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Term Structure of Sovereign Spreads - A Contingent Claim Model*

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Contents: 1. Introduction; 2. The Model; 3. The Data; 4. Results; 5. Summary and Conclusions; A. First Hitting Time Distribution.

Keywords: Emerging Markets; Sovereign Spreads; Structural Models; Default Probability.
JEL Code: G13; G15; F34; G33.

Este trabalho propõe um modelo estrutural para estimar a estrutura a termo e a probabilidade implícita de default de países emergentes que representam, em média, 54% do índice EMBIG do JPMorgan no período de 2000-2005. A taxa de câmbio real, modelada como um processo de difusão simples, é considerada como indicativa de default. O modelo calibrado gera a estrutura a termo dos spreads consistente com dados de mercado, indicando que o mercado sistematicamente sobre-estima os spreads para o Brasil em 100 pontos base na média, enquanto para México, Rússia e Turquia reproduz o comportamento do mercado.

This paper proposes a simple structural model to estimate the term structure and the implied default probability of a selected group of emerging countries, which account for 54% of the JPMorgan EMBIG index on average for the period 2000-2005. The real exchange rate dynamic, modeled as a pure diffusion process, is assumed to trigger default. The calibrated model generates sovereign spread curves consistent to market data. The results suggest that the market is systematically overpricing spreads for Brazil in 100 basis points, whereas for Mexico, Russia and Turkey the model is able to reproduce the market behavior.

*The authors would like to thank Ajax Moreira (Ipea), Luis Martins (Fidelity) and João Carlos Scandiuzzi (Pactual) for relevant suggestions and explanations; and also Paulo Levy (Ipea) and Estêvão Kopschitz (Ipea).

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1. INTRODUCTION

The strong performance and continuous progress of the investment returns in emerging markets during the last years seem to have consolidated the emerging markets role in the international portfolio, despite of the crises occurred during the second half of the 90's. As noted in the *Global Financial Stability Report* published by the International Monetary Fund (2004), the strong risk-adjusted returns in emerging securities, especially in sovereign bonds, have led many institutional investors to make strategic portfolio allocations in emerging markets. This reallocation was further increased by the improvement in the emerging markets' fundamentals and also by the exceptionally low short-term interest rates in the major financial centers. These facts created a scenario of excess of liquidity since 2001, especially in 2003, when the emerging markets sovereign spread fell from historical high levels.

Figure 1 shows the total return and the annualized daily volatility of EMBI Global, S&P500 and GBI-US from January 1998 to July 2004.

Credit spread, defined as the yield difference between a risky and a risk-free bond with similar characteristics, is related to the implied default probability and credit risk analysis of the issuer. Implied default probabilities are crucial for credit portfolio risk management or for pricing credit derivatives such as credit default swaps (CDSs).[1]

There are two broad financial approaches to assess, price, and manage credit risk: the structural and reduced-form models.

In structural models, initially proposed by Merton (1974), the contingent claims based approach (option pricing) is adopted. The risky bond is modeled as a contingent over some measure related to the economic or financial conditions of the debtor that triggers the default event – defined as when such a measure crosses a critical barrier. By making assumptions over the recovery of capital and interest rate models, default probability is derived endogenously as well as the term-structure of credit spread. Relevant extensions of Merton's model include Black and Cox (1976) and the possibility of default prior to maturity; Leland (1994) and the optimal timing for a default event; Longstaff and Schwartz (1995), introducing stochastic dynamics for the short-term riskless interest rate to price fixed and floating rate debt; Zhou (1997), proposing a jump-diffusion model that better adjust to empirical evidence of market data; and Saa-Requejo and Santa-Clara (2004), introducing a diffusion processes for the stochastic barrier.

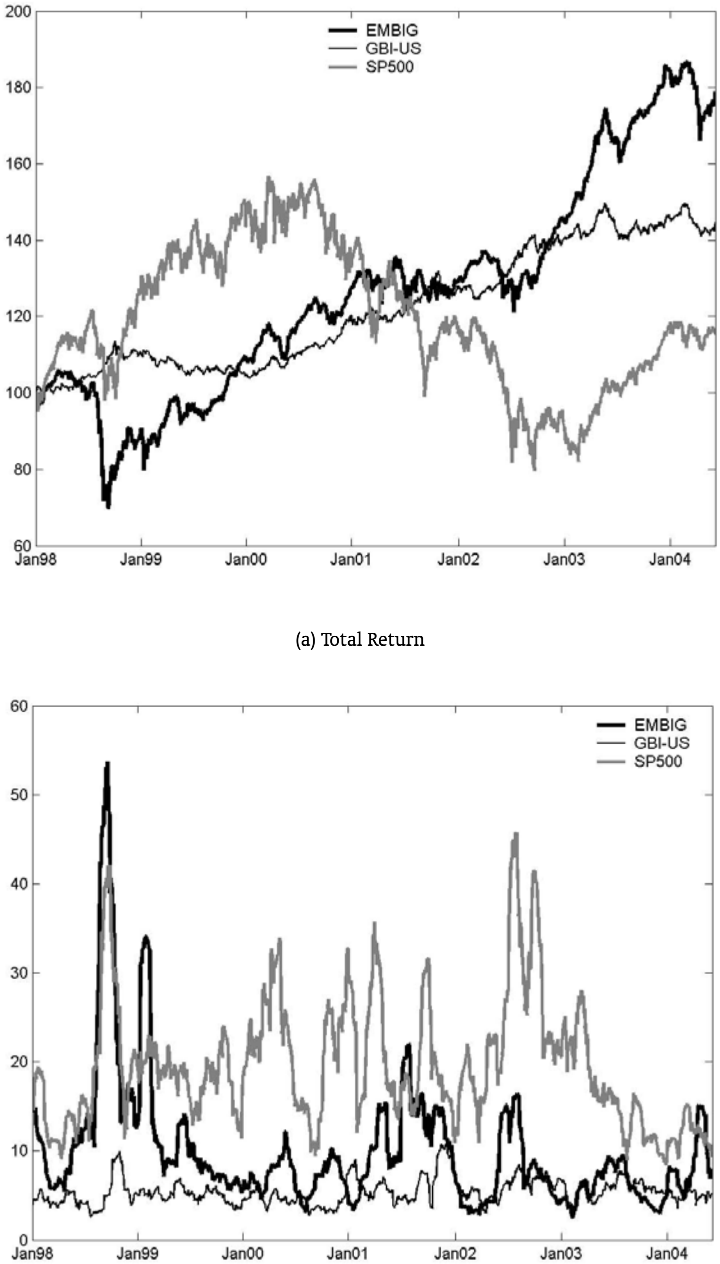
On the contrary, reduced-forms models, presented by Duffie and Singleton (1999) among others, treat default as an unpredictable event governed by a hazard rate process, where the credit spread is not explicitly related to the financial state or economic conditions of the bond issuer.

The discussion over the appropriated model to evaluate credit spread is highly controversial, tending to state that structural models are better for explaining and reduced models for forecasting. Sarig and Warga (1989) empirically investigate the term-structure of corporate credit spreads and it appears to conform to the existing theoretical results of Merton's structural model. Collin-Dufresne et al. (2001) suggest that liquidity proxies drive credit-spread changes more than the structural variables. Huang and Huang (2002) show that the class of structural models explains about 60-80% of the spread on corporate bonds rated by Moody's as speculative-grade ratings Ba, and roughly 100% for those rated B, i.e., high-risk obligations. Hund (2002) points out the difficulty of reconciling the behavior implied by the structural models with the realities observed in the credit spread market, and Delianedis and Geske (2001) attribute this empirical finding to market incompleteness.

The literature of credit risk models applied to sovereign risk is not straightforward. Cantor and Packer (1996) find that sovereign ratings are broadly consistent with macroeconomic fundamentals and

[1] According to British Bankers' Association – Credit Derivatives Report (2001/2002), the credit derivatives market is the fastest-growing segment of the OTC derivatives market, especially after the Asian and Russian crises. This market grew from US$ 40 billion outstanding notional value in 1996 to an estimated US$ 4.8 trillion by the end of 2004. In 2002, CDSs accounted for roughly 45% of the overall credit derivative market while sovereign CDSs represented around 8% of the CDSs market.

Figure 1: Index Performance



(a) Total Return

(b) Volatility (% p.a.)

spreads. Martins (1997) uses the default risk-premium obtained from US speculative long-term corporate bonds to price Brady bonds. Lehrbass (1999) develops a structural model based on an equity index of the borrower country expressed in the lender's foreign currency to analyze DM-Eurobonds issued by emerging economies. Wiggers (2002) adopts a structural model following the optimal endogenous default approach, where the domestic output in lender's foreign currency represents the default index. Hui and Lo (2002) present a structural model based on foreign exchange rate to explain sovereign spreads in South Korea and Brazil. Duffie et al. (2003) construct a reduced-form model for pricing sovereign debt with empirical evidence for Russia. Xu and Ghezzi (2002) develop a model that relates the term structure of sovereign spreads in emerging markets to the country's fiscal dynamics; and Moreira and Rocha (2004) introduce a two-factor structural model based on macroeconomic fundamentals and time-varying risk premium to forecast the Brazilian sovereign risk.

This paper implements a simple calibrated structural model to estimate the term structure of sovereign spreads and implied default probabilities of a selected group of emerging countries comprising more than 50% of the EMBIG index.

The indicator triggering default is considered to be the real exchange rate of each sovereign with respect to the USD dollar. Although real exchange rates do not directly represent the country's solvency or liquidity, it has the advantage of being a daily market variable promptly reflecting and capturing changes on the daily market spread, in opposition to the lower-frequency (monthly or even quarterly) fundamentals usually seen in structural models.

The assumption that real exchange rate is the default index can be supported by the reasoning that depreciation in domestic exchange rate of the sovereign issuer against the denominated sovereign bond currency (usually US dollar) puts under pressure the ability of the sovereign issuer to pay its liabilities, increasing the country risk. Such an argument is in accordance to Reinhart (2002) results, where 84% of the emerging market defaults are associated with currency crises mainly due to the considerable dollar denominated debt in such economies. Therefore, currency devaluation may exacerbate fiscal problems when the economy has an open capital account but a relatively small tradable sector. Moreover, according to Kaminsky et al. (1998), real exchange rate is one particularly useful indicator in anticipating currency crises.

After the model is calibrated, the term structure of sovereign spreads and the implied default probabilities are obtained.

The paper is organized as follows. The next Section introduces the model, Section 3 continues with the data, Section 4 presents the results, and the last Section discusses the main conclusions.

2. THE MODEL

Let S be the dynamics of the real exchange rate in the martingale equivalent measure, described by the stochastic process of Equation (1); where dz is the Wiener increment, σ_t is the volatility parameter, and λ_t is the risk-neutral time-varying drift.[2]

$$\frac{dS}{S} = \lambda_t dt + \sigma_t dz^* \tag{1}$$

In the absence of arbitrage opportunities, the complete market assumption implies that the risk-neutral drift of Equation (1) equals the short-term interest rate differential between the sovereign issuer and the US dollar market. This relation, known as the covered interest rate parity in international finance, is fairly correct for developed countries according to Frankel (1993). However, empirical evidences indicate the failure of the covered interest rate parity for emerging economies due to the existence of country risk that cannot be hedged. Therefore, we assume market incompleteness in emerging

[2] See Neftci (2000).

economies and estimate the risk-neutral/time-varying drift parameter λ_t by calibration with market data. Bates (1991) used a similar approach in order to explain the 1987 crash via calibration with market data.

The default event is triggered by the first time the exchange rate variable S crosses the default barrier α, i.e., when the exchange rate reaches a value that makes the debt's repayment unlikely. The moment of default is uncertain and has a probability distribution function (first hitting time) shown in Appendix.

The price of a default risky zero coupon bond $B(t,T)$ with a principal of $1 maturing at time T is given by Equation (2), where $P(t,T)$ is the price of a default riskless zero coupon bond with the same characteristics, w is the writedown in case of default,[3] $1\{\tau < T\}$ is the indicator function in case of a default event occurs prior to maturity, r is the default riskless instantaneous rate, and the expectation is taken with respect to the equivalent Martingale measure Q.

$$B(t,T) = P(t,T) - E_t^Q \left[w 1_{\{\tau<T\}} e^{-\int_t^T r(u)du} \right] \tag{2}$$

Equation (2) can be written as Equation (3), where $Ft(\tau < T)$ is the risk-neutral default cumulative probability function of a default event occurring before time T.

$$B(t,T) = P(t,T)\left[1 - wF_t(\tau < T)\right] \tag{3}$$

Let the spread $s(t,T)$ be the difference in yield between the risky and riskless bond. Then, the sovereign spread is given by Equation (4).

$$s(t,T) = -\frac{1}{(T-t)} \ln\left(\frac{B(t,T)}{P(T,t)}\right) = -\frac{1}{(T-t)} \ln\left(1 - w.F_t(\tau < T)\right) \tag{4}$$

3. THE DATA

An important benchmark for the analysis of risk and returns of worldwide emerging markets appeared with the introduction of the J.P. Morgan Emerging Markets Bond Index (EMBI). The EMBI is a total-return index for traded U.S. dollar-denominated Brady bonds in the emerging markets that satisfies some restrictive liquidity criteria. The J.P. Morgan EMBI Plus (EMBI+) relaxed the liquidity criterion of EMBI incorporating more instruments in its composition.

The J.P. Morgan EMBI Global (EMBIG) contains U.S.-dollar-denominated Brady bonds, Eurobonds, traded loans, and local market debt instruments issued by sovereign and quasi-sovereign entities; establishing a different criterion for eligible countries to be included in the index and admitting less liquid instrument than its predecessor EMBI+.

In order to make the country risk consistent with the assumptions of the model, we use the sovereign spread and duration implicit in the EMBIG of each country. Sovereign spread is the yield (stripped yield) difference in basis points between a risky and a risk-free instrument with similar characteristics, where the present value of the flows from the collateral has been removed since the collateral is not subject to sovereign risk.

Similarly, the collateral also affects the duration of the EMBIG. The appropriated duration for a sovereign spread that measures just the remaining risk after stripping away the collateral is the spread duration on sovereign-risk. Sovereign spread duration is defined as the percentage price change per basis-point change in the sovereign spread, and can also be interpreted as an average maturity of the index (without collateral).

[3] Writedown is assumed to be constant and we use the "Recovery of Treasury" formulations as Jarrow and Turnbull (1995).

To test the performance of our analysis, we choose the EMBIG by country of Brazil, Mexico, Russia, and Turkey, which corresponds roughly to 54% of the EMBIG composite during the study period 2000–2005. The sovereign spreads (in basis points) of the selected emerging economies since January 2000 to December 2005 are shown in Figure 2, and the composition by country of the EMBIG as in December 2005 in Figure 3.

Figure 2: Sovereign Spreads

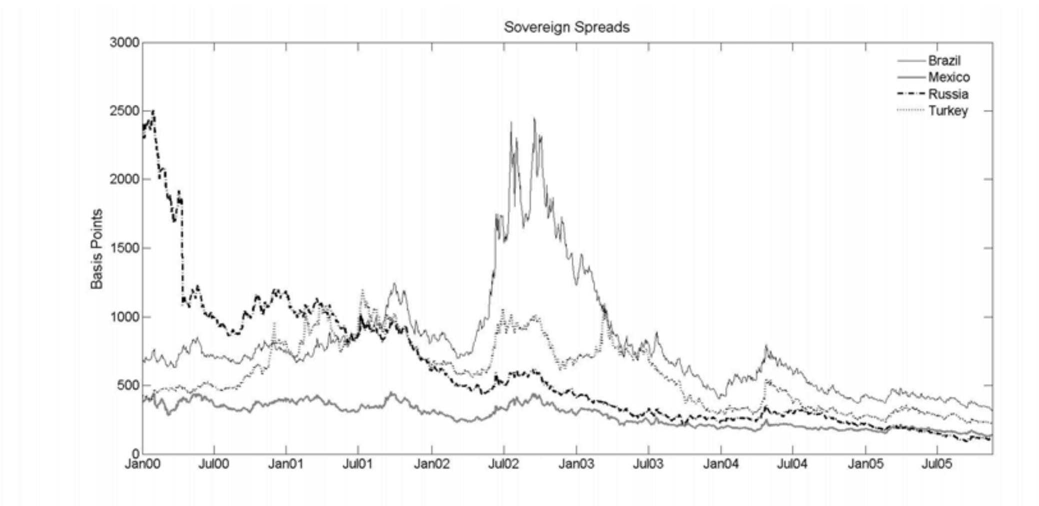
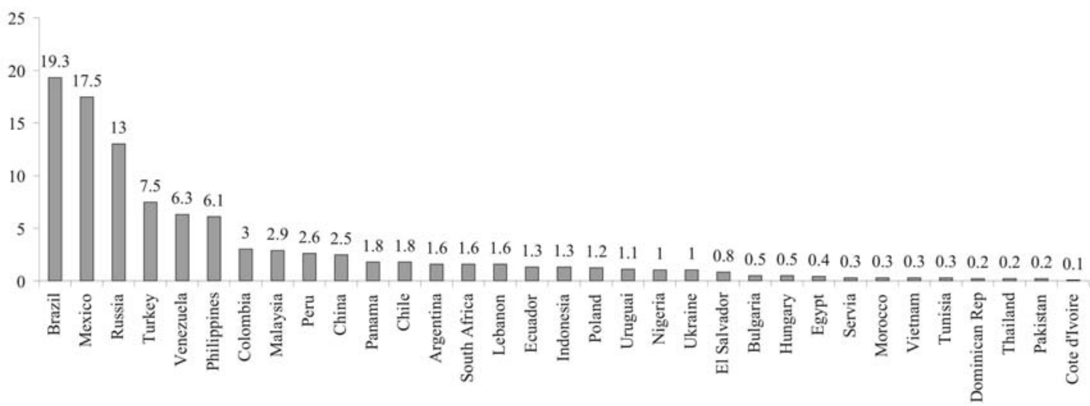


Figure 3: EMBIG Index Country Weights, %



The average defaulted debt recovery rate for sovereign bonds is taking from Moody's Investor Service (2003), giving an average value of roughly 40%. Hence, the corresponding writedown value w is 60%.

Nominal exchange rate data for each emerging country were taken from Bloomberg system and converted into real exchange rate with the consumer price indexes available at the IMF's International

Finance Statistics. Figure 3 shows the real exchange rate of the selected countries since January 1995, in USD dollar of December 2005, and the maximum rate achieved.

Figure 4: Real Exchange Rate

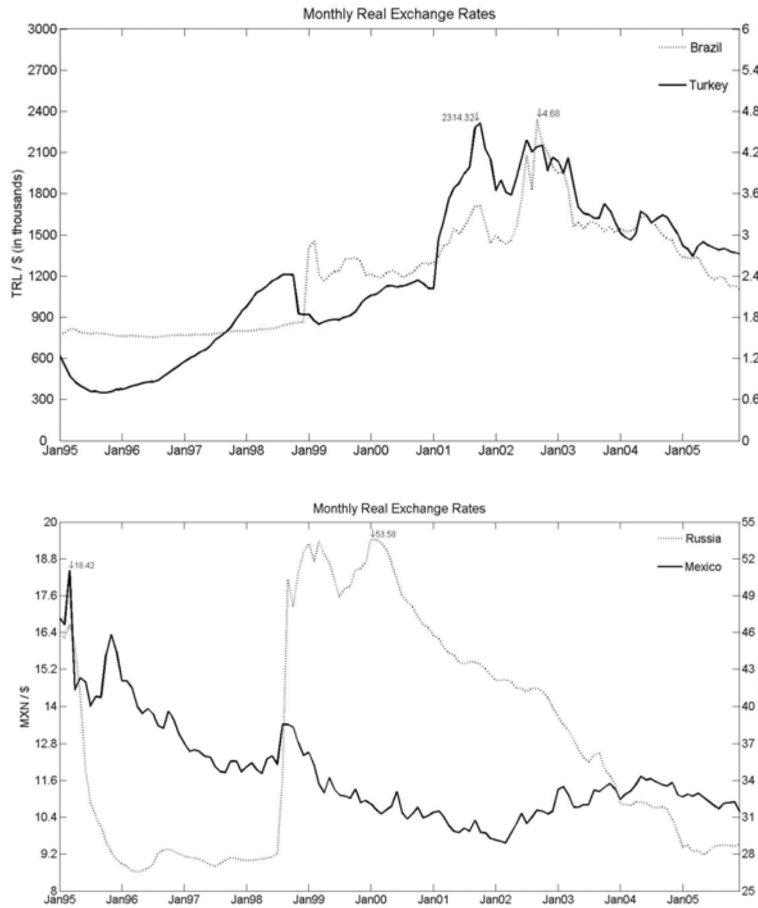


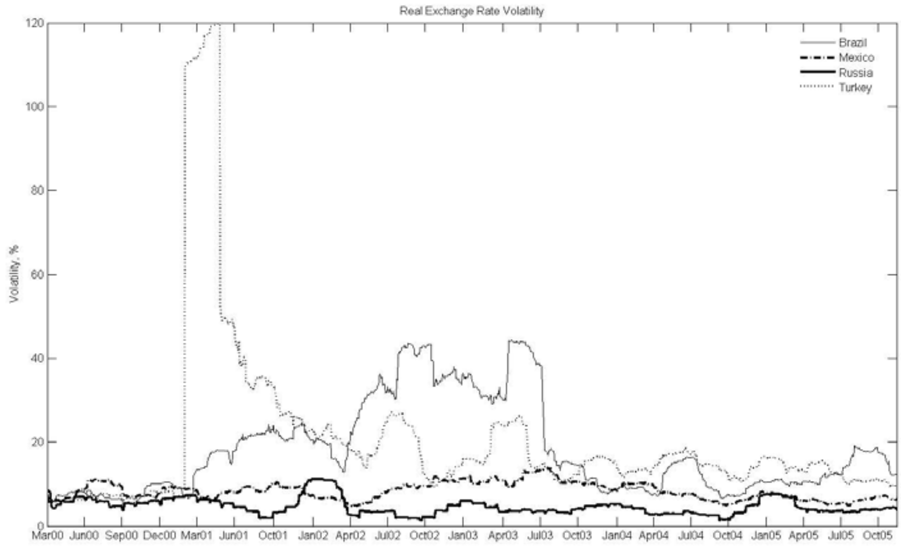
Figure 5 shows the historical annualized daily volatility parameter σ_t of Equation (1), estimated in a running window of 60 days, for the selected countries since March 2000.

4. RESULTS

The calibration process employed in this study is similar to that used by market practitioners. The risk-neutral time-varying drift parameter λ_t is calibrated with the most liquid instrument (in our case the benchmark EMBIG of each country) and then used as an input for pricing the less liquid ones (the real instruments included in the country's EMBIG). By constantly updating the calibrated parameter, we are able to incorporate all market information available up to date.

The study period begins in January 2000 and finishes in December 2005. The default barrier is estimated by minimizing the total mean square error between the sovereign spreads generated by the model and those of the real instruments for all trading days in the current month.

Figure 5: Annual Volatility



The estimated critical barrier is kept constant for the following month, while the risk-neutral time-varying drift is calibrated with the daily EMBIG sovereign spread and duration for every country.

To verify how the model fits the actual market data, Figure 6 plots the term structure of sovereign spreads generated versus the values observed in market data for some days of years 2001, 2004, and 2005. The cross-mark in the graphics represents the country's EMBIG.

The mean square errors are presented in Figure 7, and indicate the satisfactory robustness of the model during all time period, with the Brazilian 2002 election crises as exception.

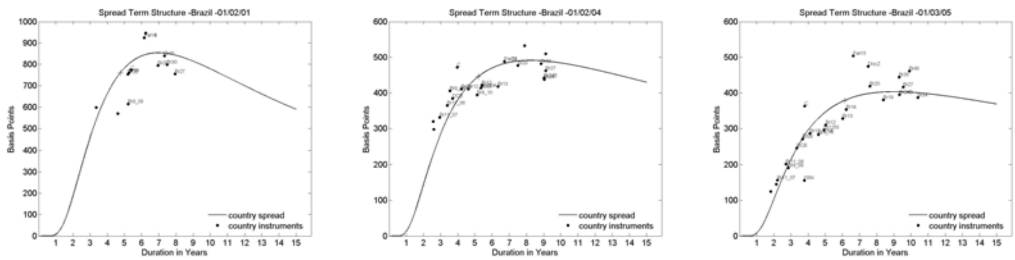
Table 1 presents the percentage errors (positive and negative)[4] implied by the model considering all instruments included in each country's EMBIG during the study period. By considering any error above 50 basis points on module as mispricing, we note that Brazil, Russia and Turkey present a pattern of instruments underpriced for the whole period on average. Thus, Brazil presents 47.91 % of its instruments underpriced, Russia 32.76 %, and Turkey 26.85 %. Such results can be explained mainly by the 2002 Brazilian elections, the Russian pos-default period of 2000–2001, and the Turkish 2001 devaluation.

By considering just the after crises period of 2003–2005, Table 2 shows the average degree (in basis points) in which the instruments' spread was overpriced. Note that instruments were underpriced by roughly 100 basis points for Brazil, while for the other sovereigns that amount was just around 50 basis points. Though not presented here, the average negative errors are of much less importance and, therefore, considered as negligible. These results evidence that the market is systematically overpricing the spreads for Brazil compared to the other emerging economies considered. As for Russia and Turkey, the pattern noticed before is within the confidence interval of the model and, hence, of little significance.

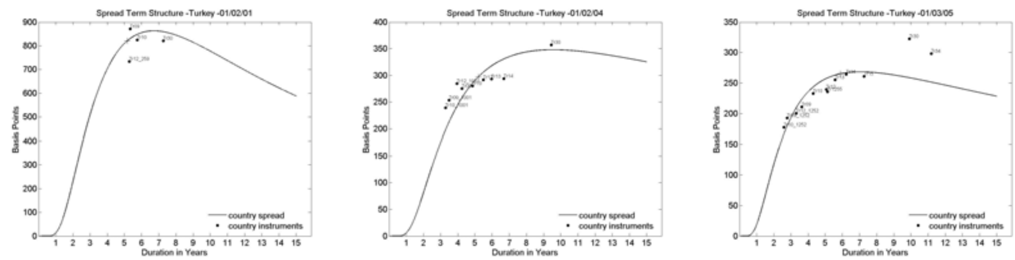
Figure 4 shows the daily-implied default probability for three and five years since May 2000.

[4] Positive error means the market is overpricing spreads comparing to the model, i.e., the instruments are being underpriced by the market.

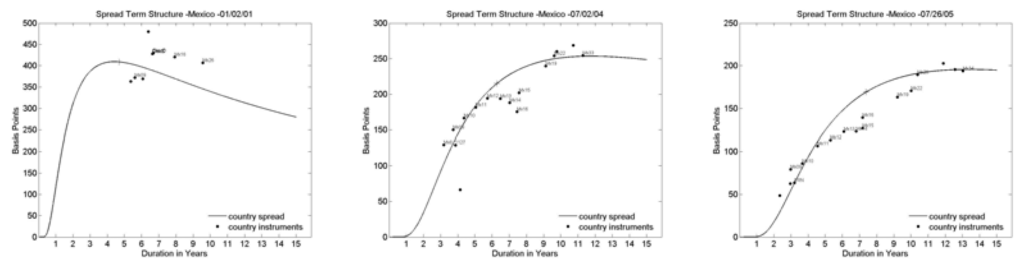
Figure 6: Model vs. Market Spreads



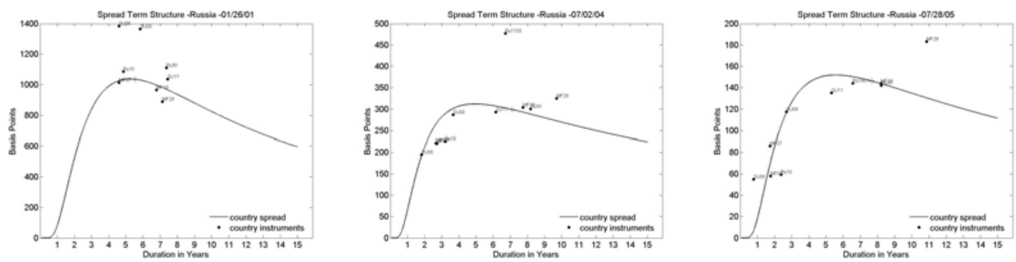
(b) Brazil



(e) Turkey



(h) Mexico



(k) Russia

Figure 7: Mean Square Errors

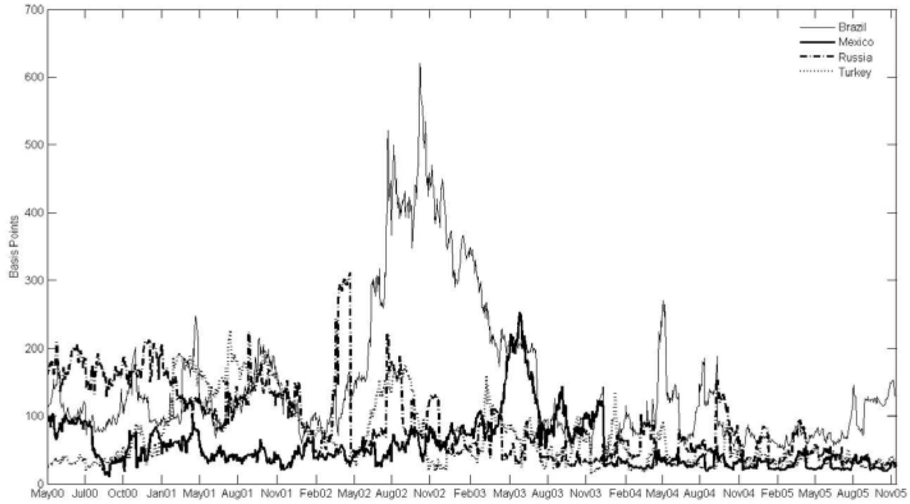


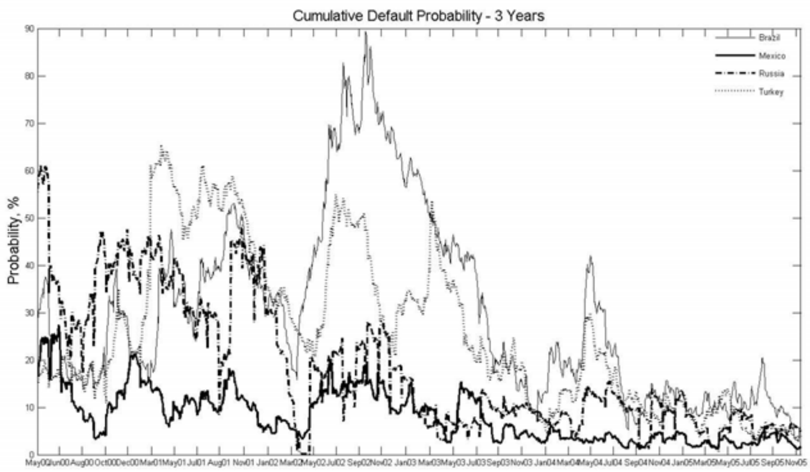
Table 1: Mispricing Errors

Percentage of Positive/Negative Error Within Interval								
		2000	2001	2002	2003	2004	2005	Average
Brazil	% post error > 50 bp	42.49%	50.91%	70.98%	54.72%	37.76%	30.61%	47.91%
	% post error > 100 bp	34.48%	36.30%	65.00%	44.13%	25.83%	15.99%	36.96%
	% neg error < -50 bp	17.78%	20.29%	8.65%	20.49%	12.26%	9.85%	14.89%
	% neg error < -100bp	11.37%	12.21%	3.19%	10.70%	4.76%	2.87%	7.52%
Mexico	% post error > 50 bp	25.62%	19.96%	17.62%	20.52%	2.62%	2.22%	14.76%
	% post error > 100 bp	12.96%	1.53%	5.09%	13.76%	0%	0%	5.56%
	% neg error < -50 bp	22.78%	14.44%	11.62%	10.51%	5.40%	1.74%	11.08%
	% neg error < -100bp	1.54%	2.39%	0.97%	2.39%	0%	0%	1.22%
Russia	% post error > 50 bp	48.30%	44.59%	37.30%	20.28%	28.69%	17.42%	32.76%
	% post error > 100 bp	35.36%	34.99%	16.79%	7.09%	11.84%	3.22%	18.22%
	% neg error < -50 bp	22.78%	14.44%	11.62%	10.51%	5.40%	1.74%	11.08%
	% neg error < -100bp	8.73%	12.23%	11.61%	0.68%	1.35%	1.80%	6.07%
Turkey	% post error > 50 bp	10.40%	56.78%	32.07%	27.50%	19.95%	14.42%	26.85%
	% post error > 100 bp	0%	37.77%	16.38%	9.20%	2.86%	0.53%	11.12%
	% neg error < -50 bp	7.54%	24.39%	22.79%	18.51%	5.64%	1.27%	13.36%
	% neg error < -100bp	2.96%	21.65%	11.35%	2.60%	1.59%	0%	6.69%

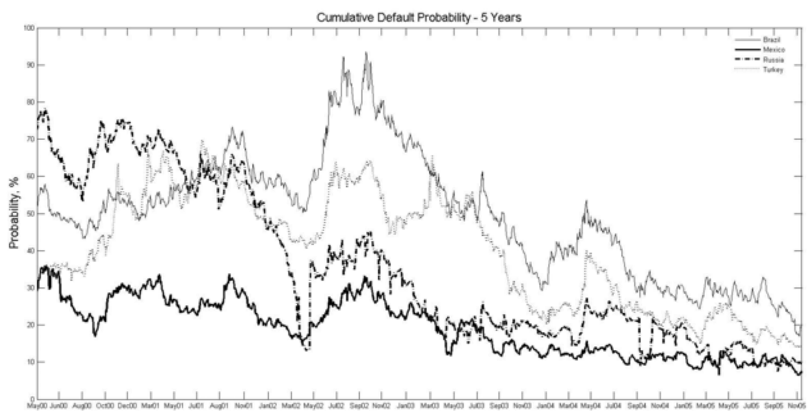
Table 2: Instruments Underpriced (basis points)

Average of Positive Error, bp (Underpriced)				
	2003	2004	2005	Average
Brazil	167	94	73	111
Mexico	118	22	22	54
Russia	47	52	38	46
Turkey	59	39	39	46

Figure 8: Implied Default Probability



(a) 3 Years



(b) 5 Years

In the first months of 2003 the implied default probability distribution decreases sharply after the turmoil of the Brazilian elections in October 2002 and Argentina crisis. Such a performance was achieved via improvements in the country-specific fundamentals combined with the large global liquidity of 2003 onward.

Table 3 shows the estimates of the implied (risk-neutral) cumulative default probability for the last day in the sample (December 5, 2005) and the issuer-weighted cumulative sovereign default rates available at Moody's Investor Service (2003).

Table 3: Cumulative Default Probabilities

Implied Default Probabilities							
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Mexico Baa1	0.00%	0.26%	1.66%	4.26%	7.62%	11.32%	15.11%
Russia Baa2	0.28%	2.40%	5.08%	7.51%	9.55%	11.25%	12.67%
Turkey Ba3	0.01%	0.08%	3.78%	8.43%	13.8%	19.3%	24.61%
Brazil Ba3	0.01%	1.01%	5.01%	11.32%	18.58%	25.90%	32.87%
Moody's data on Sovereign Default Rate (January 1985–December 2002)							
Baa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
B	7.89%	14.25%	18.33%	18.33%	22.22%	27.08%	32.69%
Investment Grade	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Speculative Grade	3.87%	7.87%	10.62%	14.19%	16.59%	19.74%	23.75%

As expected, the implied (risk-neutral) default probabilities are higher than the historical ones estimated by the credit rating agency.

5. SUMMARY AND CONCLUSIONS

This paper proposes a structural model for the estimation of the term structure of sovereign spreads, implied default probabilities, and default barriers in emerging countries that account for more than 54 % of the EMBIG index for the period 2000–2005. The real exchange rate is assumed to trigger the default event and, since it is a daily market variable, it captures changes on daily spread sooner than the low-frequency fundamentals (monthly or quarterly).

According to the model, the market is systematically overpricing the spreads for Brazil by 100 basis points on average, even considering the global liquidity period of 2003 onwards; whereas it reproduces the market behavior for Mexico, Russia and Turkey.

As expected, implied (risk-neutral) default probabilities are higher than the historical ones available at Moody's.

The use of other proxies triggering default such as sovereign equity indexes, the VIX index or other macro fundamentals, and a time varying estimated default barrier, as well as its application to sovereign CDSs are left for future research.

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A. FIRST HITTING TIME DISTRIBUTION

Let the following stochastic process with dz as a Wiener process.

$$dx = \lambda_t dt + \sigma_t dz \tag{5}$$

Following Karatzas and Shreve (2004), the first passage time density of x evaluated at $\tau > t$, i.e., $\tau = inf\{t \geq 0, x(t) > 0\}$, is given by Equation 6.

$$\pi(\tau|x_t, \lambda_t, \sigma_t) = \frac{|x_t|}{\sigma_t\sqrt{2\pi(\tau - t)^3}} \exp\left[-\frac{(x_t + \lambda_t(\tau - t))^2}{2.\sigma_t^2(\tau - t)}\right] \tag{6}$$

Through Ito's Lemma, we have that if S follows Equation 1, then $xt = ln(St/\alpha)$ follows the following stochastic differential equation:

$$dx = (\lambda_t - 0.5\sigma^2)dt + \sigma_t dz \tag{7}$$

Hence, the first passage time density of S evaluated at $\tau > t$, i.e., $\tau = inf\{t \geq 0, S(t) \geq \alpha\}$ is given by Equation 8.

$$\pi(\tau|S_t, \lambda_t, \sigma_t, \alpha) = \frac{|\ln(S_t/\alpha)|}{\sigma_t\sqrt{2\pi(\tau - t)^3}} \exp\left[-\frac{\left(\ln(S_t/\alpha) + \left(\lambda_t - 0.5.\sigma_t^2\right).(\tau - t)\right)^2}{2.\sigma_t^2(\tau - t)}\right] \tag{8}$$

The cumulative distribution function is given by Equation 9, where "$\phi(.)$" is the cumulative normal distribution function.

$$F_t(\tau < T) = \begin{cases} 1 - \phi\left(\frac{\left|\ln\left(\frac{S_t}{\alpha}\right)\right| - \left(\lambda_t - 0.5.\sigma_t^2\right).(T-t)}{\sigma_t\sqrt{T-t}}\right) + \\ +e^{\left(\frac{2.(\lambda_t - 0.5.\sigma_t^2).\left|\ln\left(\frac{S_t}{\alpha}\right)\right|}{\sigma_t^2}\right)}.\phi\left(\frac{-\left|\ln\left(\frac{S_t}{\alpha}\right)\right| - \left(\lambda_t - 0.5.\sigma_t^2\right).(T-t)}{\sigma_t\sqrt{T-t}}\right) \end{cases} \tag{9}$$