Clustering of historical floods observed on Iguaçu River, in União da Vitória, Paraná

Agrupamento de cheias históricas observadas no Rio Iguaçu, no município de União da Vitória, Paraná

Patrícia Cristina Steffen1 and Júlio Gomes1

1Universidade Federal do Paraná, Curitiba, PR, Brasil
E-mails: patriciacsteffen@yahoo.com.br (PCS), jgomes.dhs@ufpr.br (JG)

Received: July 06, 2017 - Revised: May 25, 2018 - Accepted: July 02, 2018

ABSTRACT

The occurrence of flood events has become more frequent, and, in Brazil, there are regions that suffer with the repetition of those events. In União da Vitória, located in Paraná State, Brazil, those phenomena are commonly recorded, generating series of consequences for each flood event, such as financial losses, environmental damages, home losses and deaths. So, since it is not possible to avoid the occurrence of floods, it is necessary to reduce its impacts, and in a scenario of repeated flood events, as observed in União da Vitória, it is justified the clustering of historical floods, aiming to improve the knowledge about the river basin hydrological behavior and to assist in the determination of hydrological models parameters. Clustering analysis aims to establish sets of events with similar characteristics, and, for this, based on fuzzy logic, the present study uses the fuzzy c-means method to cluster Iguaçu river floods, observed in União da Vitória, using a set of different flood severity indicators. The classification defined four clusters, according to different flood severity levels, so-called: low; medium; high; and, disaster or catastrophe. Therefore, by the analysis of similar features among different clusters of events, it is further possible to study the flood formation mechanisms, contributing to the reduction of its impacts, through real-time flood alert and forecasting systems, for instance.

Keywords: Flood clustering; Flood indicators; Flood intensity; Fuzzy c-means; Flood forecasting.

RESUMO

A ocorrência de eventos de cheia tem se tornado mais frequente e, no Brasil, existem regiões que sofrem com a repetição destes eventos. Em União da Vitória, no estado do Paraná, estes fenômenos são comumente registrados, gerando, a cada evento, uma série de consequências, como, por exemplo, prejuízos financeiros, danos ambientais, perdas de casas e mortes. Assim, uma vez que não é possível evitar a ocorrência das inundações, procura-se diminuir os seus impactos, e, em um cenário de repetição de eventos de cheia, como observado no município de União da Vitória, justifica-se a aplicação do agrupamento de cheias históricas, com o objetivo de melhorar o conhecimento sobre o comportamento hidrológico da bacia hidrográfica e auxiliar na determinação de parâmetros de modelos hidrológicos. O objetivo do agrupamento de cheias consiste em estabelecer conjuntos de eventos com características semelhantes, e, para isso, baseando-se na lógica difusa, o presente estudo utilizou o algoritmo fuzzy c-means como ferramenta para agrupar as cheias do rio Iguaçu, observadas na cidade de União da Vitória, a partir da definição de diferentes indicadores de severidade de uma cheia. Esta classificação, por sua vez, foi capaz de definir quatro diferentes grupos de eventos de cheias, com distintos níveis de severidade, assim denominados: severidade baixa; severidade média; severidade alta; desastres ou catástrofes. Desta forma, através da análise das características semelhantes entre os eventos de cada grupo, acredita-se que, posteriormente, seja possível estudar os mecanismos de formação das cheias de cada grupo e contribuir para a diminuição dos seus impactos, através, por exemplo, de sistemas de alerta e previsão de cheias em tempo real.

Palavras-chave: Agrupamento de cheias; Indicadores de cheias; Intensidade de cheias; Fuzzy c-means; Previsão de cheias.
INTRODUCTION

Recently, natural disasters such as floods, droughts, landslides and extreme temperatures have hit Brazil and affected thousands of people (EM-DAT, 2015). Among these occurrences, floods are most relevant natural disaster since they represent the largest number of occurrences and cause the largest number of deaths due to natural disasters in the country, according to the Emergency Disasters Data Base (EM-DAT, 2015).

Droughts and minor magnitude floods represent most of the hydrological period, and they limit the discharge to the main riverbed. However, when extreme floods occur, the elevation of water levels exceeds the river banks and overflow to the flooding plains (TUCCI; BERTONI, 2003; PAZ et al., 2009).

That concept represents the definition of flood applied to the present work, which is summarized as ‘flow that exceeds the drainage capacity of a river main channel’ (CASTILHO et al., 2005; TUCCI, 2012). In those cases, floods may generate consequences for the region beyond the financial losses, such as the occurrence of deaths and environmental damages, which are called intangible damages, due to the difficulty of translating those consequences into financial losses (MACHADO et al., 2005; MESSNER et al., 2006).

Thus, to reduce the impacts from those great magnitude floods, a series of control measures can be adopted. Control measures of the structural type modify the fluvial system, while control measures of the non-structural type seek to reduce the impacts through the adequate coexistence of the population with the events (TUCCI, 2012). In addition, it is important to emphasize that, in order to adopt the most adequate control measures, some aspects of the watershed must be studied to prevent possible local impacts from being transferred to the nearby regions (TUCCI et al., 1995).

Flood forecasting and warning system is an example of non-structural control measure (TUCCI, 2012). In general, the development of such type of system requires the use of hydraulic and hydrological models, which are better adjusted through a better knowledge of the basin hydrological behavior. The usual approach to apply hydraulic and hydrological models involves the steps of calibration, verification and simulation. Especially in the calibration stage, it is sought to establish a group of events that allows defining a unique set of model parameters, aiming the flood formation process dynamics in river basin is relatively well represented by the model.

Chebana et al. (2013) state that, for hydrological extreme event forecasting, frequency analysis procedures are essential and commonly applied, and in this approach, variables such as maximum water flow and maximum water level are adopted to define the magnitude of the events. However, those variables are not able to represent all the complexity involved in flood events, once they can be different in many ways. For this reason, it is questionable whether it is possible to capture the complexity of such dynamics with a single set of parameters, or whether it is necessary to define more than one set of parameters, trying to represent events or cluster of events with distinct characteristics.

Therefore, the present work assumes that the definition of different sets of model parameters, each one representing flood characteristics inherent to certain events or cluster of events, enables the improvement of hydrological models. So, it believes that it is possible to achieve this goal using clustering analysis to identify patterns in flood formation mechanism of each cluster. Certainly, considering a real-time flood forecasting and warning system, the results obtained with the present study can reduce the amplitude of flow forecasting by means of identifying to which cluster the flood best fits as the flood hydrograph evolves.

Flood indicators are initially stabilized for definition of clusters of events with similar features. According to Wang et al. (2014), the flood indicators must be able to evaluate the intensity of flood events, once the adequate characterization of those phenomena is not possible without the adoption of a set of different parameters (characteristics), in addition to maximum flow and maximum water level.

However, only the adoption of those indicators does not allow the full characterization of an extreme flood event. For this, Wang et al. (2014) suggest that a clustering method can be used to determine sets of flood events with similar characteristics, based on the adopted set of flood indicators. In this case, those clusters are distinct from each other, because of the severity of the floods that belong to each cluster.

Data clustering is based on the principle of object classification, “so that each object is similar to the others in the clustering, based on a set of chosen characteristics” (HAIR; JUNIOR et al., 2009). Therefore, as one of the multivariate statistics techniques and that it has been applying to several areas of Science, including Hydrology, the data clustering was used in this work to obtain sets of flood events with similar intrinsic characteristics.

In the literature, there are three main techniques of data clustering applied to Hydrology: Self Organizing Feature Map (SOFM), and K-means and Fuzzy c-means (fcm) algorithms. Jingyi and Hall (2004) present, initially, the Self Organizing Feature Map (SOFM) method, whose goal is to capture the topology and probability distribution of the input data, reducing the space of entrance in representative characteristics through a “self-organization” process. According to Srinivas et al. (2007), the SOFM method is widely applied in flow regionalization studies. However, according to those authors, it is not exactly a data clustering technique, due to the complexity in interpreting the results. For this reason, Lampinen and Oja (1992) proposed its use as a support for other data clustering methods.

Jingyi and Hall (2004) also present the second mentioned data clustering method, the K-means algorithm. According to the authors, despite of its simplifications, the method produces satisfactory results, including in Hydrology. Due to the acceptable results produced by the method, several applications are found in the literature. Chang et al. (2010), for example, applied the K-means method to subdivide a study area according to the flood characteristics, and to identify control points of each cluster identified. Burn and Goel (2000) and Lin and Chen (2004), respectively, applied the algorithm K-means in order to analyze the frequency of floods and to create a rainfall-runoff model. Zahmatkesh et al. (2015) used the K-means method to improve the results obtained from rainfall-runoff models for the Bronx River basin in the United States.

In the present study, among the three techniques for hydrological data clustering listed by Jingyi and Hall (2004), it was used the third method, fuzzy c-means (fcm). The fcm is an algorithm based on fuzzy logic, which aims to establish the similarities that a sample data shares with each cluster (BEZDEK et al.,
The concept of clustering based on fuzzy logic can be better understood when it is compared to the classical definition of clustering. According to Sato-Ilic and Jain (2006), in the traditional approach, when a sample data is clustered, it belongs only to one cluster, without sharing similar characteristics to the others. In clustering based on fuzzy logic, however, Sato-Ilic and Jain (2006) affirm that each sample data present distinct degrees of similarity with each cluster. In other words, the clustering is defined by the membership (similarity) degree of the sample data in relation to each one of the clusters. The sample data is assigned to the group to which it presents the highest membership degree, but it still shows a similarity to the other groups. It means by that the objects can only return values 0 (zero) or 1 to the membership function to a certain cluster in the traditional approach, while the membership function admits values in the range from 0 (zero) to 1 in clustering based on fuzzy logic. So, it allows the objects have a relation to all clusters, in a higher or lower degree, depending on the pertinence level presented in relation to each cluster.

In Hydrology, the application of clustering based on fuzzy logic is widespread. He et al. (2011), for example, applied the clustering based on fuzzy logic to evaluate flood damages. Lohani et al. (2014), in turn, applied the same concept to improve a real-time flood forecasting system.

The choice of the fuzzy c-means (fcm) algorithm as the clustering method in the present study is justified by its higher sophistication compared to other methods applicable to hydrology, and by its wide use in hydrological studies, as, for example, in Kaviski et al. (2004), Jingyi and Hall (2004), and Sadri and Burn (2011), who applied the fcm algorithm to obtain the regionalization of hydrological parameters in their study areas. Furthermore, Wang et al. (2014) developed a clustering analysis of floods to define the intensity and magnitude of the events, as well as the similar characteristics among them. Therefore, it is verified the applicability of the fuzzy c-means algorithm as a clustering method of hydrological data and, consequently, Iguazu River flood events, observed in municipality of União da Vitória, based on a set of flood severity indicators.

Due to the frequency of flood events on Iguazu River, in União da Vitória, and to the fact there is a cascade of reservoirs for electricity generation on Iguazu River, located downstream the same city, previous studies have presented hydrological models to forecast Iguazu river flow in União da Vitória. As an example, there are the studies of Mine and Tucci (1999), Mine and Tucci (2002), and Breda (2015).

Based on Foz de Areia hydroelectric reservoir, located approximately 100 km downstream from União da Vitória, Mine and Tucci (1999) presented a method for real-time forecasting of tributary flows to hydroelectric reservoirs. Due to operational limitations, both upstream and downstream from the reservoir, the authors used the combination of a linear stochastic model (ARIMA model) and a deterministic rainfall-runoff model (IPH II model), using a traditional approach to define the model parameters.

Mine and Tucci (2002) studied the management of energy production in hydroelectric power plants, choosing Foz de Areia power plant as the study area. The main goal was to maximize its energy production, keeping both favorable conditions for the reservoir and the safety of upstream population, as well as helping flood control in União da Vitória through the power plant operation. The authors applied the same hydrological models used in a previous study (Mine; Tucci, 1999), ARIMA and IPH II models, and again followed a traditional approach for hydrological studies.

On the other hand, Breda (2015), based on the flood occurred in União da Vitória in 2014 year, the third largest monitored flood in the city, applied a hydrological model with flood hydrograph splitting to perform the flow forecast. For this, Breda (2015) also used IPH II model, among others.

Thus, it observed that none of cited studies used clustering analysis of flood events neither to understand the river basin hydrological behavior in events of different magnitudes, nor to determine sets of distinct parameters, each one related to a specific cluster, considering all possible identified clusters.

Therefore, based on the presented considerations, this study aims performing a clustering analysis of historical floods, observed on Iguazu River, in União da Vitória, Paraná state, Brazil, by means of defining flood indicators and using fuzzy c-means algorithm. Thus, it expects that such kind of analysis can contribute to the development of more efficient flood forecasting and warning systems, due to both a better knowledge of the river basin and a specific calibration of the hydrological and/or hydraulic model for each cluster, identified by the clustering analysis.

STUDY AREA

The study area includes the drainage area of Iguazu river basin, defined by a cross section of interest, located approximately at 26° 13’ 44” S and 51° 04’ 58” W in União da Vitória, Paraná state, Brazil. The city rises on Iguazu River banks and there are flood records since 1891, reinforcing the importance of flood studies in the region.

Iguazu River is about 1.320 km long, crossing over most of Paraná state (PEREIRA; SCROCCARO, 2010), and it belongs to the Paraná River basin. The Iguazu River is divided into three stretches: upper, middle and lower Iguazu, respectively represented by numbers 2, 11 and 12 in Figure 1, that also shows the location of União da Vitória in the middle stretch of Iguazu River.

The flow data, used in the present study, were recorded at União da Vitória gauging station (code ANA 65310000).

METHODS

The development of the present study required the execution of the following steps:

1. Collecting flow and water level data;
2. Filling gaps in flow and water level data;
3. Definition of the overflow threshold;
4. Selection of annual flood events;
5. Definition and calculation of flood indicators;
6. Data pretreatment;
7. Data clustering;
8. Result analysis.
Collecting flow and level data

Flow and water level data were collected from the União da Vitória gauging station (code ANA 65310000). However, it was necessary to correct flow data from 1980 to 2015, using water level observed from the R5 Porto Vitória gauging station (code ANA 65365800), as explained in the following.

The operation of the Foz de Areia Hydroelectric Power Plant began in 1980, under the responsibility of the Companhia Paranaense de Energia (COPEL). The plant reservoir created a backwater effect over the water levels observed in the União da Vitória gauging station (code ANA 65310000). Therefore, a family of discharge curves, as shown in Figure 2, represents its relation between water level and discharge. As one can see, each discharge curve refers to a specific water level observed in R5 Porto Vitória gauging station (code ANA 65365800), located at upstream from the reservoir and downstream from the União da Vitória gauging station.

In order to correct flow data, it was used a computer program available by COPEL, that calculates flow in União da Vitória from the water levels observed in União da Vitória and R5 Porto Vitória gauging stations, considering the family of discharge curves, presented in Figure 2.

Table 1 presents the source of hydrological data used for developing the present work, after flow data correction.

It is necessary to mention that the cited gauging station codes correspond to those established by the HidroWEB platform, under the responsibility of the National Water Agency (ANA).

However, the data were collected from the Instituto das Águas do Paraná database, as mentioned in Table 1. For further details about the collection of data carried out for the study, it suggests to consult Steffen (2017).

Filling gaps in flow and water level data

Data gaps were observed only in water level data from R5 Porto Vitória gauging station (code ANA 65365800) in 1980 to 2015 period, which were used to correct flow data from União
da Vitória gauging station (code ANA 65310000), as mentioned previously. Water level duration curves for 1980 to 2015 common period were used to fill those gaps.

Basically, it was adopted the hypothesis that water level duration curves shows a strong relationship because the gauging stations are close to each other. In other words, it was assumed that water levels observed in both stations at the same time present the same duration. So, concerning to any day that shows a gap in water level in R5 Porto Vitória, the procedure to fill the gaps is presented as follows.

First, it was defined the duration (p\_i,UV) for the observed water level (h\_i,UV) in União da Vitória from its duration curve. Based on the adopted hypothesis, the duration (p\_i,PV) for the water level (h\_i,PV) in R5 Porto Vitória was made equal to the duration (p\_i,UV) for the observed water level in União da Vitória. Finally, the water level (h\_i,PV) in R5 Porto Vitória was defined from its own duration curve.

Equation (1) resumes the procedure applied to fill the gaps in R5 Porto Vitória water levels.

\[
\begin{align*}
    h_{i,UV} & \rightarrow p_{i,UV} = p_{i,PV} \rightarrow h_{i,PV}
\end{align*}
\]

In addition, it is necessary to say that the gaps observed in water level data from R5 Porto Vitória gauging station (code ANA 65365800) corresponds to 23 consecutive days only, from 1982 July to August 1982. Probably, such short time period did not produce any meaningful effect on the results obtained in this work.

### Definition of the overflow threshold

The overflow elevation coincides with the expropriation elevation defined for the Iguazu riverside area, in União da Vitória, as performed by COPEL. That elevation correspond to 744.50 m above sea level.

The expropriation level, or overflow level, corresponds to 4.89 m water level in União da Vitória gauging station (code ANA 65310000), once the “zero” of the limimetric ruler is equal to 739.61 m above sea level.

However, in order to estimate the flow rate corresponding to the overflow level, it is important to restate that União da Vitória water levels are influenced by the R5 Porto Vitória water levels, as mentioned previously. That fact leads to exist a family of rating curves for União da Vitória gauging station (code ANA 65310000), as shown in Figure 2, causing the overflow elevation (744.50 m) to have a corresponding flow rate in four discharge curves. In other words, 744.50 m elevation corresponds to four different flow values in Figure 2. Once the overflow threshold seeks to establish a unique value that represents an overflow situation, it was adopted 1387 m\(^3/s\) as the threshold flow value, which corresponds to the lowest flow rate among the four possible values, considering the 744.50 m elevation and the family of rating curves.

Therefore, the expropriation elevation provided two reference values for the present study, 4.89 m and 1387 m\(^3/s\), representing, respectively, threshold values for water level and flow rate.

### Selection of annual flood events

It was selected the largest annual flood event observed in each year, considering a period of 85 years, ranging from 1931 to 2015. This procedure was adopted in respect to the classical frequency analysis methods, that uses annual data series; however, it does not represent a limitation of the clustering analysis. The definition of the largest annual flood event happened regardless of whether, or not, there was overflow of the main channel.

Each selected event always contains the maximum annual flow, except for 1970 and 1980 years. In those years, the annual maximum flows occurred on the December, 31\(^{st}\), and did not correspond to the hydrograph peaks. In both cases, the flood hydrographs were considered as belonging to the following years (1971 and 1981), and may, or may not, represent the largest annual observed flood event. For 1970 and 1980 years, the flood events correspond to the second largest annual observed flow rate.

In order to determine the duration of each annual flood event, that is, to define the beginning and the ending points of each event, the minimum flows located to the left and to the right in relation to hydrograph peak were identified. Those points were adopted as, respectively, the beginning and the ending points of each event, allowing the calculation of the flood duration.

### Definition and calculation of flood indicators

Seven flood indicators were defined as variables for the clustering analysis, basically representing characteristics of the flood hydrograph, but also implicitly related to social and economic impacts. Such number of variables requires the use of multivariate statistics for the clustering analysis, which can be defined as “a set of procedures to analyze the association between two or more sets of measurements that were made on each object in one or more samples of objects” (LATTIN et al., 2011). The chosen flood indicators, as well as their definitions and defining equations, are presented in Table 2.

In Equations from (2) to (6), \(n\) is the total number of time periods over the overflow threshold; \(V\) is the overflow volume in \(i\)-th time period; \(t_e\) is the time period over the overflow threshold in the \(i\)-th period (days); \(t_s\) is the starting time of the first overflow; \(t_f\) is the starting time of the flood hydrograph; \(Q\) is the flow rate at the starting time of the flood hydrograph (\(m^3/s\)); \(Q_{ref}\) is the reference flow (\(1387 m^3/s\)); \(t_{max}\) is the hydrograph peak time; \(t_{peak}\) is the ending time of the overflow period where the peak discharge is included; \(Q_{peak}\) is the peak discharge (\(m^3/s\)).

In Steffen (2017), one can see some figures that explain with more details each one of the flood indicators presented in Table 2.

In addition, it is important to emphasize a peculiarity in water level data set. Instead of the common hydrological operation
with two daily observations, União da Vitória gauging station (code ANA 653100000) presents two measurements only after 1989. Before 1989, daily water level data were available at 7 am, only.

Therefore, in the present study, after 1989, the maximum daily water level was adopted to generate the flow data, since the objective of this work is to identify the clustering based through the flood severity.

Data pretreatment

Due to their physical and, consequently, measurement differences, it was applied a pretreatment to the flood indicators, using a normalization method. Equation (7) (WANG et al., 2014) presents the normalization method applied to the flood indicators, where \( x_j \) and \( x_j' \) are, respectively, the treated and non-treated values of the \( i \)-th observation of the \( j \)-th indicator, and \( x_j'_{\min} \) and \( x_j'_{\max} \) are, respectively, the minimum and maximum values of the \( j \)-th indicator.

\[
x_j' = \frac{x_j - x_j'_{\min}}{x_j'_{\max} - x_j'_{\min}}
\]

The \( x_j \) elements assemble the matrix \( X \), whose elements are defined from Equation (7) and belong to the \([0, 1]\) range. Then, those elements represent dimensionless values and present the same order of magnitude.

Data clustering

The clustering method adopted in this study, the fuzzy c-means (fcm) algorithm, establishes the similarities that a sample data shares with each cluster.

Aiming to minimize its objective function \( (F_c) \), the fcm is characterized by being an iterative method that follows the sequence presented as follows, where the iterations are denoted by \( (p) = 0, 1, 2, \ldots, n \):

1. The number of clusters \( (c) \) and the accuracy degree of the method \( (\phi) \) are assumed. In the following section, the method applied to define those parameters \( (r \) and \( \phi ) \) is presented, regardless of the fcm method;
2. Randomly, the initial fuzzy partition matrix \( (U^0) \) is determined, which is composed by \( \epsilon \) lines and \( I \) columns, where \( \epsilon \) is the number of clusters and \( I \) is the total number of observations;
3. The matrix contained the centroids of the \( \epsilon \) clusters \( (V) \) is calculated from Equation (8), where \( v_{ij} \) is the coordinate of the \( j \)-th flood indicator of the \( k \)-th centroid; \( x_j \) is the treated value of the \( i \)-th observation of the \( j \)-th flood indicator; \( u_{ik} \) is the fuzzy partition matrix \( (U) \) element of the \( i \)-th observation of the \( k \)-th cluster; \( c \) is the total number of clusters; and, \( r \) is the accuracy degree of the method:

\[
v_{ij} = \frac{\sum_{k=1}^{\epsilon} u_{ik} x_j}{\sum_{k=1}^{\epsilon} u_{ik}} \tag{8}
\]

4. The Euclidian distance matrix from the sample data to the centroids of the \( \epsilon \) clusters \( (D) \) is calculated from Equation (9), where \( d_{ik} \) is the Euclidian distance of the \( i \)-th observation to the \( k \)-th cluster; \( x_j \) is the treated value of the \( i \)-th observation of the \( j \)-th flood indicator; \( y_k \) is the coordinate of the \( j \)-th flood indicator of the \( k \)-th centroid; and, \( f \) is the total number of flood indicators:

\[
d_{ik} = \sqrt{\sum_{j=1}^{f} (x_j - y_k)^2} \tag{9}
\]

5. The objective function value \( (F_c) \) is calculated from Equation (10), where \( r \) is the accuracy degree of the method; \( d_{ik} \) is the Euclidian distance from the \( i \)-th observation to the centroid of the \( k \)-th cluster; \( e_{ik} \) is the adherence level of the \( i \)-th observation of the \( k \)-th cluster of the fuzzy partition

---

Table 2. Definition of the flood indicators.

<table>
<thead>
<tr>
<th>Flood indicator</th>
<th>Unit of measure</th>
<th>Description</th>
<th>Defining equation</th>
<th>Equation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak discharge (( I_0 ))</td>
<td>(m³/s)</td>
<td>Maximum flow of the selected annual event</td>
<td>Data set</td>
<td>---</td>
</tr>
<tr>
<td>Peak water level (( I_0 ))</td>
<td>(m)</td>
<td>Maximum water level of the selected annual event</td>
<td>Data set</td>
<td>---</td>
</tr>
<tr>
<td>Total overflow volume (( I_2 ))</td>
<td>(km³)</td>
<td>Total volume of water over the overflow threshold</td>
<td>( I_2 = \sum_{i=1}^{n} V_i )</td>
<td>(2)</td>
</tr>
<tr>
<td>Total time over the overflow threshold (( I_4 ))</td>
<td>(day)</td>
<td>Elapsed time of the selected flood event over the overflow threshold</td>
<td>( I_4 = \sum_{i=1}^{c} H_i )</td>
<td>(3)</td>
</tr>
<tr>
<td>Elapsed time between the starting time of the event and the overflow threshold exceeding (( I_6 ))</td>
<td>(day)</td>
<td>For the selected annual event, this is the elapsed time from the starting time of the event to the starting time of overflow</td>
<td>( I_6 = \frac{Q_{\text{ref}} - Q_{\text{th}}}{t_{\text{max}} - t_0} )</td>
<td>(5)</td>
</tr>
<tr>
<td>Average ascension rate of flood hydrograph (( I_7 ))</td>
<td>(m³/s/day)</td>
<td>Ratio of the difference between the reference flow and the initial hydrograph flow, and the ( I_7 ) indicator</td>
<td>( I_7 = \frac{Q_{\text{max}} - Q_{\text{ref}}}{t_{\text{max}} - t_0} )</td>
<td>(6)</td>
</tr>
<tr>
<td>Average recession rate of flood hydrograph (( I_8 ))</td>
<td>(m³/s/day)</td>
<td>Ratio of the difference between the peak and the reference flows, and the elapsed time between them, taken along the hydrograph recession reach</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
matrix \((U)\); \(c\) is the total number of clusters; and, \(I\) is the total number of observations of the sample:

\[
F_{ck} = \frac{1}{I} \sum_{k=1}^{c} \sum_{i=1}^{I} (u_{ki})^2 (d_{ik})^2
\]  \hspace{1cm} (10)

6. The fuzzy partition matrix \((U)\) is updated from Equation (11), where \(u_{ki}^{(p+1)}\) is the adherence level of the \(i\)-th observation of the \(k\)-th cluster of the fuzzy partition matrix \((U)\); \(c\) is the total number of clusters; \(d_{ik}^{(p)}\) and \(d_{ik}^{(p+1)}\) are the Euclidian distances from the \(i\)-th and \(j\)-th observations, respectively, to the centroid of the \(k\)-th cluster, obtained from the \((U)\) matrix; and, \(r\) is the accuracy degree of the method:

\[
u_{ki}^{(p+1)} = \left[ \sum_{j=1}^{c} \left( \frac{d_{ik}^{(p+1)}}{d_{ik}^{(p)}} \right) ^{-2} \right]^{-1} r
\]  \hspace{1cm} (11)

7. If \((U)\) does not differ from \((U)\) more than an established limit (the maximum error tolerated), as presented in Equation (12), the iterative process is finished; otherwise, steps from 3 to 7 are repeated:

\[
u_{ki}^{(p+1)} \leq \epsilon_{\text{max}}
\]  \hspace{1cm} (12)

In the present study, the maximum error tolerated \((\epsilon_{\text{max}})\) was adopted equal to \(10^{-4}\), and the error obtained at each new iteration \((p+1)\) \((\epsilon_{\text{max}})\) was calculated from the Equation (13), where \(u_{ki}\) is the element of the \(i\)-th observation of the \(k\)-th cluster of the fuzzy partition matrices \((U)\) and \((U)\):

\[
u_{ki}^{(p+1)} = \max_{u_{ki}} \left\{ \| u_{ki}^{(p+1)} - u_{ki}^{(p)} \| \right\}
\]  \hspace{1cm} (13)

Definition of \(r\) and \(\epsilon\) parameters

The parameters \(r\) and \(\epsilon\) are, respectively, the accuracy degree of the fuzzy c-means algorithm and the defined number of clusters. According to Ross (1995), the appropriate interval for \(r\) is \(1.25 \leq r \leq 2.00\). The number of clusters \((c)\), in turn, is defined by the researcher, according to the data used, and it can range from 2 to the total number of indicators.

To define those parameters, it was adopted the method presented by Bezdek et al. (1984). That method is based on the estimation of three new parameters, \(F_{j}\) and \(H_{j}\), obtained from Equations (14) and (15), and \((1 - F)\), which is the complement of \(F_{j}\). In Equations (14) and (15), \(u_{ki}\) is the element of the \(i\)-th observation of the \(k\)-th cluster of the final fuzzy partition matrix \((U)\); \(a\) is the logarithm base, equal to 10; \(c\) is the number of clusters; and, \(I\) is the number of sample observations. Bezdek et al. (1984) stated those parameters define the ideal number of clusters \((c)\) and the accuracy degree \((r)\), when \(F_{j}\) approaches 1, and \((1 - F)\) and \(H_{j}\) approach zero.

\[
F_{j} = \frac{1}{I} \sum_{i=1}^{I} \left( \frac{u_{ki}}{a} \right)^2
\]  \hspace{1cm} (14)

\[
H_{j} = -\frac{1}{I} \sum_{i=1}^{I} \left( \frac{u_{ki} \log_{a} (u_{ki})} {a} \right) / I
\]  \hspace{1cm} (15)

It is necessary to highlight that Equations (14) and (15), used to calculate \(F_{j}\), \(H_{j}\) and \((1 - F)\), do not belong to the fuzzy c-means method, but rather they are a recommended initial step for the data clustering algorithm, allowing the definition of both the number of clusters \((c)\) and the fcm method accuracy degree \((r)\).

Results analysis

The results interpretation has begun with the calculation of the seven flood indicators for each event of the data set, followed by the identification of the most severe events, according to each one of the indicators.

This classification aimed to obtain the ten most severe events, according to each one of the used indicators. So, it is important to emphasize that, for \(I_{j}\), \(I_{j}\), \(I_{j}\), and \(I_{j}\) indicators, the classification followed the descending order, while for \(I_{j}\) and \(I_{j}\) indicators, it followed the ascending order, once the lower those values \((I_{j}\) and \(I_{j})\), the greater is the severity of the event.

Using the seven flood indicators \((I_{j}\) to \(I_{j})\), calculated for each observation (each annual flood event), it was made the data pretreatment, by the normalization method, to further application of the fuzzy c-means algorithm.

The ideal number of clusters \((c)\) and the accuracy degree \((r)\) were defined by trial and error, with \(r\) ranging from 3 to 7 and \(r\) ranging from 1.25 to 2.00, in both cases using a 0.25 variation. For each combination of \(c\) and \(r\), the \(F_{j}\), \(H_{j}\) and \((1 - F)\) parameters were calculated. So, the ideal values of the number of clusters \((c)\) and of the accuracy degree of the method \((r)\) were reached from the maximum value of \(F_{j}\) and the minimum values of \(H_{j}\) and \((1 - F)\) obtained among the attempts.

Finally, after the identification of the ideal values of number of clusters \((c)\) and accuracy degree, it was analyzed the results from the application of the fuzzy c-means algorithm. The analysis of results followed this sequence: identification of the defined clusters; verification of the maximum adherence level of the events to their clusters; analysis of the cluster centroids; identification of each cluster severity level; and, event frequency analysis, according to their severity level.

RESULTS AND DISCUSSION

After the estimation of the flood indicators, they were ranked according to the severity level. Table 3 presents the ranking of the top ten highest flood events exhibited by their year of occurrence, according to each flood indicator. Some of the main events are highlighted in Table 3.

According to the Table 3, it is possible to realize that the ranking changes according to the flood indicator. For example, the 1983 event is known as the most severe event in the study area, but it is shown as first in the ranking only for \(I_{j}\) and \(I_{j}\) indicators. For the \(I_{j}\) indicator, the 1983 event is not highlighted as first in the ranking, but rather as the second one. Furthermore, for the
other flood indicators, the 1983 event is not even ranked among the top ten higher events. On the other hand, the 2010 event, which is ranked as the eighth position for $I_4$ and $I_1$ indicators and it was not even classified among the top ten events for the other indicators, occupies the first position for $I_4$ and $I_1$ indicators.

In addition, according to the Table 3, it is possible to notice a change in the ranking of the highest events for 1957 and 1976 events. The 1957 event appears in the first position only for $I_1$ indicator, and its ranking position varies for the other indicators. In a similar way, 1976 event occupies to the first position only for $I_1$ indicator, and, according to the other indicators, it is not ranked among the top ten flood events. It is possible to verify, once more referring to the Table 3, other changes in the ranking of the events, for example, 1935, 1992 and 2014 events.

If, instead of only presenting the ranking of the top ten highest flood events, it was presented the complete ranking, with all the 85 events analyzed, it would be possible to affirm that such variation in the ranking is also observed for the complete data set of the flood events.

After the estimation of the seven flood indicators ($I_1$ to $I_7$) for each observation (each annual flood event), the data pretreatment (normalization method) was applied prior the use of the fuzzy c-means method. The ideal values of the number of clusters ($c$) and the accuracy parameter ($\rho$) were defined by trial and error, as mentioned previously. Table 4 shows $F_c$, $(1 - F_c)$ and $H_c$ values, which are the recommended parameters for the definition of $\rho$ and $r$. According to the results, the suggested $\rho$ and $r$ values were, respectively, equal to 1.25 and 4, which combination corresponds to the maximum value of $F_c$ and to the minimum values of $(1 - F_c)$ and $H_c$.

Once the goal of the fuzzy c-means algorithm is to minimize its objective function ($F_c$), Table 5 presents the $F_c$ values, calculated for each iteration, using from Equation (10) and adopting $\rho$ equal to 4 and $r$ equal to 1.25.

In Table 5, it is possible to notice that the objective function achieved a stop criterion equal to 10⁻² at the iteration 18 ($F_c$ equal to 6.11). However, the process was only interrupted at the iteration 36, because the maximum error tolerated was adopted equal to 10⁻³.

Table 6 presents the clusters identified by the application of the fuzzy c-means algorithm, considering $\rho$ equal to 1.25 and $r$ equal to 4, to the data set of normalized flood indicators.

Considering the clusters presented in Table 6 and the observed flood events, it can be said that Cluster 1 is composed, mostly, by events without overflow, except for the 1931 year. The other clusters (2, 3 and 4) are composed by the events with overflow, which severity levels differ among themselves.

The composition of the clusters, presented in Table 6, was determined from the fuzzy partition matrix ($U$), generated at the end of the process. The fuzzy partition matrix ($U$) defines the adherence level of each selected event to each generated cluster. The maximum adherence level of an event to a given cluster determines the group to which the event belongs. For example, it was identified that the highest degree of membership of the 1931 event was to Cluster 1, so the event belongs to this cluster, while the highest degree of membership of 1932 event was to Cluster 2, placing this event in cluster 2. Figure 3 shows the maximum adherence degree of each event.

The adherence degrees, presented in Figure 3, can explain, for instance, the insertion of an event with extraverasion, as the case of the 1931 event, in Cluster 1, a cluster of events without...
extravasation. Observing Figure 3, one can noticed that the 1931 event has a relatively low adherence degree (close to 0.50), while most of the events presented membership degrees of near 1 (maximum value for adherence degree).

Furthermore, from Figure 3, among the 85 events, there were only 12 cases in which the highest membership degree had values lower than 0.90, and there were only 3 situations in which the highest membership degree was lower than 0.50, including the 1931 event, mentioned previously.

It is important to highlight that maximum membership degree (adherence degree) lower than 0.50 may indicate that the classification of the event in the cluster is not well defined. Besides, it is possible that such uncertainty has some relation with the definition of the initial fuzzy partition matrix, that it was randomly estimated, which could make difficult to the method converges. In the present study, since the maximum adherence degree to a given cluster was close to 1 for most of the events, the clustering presented in Table 6 was accepted.

Table 7 presents the non-normalized coordinates of the Clusters 1 to 4 centroids, resulted from the application of fcm method.

In Table 7, Cluster 1 represents the set of events without extravasation, identified by the lowest values of the indicators centroids. Also, from Table 7, it is also possible to notice that there is a gradual increase in the centroid values for the indicators: peak discharge ($I_1$); peak water level ($I_2$); total overflow volume ($I_3$); time over the extravasation threshold ($I_4$); and, average rate of flood hydrograph recession ($I_5$). In other words, Cluster 4 presented higher values than Cluster 3, which, in turn, presented higher values than Cluster 2, and so on. However, that was not observed for indicators $I_6$ and $I_7$, which did not present a well-defined trend.

The variation of the centroid coordinates in each cluster can be better observed in Table 8, through its normalized values, allowing a graphical representation of easy visualization, shown in Figure 4.

In addition, it is important to note that the variables $I_i$ and $I_j$ represent quantities inversely proportional to the severity level. In other words, the lower their values, the more critical the events, unlike the other flood indicators. Moreover, in the performed analysis, when the events did not show extravasation, a value equal to zero was assigned to those variables, making the result of the coordinates $I_i$ and $I_j$ of the Cluster 1 centroid also approached zero, which would represent a catastrophic event for those indicators. That could be solved by using the complement in relation to the unit from the value obtained for those indicators in the normalization process, or simply assigning a value equal to 1 when the event did not show extravasation. However, it would not affect the results of the clustering analysis and, for this; this procedure was not adopted in the present work.

Thus, considering the centroid coordinates, severity levels were associated to the 4 clusters, defined by the cluster analysis. For this, the general coordinate behavior of the centroids was used as criterion. In this way, the clusters were named as:

- Cluster 1: low severity;
- Cluster 2: medium severity;
- Cluster 3: high severity;
- Cluster 4: maximum severity;

Table 6. Flood clustering (for $c = 4$ and $r = 1.25$).

<table>
<thead>
<tr>
<th>Cluster 1</th>
<th>Cluster 2</th>
<th>Cluster 3</th>
<th>Cluster 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1931</td>
<td>1962</td>
<td>1979</td>
<td>1932</td>
</tr>
<tr>
<td>1933</td>
<td>1963</td>
<td>1980</td>
<td>1938</td>
</tr>
<tr>
<td>1934</td>
<td>1964</td>
<td>1981</td>
<td>1946</td>
</tr>
<tr>
<td>1936</td>
<td>1967</td>
<td>1988</td>
<td>1947</td>
</tr>
<tr>
<td>1940</td>
<td>1968</td>
<td>1996</td>
<td>1950</td>
</tr>
<tr>
<td>1941</td>
<td>1969</td>
<td>1999</td>
<td>1954</td>
</tr>
<tr>
<td>1942</td>
<td>1974</td>
<td>2004</td>
<td>1955</td>
</tr>
<tr>
<td>1943</td>
<td>1977</td>
<td>2007</td>
<td>1971</td>
</tr>
<tr>
<td>1944</td>
<td>1978</td>
<td>2008</td>
<td>1972</td>
</tr>
<tr>
<td>1945</td>
<td>1985</td>
<td>2011</td>
<td>1982</td>
</tr>
<tr>
<td>1949</td>
<td>1986</td>
<td>2012</td>
<td>1984</td>
</tr>
<tr>
<td>1951</td>
<td>1991</td>
<td>2015</td>
<td></td>
</tr>
<tr>
<td>1952</td>
<td>1994</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1956</td>
<td>2002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1958</td>
<td>2003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1959</td>
<td>2006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Centroids coordinates of Clusters 1 to 4 (non-normalized values for $c = 4$ and $r = 1.25$).

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$I_1$ (m$^3$/s)</th>
<th>$I_2$ (m)</th>
<th>$I_3$ (km$^3$)</th>
<th>$I_4$ (day)</th>
<th>$I_5$ (m$^2$/s/day)</th>
<th>$I_6$ (m$^2$/s/day)</th>
<th>$I_7$ (m$^2$/s/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1035.0</td>
<td>4.2</td>
<td>0.0</td>
<td>0.1</td>
<td>0.2</td>
<td>2.3</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>1657.9</td>
<td>5.4</td>
<td>0.3</td>
<td>12.9</td>
<td>23.6</td>
<td>39.5</td>
<td>32.9</td>
</tr>
<tr>
<td>3</td>
<td>2165.3</td>
<td>6.3</td>
<td>1.0</td>
<td>25.8</td>
<td>12.0</td>
<td>135.2</td>
<td>74.7</td>
</tr>
<tr>
<td>4</td>
<td>3463.4</td>
<td>8.2</td>
<td>4.5</td>
<td>54.3</td>
<td>19.1</td>
<td>88.4</td>
<td>106.9</td>
</tr>
</tbody>
</table>

Table 8. Centroids coordinates of Clusters 1 to 4 (normalized values for $c = 4$ and $r = 1.25$).

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$I_1$</th>
<th>$I_2$</th>
<th>$I_3$</th>
<th>$I_4$</th>
<th>$I_5$</th>
<th>$I_6$</th>
<th>$I_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>0.16</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.24</td>
<td>0.33</td>
<td>0.03</td>
<td>0.15</td>
<td>0.40</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>0.45</td>
<td>0.10</td>
<td>0.29</td>
<td>0.21</td>
<td>0.27</td>
<td>0.40</td>
</tr>
<tr>
<td>4</td>
<td>0.63</td>
<td>0.71</td>
<td>0.46</td>
<td>0.61</td>
<td>0.33</td>
<td>0.17</td>
<td>0.57</td>
</tr>
</tbody>
</table>
− Cluster 3: high severity;
− Cluster 4: disasters or catastrophes.

Figure 5 shows the time distribution of events according to their group (or severity level). In Figure 5, the vertical line represents the 1970 year that divides the period from 1931 to 2015, analyzed in the present study, in two sub periods: from 1931 to 1970 (40 years) and from 1971 to 2015 (45 years).

Table 9 provides a better understanding of the results presented in Figure 5, through the absolute and relative frequencies of events observed in each cluster, for the 1931 to 1970 and from 1971 to 2015 sub periods.

From Figure 5 and Table 9, one can see that there was a decrease in the absolute frequency of low severity events (Cluster 1) for the 1971 to 2015 period in relation to the previous period (from 1931 to 1970). There were 23 low severity events between 1931 and 1970 (40 years) and only 10 events with the same severity level between 1971 and 2015 (45 years). On the other hand, there were 9 events classified as high severity or disaster level (Clusters 3 and 4) in the period 1931 to 1970, while 21 events classified at the same severity levels were observed in the period from 1971 to 2015. Those variations cannot be explained simply by the difference in size of the two periods analyzed.

In addition to the higher absolute frequency of events classified as high severity (Cluster 3) or disaster (Cluster 4) after 1970, the most catastrophic (highest impact) floods, historically observed in the study region (1983, 1992 and 2014 events), also occurred in the period from 1971 to 2015 period.

Table 9. Absolut and relative frequencies of flood events in each cluster, observed in 1931 to 1970, and 1971 to 2015 sub periods.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolut frequency</td>
<td>Relative frequency</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(%)</td>
</tr>
<tr>
<td></td>
<td>Absolut frequency</td>
<td>Relative frequency</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(%)</td>
</tr>
<tr>
<td>1</td>
<td>23</td>
<td>57.5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>20.0</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>17.5</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5.0</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>100.0</td>
</tr>
</tbody>
</table>

CONCLUSION

The recurrence of extreme events has made it critical to study natural disasters such as floods. In União da Vitória, this type of phenomenon, historically, has been observed frequently. This work presented a differentiated approach to study those events, since the classical studies, based exclusively on the analysis of discharge and water levels, do not clearly demonstrate the reality that this type of phenomenon represents for society, such as the duration of the critical situation and the time available for the adoption of an emergency action.

The flood indicators used in this work showed that there is a variation in the ranking of the most critical events, considering each indicator specifically. For example, the flood event observed in the 1983 year is not the most critical event for some indicators, although it is the most critical event when the indicators are peak discharge and water level.

The fact of the ranking of the most critical events is a function of the flood indicator used for the analysis, and the possibility of defining more than one flood indicator to represent the flood severity, justifies the application of this new approach, characterized by the use of multivariate statistics and flood clustering analysis.

From using the fuzzy c-means (fcm) clustering method and defining a set of seven flood indicators (analyzed variables), it was possible to classify the maximum annual flood events observed.
in União da Vitória in the 1931 to 2015 period. Four clusters were identified, representing different severity levels: low severity (Cluster 1); medium severity (Cluster 2); high severity (Cluster 3); and disasters or catastrophes (Cluster 4).

Through the chronological analysis of the occurrence of flood events and knowing to which cluster each one belongs, it was verified that there was an increase in the frequency of more severe events (Clusters 3 and 4) and a decrease in the frequency of low severity events (Cluster 1), considering most recent period (from 1971 to 2015).

In addition, it is important to highlight that the researcher, regardless of fcm method, must define the number of clusters. In the present study, a specific method was used to determine the number of clusters, which fitted well to the fcm. However, it is suggested, for future studies, the adoption of other methods to determine the number of clusters, so that it is possible to compare the results and, possibly, to define the best method to estimate the ideal value for this parameter.

Besides determining the number of groups, the estimation of the initial fuzzy partition matrix (U^0) could be better evaluated to avoid randomness in its definition and to skip from initial values that could lead to non-convergence of the fcm algorithm.

Moreover, analyzing only the clustering method, other methods capable of clustering and identifying similar characteristics of historical flood events could be studied, allowing, then, the identification of the best clustering method applied to hydrological events.

On the other hand, there is still the possibility of studying new flood indicators, besides those described in this work, capable of a better characterization of the hydrological events and allowing the comparison among the clustering results.

Moreover, it is believed that, based on the use of flood clustering techniques, it is possible to advance in the knowledge of the mechanisms that rule the rainfall-runoff transformation process in a given river basin, as well as in the calibration of hydrological models; where one should try to calibrate the model for each flood cluster. Thus, a set of parameters would be defined for each identified cluster. In this way, the flow prediction model should be able to predict to which cluster the future event will belong to use the most appropriate parameter setting.

Clearly, at the beginning of the flood, it would not be possible to identify safely to which cluster the future event belongs to, so the flood forecasting system would establishing an amplitude of forecasts from the different sets of parameters. However, in a context of real-time forecasting, as the event develops itself and it is observed, the number of clusters to which it possibly belongs would decrease, leading to a reduction in the amplitude of the predictions.

Finally, it is worth mentioning that the present work used the União da Vitória as a study case, but the fuzzy c-means algorithm does not apply only to this region. Therefore, regardless of the location, since it is not the purpose of the method to evaluate the mechanism of flood formation, but rather to collaborate with its better understanding, it is noticed that the algorithm for the historical floods clustering is applicable for any place, if historical water level and flow data are available for the analysis.

REFERENCES


RBRH, Porto Alegre, v. 23, e38, 2018
Clustering of historical floods observed on Iguaçu River, in União da Vitória, Paraná


Authors contributions
Patrícia Cristina Steffen: Article writing, data analysis and interpretation, general design, final review.
Júlio Gomes: Guidance, general design, proofreading, final review of translated article.