

Optimum design of plates structures under random loadings

Projeto ótimo de estruturas de placas submetidas a vibrações aleatórias

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Resumo

Problemas de otimização estrutural, envolvendo carregamento estático, já vêm sendo estudados há algum tempo e, de certa forma, esse tipo de problema já está bem definido na literatura especializada, porém problemas envolvendo carregamento dinâmico ainda são poucos estudados e problemas envolvendo carregamento dinâmico não determinístico menos ainda. O presente trabalho apresenta a formulação do problema de otimização de placas submetidas a carregamentos randômicos. Para a modelagem da estrutura, utilizou-se o elemento de placa à flexão AST6, que fornece essas matrizes explicitamente. Uma redução dinâmica das matrizes de massa e rigidez foi utilizada para reduzir o custo computacional do problema. A solução do problema foi obtida utilizando o Método dos Pontos Interiores e, para a análise de sensibilidade das matrizes de massa e rigidez da estrutura, foi utilizado o método semianalítico. Três exemplos são apresentados para demonstrar a confiabilidade do processo. O primeiro exemplo é de uma placa isotrópica e os dois outros são problemas envolvendo placas do tipo sanduíche. Em todos os exemplos, obteve-se um projeto melhorado, em relação à geometria inicialmente proposta.

Palavras-chave: Otimização, sensibilidade, placas, cargas aleatórias.

Abstract

Structural optimization problems involving static loading have been studied for some time and in a way these kinds of problems are widely encountered in literature, but problems involving dynamic loading still have few studies not to mention problems involving nondeterministic dynamic loading. This paper presents a formulation of the optimization problem of plates subjected to random loadings. For the modeling of the structure, it used the bending plate element AST6 that explicitly provides these matrices, and dynamic mass reduction and rigidity matrices were used to reduce the computational cost of the problem. The solution was achieved using the Interior Point Method and the sensitivity analysis of mass, and for the structure's stiffness matrixes the semi-analytical method was used. Three examples are presented to demonstrate the reliability of the process. The first example is an isotropic plate and the other two are problems involving sandwich plates. In all examples, an improved design in respect to the initially proposed geometry was obtained.

Keywords: Optimization, sensitivity, plates, random loading.

1. Introduction

Random dynamic loadings are only rigorously defined in statistical terms, consequently, the analysis of structural systems subjected to such loadings must be performed through statistical methods. They can be produced by natural phenomena such as wind and earthquake or by man induced phenomena like traffic or vibration of aerospace structures due, for example, to the launching of a rocket. Although the analysis of structures under random loading is well established, the correspondent optimisation problem has been only recently tackled. Some of the most important recent works in

the area are quoted in the following lines. Kin and Wen (1990), present an analysis of the structural reliability taking into account the combined effect of several random loadings. Neubert (1993) studies the maximisation of structural damping in order to diminish the random vibration of the structure. Lipton (1994) considers the optimum distribution of plate stiffeners in plates submitted to multiple case of random loadings Alves et al. (2000) presents a thorough development of the equations for the analytical sensitivity analysis of the structural response of structures subjected to random load-

ings and validates the formulation comparing the results obtained with these equations with the ones calculated by means of the finite difference method. Alves et al. (2002) presents the complete formulation to the problem involves optimisation with random loading. Alves and Vaz (2010) presents application for truss structures. In this work, homogeneous and sandwich plates under random dynamic loadings are optimised. The AST6 finite element is used for the structural discretisation and the interior point algorithm by Herskovits (1995) is applied in the optimisation process.

2. The optimisation problem

The proposed optimisation problem consists of the minimisation of the structural mass (volum) of an homogenous or a sandwich plate subjected to the condition

that the probability that the displacement and/or acceleration at a given point of the plate should not exceed, respectively, given bounds for the displacement and/

or acceleration must be less or equal than a given bound for the probability. This problem is translated as:

$$\text{Min } \sum \rho_i A_i h_i \tag{1}$$

$$\text{Subjected to: } \Pr(u_i > u_{\max}) < P_{d_{\max}}$$

$$\Pr(\ddot{u}_i > \ddot{u}_{\max}) < P_{a_{\max}}$$

$$h_i < h_i < h_u$$

In Eq.1, ρ_i , A_i and h_i are, respectively, the mass density, area and thickness of the i^{th} plate finite element; u_i and \ddot{u}_i are, respectively, the vertical displacement and acceleration at the j^{th} nodal point, $u_{j,\max}$ and $\ddot{u}_{j,\max}$, respectively, the

bounds for vertical displacement and acceleration at the j^{th} nodal point and $P_{d_{\max}}$ and $P_{a_{\max}}$ are given bounds for the probabilities (for example 1%, 2% or 5%) associated with the displacement and acceleration respectively. Also in

Eq.1, $P_d(u_i > u_{j,\max})$ is the probability distribution function associated with the probability of the vertical displacement u_i be greater than the given bound $u_{j,\max}$,

$$P_d(u_i > u_{j,\max}) = \int_{u_{\max}}^{\infty} p(u_i) du \tag{2}$$

Where $p(u_i)$ is the probability density function of u_i . A corresponding definition

holds for $P_a(\ddot{u}_i > \ddot{u}_{j,\max})$.

The probability density function

adopted in this work is the Gaussian distribution:

$$p(u_i) = \frac{1}{\sqrt{2\pi} \sigma_{ui}} e^{-\left[\frac{(u_i - \ddot{u}_i)^2}{2\sigma_{xi}^2}\right]} \tag{3}$$

The ergodicity and zero mean as-

sumptions transform Eq.3 into.

$$p(u_i) = \frac{1}{\sqrt{2\pi} \sigma_{ui}} e^{-\left[\frac{(u_i)^2}{2\sigma_{xi}^2}\right]} \tag{4}$$

In Eqs.3 and 4 σ_{ui} is the standard deviation of u_i . From the theory of the

stochastic response of linear Multi Degree of Freedom systems σ_{ui} is obtained by the

following expression:

$$\sigma_{ui}^2 = \frac{1}{2\pi} \mathbf{B} \int_{-\infty}^{+\infty} \mathbf{S}_y(\Omega) \mathbf{d}\Omega \mathbf{B}^T \tag{5}$$

Where \mathbf{B} is the matrix whose terms are the n_j mode components and $\mathbf{S}_y(\Omega)$

is the spectral density matrix of the generalised co-ordinates. This matrix is

defined as:

$$\mathbf{S}_y(\Omega) = \mathbf{H}(\Omega) \mathbf{S}_p(\Omega) \mathbf{H}(\Omega) \tag{6}$$

In this equation $\mathbf{S}_p(\Omega)$ is the spectral density matrix of the generalised loads and

$\mathbf{H}(\Omega)$ is a diagonal matrix whose generic term is the complex frequency response

function of normal mode n given by:

$$\mathbf{H}_{n=} [(K_n - M_n \Omega_n^2) + 2i \xi_n M_n \Omega_n] \tag{7}$$

Where K_n , M_n , Ω_n and ξ_n are, respectively, the generalised stiffness, mass, frequency of the exciting force

and damping ratio of normal mode n . The design variables considered are the plate thickness in the isotropic plate and

the sheet and core thickness in the case of a sandwich plate.

3. Sensitivity analysis

The gradients calculation to obtain the search direction is performed through the analytical method for the constraints. Alves (2000) presents a

thorough development for the sensitivity analysis of the structural response due to random loading. The developed analytical equations are validated by

a comparison with finite difference solutions. In the next section an example of the sensitivity analysis of a plate structure is presented.

4. Examples of optimization

Example 1. Isotropic Plate

In this example, the isotropic plate displayed in Figure 1 is optimised. The plate properties are the same as

indicated in the previous example. The following constraint is imposed: the probability that the displacement in

the centre of the plate, u_c , is less than 1mm, should be less than or equal to 1%. Therefore:

$$Pd(u_c > 0.001) = \int_{0.001}^{\infty} p(u_c) du \leq 1\% \tag{8}$$

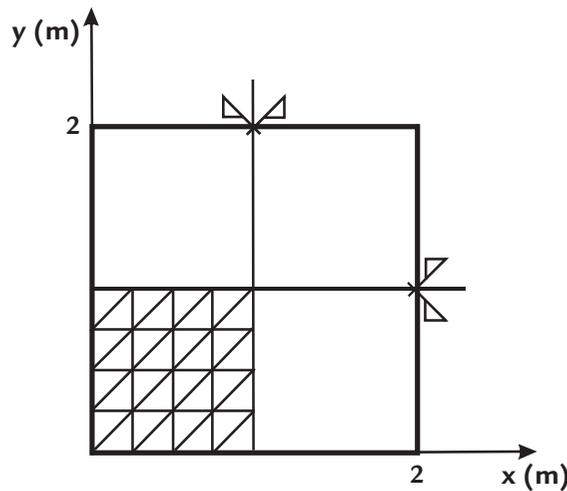


Figure Plate of analysis.

Two cases are considered regarding the number of variables:

1. Only one design variable (constant plate thickness) for all the mesh elements in Figure 2A.

2. Four design variables (four different plate thickness) as indicated in Figure 2B.

objective functions and constraints are given in tables 1 and 2 for cases 1 and 2, respectively. The thickness configuration for the optimum design of case 2 is displayed in Figure 3.

	Design Variable (m)	Obj. Fun. (m ³)	Const. (%)
Initial	1.00x10 ⁻²	1.00 x10 ⁻²	0.27
Final	8.29x10 ⁻³	8.29x10 ⁻³	1.0

Table 1 Design variables, objective function and constraints in model 1.

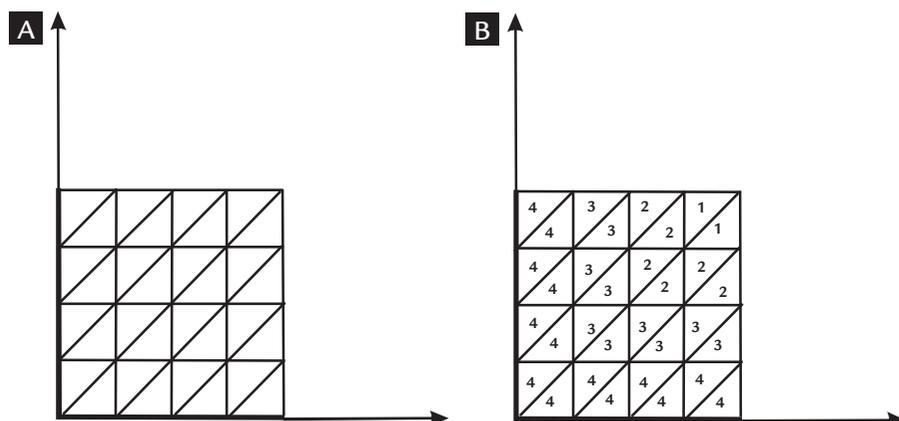


Figure 2
A) Optimization for 1 project variable.
B) Optimization for 4 projects variables.

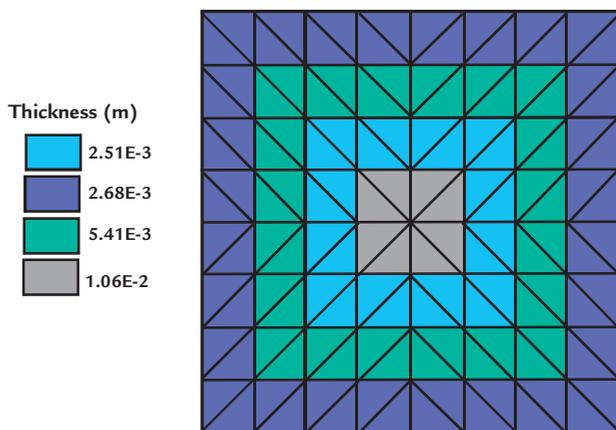


Figure 3
Thickness Distribution throughout the plate considering 4 design variables.

Design Variable (m)	Initial	Final
h_1	1.0×10^{-2}	1.06×10^{-2}
h_2	1.0×10^{-2}	2.61×10^{-3}
h_3	1.0×10^{-2}	6.41×10^{-3}
h_4	1.0×10^{-2}	2.68×10^{-3}
Obj. Fun.(m ³)	1.0×10^{-2}	4.33×10^{-3}
Const.(%)	0.27	1.0

Table 2
Design variables, objective function and constraints in model 2.

Example 2. Sandwich Plate

The sensitivity analysis and the optimisation of a sandwich plate, Figure 4, are presented here. This type of structure is largely employed in satellite structures as they present high bending

rigidity and low weight. The plate is modelled with a 4x4 mesh as indicated in Figure 1 and the random loading is the same as in Example 1. The material properties are given in Table 3.

Two design constraints are considered associated with the displacement u_c and the acceleration \ddot{u}_c at the centre of the plate as expressed by the following equations:

$$Pd(u_c > 0.001) = \int_{0.001}^{\infty} p(u_c) du \leq 2\% \tag{9}$$

$$Pa(\ddot{u}_c > 11.5) = \int_{11.5}^{\infty} p(\ddot{u}_c) du \leq 10\% \tag{10}$$

Results for the sensitivity analysis of the standard deviation of the displacement with respect to the design variables taking into account as design variables the thickness of the upper and lower sheets and the core thickness are given in Table 4. These results indicate the fair convergence of the sensitivity values calculated

by Finite Difference Method (FDM) compared to the values calculated by Analytical Method (AM). Finally, these conclusions support the reliability of the optimisation method for sandwich plates.

The optimisation is initially performed considering the thickness of the sheets as the only design

variable. The results are presented in Table 5. Then, both the thickness of the sheets and of the core of the sandwich plate are taken as design variables leading to the results of Table 6. It must be noted from Table 7 that the first constraint becomes active when the core thickness is a design variable.

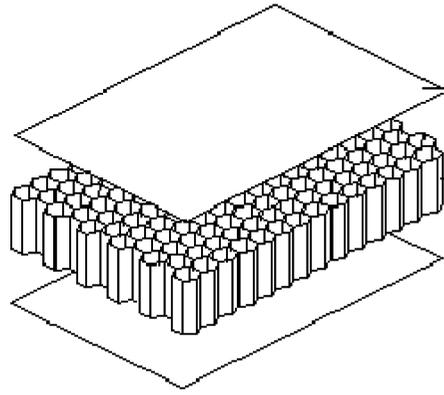


Figure 4
Sandwich panel.

Aluminium 2024-T3	Material	Isotropic
	Elasticity Modulus(N/m ²)	E=6.80E+10
	Shear Modulus(N/m ²)	G=2.56E+10
	Poisson Ratio	ν=0.33
	Density (N/m ³)	ρ=2700.0
Aluminium HoneyComb	Material	Ortotropic 2D
	Shear Modulus (N/m ²)	G ₁₂ =1.0e+6 G ₁₃ =2.206E+8 G ₂₃ =1.117E+8
	Poisson Ratio	ν=0.33
	Density(N/m ³)	ρ=36.8

Table 3
Material characteristics.

	AM	FDM (Δx=10⁻²)	FDM (Δx=10⁻⁴)	FDM (Δx=10⁻⁶)	FDM (Δx=10⁻⁸)
$\partial\sigma/\partial h_s$ (x10 ⁻¹)	6.0567	13.3341	5.9732	6.04506	6.05668
$\partial\sigma/\partial h_h$ (x10 ⁻⁵)	7.12480	8.1256	7.8654	7.22467	7.12568

Table 4
Sensitivity analysis results
for the Sandwich Plate.

Design Variable (m)	Initial	Final
h_s	3.0 x10 ⁻³	2.602x10 ⁻³
Obj. Func.(Kg)	16.568	14.42
Const. (%)	Initial	Final
1	0.128	0.545
2	6.78	8.87

Table 5
Design variables, objective function
and constraints in the sandwich plate.

Design Variable (m)	Initial	Final
h_s	3.0 x10 ⁻³	2.681x10 ⁻³
h_h	1.0 x10 ⁻²	7.965x10 ⁻³
Obj. Func. (Kg)	16.568	14.771
Const. (%)	Initial	Final
1	0.128	2.0
2	6.78	6.63

Table 6
Design variables, Objective function
and constraints in the sandwich plate.

Example 3. Sandwich plate with random loading with variable PSDF

The sandwich plate of the previous example is now submitted to a random load with the Power Spectral Density

Function (PSDF) indicated in Figure 5 and applied at the centre of the plate.

Now the material of the sheets is

constituted by orthotropic carbon fibres with their properties given in Table 7.

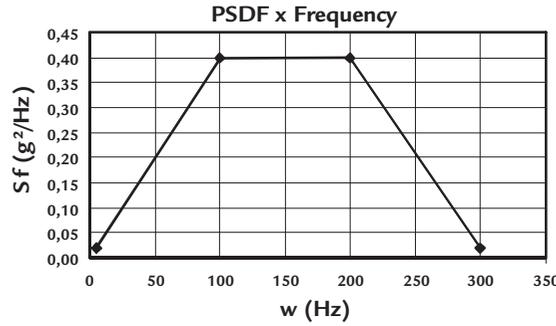


Figure 5
Plot of the variable PSDF versus exciting frequency.

Carbon Fibre	Material	Ortotrópico 2D
	Elasticity Modulus in the longitudinal direction (E1) (N/m²)	200.0E+9
	Elasticity Modulus in the lateral direction (E2) (N/m²)	14.5E+9
	Shear Modulus (N/m²)	G ₁₂ = 4.9E+9 G _{1z} = 4.9E+9 G _{2z} = 4.9E+9
	Poisson Ratio	n=0.3
	Density (N/m³)	γ=1650.

Table 7
Mechanical proprieties of the carbon fibre.

The imposed constraints for this problem are:

$$Pd (u_c > 0.008) = \int_{0.008}^{\infty} p(u_c) du \leq 2\% \tag{11}$$

$$Pa (\ddot{u}_c > 10) = \int_{10}^{\infty} p(\ddot{u}_c) du \leq 10\% \tag{12}$$

The first optimisation is considered with only the sheet thickness as design variables. The results are shown in Table 8.

Both the thickness of the sheets and of the core are considered as design vari-

ables in the second optimisation whose results are given in Table 9. It can be observed, as in the previous example, that the first constraint becomes active when both thicknesses are the design variables.

On the other hand, the first optimisation leads to a better design. The reason for this is that the mass density of the core is smaller than that of the sheets, thus having negligible influence on the total mass.

Table 8
Results for example 3 (11 steps).

Design Variable (m)	Initial	Final
h _s	2.0 x10 ⁻³	1.632x10 ⁻³
Obj. Fun.(Kg)	6.968	5.854
Const. (%)	Initial	Final
1	0.297	1.465
2	5.004	7.999

Table 9
Design variables, objective function and constraints for the sandwich panel (10 steps)

Design Variable(m)	Initial	Final
h _s	2.0 x10 ⁻³	1.803x10 ⁻³
h _h	1.0 x10 ⁻²	8.598x10 ⁻³
Obj. Fun.(Kg)	6.968	6.267
Const. (%)	Initial	Final
1	0.297	2.002
2	5.004	5.24

5. Conclusions

The following conclusions are drawn from the analysis of the results of the examples:

A better design in relation to the initial design is obtained in all the analysed cases. In Example 1, with 4 design variables, the variable less altered is variable 1 (h). This is to be expected as the elements near the point of application of the load must be more rigid. This example is illustrative since the real plate has a

constant thickness.

In the sandwich plate example the optimisation process is faster when two design variables are used instead of one. It must be noted that in this case there is an inversion relative to which constraint becomes active when compared to the first case with one design variable. The acceleration constraint tends to be active when one design variable is taken into account whereas the displacement

constraint is active when two variables are considered. It must be noted that, in these examples, the best design is obtained when only the sheet thickness is taken as design variable. This is due to the fact that the material of the core is much lighter the sheet material. If the mass densities of the sheet and of the core are the same, the best design is obtained with both thicknesses as design variables.

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