et al., 2013; Bueno, 2015), Oncorhynchus mykiss (Milne et al., 2015), and Larimichthys crocea (Cai et al., 2016).

In fish ecology, bioenergetic models have been used primarily to calculate the consumption of feed based on temperature and growth data (Kitchell et al., 1977; Hanson et al., 1997), subsidizing the development of computer software, as Fish Bioenergetics, version 3.0 (Hanson et al., 1997). However, these software are very generalist because they use the same metabolic rate, regarding the natural food chain (plankton and wild fish) and disregarding oscillations in temperature and body energy retention rates in different life stages and species. This may generate inaccurate and little precise values to estimate wastes produced by the animal metabolism.

However, the application of the bioenergetic models for aquaculture takes into account all of these factors, as exemplified by the free software Aquability . These models are accurate in the development of ideal strategies for feed and waste calculation (Cho & Bureau, 1998; Strand, 2005), and can be improved and become effective tools for farmers and for agencies funding and monitoring the activity.

Use of nutritional bioenergy to estimate aquaculture waste

The production of wastes from aquaculture can be estimated by simple principles of nutrition and bioenergetics, as observed in Cho & Bureau (1998), which adopt a “biological” approach, instead of a “chemical” one. Ingested food is digested and provides proteins, lipids, and carbohydrates, which are sources of energy and nutrients potentially available for animal maintenance, growth, and reproduction. The
rest of the feed (not digested) is excreted in feces as solid waste (SW).

The by-products of the metabolism, such as ammonia, urea, phosphates, and carbon dioxide, are excreted as dissolved waste (DW), mainly through the kidneys. The total waste (TW) from fish feed during farming is composed by solid and dissolved wastes, along with the waste of apparent feed loss (AFL) during feeding, in which TS = SW + DW + AFL.

However, the SW, DW, and AFL outputs are biologically estimated by: SW = [food consumed × (1 - apparent digestibility coefficient, ADC)]; and DW = [(food consumed × ADC) - nutrients retained by the fish].

Therefore, the DW may be calculated by the difference between the digestible and retained nutrients in the carcass. The precise estimate of total SW requires a reliable calculation of the wastes of AFL. Therefore, the estimation of AFL is almost impossible. However, the best estimates can be made based on the energy requirements and the expected gain, as in Cho (1992), in which the energy efficiency (energy gain/consumption) indicates the degree of AFL for a particular operation. In this context, the requirement and quantity of theoretical feed (QTF) can be calculated based on the nutritional energy balance (QTF = gain + excrete), which includes heat loss.

The amount of feed input that exceeds the QTF is assumed as AFL, and all nutrients that encompass AFL must be included in the quantification of solid waste. According to Cho (1992), this approach can lead to a relatively conservative estimate.

However, biological procedures based on the ADC for SW and on comparative analyzes of carcasses for DW provide reliable estimates, and the biological methods are flexible and able to adapt to a variety of conditions and farming environments (Bureau & Hua, 2010).

Mathematical models for the analysis of the carrying capacity of reservoirs

One of the main strategies adopted by the managing and monitoring agencies is the use of hydrodynamic models that calculate the carrying or nutrient load capacity of a particular water body, as well as the effect of fish farming.

Based on these aspects, several mathematical models were proposed: of Dillon & Rigler (1974), of Vollenweider (1975), Mike application, ECO Lab module (DHI Water and Environment), “Variáveis que Interagem de Modo Seminquantitativo” (Visq), Structural Thinking Experimental Learning Laboratory with Animation (Stella), Qualres, Ecopath Modeling, “Pegada Ecológica”, Delph 3D, and 3D Water Modeling System (Mohid), which are tools that simulate the dynamics of the variables that occur in the aquatic environment.

In general, these models are based on the direct relationship between P increase and algae growth. However, when these models are used to determine the carrying capacity for fish production, specific zootechnical and limnological factors are not always considered, which can under- or overestimate the real contribution of effluents from fish production. This shows the importance of the integration of bioenergetic models to determine the wastes from fish farms and to assist in the input of data for hydrodynamic modeling.

Integration between the mathematical models for the definition of carrying capacity

To define the carrying capacity, the Dillon & Rigler model (1974) was applied, considering the P concentration (mg m⁻³) in water as a function of the annual P load (La, in mg m⁻² per year), the P retention coefficient (Rp), average depth (z, in meters), and water residence time of the reservoir (ρ, in years). P concentration is given by the equation: 

\[ [P] = \frac{La (1 - Rp)}{(z \times \rho)}, \]

in which \( z \) is calculated by the ratio between the volume and the area of the body of water; \( \rho \) is calculated by the ratio between the average and maximum flow volume of the reservoir; and Rp is the P retention coefficient from the study by Larsen & Mercier (1976), with modifications by Canfield & Bachmann (1981), obtained by the following equation:

\[ Rp = \frac{1}{(1 + 0.614 \times \rho^{0.49})}. \]

The parameter for P content is \( \Delta[P] \), which is the increase in the concentration of P in water for a given \( La \). The following equation shows the relationship between these parameters: 

\[ La = \frac{(\Delta[P] \times z \times \rho)}{(1 - R)}, \]

in which \( \Delta[P] \) is given by subtracting the current P content in the water of the reservoir by the...
maximum concentration allowed by Resolution No. 357/2005 of Conselho Nacional do Meio Ambiente (Conama) (Brasil, 2005). From the Δ[P] permitted, the maximum La allowed is calculated, i.e., the amount of P that can be added to water.

Currently, Agência Nacional das Águas (ANA) takes into account the multiple uses of the reservoir in the issuance of permits for aquaculture activities, based on Resolution No. 357/2005 of Conama (Brasil, 2005). According to legislation, the limit for the total P is 30 mg m\(^{-3}\), in lentic environments, and 50 mg m\(^{-3}\) in intermediary environments, with water residence time varying from 2 to 40 days and permanent tributaries for lentic environments.

Based on these numbers and to standardize the amount of P allowed for aquaculture activities, the maximum amount of P allowed is limited to the fraction of 1/6 for lentic environments (30 mg m\(^{-3}\)), i.e., the maximum load to be discharged by aquaculture is of 5 mg m\(^{-3}\) P per year (ANA, 2009).

The remaining 5/6 would be reserved for other uses with P contributions to water, such as the dilution of domestic and industrial sewages, besides the amount of natural P. It should be noted that, in specific cases, in which there is a prior study of the reservoir, analyzing its multiple uses, the ability of the hydric body for aquaculture activities may vary; however, it will depend on the analysis and approval of the regulatory agency.

Therefore, La was calculated in function of a Δ[P] of 5 mg m\(^{-3}\). Then, the P load allowed in all the reservoir (Lr) was determined, in mg per year, using the La, representing the maximum load of P allowed per square meter, multiplying the value obtained by the water surface area (A, in m\(^{2}\)) from the reservoir, according to the equation: \(\text{Lr} = \text{La} \times A\). The water permanence of 90% was used, which is obtained by the equation: \(\text{Lr} = (\Delta[P] \times V_{90} \times \rho) / (1 - \text{Rp})\), in which \(V_{90}\) is the volume at 90% of water permanence (ANA, 2009). Then, \(\text{Lr}\) was converted into the annual authorized fish production. For this purpose, the amount of P in water for each ton of fish produced must be estimated.

**Example of the application of the proposed methodology**

The Fish-PrFEQ factorial bioenergetic model of Cho & Bureau (1998) is used to simulate the P load released by fish production (Pa). Different levels of total P in the feed (0.8, 1.0, and 1.5%) are considered for tilapia under different water temperatures (21, 25, and 29ºC, respectively) (Table 1). With Pa, the total P load allowed throughout the reservoir (Lr) is calculated by the equation that multiplies the L value (which represents the maximum load of permitted P per square meter) by A (total surface area, in m\(^{2}\)), i.e., \(\text{Lr} = L \times A\). Then, having \(\text{Lr}\) and \(\text{Pa}\), the authorized fish production is calculated (B, in mg per year) based on the allowed P load in the reservoir (Lr, in kg per year), with \(B = L_r / Pa\).

Specifically, a simulation of the application of this methodology was carried out in the reservoir of Ilha Solteira, in Paraná river, in the state of São Paulo, Brazil (Table 2).

**Table 1.** Estimation of solid waste (total P) in the production of Nile tilapia (*Oreochromis niloticus*), at different temperatures, in cages, by the bioenergetic nutritional model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>0.8% P in the feed</th>
<th>1.0% P in the feed</th>
<th>1.5% P in the feed</th>
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<tr>
<td></td>
<td>21ºC</td>
<td>25ºC</td>
<td>29ºC</td>
</tr>
<tr>
<td>Expected feed conversion(^{(1)})</td>
<td>1.4</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Total P (kg) – Pa(^{(2)})</td>
<td>3.6</td>
<td>3.3</td>
<td>4.4</td>
</tr>
</tbody>
</table>


---

\(^{(1)}\)Diet used for the model: 90% dry matter (DM), 69% digestible DM, 35% crude protein (CP), 31% digestible CP, 16 MJ kg\(^{-1}\) gross energy, and 11 MJ kg\(^{-1}\) digestible energy. 

\(^{(2)}\)Pa, load of P released (kg) for each ton of fish produced. Source: Bueno (2015).
Table 2. Application of the new methodology of the bioenergetic nutritional model used to determine the carrying capacity and to manage the maximum fish production of the reservoir of Ilha Solteira, in the state of São Paulo, Brazil.

<table>
<thead>
<tr>
<th>Item</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of reservoir</td>
<td>Lentic</td>
</tr>
<tr>
<td>Blocked river</td>
<td>Rio Preto</td>
</tr>
<tr>
<td>Basin/region</td>
<td>Paraná</td>
</tr>
<tr>
<td>Water level (n.a.)</td>
<td>Operating minimum</td>
</tr>
<tr>
<td>Maximum value</td>
<td>328.00</td>
</tr>
<tr>
<td>Minimum value</td>
<td>314.00</td>
</tr>
<tr>
<td>Maximum area</td>
<td>1,195.20</td>
</tr>
<tr>
<td>Minimum area</td>
<td>638.20</td>
</tr>
<tr>
<td>Maximum volume</td>
<td>21,060.30</td>
</tr>
<tr>
<td>Minimum volume</td>
<td>8,232.40</td>
</tr>
<tr>
<td>Average depth at the minimum value considered (m)</td>
<td>12.90</td>
</tr>
<tr>
<td>Average flow rate (m³ s⁻¹)</td>
<td>5,222.62</td>
</tr>
<tr>
<td>Residence time (td, years)</td>
<td>0.13</td>
</tr>
<tr>
<td>Residence time (td, days)</td>
<td>46.67</td>
</tr>
<tr>
<td>Δ[P] (mg m⁻³)</td>
<td>5.00</td>
</tr>
<tr>
<td>Coefficient of retention of P (R)²)</td>
<td>0.56</td>
</tr>
<tr>
<td>Species</td>
<td>Oreochromis niloticus</td>
</tr>
<tr>
<td>Maximum P load in the reservoir (kg per year)</td>
<td>726,531.16</td>
</tr>
<tr>
<td>Maximum fish production (Mg per year) – 0.8 P at 25°C</td>
<td>219,182.13 (obtained by the model)</td>
</tr>
<tr>
<td>Maximum fish production (Mg per year) – 1.0 P at 25°C</td>
<td>145,188.21 (obtained by the model)</td>
</tr>
<tr>
<td>Maximum fish production (Mg per year) – 1.5 P at 25°C</td>
<td>96,035.45 (obtained by the model)</td>
</tr>
</tbody>
</table>

\[ L_r = \left( \frac{[\Delta[P] \times V_{min} \times (1/td)]}{(1 - R)} \right) \], according to Dillon & Rigler (1974). \[ R = 0.761 \times \{1 - \exp[-0.0282 \times (td \times 365)]\} \], according to Straskraba (1996).

Concluding remarks

The use of factorial bioenergetic models, integrated with the hydrodynamic model, aids in determining the waste load and in adjusting the values used to calculate the carrying capacity of the reservoir for fish production.

The approach presented allows monitoring and managing aquaculture enterprises installed in lakes and reservoirs, besides improving the analysis methodology used for licensing each aquaculture enterprise, considering water quality parameters, feed nutritional quality, and peculiarities of each species (feeding habits, genetics, and growth stages). In this way, it becomes possible to encourage producers and the industry to use feed with lower environmental impact and management techniques that promote aquaculture sustainability.

Regarding fish production, in loco joint inspection actions (production reports) are recommended, as well as programs for monitoring the quality of water and sediments for the control of the carrying capacity of lakes and reservoirs.

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Mathematical modeling for the management of the carrying capacity of aquaculture enterprises


