Balance of longwave radiation employing the rate of solar radiation for Santa Maria, Rio Grande do Sul, Brazil

Saldo de radiação de ondas longas empregando a razão de radiação solar para Santa Maria/RS

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ABSTRACT - New coefficients were determined for the weighting term for cloudiness in the Brunt-Penman equation using the rate of solar radiation ($R_K$) in place of the rate of sunshine duration ($n/N$). The coefficients in the Brutsaert method proposed for daytime in southern Brazil were also tested and adjusted, and the method was selected which gave the more accurate daily results in relation to the original Brunt-Penman equation, for Santa Maria in the state of Rio Grande do Sul, Brazil (RS). Meteorological data covering 2,472 days obtained from the automatic and conventional weather stations in Santa Maria were used. The adjusted equations were tested with the remaining 1/3 of the data. The Brunt-Penman equation modified by the term for cloudiness weighted both for solar radiation incident on the surface with no cloudiness ($R_{K,R}$) and for solar radiation incident at the top of the atmosphere ($R_{K,K}$), were those that resulted in the best statistical indices relative to the original Brunt-Penman equation. In those equations the boundary conditions, $0.3 \leq R_{K,K} \leq 1$ or $R_{K,R} \geq 0.22$, were imposed. Although having similar statistical indices, a sensitivity analysis showed that the Brutsaert equation and other weightings for cloudiness resulted in larger deviations when compared to the original Brunt-Penman equation, in addition to having greater complexity for practical application.

Key words: Solar radiation. Atmospheric transmissivity. Brunt-Penman equation. Emissivity of the atmosphere.

RESUMO - Foram determinados novos coeficientes do termo que pondera a nebulosidade na Equação de Brunt-Penman utilizando-se a razão de radiação solar ($R_K$) no lugar da razão de insolação ($n/N$) e foram testados e ajustados coeficientes das equações do método de Brutsaert propostas para o período diurno no sul do Brasil e verificou-se qual das metodologias tem resultados mais corretos em relação à Equação de Brunt-Penman original, para Santa Maria/RS, para o período diário. Foram utilizados dados meteorológicos obtidos nas Estações Meteorológicas Automática e Convencional de Santa Maria em 2.472 dias. Os coeficientes foram ajustados pelo método de regressão linear ou não-linear, dependendo do modelo, utilizando-se 2/3 dos dados. As Equações ajustadas foram testadas com os demais 1/3 dos dados. A equação de Brunt-Penman modificada com o termo da nebulosidade ponderada tanto pela radiação solar incidente na superfície na ausência de nebulosidade ($R_{K,R}$) como pela radiação solar incidente no topo da atmosfera ($R_{K,K}$), foram aquelas que resultaram nos melhores índices estatísticos comparativamente à equação original de Brunt-Penman. Nessas equações foram impostas as condições de contorno: $0.3 \leq R_{K,K} \leq 1$ ou $R_{K,R} \geq 0.22$. Embora com índices estatísticos semelhantes, uma análise de sensibilidade mostrou que a equação de Brutsaert e demais ponderadoras da nebulosidade resultaram em maiores desvios quando comparadas com a Equação original de Brunt-Penman, além de apresentarem complexidade superior para a aplicação prática.

INTRODUCTION

Net radiation at the surface ($Q^\circ$) is the main input variable in the calculation of evapotranspiration, being of great importance in modelling and climate forecasting and in planning the use of water resources (KJAERSGAARD; PLAUBOR; HANSEN, 2007; SANTOS et al., 2011; SRIDHAR; ELLIOTT, 2002).

When calculating $Q^\circ$ the balance of both shortwave ($K^\circ$) and longwave ($L^\circ$) radiation are included. Accuracy depends on the quality of the measurements, adjustment of the coefficients or estimates of other input terms of the models (KJAERSGAARD; PLAUBORG; HANSEN, 2007). The calculation of $L^\circ$ is more challenging as it is a function of surface temperature and the effective temperature of the atmosphere, being dependent on atmospheric properties, mainly cloudiness and humidity ($\varepsilon$) (KRUK et al., 2010; SEDLAR; HOCK, 2008), and its direct measurement requires the use of special filters. Due to these factors, measuring $L^\circ$ becomes complex and expensive (DUARTE; DIAS; MAGGIOTTO, 2006; FIETZ; FISCH, 2009; SAMAN et al., 2007; SRIDHAR; ELLIOTT, 2002). Additionally, Savage and Heilman (2009) and Blonquist, Tanner and Bugbee (2009) cite authors who question the accuracy of net radiometers.

Despite the complexity, Brunt (1932) proposed an empirical equation for $L^\circ$ based on the air temperature ($T$) and $\varepsilon$. Brunt (1939) later included a weighting-term for cloudiness ($n$). Penman (1948) replaced $m$ by the ratio of sunshine duration ($m/10 = 1 – n/N$), which then came to be called the Brunt-Penman equation, and which can be used for a uniform, flat surface (HELDWEIN et al., 2012). This methodology has been used in studies in Santa Maria to estimate $L^\circ$ (SILVA et al., 2011; SILVA et al., 2008; PIVETTA et al., 2011; TAZZO et al., 2012) and has produced good results at various locations (BILBAO; DE MIGUEL, 2006; CROWFORD; DUCHON, 1999; LHOMME; VACHER; ROCHELEAU, 2007; PÉREZ-GARCIA, 2004; SOBRINHO, 2011; SRIDHAR; ELLIOTT, 2002).

With easier access to data from automatic stations, which measure global solar radiation incident on the surface ($K^\circ$) and not $n$, Allen et al. (1998) replaced $n/N$ by a function of the ratio $K^\circ$: to the global solar radiation that would fall on the surface in the absence of clouds ($K^\circ_{\downarrow}$). In Santa Maria RS, in a pre-analysis of the data which took as a reference the original Brunt-Penman equation, there was a significant deviation from a 1:1 straight line [$L^\circ = 0.4164 + 0.8934L^\circ_{\downarrow}$], with $L^\circ_{\downarrow}$ being calculated using the coefficients of Allen et al. (1998)], indicating a need to fit the equations to the local conditions.

There are relatively simple, alternative methods for the determination of $L^\circ$. Brutsaert (1975) derived an equation for the effective emissivity of a clear sky ($\varepsilon$) in the calculation of longwave radiation incident on the surface ($L^\circ_{\downarrow}$), where the weighting for cloudiness is a function of the cloud cover fraction. Duarte, Dias and Maggiotto (2006) adjusted the coefficients of the equations for daytime on an hourly basis using the rate of radiation in Ponta Grossa in the state of Paraná (PR), getting good results compared to measured values.

The aims of this work were (i) calculate new coefficients of the weighting term for cloudiness of the atmosphere in the Brunt-Penman equation using the rate of solar radiation ($R^\circ$) in place of $n/N$, and readjust the coefficients for weighting cloudiness of the atmosphere in the equations proposed by Duarte, Dias and Maggiotto (2006) and (ii) determine which methodology gives the more correct results in relation to the original Brunt-Penman equation, for Santa Maria, on a daily basis.

MATERIAL AND METHODS

Meteorological data for global solar radiation incident on the surface ($K^\circ_{\downarrow}$, MJ m$^{-2}$ d$^{-1}$), air temperature ($T$, K) and relative humidity ($\text{RH}$) were obtained from the automatic weather station belonging to the 8th Meteorological District of the National Meteorological Institute (8th DISME/INMET), located in the Department for Plant Science of the Federal University of Santa Maria (29º43' S, 53º43' W, at an altitude of 95 m). The values were obtained on an hourly basis and from these the daily average was calculated, with the exception of the accumulation of $K^\circ_{\downarrow}$ during the day. The data for actual sunshine duration ($n$, h) were obtained from the main conventional station also belonging to the 8th DISME/INMET, located by the side of the automatic station. Only days with no missing data were used.

The original Brunt-Penman equation (Equation 1) was taken as the reference, following the proposition of Doorembos and Pruitt (1975) for wet climates, but with the constants related to the effect of air humidity corrected for the use of partial vapour pressure in hPa:

$$L^\circ = -\varepsilon\sigma T^4 \cdot (0.56-0.0779\varepsilon)(0.1+0.9n/N)$$  (1)

where $\varepsilon$ is the emissivity of the surface, $\sigma$ is the Stefan-Boltzmann constant (4.903 10$^{-8}$ MJ m$^{-2}$ d$^{-1}$ K$^{-4}$) and $N$ is the length of the astronomical day, i.e. the maximum possible sunshine duration (h). Doorembos and Pruitt (1975) considered $\varepsilon$ as equal to 1.00, however, according to Gates (2003), for most plants and animals $\varepsilon$ varies between 0.95 and 0.98 for longwave radiation. In this study, a value of $\varepsilon$ of 0.95 was assumed. The term $-\varepsilon\sigma T^4$ refers to the energy emitted by the surface ($L^\circ_{\downarrow}$), and the other terms in parentheses refer to the subtraction effect on this emission by the atmosphere. $N$ was calculated according to Allen et al. (1998).
Allen et al. (1998) replaced the function for the rate of sunshine duration in equation 1 with the relation $a + b R^r$. Local calibration of $a$ and $b$ was done by linear regression between the original values of the Brunt-Penman equation (left-hand side of Equation 2) and the rate of solar radiation ($R^r$):

$$0.1 + 0.9n/N = a + b R^r$$  \hspace{1cm} (2)

$R^r$ can be calculated in relation to the global solar radiation incident on the surface on cloudless days ($K_{w,r}$, MJ m$^{-2}$ d$^{-1}$) or in relation to the solar radiation incident on the top of the atmosphere ($K_{w}$, MJ m$^{-2}$ d$^{-1}$):

$$R^r = K_w / K_{w,r} \downarrow$$  \hspace{1cm} (3)

$$R^r = K_w / K_{w,r} \downarrow$$  \hspace{1cm} (4)

Allen et al. (1998) used equation 3 for quantifying $R^r$, recommending the sum of the adjustment coefficients for the Ångström-Prescott equation (assuming $n/N = 1$) multiplied by $K_w$ for calculating $K_{w,r}$, so representing the average conditions of the local atmospheric transmissivity under a cloudless sky. Buriol et al. (2012) updated these coefficients for Santa Maria. In the absence of these coefficients, Allen et al. (1998) recommend using the equation $K_{w,r} = 0.75 + 2 \times 10^{-5} z$ $K_w$, where $z$ is the local altitude. This equation considers that at sea level, a condition similar to Santa Maria (95 m altitude), atmospheric transmissivity is equal to approximately 0.75 throughout the year. In this work, equations 3 and 4 were tested to determine $R^r$, with the values for $K_{w,r}$ determined from the local Ångström-Prescott coefficients (BURIOL et al., 2012), by the alternative equation given by Allen et al. (1998) and from a cosine curve adjusted for the three maximum values for $R^r$ in each month and the Julian day ($D$). Although it is unclear in Allen et al. (1998), according to ASCE-EWRI (2005), a boundary condition of $0.3 \leq R_{k,r} \leq 1.0$ should be imposed, corresponding to $R_{k,r} \geq 0.22$, assuming an average transmissivity for the atmosphere of 0.75.

To obtain $L^*$ from the Brutsaert equation for the emissivity of a clear sky and weightings for the cloud cover fraction ($c$), the longwave radiation emitted by a clear sky to the surface is first calculated from the air temperature ($L_\downarrow$), seeing that within any 24 hour period the values for the air and surface temperatures tend to converge:

$$L_\downarrow = -c_s \sigma T^4$$  \hspace{1cm} (5)

$L_s$ is the effective emissivity of a clear sky, derived physically by Brutsaert (1975) with its coefficients adjusted for hourly daytime data in Ponta Grossa, PR (DUARTE; DIAS; MAGGIOTTO, 2006):

$$c_s = 0.625 (c/T^{1.31})$$  \hspace{1cm} (6)

In equation 6, $e$ should be expressed in Pa:

$$e = 610.8110_T^{3.5} (T - 273) (237.5 + T - 273) \times UR/100$$  \hspace{1cm} (7)

To allow for the effect of cloud cover throughout the day, Duarte, Dias and Maggiotto (2006) adjusted and tested Equations 8 and 9:

$$L_\downarrow = L_\downarrow (1 + 0.242 c_{0.58})$$  \hspace{1cm} (8)

$$L_\downarrow = L_\downarrow (1 - c_{0.671}) + 0.990 c_{0.671} \sigma T^4$$  \hspace{1cm} (9)

where $L_\downarrow$ is the longwave radiation emitted by the sky under any cloud conditions (MJ m$^{-2}$ d$^{-1}$). As it is difficult to find values for $c$ taken throughout the day, Crawford and Duchon (1999) suggested the relation:

$$c = 1 - R_{k,r}$$  \hspace{1cm} (10)

which was used by Duarte, Dias and Maggiotto (2006) and in this work.

$L^*$ is given by equation 11:

$$L^* = L_\uparrow + L_\downarrow = -c_s T^4 + L_\downarrow$$  \hspace{1cm} (11)

Calibration of the coefficients of Equation 2 and local readjustment of the coefficients in equations 8 and 9 were done with two thirds of the 2,472 days of available data from the automatic weather station, selected for two days in every three throughout the series. The remaining days were used to test the performance of the equations by regression analysis, using the equation coefficients as accuracy indicators ($a = 0$ and $b = 1$), and the coefficient of determination ($R^2$) and the root mean square error (RMSE) as indicators of data dispersion around the average (best results corresponding to $R^2 \approx 1$ and RMSE = 0). Analyses were carried out using the R software (R DEVELOPMENT CORE TEAM, 2012).

RESULTS AND DISCUSSION

Modified Brunt-Penman equation

Tests of the estimates of $L^*$ with $R_{k,r}$ (Equation 5) obtained with $K_w$ determined from the monthly coefficients of Ångström-Prescott adjusted to Santa Maria (BURIOL et al., 2012) with the modified equation 1, resulted in smaller deviations, better fits and in slopes closer to 1.00 when compared to the values estimated by equation 1 in relation to the other methods of calculating $R_{k,r}$ (data not shown). Statistical Estimates of $L^*$ obtained using the alternative equation proposed by Allen et al. (1998) - for Santa Maria (altitude of 95 m), $R_{k,r} = (0.75 + 2 \times 10^{-5} z) K_w$ - or the cosine equation (Figure 1), were very close to those of the Ángström-Prescott monthly coefficients (BURIOL et al., 2012), although, as can be seen in Figure 1, these result
in significantly greater average values for the transmissivity of clear skies \( (T_{\text{clear}}) \) (p<0.01 by t-test, averages of 0.75, 0.74 and 0.78 for the daily estimates respectively).

**Figure 1** - Variation in the three largest monthly values of maximum transmissivity for global solar radiation under a cloudless sky \( (T_{\text{clear}} = K_{\|} K_{\perp}^{-1}) \); points of the atmosphere in Santa Maria as a function of the Julian day \( (D) \), and those estimated by the cosine equation (continuous line), the alternative equation suggested by Allen et al. (1998) (long-dashed line) and the sum of the coefficients \( (a + b) \) of the Ångström-Prescott equation as determined by Buriol et al. (2012) (short-dashed line). \( T_{\text{med}}, \ T_{\text{max}}, \ T_{\text{min}} \) are respectively the annual mean, maximum and minimum values for transmissivity. \( z \) is the local altitude (95 m)

Despite the lower physical coherence, the alternative equation by Allen et al. (1998) resulted in estimates of \( L^* \) quite close to those of Equation 1, with a non-significant linear regression (p > 0.05), a slope equal to 0.9989, \( R^2 \) equal to 0.99 and RMSE of 0.62 MJ m\(^{-2}\) d\(^{-1}\), almost the same statistical indices obtained with the local coefficients of Buriol et al. (2012). Therefore, due to the convenience of using the alternative equation of Allen et al. (1998), which assumes a fixed value throughout the year, and to the statistical results indicating the same quality as when using local coefficients, it was considered appropriate to use the equation \( R_{K,\perp} = 0.75 \ K_{\perp}^{-1} \) in the other analyses.

In Figure 2a it can be seen that the coefficients adjusted for \( R_{K,\perp} \) in place of the rate of sunshine duration (modified Equation 1) differ from those specified by Allen et al. (1998): -0.2614 against -0.35 and 1.2250 against 1.35. This discrepancy confirms the need for local determination of these coefficients in order to get estimates which are more consistent with Equation 1. Theoretically, the sum of the coefficients of the weighting term for cloudiness in equation 1 should be equal to 1, implying that cloudiness has no effect on the counter-radiation when there are no clouds \( (n/N = 1) \). However, the sum of the weighting coefficients of cloudiness in equation 1 modified by \( R_{K,K} \) (Figure 2a) differs from 1 by 0.0364 \( (1.2250 - 0.2614 = 0.9636) \). Lhomme, Vacher and Rocheteau (2007) also found a difference of this magnitude in these coefficients on an hourly timescale and considered it to be the result of the purely statistical nature of the relation, recommending deletion of the weighting term for cloudiness in the modified equation 1 when \( R_{K,K} = 1 \).

Alternatively the same estimation can be made considering \( K_{\perp}^{-1} \) instead of \( K_{\|}^{-1} \), which gives \( R_{K,K} \) (equation 4) and results in a slope of \( b = 1.6156 \) (Figure 2b), with practically the same value for the coefficient of determination (\( R^2 \)), RMSE and linear coefficient. The disadvantage of using \( R_{K,K} \) is the difficulty of imposing a physically valid upper limit, as is done with \( R_{K,\perp} \) and \( n/N \), where the maximum value is 1.

In testing equation 1 modified by the coefficients calculated for Santa Maria (Figure 2) from independent data (Figure 3), it is seen that fitting to the values of the original Brun-Penman equation remained high \( (R^2 \approx 0.99) \) and the error was relatively low \( (\text{RMSE} \approx 0.62 \text{ MJ m}^{-2} \text{ d}^{-1}) \), with estimated values of \( \approx -10.40 \) to \( \approx -0.55 \text{ MJ m}^{-2} \text{ d}^{-1} \). Imposition of a lower limit of \( R_{K,K} = 0.30 \) and \( R_{K,\perp} = 0.22 \), proved to be very effective in fitting the data and in the quality of the estimates obtained with the independent data. In a test of adjusted equations where these limits were not considered, 3.60% of the estimated values were seen to be above 0 MJ m\(^{-2}\) d\(^{-1}\), with a maximum of 0.63 MJ m\(^{-2}\) d\(^{-1}\). If they occur, daily positive values of \( L^* \) for Santa Maria should be close to zero, with values such as 0.63 MJ m\(^{-2}\) d\(^{-1}\) being inconsistent. Furthermore, not imposing these limits led to higher RMSE values \( (\approx 0.72 \text{ MJ m}^{-2} \text{ d}^{-1}) \), data not shown), although there is a tendency to overestimate \( R_K \) on days when \( n/N = 0 \) (respective averages for \( R_{K,\perp} \) and \( R_{K,K} \) of 0.17 and 0.13 when imposing \( R_{K,K} = 0.30 \) and \( R_{K,\perp} = 0.22 \)).

The Brutsaert equation and weightings for cloudiness corrected for Santa Maria

From comparison of the results of Equation 11 with those of equation 1, given \( 1.00 \geq c \geq 0.00 \), the quality of the estimation of \( L_{\perp} \) by equation 8 was found
Figure 2 - Relation between the weighting term for cloudiness in equation 1 \((0.1 + 0.9 \, n/N)\) and (a) the rate of radiation as determined by equation 3 \(R_{K,R}\) and (b) by equation 4 \(R_{K,K}\). \(p_a\) and \(p_b\) in parentheses indicate the level of significance of their respective coefficients to be significantly lower (coefficients of the adjustment equation: \(a = -1.8607\) and \(b = 1.1096\) \((p<0.01)\), \(R^2 = 0.86\), \(\text{RMSE} = 0.99 \, \text{MJ} \, \text{m}^{-2} \, \text{d}^{-1}\)), indicating the potential for its use with coefficients corrected for Santa Maria RS, although there was significant deviation from a 1:1 straight line (\(a = -1.3727, b = 1.2119\), \(p<0.01\) and a significant frequency of positive values for \(L^*\). Correction of the coefficients of equation 9 for Santa Maria RS was done.

Figure 3 - Comparison of the values for net longwave radiation estimated by equation 1, and by equation 1 modified with (a) the rate of radiation obtained by equation 3 and (b) by equation 4, with the calculated coefficients for Santa Maria (Figure 2). The linear coefficients of the regression equations were not significant at 5% probability. \(p_b\) shows the level of significance of the slope of the adjustment equation.
with values for $L^*$ determined by Equation 1, isolating $L_\downarrow$ in equation 11 and replacing it by equation 9 with the coefficients unknown, according to Equation 12:

\[ L^* + \sigma T^4 = L_\downarrow (1 - c') + a'c' \sigma T^4 + c' \]  

(12)

where $a'$, $b'$ and $c'$ are the adjustment coefficients. In relation to equation 9, the linear coefficient $c'$ was added. Modifications were therefore made only on the $L_\downarrow$ component of $L^*$ (referring to the emission of radiation by the atmosphere), which is more dependent on atmospheric conditions and the predominant type of cloud in the area; with the term referring to the emission of radiation by the surface and by a cloudless atmosphere, both of which are derived physically, remaining the same. In addition, the coefficients adjusted for Ponta Grossa (DUARTE; DIAS; MAGGIOTTO, 2006) refer to daytime and to hourly data, and although developed for $1.00 \geq c \geq 0.00$, the boundary conditions suggested by Allen et al. (1998) and ASCE-EWRI (2005) were also tested. Thus, by using numerical methods, adjustment of the coefficients was achieved considering the whole range of variation of $c$ (1 to 0: equation 13) and imposing $0.30 \leq R_{K,R} \leq 1.00$ as suggested by Allen et al. (1998) and ASCE-EWRI (2005), i.e. $0.70 \geq c \geq 0$ (Equation 14):

\[ L_\downarrow = L_\downarrow (1 - c^{0.7157}) + 1.0410c^{0.7157} \sigma T^4 - 2.7044 \]  

(13)

\[ L_\downarrow = L_\downarrow (1 - c^{0.9482}) + 1.0647c^{0.9482} \sigma T^4 - 2.1132 \]  

(14)

Adjustment of the data was high when generating the new coefficients ($R^2 = 0.96$ and RMSE = 0.70 MJ m$^{-2}$ d$^{-1}$ with equation 13, and $R^2 = 0.97$ and RMSE = 0.64 MJ m$^{-2}$ d$^{-1}$ with equation 14), for all significant coefficients ($p<0.01$). Compared to equation 9, there was a significant increase in the exponent for $c$ (0.671 to 0.7161 with equation 13, and to 0.9482 with Equation 14) and in the multiplying coefficient for the second right-hand side term (0.9900 to 1.0410 in equation 13, to 1.0647 in equation 14). The most significant change was in coefficient $b'$ that defines the shape of the response curve for cloudiness.

Testing equations 13 and 14 with the independent data resulted in a high adjustment to equation 1 ($R^2 = 0.98$ and RMSE equal to 0.71 and to 0.64 MJ m$^{-2}$ d$^{-1}$ respectively), with a non-significant linear coefficient ($p>0.05$) and slope $\approx 1$ (Figure 4). It was noted that the statistical indices for the estimates of equation 14 (Figure 4b) were quite similar to those of the modified equation 1, while equation 13 had lower statistical indices (Figure 4a). Moreover, not imposing an upper limit for cloudiness resulted in 2.30% of days with $L^* > 0.0$ MJ m$^{-2}$ d$^{-1}$, to a maximum of 0.48 MJ m$^{-2}$ d$^{-1}$, again pointing to the efficiency of the limits suggested by Allen et al. (1998) and Asce Ewri (2005) for $R_{K,R}$.

Comparison between methods and discussion of the results

In an overall assessment, the coefficients adjusted for the rate of radiation (Figure 2) in the modified equation 1, and in equations 13 and 14 for the calculation of $L_\downarrow$, showed similar statistical adjustments.

Figure 4 - Relation between the values for longwave balance as determined by Equation 1 ($L^*$) and Equations (a) 13 ($L^*_{SM, Eq.13}$) and (b) 14 ($L^*_{SM, Eq.14}$) for Santa Maria RS with the independent data set. The linear coefficients of the regression equations were not significant at 5% probability. $p_b$ refers to the level of significance of the slope of the adjustment equation.
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when compared to the data from equation 1 with the original coefficients. However, with no limits imposed (0.30 ≤ R_{k,r} ≤ 1.00, 0.22 ≤ R_{k,k} ≤ 0.70, c ≥ 0.00), there was an estimate of \( L^* < 0 \) MJ m\(^{-2}\) d\(^{-1}\) for both methodologies, which is unexpected for the daily climatic conditions of Santa Maria, RS. For the entire data set, \( L^* \) calculated with equation 1, resulted in a maximum value of -0.56 MJ m\(^{-2}\) d\(^{-1}\) against -0.57 MJ m\(^{-2}\) d\(^{-1}\) with the modified equation 1 with the imposed boundary conditions, 0.48 MJ m\(^{-2}\) d\(^{-1}\) by equation 13 and 0.04 MJ m\(^{-2}\) d\(^{-1}\) by equation 14. These results, arising from the different types of cloud when the sky is completely overcast, result in a registered sunshine duration of almost zero (\( n = 0 \) h), but with different variations in \( K_1 \) and \( L_1 \) (GUEYMARD; JINDRA; ESTRADA-CAJIGAL, 1995; MONTEITH; UNSWORTH, 1990).

On 63% of the days with \( R_{k,r} < 0.3 \), rainfall greater than 2.00 mm was seen, indicating the predominance of cloudiness and different classes of cloud when \( n/N = 0 \). The different types of cloud on overcast days result in a large variation in \( K_1 \) (GUEYMARD; JINDRA; ESTRADA-CAJIGAL, 1995; LHOME; VACHER; ROCHETEAU, 2007), and in deviations from linearity in the ratio of \( R_\lambda \) to \( n/N \). The greater importance of the differences in \( K_1 \) measured on overcast days is due to the low daily values seen for solar radiation, whereas on predominantly sunny days, values for \( K_1 \) measured at times of direct sunshine are much greater, the overcast periods becoming less important when making up the total daily value. In other situations, though less frequent, the formation of isolated cumulus clouds enables reflection of solar radiation off the cloud-wall towards the surface (MONTEITH; UNSWORTH, 1990), resulting in higher records for \( K_1 \) compared to days with similar cloud cover of other types. These conditions may explain some of the greatest deviations seen when testing models modified for use in relations where \( R_{k,r} \) or \( R_{k,k} \) substitute the rate of sunshine duration.

Although solar radiation shows sensitivity to the type of cloud, \( L_1 \) is dependent on the effective temperature of the sky and its emissivity, being more sensitive to the part of the sky covered by clouds and to the height of the cloud base than to cloud thickness (MONTEITH; UNSWORTH, 1990). According to Monteith and Unsworth (1990), the emission of radiation by an overcast sky is a function of the temperature of the cloud base and the atmosphere below it, making it possible to use a linear relation between \( L^* \) and \( n/N \). This however needs correcting to be maintained for \( R_\lambda \), and is achieved with the boundary condition suggested by Allen et al. (1998) and by ASCE-EWRI (2005). This linearity is also seen by increasing the exponent of \( c \) to a value close to 1.00 in equation 14, meaning there was an almost linear effect from cloudiness when the boundary conditions were imposed.

Figure 5 - Simulation of the longwave radiation balance estimated by Equations 11, 13 and 14 (\( L^*_{SM,Eq1} \) \& \( L^*_{SM,Eq14} \)), by Equation 1 modified by \( R_{k,r} \) and the local coefficients (\( L^*_{obs} \)) and by the Brunt-Penman Equation (equation 1, \( L^* \)) for temperatures \( T \) in the range of 273-303 K and relative humidity equal to 70% (a) and 90% (b), for days without cloud cover (\( c = 0; R_{k,r} = n/N = 1 \)) and completely overcast days (\( c = 1 \) or \( c = 0.7; R_{k,r} = 0.3; n/N = 0 \))
Compared to the original coefficients of Duarte, Dias and Maggiotto (2006), there is a slight trend towards an increase in linearity when the entire range of c (1 to 0) is used, corresponding to a less intense effect from the clouds (or the degree of cloudiness) in Santa Maria RS when the value of c is low, which may be linked to the different types of cloud or to the different time scales used. Another important factor is the occurrence and characteristics of night and morning mists in Santa Maria, which are then weighted by the new coefficients. Overnight mists usually dissipate early in the day, and are not weighted either by \( R_e \) or \( n \). In the early hours of the morning however, although there is no record for \( n \) when there is mist, relatively high intensities of radiation are recorded compared to periods of overcast sky.

In a sensitivity analysis, taking as reference equation 1 and an RH equal to 70% and 90%, it was found that deviations for the modified equation 1 adjusted for Santa Maria are greater when the days are sunny, and lower on overcast days (Figure 5a and 5b). On the other hand, equation 13 estimates \( L^* \) with a high fit to equation 1 on sunny days for a temperature range of 273 K to about 290 K for an RH of 70%, and to about 285 K for an RH of 90%, with increasing errors at higher temperatures.

Despite better statistical results when testing the equations (Figure 5), equation 13 overestimates \( L^* \), with deviations greater than those of equation 14, up to 292 K when the RH is equal to 70% and up to 287 K when the RH equals 90%, on days without cloud cover. For high values of \( T \), equation 14 also underestimates \( L^* \), although with less intensity than does equation 13. For overcast skies, equations 13 and 14 give very different results, with the first overestimating values for \( L^* \) over the whole studied range of \( T \), where \( L^* > 0 \) MJ m\(^{-2}\) d\(^{-1}\) for \( T \geq 279 \) K, while equation 14 underestimates \( L^* \) up to 293 K and 291 K (Figures 5a and 5b respectively).

This analysis shows that in the middle of the higher frequency range for the average daily RH in Santa Maria (60% < RH ≤ 80% on 74% of the days evaluated) and the more frequent of the average daily temperatures (288 < T ≤ 298 K on 62.5% of the days evaluated), the values estimated by equation 1, modified considering the boundary conditions (ALLEN et al., 1998; ASCE-EWRI, 2005), display the smallest deviations in relation to equation 1. The use of equation 13 would have some advantage only at the lower limit of this temperature range, while equation 14 results in greater deviations across practically the whole range of variation of \( T \). When the RH is high, Equations 13 and 14 result in even greater deviations compared to Equation 1. Therefore, the modified equation 1 (Figure 2) is the equation that estimates values for \( L^* \) as a function of \( R_e \) which are closer to those of the original Brunt-Penman equation that adjusts for the rate of sunshine duration. In that equation, the use of \( R_{s, e} \) should be preferred, as it allows for greater control of the physical quality of the observed data for solar radiation.

**CONCLUSION**

The modified Brunt-Penman equation is the one that best approximates the values for the balance of longwave radiation for Santa Maria, RS as a function of the rate of radiation, compared to the original Brunt-Penman equation:

\[
L^* = -0.95eT^4 (0.56-0.0779\sqrt{e})(1.2250R_{s, e}^{-0.2614}) \tag{15}
\]

where \( \sigma = 4.903 \times 10^8 \) MJ m\(^{-2}\) K\(^{-4}\) d\(^{-1}\) is the Stefan-Boltzmann constant, \( T \) is the average daily air temperature in K, \( e \) is the partial vapour pressure of the air in hPa and \( R_{s, e} = K_1/K_\downarrow \) \((0.3 \leq R_{s, e} \leq 1.0) \), where \( K_1 \) is the global solar radiation incident on a horizontal flat surface and \( K_\downarrow = 0.75 \ K \downarrow \) is the radiation incident on the surface in the absence of clouds.

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