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REVISTA BRASILEIRA DE ECONOMIA

DOI 10.5935/0034-7140.20230015
ISSN 1806-9134 (online)

FGV
FUNDAÇÃO GETULIO VARGAS

A note on the trade-off between efficiency and equity in public financing of higher education*

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Contents

Keywords
higher education, free public education, public policy

JEL Codes
I22, I24

Abstract · Resumo

We build a model that captures the main elements of the public higher education system to evaluate the performance of different financing mechanisms in terms of efficiency and equity. Our main finding is that the provision of direct places in public colleges and universities raises a trade-off between efficiency and equity whenever part of the education cost is financed through taxes. Alternative mechanisms, namely public provision with tuition fees covering totally or partially the educational cost, are also analyzed. We show that, compared to these alternatives, the policy of "free" higher education performs worse in terms of the trade-off between efficiency and equity. The less the taxpayer subsidizes students the lower the trade-off.

1. Introduction

In many countries, like Brazil, public colleges and universities are completely financed through general tax revenue. Such policies have important distributive effect as the resources of the entire society are transferred to a limited group of

*We are grateful for the comments and suggestions of Guilherme Stein, Ronald Hillbrecht, the editor and an anonymous referee. All the remaining errors are ours.

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[‡]This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

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[¶]This research was supported in part by The Brazilian National Council for Scientific and Technological Development (CNPq) under grant number 305358/2020-0.

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Received on April, 2022
Final Acceptance on March, 2023

individuals. The huge budget of those institutions suggests that, when there is inefficiency, the magnitude of the social welfare loss may be enormous.[1] Even within higher education policies, however, the specific design of the financing mechanisms might lead to significantly different results in terms of efficiency, opportunities, and income distribution.

In this paper, we use a simple theoretical model to analyze the levels of efficiency and equity resulting from different financing mechanisms for higher education. In particular, we are interested in studying the effects of the policy of "free" public colleges or universities, in which the government provides a certain number of direct spots in public higher education institutions but does not charge the students any tuition fee, covering the costs of such institutions completely through taxes. Alternative mechanisms, namely public provision with tuition fees covering totally or partially the educational cost, are also analyzed. We show that, compared to these alternatives, the policy of "free" higher education performs worse in terms of the trade-off between efficiency and equity.

Our main finding is that the public provision of places in colleges and universities raises a trade-off between efficiency and equity whenever part of the total education cost is financed through general taxes. The reason is that students receive a net subsidy when tax revenue is used to finance the public higher education policy. This subsidy works as a further incentive for young adults to choose higher education. As a consequence, the number of places in colleges and universities (the number of skilled workers) that clears the labor market is higher than the one that maximizes social welfare. Accordingly, any policy that decreases the amount transferred from taxpayers to students can reduce the trade-off between efficiency and equity. In particular, if the government provides places in public institutions but charges a percentage of the education cost for it, there will be an improvement compared with the "free" education policy.

Our theoretical contribution is related to the work of Azevedo and Salgado (2012), which also tackle the free public higher education system in Brazil. Their model, like ours, assumes that credit markets are imperfect. The authors show that free public universities distorts the decisions of the richer households, who could already pay for their (private) higher education. As a consequence, free universities do not solve the problem of lack of access by poor households

[1] According to data from Portal da Transparência, the total budget of the Federal University of Rio Grande do Sul (UFRGS) in 2016 was R$1.7 billion. In the same year, the total tax revenue of Caxias do Sul, the second-largest city in the Rio Grande do Sul state, with population over 500,000, was R$1.3 billion. Another example is the São Paulo University (USP), whose total budget in 2016 was 63% of the total budget of Curitiba, the fifth richest (in terms of GDP) Brazilian city.

to higher education. Our framework is, however, closer to García-Peñalosa and Wälde (2000), who develop a model that accounts for different financing mechanisms for higher education and compare the outcomes in terms of human capital efficiency levels and lifetime income inequality. We expand on their work by modeling the case where the government acts by offering a limited amount of places at public universities for free—instead of just subsiding education.

2. The environment

Consider an economy that lasts for two periods. There are two groups of individuals at the beginning of the first period, namely the *previous generation* and the *young adults*. Their population size is fixed and denoted by $\bar{N}$ for the previous generation and by N for the young adults. The previous generation lives only in the first period, in which its entire population works. Young adults, on the other hand, live in both periods and are ex-ante homogeneous.

At the beginning of the first period, each young adult must choose between pursuing higher education or starting to work immediately. By choosing to study, he dedicates the first period to college or university activities and will only be able to work in the second period, when he will then become a skilled worker. On the other hand, if the choice is not to enter into higher education in the first period, then the individual works both periods as an unskilled worker. Let H be the number of young adults that choose to study and become skilled workers, and L be the number of those who become unskilled workers. Each individual who enters into higher education incurs a fixed educational cost $E > 0$. We assume that the previous generation population $\bar{N}$ is exogenously divided into skilled and unskilled workers, given by $\bar{H}$ and $\bar{L}$, respectively.

Only one good is produced in the economy in each period $t = 1, 2$, which is denoted by Y_t. The aggregate static production function requires as inputs both skilled and unskilled labor, $Y_t = F(H_t, L_t)$. The production function $F(\cdot)$ is twice continuously differentiable, increasing and concave in each type of labor. Both inputs are essential, that is, $F(0, L_t) = F(H_t, 0) = 0$.

Let us suppose that labor markets in this economy are competitive, such that wages of both skilled and unskilled workers are equal to their respective marginal productivity. Let $w_t^H(H_t, L_t)$ and $w_t^L(H_t, L_t)$ denote such wages for skilled and unskilled workers, respectively. Given the production function characteristics, we have that the wage of a given type of worker is decreasing in the number of workers of his own type and increasing in the number of workers of the other type. Note that this happens because there is a fixed population ($L_t + H_t = N$).

One can now define the social welfare function (SW) of this economy as the present value of the sum of all productions net of the total spending in higher education (which occurs in the first period). Formally, we have

$$SW = F(H_1, L_1) + \delta F(H_2, L_2) - EH, \tag{1}$$

where $\delta \in (0,1)$ is the intertemporal discount factor that governs both the workers and the government's preferences. There is a financial sector in the economy capable of transferring goods from period $t = 1$ to $t = 2$ with a return rate of r. We henceforth assume that $\delta = 1/(1+r)$.

Definition 1. *A financing mechanism is efficient if the resulting allocation of young adults between skilled and unskilled workers maximizes SW. In this case, we call it the optimal allocation.*

When there is no efficiency, the higher the distance between the resulting allocation and the optimal one, the larger the inefficiency of the mechanism.

Definition 2. *A financing mechanism satisfies the criterion of ex-post equality when the lifetime income flows of both groups of young adults are equal.*

The lifetime income flow of an individual is the present value of the sum of his wages net of education costs, that is, $w_1^j(H_1, L_1) + \delta w_2^j(H_2, L_2) - EI_j$, for $j = H, L$, where $I_j = 1$ if $j = H$ and $I_j = 0$ otherwise. Implicitly, we are assuming linear additive discounted preferences over the net availability of the consumption good. When higher education is partially or entirely provided by the government, individuals also pay taxes in the second period.

2.1 The baseline case

We assume that government intervention in higher education is necessary due to imperfections in the capital market. Individuals cannot borrow money to finance their studies because the future return of the acquired human capital is not seen as solid collateral by potential creditors. Thus, banks and other financial institutions do not provide credit for young adults to pay the cost of their education in a private institution in the first period. This market failure creates the opportunity for public financing, which may mitigate the inefficiency caused by such a market imperfection.[2]

[2] In this note, we focus on the case of government provision of places in public colleges and universities. An alternative public policy would be for the government to act as an intermediate between students and the financial sector and loan money to young adults. It is straightforward to show that, in this case, efficiency and equity would be achieved simultaneously. The working paper version, which is available upon request, explores this alternative in detail.

Let us start, however, by assuming that capital market is perfect. In this case, all young adults who want to borrow the amount E to pay for their education can do so. Young adults choose whether to enter into higher education or to work in the first period by comparing the lifetime income flow resulting from each option. Let W^L and W^H represent the lifetime income flow of unskilled and skilled workers, respectively. With perfect capital markets, students borrow E in period $t = 1$ and then repay the amount $(1 + r)E$ in $t = 2$. Hence, the discounted income flow of both groups is given by

$$W^L = w_1^L\left(\overline{H}, \bar{L} + N - H\right) + \delta w_2^L\left(H, N - H\right); \tag{2}$$

$$W^H = \delta w_2^H\left(H, N - H\right) - E. \tag{3}$$

The equilibrium condition for the labor market is $W^L = W^H$. If, for example, the allocation of young adults between higher education (skilled jobs) and the labor market (unskilled jobs) in the first period is such that $W^L > W^H$, there will be a relative scarcity of unskilled workers, which makes their wages higher and the wages of the skilled ones lower. This, in turn, encourages some of the individuals who had decided to enter into higher education to start to work immediately in the first period.

From the above discussion, we can conclude that the existence of a perfect capital market guarantees ex-post equality. Thus, $W^L = W^H$ implies

$$\delta w_2^H\left(H, N - H\right) - w_1^L\left(\overline{H}, \bar{L} + N - H\right) - \delta w_2^L\left(H, N - H\right) = E. \tag{4}$$

Let us now analyze the efficiency of this economy when there is no market failure. Given the Definition 1, the optimal (efficient) allocation (H, L) of young adults solves the following problem:

$$\max_{H}\; F\left(\overline{H}, \bar{L} + N - H\right) + \delta F\left(H, N - H\right) - EH. \tag{5}$$

By obtaining the first order condition and using the equivalence of wages and the production function derivatives, we get the same expression as (4) as the condition which defines the efficient number of young adults in higher education H^* —and thus $L^* = N - H^*$, the optimal number of unskilled workers. Therefore, when the capital market is perfect, it is possible to achieve equality in terms of lifetime income flow and efficiency (the maximum social welfare) at the same time.

3. Public provision of higher education

From now on, we assume that the capital market is imperfect. Since young adults cannot work and study simultaneously, an imperfect capital market implies that no individual can pursue higher education. In this context, there is room for a government intervention that allows young adults to get into a college or university if they want to do so.

We model the government intervention as follows. First, the only government action is to provide opportunities for higher education. Second, the government intertemporal budget is balanced, that is, the total tax revenue is equal to the cost of the policy (both in present values). For the sake of simplicity, we assume that taxes are lump-sum. Third, although young adults cannot borrow money in the capital market, the government can. One can justify such an assumption by observing that the government's power to tax is often seen as solid collateral. Thus, the public policy is implemented in the first period and is financed through loans; in the second period, the government pays back the loans by taxing young adults. Observe that the taxation must take place in the second period because the share of young adults that choose to enter into higher education has no income while they are studying.

3.1 Free public higher education

Let us assume that the government provides, free of any charge, a fixed number K of places in public higher education institutions. In this type of financing mechanism, some colleges and universities are publicly owned. The number of places available for young adults in those institutions is chosen by the government. Students of public institutions do not pay any kind of tuition fee, such that there is no direct cost to be part of one of them. Such a policy is financed by taxes, which are paid by all citizens, including those who do not study in public universities and colleges. It is worthwhile noting that, whenever the government sets $K < N$, it is necessary to implement a selection process to determine those who will have access to higher education among the young adults who are interested in studying.[3]

To focus our analysis on the effects of different financing mechanisms, we assume that the education cost per student in a public higher education institution is the same as in a private one, namely E.[4] Therefore, the government

[3] In this simple framework, we can assume that a lottery is conducted, so those who get the chance to study are selected based purely on luck. In practice, however, selection processes—which in Brazil are called *Vestibular*—are usually based on test performances, which depend on skills and previous investments that might be correlated with family background. In that sense, the limited provision of places in public universities can give rise to an additional problem of ex-ante inequality, that is, the opportunity of enjoying the benefits of public policy is not accessible to everyone. Rather, it depends on previous heterogeneous characteristics.

[4] This assumption allows us to focus on the effects of different financing mechanisms, but it may not fit well some developing countries. In some cases, professors and staff of public colleges and universities are civil servants, receiving wages and benefits higher than the employees of the private sector. This makes public higher education more expensive than private in several countries.

incurs the total cost of EK to provide K places. This amount is borrowed from banks in the first period, creating a debt of $(1+r)EK$ to be paid back in the second period. The intertemporal budget constraint implies that the total tax revenue must be equal to this amount, which is equally divided among all the young adults in the second period. Thus, each individual pays to the government the amount $(1+r)EK/N$, whose present value is

$$T = \delta(1+r)\frac{EK}{N} = \frac{EK}{N}. \tag{6}$$

One can observe in (6) that, whenever the government sets $K < N$, we have $T < E$. In other words, if the number of places available in public institutions is lower than the entire population—and, therefore, some individuals may not have access to higher education—, the tax that each young adult pays is lower than the individual education cost. The reason is that, although not more than $K < N$ individuals will enter into a public college or university, all the young adults are taxed to finance it.

3.1.1 Economic efficiency

Let us start by supposing that the government wants to achieve maximum social welfare. Let K^A be the number of places provided when such an objective is achieved. Notice that the presence of the government changes the social welfare function (1): individuals do not pay the individual education cost, such that society no longer spends EH; however, now there is the tax T, creating the total cost of NT for society. Therefore, the new social welfare function is given by

$$SW = F(H_1, L_1) + \delta F(H_2, L_2) - EK, \tag{7}$$

where we use (6) to substitute T.

In its aim of achieving economic efficiency, the government must offer the number of places that solve the problem of maximizing the welfare function, given the size restriction of the young adult population. There is, however, an additional constraint: since each new place offered in a public higher education institution implies an increase in the total cost of the policy and, consequently, of the amount paid in taxes by society, the existence of unoccupied places is a waste of resources and a penalty on welfare. Therefore, any efficient allocation in this context occurs only if all available places are occupied, that is, if $H = K$. Hence, the problem to be taken into account by the government can be formally presented as follows:

$$\max_{K} \; F\left(\bar{H}, \bar{L} + N - K\right) + \delta F(K, N-K) - EK. \tag{8}$$

The problem (8) is equivalent to the baseline case's. Therefore, the resulting FOC for K will be the same as (4) was for H, and thus the efficient allocation under the present financing mechanism is equal to the one in the case of perfect capital market and private institutions, that is, $K^A = H^*$.

3.1.2 Ex-post equality

Assume now that the government wants to achieve equality of outcomes. In this case, it chooses K^B to equalize the lifetime income flows of both skilled workers and unskilled ones. An individual who chooses to join the labor force immediately now must pay taxes in the second period, which are used to finance public higher education. Therefore, the lifetime income flow of an unskilled worker is

$$W^L = w_1^L\left(\bar{H}, \bar{L} + N - H\right) + \delta w_2^L\left(H, N - H\right) - T. \tag{9}$$

On the other hand, an individual who proceeds to higher education, instead of paying the individual education cost E, must now pay the tax T, like all young adults. Formally, the lifetime income flow of a skilled worker (or student) is

$$W^H = \delta w_2^H\left(H, N - H\right) - T. \tag{10}$$

By (6), once the number of places K is fixed, T becomes a constant that satisfies $0 \leq T \leq E$. Therefore, the functions that represent the lifetime income flows in the benchmark case are shifted by T. In the case of the students, there is an upward shift in the function W^H because now they disburse less than before ($T \leq E$). The opposite happens in the case of unskilled workers: the function W^L suffers a downward shift, given that now they have to disburse the amount T.

If the government succeeds in achieving ex-post equality, the number of places provided must be such that $W^H = W^L$, that is,

$$\delta w_2^H\left(H, N - H\right) = w_1^L\left(\bar{H}, \bar{L} + N - H\right) + \delta w_2^L\left(H, N - H\right). \tag{11}$$

If the government chooses K^B equal to the value of H that satisfies equation (11), all provided places will be filled and there will be equality of lifetime income flows. To prove the first claim, suppose that K^B satisfies (11). Then if there are places that are not filled ($H < K^B$), there will be a relative scarcity of skilled labor, which makes w_2^H higher, resulting in $W^H > W^L$. This will encourage more young adults to enter higher education and then all places will be filled.

Observe that equations (4) and (11) are different. We no longer have a policy that satisfies simultaneously the criterion of efficiency and ex-post equality. This creates a standard trade-off between efficiency and ex-post equity. To see this, suppose that the government chooses the efficient number of places K^A and that all places are filled ($H = K_A$). Then, the lifetime income flows are such that $W^H - W^L = E > 0$, where we use (4). Furthermore, one can conclude that $K^A < K^B$. The inefficiency arises from the fact that the total cost of education EK^B is shared among all young adults, including those who do not study, which makes those who do study receive a net subsidy (recall that $T < E$).

The difference in the lifetime income flows expressed by $W^H - W^L$ also allows us to conclude that, when K^A is provided instead of K^B, some young adults want to study but are not able to do so. In fact, whenever one group's lifetime income flow is higher than the other, there will be individuals willing to migrate to the group with a higher wage. However, it may be the case that the government sets a fixed number of places which is not enough to guarantee the labor market equilibrium. This is what happens when the number of places is K^A: we have $W^H > W^L$, such that some individuals want to enter into higher education, but there are no places available.

3.2 Public higher education with tuition fees

Suppose now that students pay a tuition fee to the college or university. This payment is made only in the second period as students only earn their skilled job wage after graduation. We assume that the amount to be paid is a share (a percentage) of the total individual cost E. Thus, public higher education institutions are no longer free, but, instead, become partially subsidized: students pay for their studies, yet the tuition fee is not enough to cover the total education cost. The difference between what is paid by the student and his educational cost is covered by taxes, collected from all young adults as described in the previous section.

Let $0 < \tau < 1$ be the percentage of the individual educational cost E paid as tuition fee. Observe that $\tau = 0$ corresponds to the case of "free" education analyzed in the previous section and $\tau = 1$ to the case in which students pay the whole cost of their education, such that there is no subsidy. Now, each one of the H individuals who choose to enter into higher education must pay τE. The total revenue from tuition fees is, therefore, τEH ($\delta\tau EH$ in present value).

Given the intertemporal budget constraint of the government, the following must be satisfied:

$$N\widehat{T} + \delta\tau EH = \delta(1 + r)EK,$$

where $\widehat{T}$ is the individual tax (in present value) under this financing mechanism. We can rewrite the above expression to express the present value collected from each young adult:

$$\widehat{T} = \frac{EK}{N} - \delta\tau\frac{EH}{N}. \tag{12}$$

Notice that whenever there is at least one young adult in college or university ($H > 0$), the second term on the right-hand side (RHS) is negative.

As a consequence, we can compare (6) with (12) and conclude that $\widehat{T} < T$. This conclusion is quite intuitive, given that when part of the educational cost is financed through tuition fees charged directly from students, taxes must be lower to cover the other part. In particular, those individuals who do not benefit from this educational policy are now better off as they pay less in taxes.

Those young adults who choose to enter into higher education, nevertheless, continue to benefit from public subsidies. To see this, let us compare the individual education cost with the total disbursed by them (both in present value): $E - \left(\widehat{T} + \delta\tau E\right) = (E/N)\left[N - K - \delta\tau(N - H)\right]$. When all the places in public higher education are filled ($K = H$), RHS of this expression becomes $(E/N)(N - K)(1 - \delta\tau)$. Finally, as $\delta, \tau \in (0,1)$, we have $\delta\tau < 1$, and thus $E - \left(\widehat{T} + \delta\tau E\right) > 0$.

Under this new financing mechanism, social welfare is given by

$$\begin{aligned} SW =& F(H_1, L_1) + \delta F(H_2, L_2) - N\widehat{T} - \delta\tau EH \\ =& F(H_1, L_1) + \delta F(H_2, L_2) - EK, \end{aligned} \tag{13}$$

where we use the value of $\widehat{T}$, given by (12). It is straightforward to see that expression (13) and (7) are identical. As a consequence, the efficiency condition determines, once again, that the optimal allocation is $K^A = H^*$.

Let us verify whether this policy satisfies equality in terms of outcomes. Observe that W^L now is similar to the one of the previous section, given that an unskilled worker earns w_t^L in each period t and pays taxes in the second one. However, taxes are lower under this financing mechanism:

$$W^L = w_1^L\left(\overline{H}, \overline{L} + N - H\right) + \delta w_2^L(H, N - H) - \frac{E}{N}(K - \delta\tau H), \tag{14}$$

where we use (12) to substitute $\widehat{T}$ again.

For a given K, W^L continues to be an increasing function of H. However, the third term on the RHS of (14) is no longer constant as in (9): it is a linear and increasing function of H, which makes the shift in the curve W^L no longer parallel. Notice that, when $H = 0$, (14) is equal to (9), and so both curves approach each other for small H. On the other hand, as H increases, the last term in (14) increases as well, partially offsetting the downward shift resulting from the presence of $\widehat{T}$.

Students continue to earn wages only in the second period, but now they have to pay taxes as well as tuition fees. Thus, their lifetime income flow is

$$\begin{aligned} W^H =& \delta w_2^H(H, N - H) - \widehat{T} - \delta\tau E \\ =& \delta w_2^H(H, N - H) - T + \delta\tau E\left(\frac{H}{N} - 1\right), \end{aligned} \tag{15}$$

where T is given by (6).

It is straightforward to see that (15) is equal to (3), except for the presence of the third term on the RHS. This difference implies that now there are two opposite effects on the lifetime income flow when H increases: on the one hand, the higher the number of skilled workers the lower their wage; on the other hand, for a given K, more students lead to a higher amount collected from tuition fees, which makes taxes less necessary. In the extreme case of $H = N$, we have the same function as in the model in which places are provided with the government charging no tuition fee. As H decreases, however, W^H approaches the baseline case, given that the tuition fee partially offsets the upward shift in the curve.

Formally, the slope of W^H is given by

$$\frac{\partial W^H}{\partial H} = \delta\left(\frac{\partial w_2^H}{\partial H} + \tau\frac{E}{N}\right),$$

such that its sign depends on which of the two terms is higher in terms of magnitude. Notice that $\partial w_2^H/\partial H$ measures the marginal wage of the skilled job and it is, therefore, decreasing in the number of students H. The absolute value of this term can also be seen as a measure of labor market rigidity: the higher (respectively, lower) $|\partial w_2^H/\partial H|$, the less (more) rigid the labor market is. The second term is the marginal impact on subsidies, which is positive, increasing in E and decreasing in N.

We henceforth assume that $|\partial w_2^H/\partial H| > \tau(E/N)$. This assumption holds when (i) the labor market is flexible enough, (ii) the population of young adults is high enough, or (iii) the tuition fee τE is low enough. When such an assumption is violated, W^H may increase for some values of H. This could create multiple equilibria. In addition, if W^H is increasing for all H, then all young students will choose to study, such that $N = H$. But this implies that $L = 0$ and, therefore, $Y_2 = 0$, a contradiction.

Let $\widehat{K}^B$ be the number of places in public higher education institutions that equalizes the lifetime income flows under the policy with tuition fees. If the government wants to achieve ex-post equality, $\widehat{K}^B$ must satisfy

$$\delta w_2^H(H, N-H) = w_1^L\left(\overline{H}, \bar{L} + N - H\right) + \delta w_2^L(H, N-H) + \delta\tau E, \qquad (16)$$

such that all places will be filled by young adults.

One can notice that (16) is different from (11). The number of skilled workers that clears the labor market in the mechanism in which students pay no tuition fee is higher than the one when those who enter into higher education pay part of the educational cost directly. To see this, let us rewrite (16) as

$$\delta w_2^H(H, N-H) - w_1^L\left(\overline{H}, \bar{L} + N - H\right) - \delta w_2^L(H, N-H) = \delta\tau E. \qquad (17)$$

Now, recall that K^B satisfies (11). Thus, if $H = K^B$ the left-hand side (LHS) of the above expression is equal to zero, such that we have $0 < \delta\tau E$. Given that the w_t^j is decreasing (respectively, increasing) in H for $j = H$ ($j = L$), then we must have $\widehat{K}^B < K^B$ to satisfy (17).

Although the number of places that the government must provide to achieve ex-post equality is lower than under the policy of "free" public higher education, it continues to create inefficiency. As the optimal allocation satisfies (4), by substituting $H = K^A$ into (17) we have $E > \delta\tau E$. Thus, the fact that the LHS of (17) is decreasing in H implies that $\widehat{K}^B > K^A = H^*$. The trade-off between efficiency and ex-post equality can also be seen through the comparison between the lifetime income flows: $W^H - W^L = E(1 - \delta\tau) > 0$. The intuition behind this result is similar to the case of "free" higher education. Given that students receive a net subsidy, there is an extra incentive for individuals to enter public colleges or universities.

The difference $W^H - W^L$ also allows us to conclude that the trade-off between efficiency and equality is reduced when the government provides $H = \widehat{K}^B$ instead of $H = K^B$. Recall that, under the "free" higher education, we have $W^H - W^L = E$. Under the policy in which students pay a tuition fee, however, this difference is $E(1 - \delta\tau) < E$. This result has an important policy implication: although the introduction of tuition fees does not eliminate the income inequality between skilled and unskilled workers resulting from the provision of places in public institutions, it can reduce it. Furthermore, such a reduction is larger the higher the percentage of the individual education each student pays, τ.

Let us now analyze a particular and extreme case of the previous financing mechanism, in which students pay the entire cost of their education, that is $\tau = 1$. Notice that, under this policy, students no longer receive subsidies. This implies that there are no taxes ($\widehat{T} = 0$) and each student pays $\mathcal{T} = EK/H$, which is equal to the individual educational cost E whenever all the places are filled ($K = H$). As this financing mechanism is a particular case of the one analyzed in the previous section, it is straightforward to see that, when $\tau = 1$, SW is equal to (13) and thus equal to (1) as well. Once again, the efficient allocation is that $K = H^*$.

The lifetime income flows of skilled and unskilled workers are now

$$W^H = \delta w_2^H(H, N - H) - \frac{EK}{H} \tag{18}$$

and (2), respectively. Given that there are no taxes, the income flow of an unskilled worker is the same as in the baseline case: it continues to be an increasing function of H.

As before, the function W^H is composed of two terms, which present opposite effects on the income flow as H increases. The first one, the skilled job wage, is a decreasing function of the number of students, as we have seen. For a given K, the tuition fee paid by each student, EK/H, is also decreasing in H, since the total educational cost is shared among a higher number of individuals. Because of the minus sign in the expression, the second term of (18) is, therefore, increasing in H. We can invoke the assumption that $\left|\partial w_2^H / \partial H\right| > \tau\, E/N$ —assuming that the labor market is flexible enough, for example—to guarantee $\partial W^H/\partial H < 0$.

The number of places K^B that the government must provide if it wants to achieve ex-post equality solves the following equation:

$$\begin{gathered} \delta w_2^H (H, N-H) - \frac{EK^B}{H} = w_1^L \left(\bar{H}, \bar{L} + N - H\right) + \delta w_2^L (H, N-H) \\ \delta w_2^H (H, N-H) - E = w_1^L \left(\bar{H}, \bar{L} + N - H\right) + \delta w_2^L (H, N-H), \end{gathered} \tag{19}$$

where we use the fact that, in equilibrium, $K^B = H$. Given that (19) is equal to (1), they have the same solution $K = H^*$. Thus, we have that efficiency and ex-post equality are simultaneously achieved when there is a public provision of higher education institutions but students are responsible for paying the entire educational cost.

The above result is somehow expected, considering that the source of inefficiency in the financing mechanisms analyzed in the previous sections is the subsidy that students receive. When individuals pay for their own education, on the contrary, there is no subsidy, such that the only factor driving young adults' choices between being a student or an unskilled worker is the relative wage.

4. Concluding remarks

Several conclusions and policy implications can be drawn from our analysis. First, policymakers must be aware of the inevitability of the trade-off between efficiency and equity originating from public policies for higher education. This implies that they must select which criterion is the most important to be pursued. Second, one of the costs of subsidizing students through taxes is inefficiency. The policy of "free" higher education is the worse on this criterion because the subsidy is maximum when students do not pay any tuition fees. Therefore, regardless of the criterion the policymakers want to achieve, there are alternatives better than "free" education. In particular, charging students a tuition fee that covers at least part of their education costs increases society's welfare.

Our model can be extended in several ways. One possibility is to assume that the quality of public and private higher education institutions is different.

This would depict the Brazilian scenario, for example, in which public universities are in general better than private ones. This would also allow us to build a model with competition between the two types of higher education institutions. In this context, another possible extension is to allow the family's wealth to be used to pay tuition fees. Another common characteristic of many countries is that college students work part-time—sometimes full-time, with their classes in the evening. This is another interesting extension that makes our model closer to developing economies, such as Brazil.

References

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