Brazilian Journal of Political Economy, vol. 37, nº 4 (149), pp. 734-754, October-December/2017

The impacts of the early 20th century physics and mathematics crisis on contemporary economics discourse

A influência da física e da matemática na constituição da economia no início do século XX

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RESUMO: Por meio da discussão de vários autores sobre o processo de matematização da economia, procuramos esclarecer o progresso deste processo no mundo. Mirowski (1991) analisa esse fenômeno e acha que ele começou mais fortemente a partir de 1925. No entanto, ele não mostra em seu artigo como a matemática e a física qualitativamente impactaram a economia. Este é o objetivo principal deste artigo: descrever, de maneira sistemática, como a matemática e a física influenciaram a constituição da economia, culminando na adoção do equilíbrio geral como metateoria e a generalização do método hipotético-dedutivo.
PALAVRAS-CHAVE: Matematização; economia; processo; discurso; equilíbrio.

ABSTRACT: Through discussion of several authors, dealing with the process of mathematicization of the economy, we seek to clarify the progress of this process in the world. Mirowski (1991) analyzes this phenomenon and finds that it began more strongly from 1925. However, he does not show in his article as mathematics and physics qualitatively influenced the economy. This is our primary goal in this paper: an account, in a systematic way, of how mathematics and physics have influenced constitution of economics, culminating in the adoption of general equilibrium as metatheory and the generalization of the hypothetical-method deductive.
KEYWORDS: Mathematicization; economics; process; discourse; equilibrium.
JEL Classification: B1; B4.

INTRODUCTION

Today's mainstream economics deems negatively articles that are not mathematically formalized less scientific.[1] This, however, has not always been the case. According to Mirowski (1991), the main inflection point at which mathematical

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[1] We consider mainstream economics to be the school that accepts the general equilibrium theory and

 http://dx.doi.org/10.1590/0101-31572017v37n04a05

formalization became prevalent in economics publications occurred in the 1930s. We penned this article in an attempt to understand how this change unfolded. The article, then is an effort to systematize the debate by reviewing the literature produced by the main authors who address the topic.

Our investigation begins with an analysis of Philip Mirowski's (1991) article on some of the inflections economic discourse experienced over history, enabling the establishment of a more mathematicized economic discourse. Next, we review papers devoted to the main influences and impacts changes had in mathematics and physics had on economic discourse. We rely mainly – and comparatively, insofar as possible – on the works of authors such as Bruna Ingrao and Giorgio Israel (1990), Lionello F. Punzo (1991), Michel Beaud and Gilles Dostaler (1997), and E. Roy Weintraub (2002). Based on this review, we analyze the main changes that physics and mathematics experienced in the early 20th century. We then note the importance of mathematicians David Hilbert and John von Neumann, of the philosophers and scientists of the Vienna Circle, of participants at Bourbaki seminars – to include mathematician Gerard Debreu – for the advance of the mathematicization of economic discourse. If the empirical-deductive method prevailed in economic theory production until the so-called marginalist revolution in the late 1800s, the hypothetical-deductive method, as defined by Luiz Carlos Bresser-Pereira (2009), became prevalent after the revolution that mathematics and physics experienced in the same period.

We base our text on Bresser-Pereira (2009, 2016), who distinguishes "methodological sciences" such as mathematics, statistics, econometrics and game theory, which have no object of study but serve as an instrument for logical reasoning, of the "substantive sciences", which have an object. The latter are subdivided into natural sciences, such as physics and biology, and social sciences, such as economics and sociology. However, these definitions emphasized by Bresser-Pereira came to prevail fundamentally after the crisis experienced in mathematical physics occurred in the early 20th century. This is because before that time both physics and mathematics were confused within the so-called natural sciences. After this crisis, there was an increasing tendency in the separation of these two sciences. Mathematics has become a predominantly methodological science whose criterion of validity has become logical consistency, and physics has continued its course as a substantive science.

For Bresser-Pereira, both economics and physics are substantive sciences and require empirical proof, based on real assumptions. While the most appropriate method for physics can be called empirical-deductive, in which its experiments can be controlled, the appropriated method of economic science must be the historical-deductive. The difference is that, unlike physics, where the behavior of atoms is predictable, while in economics the reaction of economic agents to expectations are less predictable. The solution found by the neoclassical school to this problem

rational expectations as assumptions. It includes neo-classical and neo-Keynesian economists, for whom the nominal variables (money) do not affect real variables (income and employment) in the long run.

– to have as an axiom the homo economicus and adopt the hypothetical-deductive method allowed that economics become a mathematical science, but opened room for the construction of empty theories of reality.

INFLECTION POINTS IN ECONOMIC DISCOURSE

According to Mirowski (1991), the academic economic discourse experienced two inflection points. The first of these occurred around 1870, as the marginalist school consolidated. Members of this school – a group of researchers whose background was essentially in engineering – attempted to come up with a project to ensure a scientific status for political economy, which they deemed insufficient at the time. They included William Stanley Jevons, Leon Walras, Francis Ysidro Edgeworth, Irving Fisher, Vilfredo Pareto, to name a few. These authors drew inspiration from a physics metaphor: "*equilibrium in a field of force*". Based on this thinking, they likened potential energy to utility. According to Mirowski, this is why many authors, even if unaware or one another's activities, copied the mathematics of physics "literally term for term and dubbed the result mathematical economics".

In the meantime, the marginalist school's discourse, associated with rational mechanics, faced heavy resistance. Mirowski tried to assess the evolution of the incorporation of the language of mathematics into economics by reviewing scientific production, particularly by analyzing the leading academic reviews. However, because publication of economic reviews lacked continuity until 1890, he faced difficulties checking for the actual impacts of the marginalist revolution or the rise of the neoclassical school on the economic discourse from 1870 to 1887.

Mirowski concludes that the mathematicization that the "neoclassical marginalist" theory desired made little headway until the early 1920s. But in 1925-1936 came what Mirowski referred to as a second quantum leap. It was the second inflection point in economic discourse. Unlike the previous inflection, the world's main economics reviews already had continuity in this latter period. For this reason, Mirowski selected four of them for his study: *Revue D'Économie Politique* (RDP), *Economic Journal* (EJ), *Quartely Journal of Economics* (QJE), and *Journal of Political Economy* (JPE). His goal was to find out when the switch occurred from a less to a more mathematicized discourse. Mirowski studied the analytical sample covering the 1887-1955 period, and collected qualitative data on the articles published in the relevant periodicals. The author found that, from 1887 to 1924, the participation of mathematical discourse in the reviews studied was very low. Until 1924, they seldom devoted more than 5% of discourse to mathematical discourse. The shift in economic discourse took place, then, between 1925 and 1936. Among the periodicals studied in this period, the QJE led the shift, at 25% of all pages featuring mathematical economic discourse. For the JPE, the process took a while more, and only reached the same levels as the QJE in the 1950s. The EJ and the RDP reached 10% after World War II.

For now, it is important to emphasize that "examination of the character of the papers before and after the second 'rupture' indicates clearly that the newly-achieved level of mathematical discourse was narrowly associated with the neoclassical research program" (Mirowski, 1991, p. 150). It is not Mirowski's intent to provide an in-depth analysis of the causes of the second "rupture", at least as far as his 1991 paper is concerned. It is, however, one of our objectives. To this end, we will carry out a historical analysis of this inflection point associated with transformations had in mathematics.

THE INFLUENCE OF MATHEMATICS AND PHYSICS ON ECONOMIC DISCOURSE

For most of the authors studied in this paper, the so-called neoclassical Marginal revolution drew inspiration from classical mechanics. As Ingrao and Israel (1990), Punzo (1991), Weintraub (2002) point out, the Newtonian model, which served Leon Walras, W. S. Jevons, Pareto, and others, was based on observation. And, because it was based on observation, it limited the development of mathematical economics at the time, because not every economic theory found support in observable characteristics. As seen in Mirowski (1991), the advance of mathematical discourse over economics only increased significantly since the 1930s. The question we ask is: what might have enabled this advance? The hypothesis we explore here is that the advance was only possible due to the changes that mathematics and physics underwent as a result of a crisis the two sciences experienced in the early 19th century, leading economists to apply the hypothetical-deductive model *in lieu* of the historical-deductive one.[2] During this crisis, there was a departure from the conception of rigor connected to correspondence with reality. The idea was replaced by the notion of rigor connected to formal mathematical proof.

The mathematics and physics crisis of the early 20th century

To understand the process of mathematicization of economic discourse we must first understand the changes that physics and mathematics underwent in the early 20th century.

For Ingrao and Israel (1990) it was Vito Volterra who first presented the two faces of the crisis in 1907. One aspect was Einstein's theory of general relativity,

[2] The hypothetical-deductive method allows for a "precise and quantifiable hypothesis. Insofar as the method starts from a principle - the *homo economicus* whose behavior is completely predictable – complemented with a few additional assumptions, this method allows a precise and mathematical theory." The historical-deductive method, in its turn, "does not part from simple assumptions, but rather from the observation of a complex and changing reality. Both are deductive, but while one is hypothetic – starting from an assumption – the other is historical – starting from observed sequences of facts and keeping close to them during the deductive process." (Bresser-Pereira, 2009).

which challenged fundamental conceptions of classical mechanics, such as absolute space and time, "simultaneous events", etc. Even more important, however, was the development of quantum physics, which challenged the continuous representation of phenomena and launched the hypothesis of leap energetic variation. Quantum physics showed, on the microscopic level, that a particle's position and speed could not be simultaneously determined. This brought down the notion that knowledge of a particle's position and speed could determine its future evolution. For this reason, the analogy with classical mechanics lost the key role it had held and gave way to mathematical analogy.

Based on these ideas, quantum physics also questioned the fundamental role of infinitesimal calculus and the centrality assigned to mathematical representations of phenomena by means of differential equations. These changes stemming from physics had a significant impact on mathematics. Previously, according to Ingrao and Israel, mathematics had been a tool to describe the laws of physics and, insofar as possible, predict the behavior of observed processes in numerical terms. Since the changes seen in the early 1900s, mathematics took on a role that cast experimentation aside. Experimentation, therefore, became uncoupled from theorization. One consequence of the rising scientific approach was the demise of classical science's unified edifice, leading to the fragmentation of scientific work, that is, a process of increasing specialization paradoxically made it more uncertain.

E. Roy Weintraub (2002, pp. 31-32) tells the tale of how mathematics influenced economics throughout the 20th century. He notes that Francis Ysidro Edgeworth (1845-1926) conceived of mathematics as an intellectual framework where physics might develop. Therefore, political economy was to imitate the fundamental model of physics and the manner of the imitation was to embrace the physics model's mathematical framework. However, as seen earlier, the model of physics had started to become obsolete in the early 1900s, after facing the challenge coming from Einstein's theory of general relativity and, most of all, from quantum physics. Euclidean geometry, which was regarded as the science of space, was heavily challenged, as Einstein's theory had light beams follow curved, rather than straight, paths. In the light of this, Edgeworth turned his attention to pure and applied statistics as an instrument for unravelling the secrets of human behavior. Like Edgeworth, Vilfredo Pareto (1848-1923) believed that mathematical argumentation created basic propositions within mathematical logic and derived the implications of such propositions. Mathematical argumentation might lead to conclusions, but those conclusions would normally be qualitative rather than quantitative (Weintraub, 2002, p. 37). As a result, the emerging "new mathematics" did not match Pareto's thinking. The changed mathematics influenced Pareto, who faced difficulties applying the experimental method to mathematical economics. Pareto might have dismissed economics as a result of the stalemate, or even embraced a "totally normative" method. He chose to study sociology, in an attempt to make contributions to economics.

What Pareto's difficulties show is that there was a barrier to the development of mathematical economics in the early 20th century. This is perhaps why publication of more mathematicized economics articles, such as those noted by Mirowski (1991), was scarcer in this period. A new analytical method was needed.

Ingrao and Israel (1990, p. 170) note that the crisis emerged in general equilibrium economics around 1910, or the Walrasian model's crisis stemming from the emulation of classical mechanics.[3] This crisis was due to the realization of the impossibility of unifying classical mathematical physics and "mathematical economics" in the same direction we have indicated in connection with Pareto. The leading proponents of physics reductionism, in particular those who expected to extend the method of physics to other sciences, declared that new attempts at unification were impossible.

On the other hand, the new transformations that determined the supremacy of the new reductionism based on mathematical analysis, on formal proof, over the classical reductionism, had an impact on the general equilibrium model. It was no longer about reducing phenomena to mechanical laws, as Walras had done, but about formally unifying different laws by means of mathematical frameworks devoid of empirical content. The ties between laws and empirical reality became looser.

We can therefore say that in this period mathematics ceases to be a natural (substantive) science because it no longer required experimental proof of its laws, but merely the justification of mathematical logic. Better yet, to use the definitions provided in Bresser-Pereira (2009), we might understand that mathematics became a methodological science whose objective lies associated with logical-deductive correspondence, and not with reality. Even from the viewpoint of Ingrao and Israel, however, this is controversial: views connected with the Vienna Circle in the 1930s led to the preservation of characteristics linked to empirical proof in both mathematics and economics, as we will see ahead.

Given, however, that the mathematical models applied to physics no longer needed to conform to reality, the way was clear to overcome the limitations facing attempts to unify the method of physics with other sciences, including mathematical economics. What was sought for economics, as had been the case with the Marginal "revolution" of the late 1800s, was to give it science status.

According to Ingrao and Israel (1990, pp. 172-173), the two streams – one associated with the old reductionism, another with the new – came together in the United States and gave rise to the most significant recent developments, as they attempted to salvage the paradigmatic course of the "Walrasian program". The two merging streams, according to the authors, were those of the "anti-formalists" and the "formalists" (incoming from mathematics and mimetized into economics). As for the influence this had on economics, the literature seems to show some disagreement between Ingrao and Israel (1990) and Weintraub (2002), as the latter understands that the influence of anti-Formalist mathematics prevailed in economic theory, rather than a merger of the two streams as the former authors propose.

[3] For Weintraub (2002), this crisis began in the late 1800s.

The anti-Formalist and Formalist schools

The crisis of mathematical physics of the early 20th century led mathematics to split into the "anti-formalists", who favored the mathematics development in association with experimental questions, and the "formalists", who would rather develop mathematics free from all restraints, except for formal rigor (Weintraub, 2002, pp. 46 and following).

The presence of these two streams in mathematical physics was observed by several authors, including Italian mathematician and physicist Volterra (1860-1940). According to him, the former stream's analyses were based on a problem's empirical characteristics, while the latter focused harder on analyzing logical-mathematical reason. For Formalist mathematicians, scientific rigor was determined by an approach's formal rigor. For Volterra, however, rigor in science was not assured by axiomatization; an "axiomatized" science was not necessarily rigorous. Volterra's main concern and school lay with the limits of axiom choices; for him, unrestricted axiom choice did not force scientists to limit themselves to observed reality. Along with his disciple, mathematician Griffith Evans, he claimed that "[...] mathematical models are not free, but rather tightly constrained by the natural phenomena that those mathematical constructions must model" (Weintraub, 2002, p. 71). Rigor, therefore, is associated not with freedom in ideas, axioms and abstract structures, but with fundamentals directly and specifically associated with basic physical reality. That is, ideas should be based on reality directly apprehended through experimentation and observation. Nowadays, instead, a study's scientific rigor tends to be associated with the logical consistency of its formal mathematical development.

The previously mentioned Griffith Conrad Evans (1887-1973) was another mathematician who associated rigor and correspondence with reality. For him, rigor was linked with the connection between conceptual categories and the physical fundamentals of reality. And, in his opinion, the economic theory Jevons and Walras developed failed to meet this requirement. For Weintraub, this notion from Evans is marginal to the community of economists and converged with Volterra. For both Evans and Volterra, the discussion revolved not around formalism vs. anti-formalism, but rather around rigorous vs. non-rigorous. And for them, rigor meant finding basis in reality, in natural phenomena. As seen in Mirowski (1991), the significant increase in publication of articles with a more mathematicized discourse took place around the 1930s. And, according to Weintraub (2002, pp. 66-70), mathematical economists in the 1940s left practically no room for Evans's ideas.

For Weintraub (2002, p. 72), in the same vein as Punzo (1991), this controversy surrounding mathematical formalism lies on a misunderstanding of the history of mathematics, of the history of economics, and of the history of the relation between these two, in the presence of confusion about the definition of the concepts of rigor, axiomatic, and formalism. Do they have the same meaning? Might one hold them equal, as in "formal = abstract = axiomatized = mathematical"? Might one then add "= rigorous economics" to the equation?

While attempting to answer the foregoing questions, Weintraub (2002, pp. 75--76) takes note of Ken Dennis's criticism. Dennis emphasizes that logical rigor is missing from mathematical economics because its formal mathematical apparatus fails to capture or express the economic contents of the theory, and the contents of the theory miss the formal expression meanings capable of being rigorously established and critically verified. For Dennis, an argument is rigorous if, and only if, it proceeds from assumptions to conclusions and attempts at every step to comply with the rules of formal logic. In addition, for Dennis as well as Mirowski (1991), logical form goes beyond mathematics and logic in economics stems from language rather than mathematics. Only by studying economic discourse might one unravel the complexity of rational thought. With this in mind, Weintraub asks why many economists understand that formality sets good economic analysis from bad.

In economics, this entire discussion about anti-formalists and formalists is mainly connected with changes had in the general equilibrium theory. In this sense, Punzo (1991, pp. 15-16) argues that the differences between intuitionalists (anti-formalists) and formalists are respectively reproduced in the models of Walras and Debreu. In the case of the Formalist approach, he points out that general equilibrium can only be achieved when economically relevant entities are endogenously determined. Walras's classical approach, on the other hand, was based on a single economy. As a result, according to Punzo, generation of a general equilibrium could not be counted on because the fact embedded a methodological inconsistency due to the presence of two rival principles in a state of unstable equilibrium: reductionism (an economy made up of selfish individuals – microeconomics) and biologistic holism, wherein the endogenous price formation process was replaced by a systemic view describing a resource allocation process globally regulated by the principle of scarcity. Punzo refers to this approach as functionalist (associated with classical mechanics). When this functionalism met with crisis, then emerged modern functionalism, which found support in mathematical formalists. This modern functionalism enjoyed the collaboration of the Vienna Circle, one of whose main propositions was "the codification of the laws governing functional relationships". Because formal models required validation, they looked for this validation in proof of model consistency, as only such models might explain the endogenous variables.

Therefore, when we refer to the general equilibrium theory, as in mathematics, we may also distinguish between anti-Formalist and Formalist authors. In the 19th century, there were models based on real descriptions of the economy. Perhaps as a result, those models were more concerned with paying some sort of social "bill" than with equilibrium. Causal relations lacked formal mathematical rigor (Punzo, 1991, p. 17).

The new reductionism and the influence of mathematician David Hilbert

According to Weintraub (2002, p. 80), the fundamental shift that allowed mathematicization to gain ground over economic discourse took place first in mathematics and was only later transplanted into economics. In the vicinity of 1900, questions flourished regarding mathematical paradoxes in theory, arithmetic and logic. These questions made room for the development of Hilbert's formalism in 1918-1922.

On the other hand, at about the same time, development of non-Euclidean geometry led to the acknowledgement of the crisis of intuitionism (anti-formalism) as the basis for truth. This added great momentum to mathematical developments in the 20th century, culminating with Hilbert's renunciation of Euclidean geometry studies that assigned intuitive and empirical content to mathematical models.

Around 1900, the European, Anglo-Saxon world of mathematics to which David Hilbert belonged was still characterized by a mixture of geometry and applied mechanics, upholding inconsistent views of mathematical truth based on logic and nature. These notions were not as mightily defended in continental Europe. It was Hilbert who described the new axiomatic path for mathematics and, albeit late, the community of mathematics started to change (Weintraub, 2002, p. 84).

In economics, there is some disagreement regarding the nature of changes in mathematics and in mathematical economics. For example, one cannot accurately pinpoint the beginning of the process by means of which economics based on the classical mathematical physics model changed into the new model based on quantum physics, or to which point this took place, or whether the model adopted in economics was Formalist or anti-Formalist. One can, however, get some sense of it. Let us focus on this debate.

In the late 1910s, Hilbert and other mathematicians developed mathematical frameworks for physics fields like radiation, thermodynamics, gravitation, etc. According to Weintraub (2002, p. 89), these replaced the late 19th century's reductionist models. It was about creating a new mathematical reductionism framework that required the axiomatization of mathematical theories, more specifically of set theory and arithmetic. Both needed to contain consistent and complete systems. The addition of axioms or assumptions should enable the system to become complete, so that there might be theoretical certainty about what was true or false. The system's completeness became linked with the ability to decide on propositions, or on mathematical proof. The notion of rigor started to change with Hilbert; if theory previously had to be connected with real phenomena in order to be rigorous, from this point on rigor drifted in the direction of mathematical consistency, the criterion that began dictating the agenda of the so-called Queen of Sciences. As noted earlier, such a change led to a split between anti-formalists and formalists. With Hilbert, therefore, the image of mathematics started to change as the notion of rigor changed. The kind of mathematical proof that formalists advocated became the prevalent criterion, based on the hypothetical-deductive method. As argued by Weintraub (2002), Punzo (1991), Ingrao and Israel (1990), the "fundamental physics model" was the prevalent criterion of truth under the old reductionism. Reconciling this with Bresser-Pereira (2009), the old reductionism uses an empirical-deductive method, while the new reductionism's method is hypothetical-deductive. For Bresser-Pereira, all mathematics rely on the hypothetical-deductive method. A discrepancy exists, then, between the thinking of Professor Bresser and the named authors, as for the former physics even now uses the empirical-deductive method and for the latter, according to our interpretation, physics started to use a hypothetical-deductive method after the mathematical physics revolution of the early 1900s. According to Bresser-Pereira (2009), therefore, in the physics, the historical-deductive method continues being the main method. The physicist is free to use deduction more widely because neutrons and protons act in a predictable way, and, so, his empirical models are accurate, something that does not happen to the economic models. Instead of living with the incertitude, which is inherent to economic behavior, neoclassical economists opted for the use of the adoption of the hypothetical-deductive method which allowed to the mathematicization of economics, but at a very high cost.

According to Ingrao and Israel (1990, pp. 182-184), the German city of Göttingen was the leading center in the development of this new approach to mathematics. And Hilbert was the leading mathematician at the helm of this development. His 1899 paper "Grundlagen der Geometrie" carried the main tenets of this axiomatizing trend. In the paper, Hilbert points out that a mathematical theory is a complex set of theorems obtained by means of deductive logic. In conclusion, he argued that a mathematical entity was determined by axioms. Hilbert regarded only axioms and theorems as significant elements of theory. What about substantive content? Irrelevant as concerns the logical structure. For him, the terms "point", "line", and "plane" might well be substituted with "chair", "table" and "beer mug" without harming a theory's validity. In fact, he believed that the axiomatic method allowed mathematics complete freedom of movement, and convinced a large portion of the mathematics community of the soundness of the new paradigm. To overcome the fundamentals crisis mathematics was undergoing, he established a rigorous program intended to demonstrate the non-contradictory nature of the core of mathematics, that is, arithmetic.

In what manner would the new reductionism influence economic theory? For Weintraub (2002, p. 94), by embracing the Formalist project, many economists were following a misleading path. One must then examine Kurt Gödel's view to find out what happened to Hilbert's Formalist and non-Formalist program. The Formalist project survived even though it implied the loss of mathematical certainty. Such a loss stems from Gödel's theorem of incompleteness, developed in the early 1930s and presented at the Vienna Circle. This theorem proved that set theory could not be complete. This was a massive strike against Hilbert's Formalist program, which argued that all scientific knowledge could be axiomatically developed and formalized. That is, Hilbert's program emphasizing that particular mathematical results would be consistent was proven impossible.

The "misleading" Formalist ideas would become very relevant to economics, as one of the main authors they influenced was mathematician John von Neumann, whose logic was directly connected with Hilbert's Formalist program.

General Equilibrium and the Vienna Circle

Some authors hold that the influence of mathematics on economics became more intense because of the development of the general equilibrium theory, as suggest Ingrao and Israel (1990, p. x), who argue that "the problem of mathematicization is no secondary feature of general economic equilibrium theory but rather one of the basic reasons for its creation and development".

For Punzo (1991, pp. 1-2), the Marginal revolution in economics was not just the one that Jevons, Walras, Marshall, Menger, etc. brought about; it deepened since 1924 due to some economists that attended the Vienna Circle, a group whose most active members included philosophers Moritz Schlick (1882-1936), Rudolf Carnap (1891-1970) and Otto Neurath (1882-1945), who was also interested in social sciences. Discussions at the Vienesse "Mathematical Colloquium" also included economists Morgenstern (1902-1977) and Karl Schlesinger (1889-1938), mathematicians Menger (1902-1985), Abraham Wald (1902-1950) and Von Neumann (1903-1957); and logician Gödel (1906-1978). These scholars upheld the convenience of formalizing social sciences, as Ingrao and Israel (1990, p. 188) note. The Circle re-examined Walras's general equilibrium model. The matters that concerned the members of the Circle had to do with what should be understood as a state of equilibrium, the meaning of the existence of such a state, and how to prove its existence.

One of the meanings of equilibrium stemmed from classical mechanics and concerned two opposing forces. Another meaning came from the idea of the possible reconciliation of individual choices. The latter prevailed and stands even now in economics.

The importance of John Von Neumann for the general equilibrium theory

According to Ingrao and Israel (1990, pp. 184-186), Von Neumann became: "the ideal scientist to embody the new mathematical paradigm", culminating in the replacement of the old mechanicist determinism-based reductionism for another based on the idea of mathematical analogy, or mathematical centrality, wherein a purely hypothetical-deductive scheme prevailed, as noted earlier. The "new" mathematics that Von Neumann led was based on functional analysis techniques, measure theory, convex analysis, topology, and the use of the fixed-point theorem. The point of convergence between Von Neumann and the Vienna Circle is the proof of logical consistency of the general equilibrium model (Punzo, 1991, p. 9).

However, Von Neumann was not alone contributor to the modern general equilibrium theory. The contribution came from a group of mathematicians and economists, chief among which Wald, who was strongly influenced by the Vienna Circle's ideas.

G. Cassel was the first to attempt to solve the problem of the existence of equilibrium, posing two problems: 1) the primal problem, which concerns the constitution of supply and demand of goods; and 2) the dual problem, which concerns the price-cost relationship. Wald went a step further by assuming that all goods produced sought positive prices and stating that they would only be produced from the moment that equality between production cost and sale price was assured in each process. Wald understood that these two questions lay beyond the scope of formal proof. As a result, according to Punzo (1991), "only half of the leap necessary to go from the classical general equilibrium theory of the founders to its modern version was made by Wald". It was Von Neumann who made the model complete by introducing two rules: the competitive prices rule, and the efficient techniques choice rule. This was due to the fact that both the set of goods and the techniques used were to be endogenously determined. With Von Neumann's novel ideas, each instance of equilibrium could describe an economy. Therefore, the "one-to-one" correspondence between an economy and its model was lost and, as a result, equilibrium did not have to be realized in a single economy, but several of them. This new development led to the notion of various instances of equilibrium within the model, which, as a result, was born complete.

John Von Neumann presented the formal resolution for general equilibrium at a 1932 Princeton University seminar on his economic growth model. Publication regarding the model only came out in Germany, in 1937, titled: "Über ein Okönomishes Gleichungssystemund eine Berallgemeinerung des Brouweschen Fixpunksãtzes". The study contained the ideas that provided the basis for the formulation of modern general equilibrium theory. Weintraub (2002, pp. 95-96) regards the article as the most important in mathematical economics for four reasons, as it was the genesis of (1) the modern existence of proof in general equilibrium models; (2) linear programming and the duality gap system; (3) the turnpike theory, and (4) the fixed-point theory, which, according to Beaud and Dostaler (1997, p. 70), is connected to the minimax concept and involves the field of algebraic topology. In 1911, the mathematician Brouwer[4] made proof of the fixed point theorem, which is used in physics and was extended to economics by mathematician S. Kakutani in 1941, serving both game theory and the general equilibrium theory, and being instrumental for the proof of the existence of Debreu's equilibrium theorem in the 1950s. According to Weintraub (2002), Von Neumann's main contribution to economics, despite having been published in 1937, emerged from the discussion of Hilbert's Formalist program of the 1920s, and was only translated into English for the *Review of Economic Studies* in 1946-1947. For this reason, Von Neumann's main work, with its influence on economics, did not involve discussion of Gödel's Theorem of Incompleteness, as Von Neumann only came into contact with this theorem in the late 1930s, when he recognized the inconsistency of Hilbert's Formalist program.

According to Ingrao and Israel (1990) Morgenstern later added to Von Neumann's theoretical development. Morgenstern's 1928 doctoral thesis criticized economists for using primitive mathematical techniques and suggested applying game theory to social behavior. He emphasized that the complexity of possible interactions could only be examined by means of extensive use of mathematics and the necessary logical rigor, in addition to analysis by means of formal structures specially created to address the problem. These concerns of Morgenstern's led him, together with Von

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4 Prado (2007) is a Brazilian economist who discusses the irrelevance of the theorem, as Brouwer himself proclaimed.

Neumann, to attempt to create a new mathematical language to handle specific market situations. Although collaboration between the two began in 1939, it was only in 1944 that its results first emerged, with the publication of the book *Theory of Games and Economic Behavior*. Being allied with the development of game theory, Von Neumann and Morgenstern's book strictly axiomatized economics in an attempt to find the principles driving the rational behavior of participants in the economy. Even as they recognized that social phenomena were no less complex than physical ones, they demanded invention of new mathematical tools, as had been the case in physics. The book's first criticism of Walrasian equilibrium has to do with its limitations under market circumstances within the framework of utility maximization. The framework disregarded all intermediate situations between perfect competition and pure monopoly. The question was associated with the Walrasian method's inability to represent choices and behavior in a decentralized market where market forces were in operation. In such a market, the Walrasian model's failure was due to its inability to describe the influences of conscious agents' individual choices and behaviors on other agents' behaviors. In fact, agents only had this interaction in mind when attempting to maximize utility. As a result, the Walrasian theory of equilibrium's status was undermined, as it mirrored classical mechanics by limiting itself to a single observed moment of agent interaction. Hence the need for the game theory substitution Von Neumann and Morgenstern suggested, which would provide a solution to the problem. In this respect, by means of axiomatized mathematical analysis of possible game strategies and optimal results, the two authors attempted to describe a number of individual interaction processes (Ingrao and Israel, 1990, pp. 194-197).

Weintraub (2002), in his turn, argues that it makes little sense that Hilbert's Formalist program was incorporated into economics, as he sought for a proof of consistency of arithmetic, logic and set theory. It was therefore the non-Formalist mathematical program that influenced economics. This program lay with a school that was connected to others besides Hilbert. Still, Hilbert's Formalist program had the most influence on authors seeking for a proof of general equilibrium. Gödel showed that it was impossible to be certain of the fundamentals of knowledge based on logic or mathematics because set theory was incomplete. Even so, the theorem made room for what became known as relative certainty, given that it was proved possible to maintain "consistency relative to an extended set of postulates or axioms: if a proposition P was undecidable in system A, appending P to A (extending the axiom system) could assure P's truth" (Weintraub, 2002, p. 98). Such a P would be relatively true for any system, as the consistency of the framework to which the system belonged would be relative as well. The example two-person game theory were to be formalized, then, for the conclusions to be true, the assumptions would also have to be true. What was regarded as truth was associated with the fit between a known theoretical model and a physical model. Behind this concept lay the idea of a reductionist mechanics used to make rigorous scientific arguments developed by Volterra, Evans, Edgeworth and Pareto. On the other hand, Hilbert's axiomatic approach emerges in opposition to this image of mathematical rigor. This approach contributed to the formation of a new image of mathematics, one that led to the appearance of mathematical economics. "To preserve the relationship between rigor and truth, economists began to associate rigor with axiomatic development of economic theories, since axiomatization was seen as the path to discovery of new scientific truths" (Weintraub, 2002, pp. 97-98).

For now, we can see that the divergences regarding mathematical formalism derive from the divergences and signification changes in scientific knowledge had in the early 20th century.

Bourbaki and general equilibrium as meta-theory

To better understand this point, we borrow the definition of meta-theory as found in Punzo (1991, p. 3). For this author, meta-theory is better understood as a set of instructions for the selection of indefinite terms, how to combine these terms into properly elaborated formulas, and, finally, how to obtain true propositions by means of deductive reasoning in the form of theorems. With this definition in mind, Walras's equilibrium theory is not a meta-theory, as economics was still under the influence of classical mechanical intuitionism at the time. The theory of general economic equilibrium only became a meta-theory after changes that occurred in mathematics, whose influence became stronger since the Vienna Circle.

Beginning with the influence from Hilbert's axiomatic approach, which involved the analysis of formal systems, the so-called modern general equilibrium appeared in economics via the application of Wald's and Von Neumann's axiomatic method. This new general equilibrium, however, was developed at a metatheoretical level whose rules were justified and whose existential affirmations were established to validate the model and the endogenous variables' theoretical explanation. By introducing the idea that equilibrium would not occur in a single economy, and that multiple realizations were possible, Von Neumann further limited the room for the applicability of mathematics by means of formal rules. This idea, which in the modern general equilibrium theory uses rules and the realization of several equilibria, would become more complete with the influence of Bourbaki. After all, according to Punzo (1991, pp. 2-5), the new reductionism based on Hilbert's axiomatic method included a principle of hierarchical interdependence between several theories behind which lay the meta-theory's unifying singularity. The author argues that the scholars who presented Bourbaki seminars might be deemed followers of Hilbert, as they derived the conception of a set of theories that were added to the mathematical model and unified by certain construction principles. These principles may be considered to be meta-theoretical, as they defined models as logical structures. Such structures were to be understood in terms of their own logic, rather than reality. In the case of the general equilibrium, the models were unified by general laws.

Bourbaki was the collective pen-name of a group of mathematicians that tried, in the 1930s, to reintroduce rigor into calculus teaching in France, rewriting classical French mathematics treatises. In 1939, concerted efforts of their founders produced the "Theory of Sets", the first in a series of volumes that would for a major book project titled: "Elements of Mathematics", which showed a working plan and how Set Theory connected with other mathematical areas: general algebra, general topology, classical analysis, topological vector intervals, and integration. The central idea that pervades the work of the group of scholars and droves the production of books about these six areas of mathematics was, a priori, to have a general theory supporting the development of theories and proofs before moving on to application, that is, from the generic to the specific (Weintraub, 2002, pp. 104-107).

The Bourbaki group organized seminars to discuss the development of their ambitious project, and it was through these "Bourbaki seminars" that French mathematicians resumed relations with the mathematics community after World War II. With its great intellectual prowess, French mathematics became increasingly noticeable to US mathematicians. Bourbaki's ideas avoided the debate on formalism, idealism and anti-formalism, and moved in the direction of developing an axiomatic approach centered on the concept of structure, enabling mathematicians to develop theories based on certain standards acceptable under such a concept (Weintraub, 2002, p. 110). The group also argued that several higher-order structures had to be considered. Even so, when Bourbaki mentioned Gödel's theorem of incompleteness, it was in such a manner that, in Weintraub's opinion, the questions arising from that theorem were avoided rather than faced.

According to Weintraub (2002, pp. 112-113), Bourbaki adopted formalism to avoid philosophical difficulties. For the purposes of our historical analysis, then, how are Bourbaki and economics connected? The link was Debreu, one of those responsible for the creation of a pure theory of economics. Before World War II, Debreu was preparing to get his bachelor's degree in physics and mathematics. During the war, he entered the "École Normale Supérieure", where, under the influence of Bourbaki member Henri Cartan, he developed a sound mathematical background.

When Debreu first got in touch with economics textbooks, he was disappointed in arguments that he regarded as somewhat "loose". He later had access to other economics papers by which he was influenced, in particular *A la Recherche d'une Discipline Économique*, by future Nobel laureate in economics Maurice Allais. His contact with this work took place by coincidence in 1943, when it was still circulating as a draft. Years later, other economics works would also impact his training, such as John Von Neumann's "A Model of General Economic Growth" (1937) and "Theory of Games and Economic Behavior" (1944), the latter of which, as noted earlier, with Oskar Morgenstern as a co-author. Soon after World War II, many French mathematicians migrated to the US, including Debreu, who was allocated at the University of Chicago (with the Cowles Commission). These mathematicians, however, had little knowledge of economics, and, being introduced to economic research, tended to develop it as they would in mathematics, sidestepping the empirical aspect (Weintraub, 2002, pp. 122-123).

At the University of Chicago, via the Cowles Commission, where he became a permanent member in 1950, Debreu was able to spread Bourbakist thinking, which became prevalent in Chicago and would greatly influence Tjalling Koopmans, the Commission's intellectual leader and who, together with Debreu, became an advocate of the new approach. Later, in 1955, the Cowles Commission moved to Yale, which had the biggest economics department in the US at the time. This helped disseminate Bourbaki's ideas to other economics departments in the United States and worldwide. As Weintraub (2002, pp. 118-121) points out, in "Theory of Value", one of Debreu's main works, he embraced a method analogous with the Bourbaki theory of sets. The monograph attempted to establish an analytical mother structure from which all economics works should depart: general equilibrium. Beginning with Debreu's theoretical development, the model's purpose became identifying the essence of the equilibrium system. Debreu's initial concern, like Bourbaki's, was to justify the initial identification of structures.

The problem that these ideas generated was that such a meta-theoretical mechanism might, at best, imitate a computing algorithm: like a calculating device, where the model in which the algorithm is embedded is incomplete, and the determination of values is logically-deductively designated, dispensing with the need for any correspondence with reality.

Despite the many problems with this approach, Beaud and Dostaler (1997, pp. 71-72) understand that Arrow and Debreu proved that competitive equilibrium existed, given the restrictive assumption that every individual initially had some positive quantity of all goods available for sale. This became known as the theorem on the existence of competitive equilibrium. In spite of the very strong assumption, they understood that the competitive model is a reasonable description of reality. However, neither the stability nor the uniqueness of equilibrium have been independently proven, and even the proof of existence is challenged, as we will see ahead, with Weintraub (2002). Debreu himself noted that demonstrating the uniqueness and stability of general equilibrium requires assumptions that are far too restrictive. In addition, the existence of general equilibrium that Debreu supposedly proved involves a theory that excludes money and uncertainty, both of which are crucial to a market economy.

Arrow and Debreu's "inexorable" proof of the existence of general equilibrium

According to Weintraub (2002, pp. 184-186), since the 1954 publication Arrow and Debreu's article "On the Existence of an Equilibrium for a Competitive Economy", the idea of general equilibrium increasingly became part of the accepted corpus of knowledge in economics, practically exempted from proof, a proof unto itself. We ask whether acceptance of the article in Academia was immediate, or took place over a period of years. The route Weintraub chose to answer this was to observe how undergraduate and graduate micro-economics textbooks incorporated the results, as well as to attempt to show how Arrow and Debreu proceeded to get the article published in *Econometrica*. Weintraub's discussion begins in the 1940s, prior to publication of Arrow and Debreu's proof of equilibrium, and points out that economists at the time, such as his father, Sidney Weintraub, noted the importance of analyzing the tendency to equilibrium in specific markets. But they were reluctant to agree with the view that all markets must be in equilibrium at the same time. Admitting the tendency to equilibrium in a specific market lies far distant from endorsing the idea that all markets may be in equilibrium simultaneously.

Before 1930, proof of the existence of general equilibrium was based on the idea that a number of equations existed equal to the number of unknowns. Resolving this was regarded as a problem more for mathematics and economics, and dated back to Walras's time. This occurred frequently in textbooks and even in Hicks's "Value and Capital". Although few economists in the 1930s understood that mathematical proof of the existence of general equilibrium was a difficult problem to solve, and despite even awareness of the works of Wald and Von Neumann, it appears, according to Weintraub, that those analyses have not "crossed over, as it were, into mainstream economics", being relegated to the "background of mathematical economics". In this context, Weintraub's main concern was to show the transition involved in moving past the understanding that the proof of equilibrium consisted in matching the number of equations and unknowns, which a larger community regarded as correct, and replacing it with Arrow and Debreu's proof of existence based on a fixed-point theorem. In this respect, Weintraub mentions another important paper from the period, George Stigler's "The Theory of Price" (1946). In it, Stigler briefly touches upon general equilibrium in an introductory sub-section. According to him, the most that can be said of general equilibrium is that it is more inclusive than partial equilibrium, but is never complete. On the other hand, in the 1952 revised edition of the same paper, Stigler adds a final chapter on general equilibrium, mentioning that advances had been made in this sense in the immediately preceding years (Weintraub, 2002, p. 187).

This shows that, although Arrow and Debreu's article was published at a later time (1954), the influence of modern general equilibrium on economics was rising. So much so that, in 1958, the first edition of James Henderson and Richard Quandt's textbook "Microeconomic theory: A Mathematical approach" contained a sub-section (titled multimarket equilibrium) that emphasized the "new proof" of equilibrium in little detail. These changes in microeconomic theory learning were progressive and, as Weintraub (2002, p. 189) remarks: "validity of their proof had gained widespread acceptance within the community of economists, although the details were not presented to students in their microeconomics textbooks". In the preface to the second (1971) edition, Henderson and Quandt pointed out the proof's difficulty and presented Brouwer's fixed-point theorem instead of Arrow and Debreu's proof. This theorem, however, only proves the existence of equilibrium for a restricted case. For the general case, they only offered a rough sketch of Debreu's proof via Kakutani's fixed point theorem in "Theory of Value" (1959).

Therefore, starting in 1958, Henderson and Quandt started to teach their economics students that Arrow and Debreu had proved the existence of general competitive equilibrium. And a short while after the publication of Arrow and Debreu's article in *Econometrica*, validity of this proof had become accepted as truth in economics academia, even though, as Weintraub notes, the details were not presented to students. If economists are in agreement to assume that graduate textbooks reflect the consensus about a subject's contents according to a prevalent paradigm, then one might say that Arrow and Debreu's proof was accepted a few years after its publication, as in 1958 it was already part of one of the main microeconomic theory textbooks used by Doctor of Economic candidates in the US.

Although Arrow and Debreu's proof of equilibrium was, since its publication in *Econometrica*, taken by many authors as a demonstrated truth, Weintraub shows how the result was achieved without initially enjoying unanimous acceptance. Indeed, what Weintraub questions is the meaning of proof according to different traditions, thereby showing that not everyone immediately accepted Arrow and Debreu's demonstration. In this sense, Weintraub brings up the two reviews *Econometrica* requested on the paper, by Professor William Baumol, of the Economics department of Princeton University, and Cecil Glenn Phipps, from the University of Florida's mathematics department. The former approved publication of the article, but the latter did not. Baumol approved the article with minor suggestions, and understood that it was a good article to publish. Phipps completely rejected the article, questioning the proof of the axioms in it. Despite Phipps opposing review, the article was published because *Econometrica* associate editor Nicholas Georgescu-Roegen believed that one favorable review was sufficient, in the light of Debreu's and Arrow's academic prowess. So, therefore, confirmation of the accuracy of the paper's mathematics could be provided at a later time. When the article was approved, Phipps sent a critical letter to *Econometrica* with a request for publication. In order to determine whether or not to publish the letter, the review requested the opinions of authors Arrow and Debreu, as well as those of other mathematical economists: Ragnar Frisch (editor at *Econometrica*), Lionel Mckenzie, Hukukane, Nikaido, Koopmans and Georgescu-Roegen. Their attitudes ranged from frontal opposition against and disqualification of Phipps's letter to a more sympathetic view. The article was finally published without change and Phipps's letter to the editors never became public (Weintraub, 2002, pp. 195-207).

In addition to the rhetorical reasons for the dissemination of the Hilbertian/Bourbakist ideas via the modern theory of general equilibrium, the process of mathematicization of economic discourse had a political and ideological boost. In 1957, an unusual event to place for the US: the Soviet Union managed to place a satellite, the Sputnik, into orbit, while the Americans failed in their attempts. This led to public disquiet as it showed that the US were lagging in technology behind the former USSR as concerns engineering and missile launching (Weintraub, 2002, p. 246), and regarded also as a scientific and technological lag of potentially devastating consequences. In order to overcome this technology gap, the US government reevaluated its education system, adding emphasis to mathematics and engineering.

The mathematics that US schools emphasized, however, was not the applied kind, but the mathematics associated with "elegant proof", emphasizing the most abstract of theorems. Weintraub, however, saw little future for himself in the 1960s' mathematics, which followed Bourbakist ideals in the US and had little application. He then decided to pursue application of the mathematical tool kit in economics. Weintraub notes that economics underwent a split from 1930 to 1950. This led the science to divide between mathematical and non-mathematical economists. The former were misleadingly termed Formalist economists. For Weintraub, this was because the aspect of mathematics that was used in economics was not the side of mathematical proof (Formalist) but the non-axiomatic side of Hilbert's program (Weintraub, 2002, p. 255). This claim from Weintraub may appear a little contradictory, but is actually not. The author is trying to show that doubt exists surrounding Debreu's proof of existence of general equilibrium. As a result, not even general equilibrium, which is the main guide for the production of models in neo-classical economics, could be proved with certainty. Furthermore, problems created by quantum physics, which proved that energy is not processed continuously, but in leaps, ended up raising questions about the validity of differential and integral calculus, which require continuous functions. With all of this, and in the absence of Formalist proof, Weintraub maintains that whet ended up prevailing in microeconomics was a non-Formalist mathematical influence. We differ from this author because microeconomics would be more mathematicized according to the economic definition of mathematical formalization given in the beginning of this work, not according to a definition from frontier mathematics or physics themselves.

CLOSING REMARKS

This essay attempts to review some of the main influences mathematics and physics have had in economic discourse in the 20th century. To this end, we reviewed references from some of the main history of economic thinking authors who cover this subject.

In Mirowski (1991), we find that two moments of rupture exist in economic discourse; one in 1870-1887, with the so-called "Marginal revolution"; another in 1925-1936. The former rupture, marking the beginning of the Marginalist approach in economics, was unable to drive economist discourse toward mathematicized language, although this was one of the objectives for the stream's founders, like JeVons, Walras and Edgeworth; according to Mirowski's research, the rupture could not bring mathematicized articles to answer for a substantive share of articles in the period's main periodicals. The latter rupture, on the other hand, is more significant because it shows a qualitative and quantitative change in economic discourse. The change is qualitative, as the marginalists lacked the mathematical tools to prove the existence of general equilibrium. These only became available after the "revolution" mathematical physics experienced in the early 20th century. This revolution changed the meaning of rigor. Previously, a theory had to correspond to a physical model in order to be rigorous. Afterward, the conception of rigor shifts into the domain of logical coherence. This enabled theory production to become unlimited in physics and mathematics. The same occurred in economics, as theoretical production whose criterion of truth is correspondence with reality limits the production of models, and hypothetical-deductive coherence does not. Therefore, with the revolution in mathematical physics and its influence on economics, a change takes place that is both qualitative, since the focus of rigor shifts form observation to logical coherence, and quantitative, as the construction possibilities for the existence of equilibrium may occur in more than one way.

Mirowski's (1991) text, however, does not explain these ruptures. It is authors like Ingrao and Israel (1990), Punzo (1991) and Weintraub (2002) who review the history of economic thinking in search of answers to explain the mathematicization of economic discourse. We understand that this process took place with the increasing use of the hypothetical-deductive method in economic discourse, as Bresser-Pereira (2009) defines it.

Ingrao and Israel, Punzo, and Weintraub all agree that one must cover the history of physics and mathematics in order to understand the changes that economic discourse experienced in the 20th century, as they argue that those sciences are crucially influential over economics. For this reason, their texts begin by narrating the crisis that mathematical physics faced in the early 1900s. They stress the transformations caused chiefly by quantum physics, and through non-Euclidean geometry in mathematics, that changed the previous, observation-based criterion for rigor and started to rely on mathematical analysis for hypothetical-deductive theory development. The applicability of math was previously limited by the need for correspondence with reality, but, since those changes, it becomes unlimited with only formal proof as criterion for rigor. Meanwhile, the stream that relied on the fundamental physical model did not completely disappear, and was represented in the Vienna Circle in the 1920s and 1930s.

The influence that these two streams, which authors call anti-Formalist and Formalist, had on economic discourse is intrinsically connected with the discussion over general equilibrium. The authors agree that today's general equilibrium differs from that of Walras and Cassel. Walras's general equilibrium was an equilibrium associated with classical mathematical physics thinking, which had observation as basis for a rigorous general model. On the other hand, for Ingrao and Israel and for Punzo, the modern general equilibrium coming from Wald, Von Neumann and Arrow-Debreu, all of whom Hilbert influenced, has both the Formalist aspects of modern mathematics and the anti-Formalist ones of classical mechanics. For Weintraub, in his turn, even though he does consider the strong influence of the Formalist mathematic stream on economics after the Vienna Circle, the anti-Formalist aspects prevailed in economic discourse.

On the other hand, for Ingrao and Israel, after the paradigm shift within the general equilibrium theory, formal development and the interpretative paradigm travelled on different paths. The theory needed for the marketplace's professional economists, such as a heuristic model of competition, broke away from the development of the mathematical approach. "The two separate lives did, of course, meet and intermingle during certain periods of fruitful exchange, but long stretches of their histories ran on separate lines" (1990, p. 175).

By means of what has been stated in this paper, I believe that I got to show the connection between change in the forms of rigor of mathematical physics and the adoption of the hypothetical-deductive method by economics. Importance of history for establishing the link is critical for the mainstream understanding of the economy today. This mainstream uses highly abstract theories to prescribe economic policies, which creates discrepancies between their predictions and what actually happens. This is because in economics, a social science, experiments are not controlled. Although many experimental economists think that there is possibility of control experiments, conducting controlled experiments is only possible within laboratories for which physics, chemistry are better suited sciences. Economics is a social science whose most appropriate method is the historical-deductive where you can work with open models and rationality whose transitivity and completeness of preferences does not constitute the best prerequisites for companies practice their pricing policy today. Rather, the marketing departments of companies emphasize in their advertising campaigns rationally consumer emotion. One of the common asked questions in economics is why economists are wrong? An answer that we can conclude is that because they use hypothetical-deductive models in their predictions. These models do not have great predictive power because they are laboratory models and not reproduce the actual economic relations in society. Economics is different from mathematical physics, such as former Brazilian Minister of Economy Delfim Netto says, "in economics atoms think". The hypothetical-deductive models are closed and must typically be applied to methodological sciences. In economics, the predictive models should be open and use the historical-deductive method described by Bresser-Pereira (2009).

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