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Start position of a sloping hydraulic jump

Posição de início do ressalto hidráulico afogado

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ABSTRACT

Although sloping hydraulic jumps are frequent in energy dissipators, there are few studies regarding this type of phenomenon. Since the jump is accompanied by violent impacts and sudden variations of velocity and pressure, it is important to know the region where the phenomenon will occur, in order to make a safe and economic design possible. In this paper, a methodology for the determination of the start position of the sloping jump along a Creager spillway is introduced. It was verified that this position is a function of both the incident Froude number and the submergence factor.

Keywords: Sloping hydraulic jump; Start position of jump; Submergence factor.

RESUMO

Apesar da frequência com que ressaltos hidráulicos afogados ocorrem nas estruturas de extravasamento de barragens, são poucos os estudos que abrangem este tipo de fenômeno. Sendo o ressalto hidráulico acompanhado de impactos violentos e variações bruscas de velocidade e pressão, é importante que se conheça a região em que o fenômeno incidirá, a fim possibilitar um dimensionamento seguro e econômico. Neste trabalho, é proposta uma metodologia para a determinação da posição inicial do ressalto afogado ao longo da calha de um vertedouro do tipo Creager. Verificou-se que esta posição é função do número de Froude incidente e do grau de submergência imposto.

Palavras-chave: Ressalto hidráulico afogado; Início do ressalto; Grau de submergência.

INTRODUCTION

The intrinsic macroturbulence of the hydraulic jump generates high levels of energy dissipation in the phenomenon, which explains its fundamental importance in structures such as dams for energy generation. Since the 1950s, there has been a significant increase in the number of researches about the hydraulic jump, when the studies of Rouse et al. (1959), Schröder (1963) and Rajaratnam (1965) were published. Dai Prá (2011) presents a review with more than 40 studies detailing the jump behavior. It is worth mentioning that, lately, most investigations have been attempting to explain the internal characteristics of the jump, such as the velocity distribution along its development, the pressure fluctuations within its interior and the effects of aeration in its behavior. The applications of the hydraulic jump with respect to open channel flows include the energy dissipation downstream of spillways and other hydraulic structures; the maintenance of high depths of water in water distribution channels; the indication of special flow conditions; among others.

The sequent depths (Y_1 and Y_2, shown in Figure 1) are relevant in the description of the hydraulic jump, affecting the determination of the type of jump and characteristics such as roller and jump lengths. Teixeira (2003) and Trierweiler (2006) present a series of studies that establish expressions to determine the conjugate depths in horizontal channels. In spite of that, the most widely used expression is the one derived by Bélanger (1828), written as Equation 1.

$$\frac{Y_2}{Y_1} = \frac{1}{2} \cdot \left[\sqrt{1 + 8 \cdot Fr_1^2} - 1 \right] \tag{1}$$

In which,

Y_1 is the sequent depth upstream of the jump (m);

Y_2 is the sequent depth downstream of the jump (m);

Fr_1 is the incident Froude number (dimensionless).

It is known that the hydraulic jump can occur in sloping channels, which leads to the consideration of the forces due to gravity in its resolution. Although the law of conservation of momentum is theoretically applicable to this type of jump, in practice, the number of available solutions is limited. According to French (1985), there is a series of difficulties to the procurement of solutions of this type of phenomenon, e.g.: the term referring to the vertical component of the fluid weight is generally poorly quantified, because the length and shape of the jump are not well defined; the specific weight of the fluid in the control volume is not constant due to air entrainment; the pressure terms cannot be accurately quantified. Rajaratnam (1967) emphasizes that, if the law of conservation of momentum is analyzed parallel to the channel bottom, the component of the fluid weight within the control volume of the hydraulic jump must be considered.

Some recent researches have been directed towards studying the hydraulic jump using numerical and computational methods. Some examples are: Bayon et al. (2016), Jesudhas et al. (2018), Valero et al. (2018) and Witt et al. (2018). However, the numerical models yield only mean values of hydraulic phenomena such as the hydraulic jump, not being capable of describing fluctuating and extreme values of pressure and velocity. In physical modeling, some studies have been trying to describe the behavior of the jump on rough channels, as in Hassanpour et al. (2017), Felder & Chanson (2018) and Palermo & Pagliara (2018), while other researchers have been investigating the loads applied on the stilling basin due to the jump, as can be seen in Li et al. (2015), González-Betancourt (2016) and Dai Prá et al. (2016). Regarding the positioning of the sloping jump, researches that propose an estimate criterion were not found. There is, though, a classification of the jump as to its degree of submergence, being more commonly applied in sloping channels, and not in Creager spillways with toe curves. The results herein included addressed these aspects.

HYDRAULIC JUMP BACKGROUND

The hydraulic jump downstream of sloping channels, as presented by Rajaratnam (1967) and Peterka (1974), follows a classification attributed to Kindsvater (1944), who suggested the existence of five types of jump, as shown in Figure 2. The A-jump is formed when the downstream sequent depth (Y_2), calculated using the Bélanger's equation, is equal to the one occurring downstream of the jump (T_w). This type of jump is formed completely on the horizontal channel downstream of the spillway, starting at the joint of the spillway toe with the stilling basin. The classic jump is the same case of the A-jump, except that it occurs on a horizontal channel, instead of downstream of a spillway. The B-jump is formed when the jump is partially over the stilling basin and partially over the spillway, submerging the toe of the spillway. The C-jump, when the end of the roller is exactly above the start of the stilling basin. The D-jump is characterized by the formation of the roller entirely over the spillway.

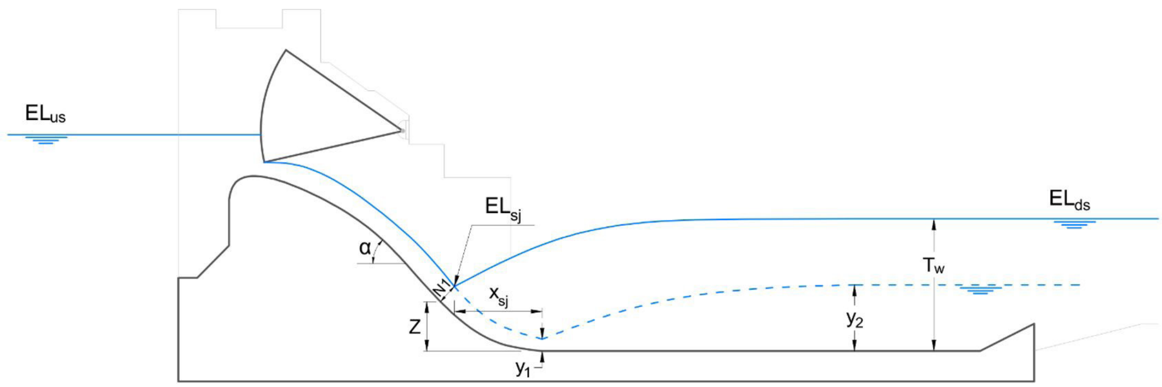

Figure 1. Definition sketch for the sloping jump.

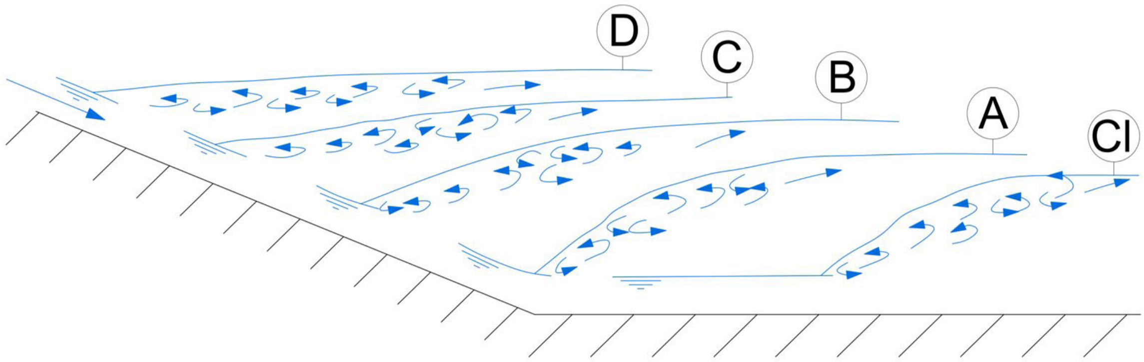

Figure 2. Types of sloping channel jumps (Hager, 1992).

The B-jump, also called sloping jump in this paper, occurs when there is an increase in the water depth downstream of the jump, such that the inclined channel upstream of the apron acts as the dissipation structure.

Unlike the sloping jumps, the submerged jumps are the ones occurring downstream of sluice gates. In order to measure the degree of submergence of this type of jump, Rajaratnam (1967) introduced the submergence factor S^*(dimensionless), given by Equation 2. For Marques et al. (1999), the submergence factor S (dimensionless) is expressed as in Equation 3.

$$S^* = \frac{T_w - Y_2}{Y_2} \tag{2}$$

$$S = \frac{T_w}{Y_2} \tag{3}$$

In which:

T_w is the tail water depth downstream of the jump (m).

According to Peterka (1974), an increase in the tail water at the end of the basin leads to an increase of water depth in the start of the jump, "submerging" part of the spillway. However, a vertical increment ΔY_2 in the tail water depth does not cause an equal increment in the upstream water depth, differently from what could be assumed. The increment ΔY_1 caused upstream of the jump is several times ΔY_2. For a second increment on the tail water, the upstream increment $\Delta Y_1^{'}$ is still higher than $\Delta Y_2^{'}$, but to a lesser degree. This effect continues until the tail water reaches 1.3 times the conjugate depth Y_2. After this, the relationship between the increments observed upstream and downstream of the jump is linear.

Regarding sloping jumps, it is worth mentioning the studies carried out by Elevatorski (1959), which summarizes the characteristics observed in this type of phenomenon, and Rajaratnam (1967), which gives a generic review on the subject, including other types of jumps over sloping channels as well. Hager (1988) established the descriptive relations of the sloping jump, such as the expression for the conjugate depths Y_2 / Y_1, the roller length and the efficiency of the jump. In general, authors focus on the external characteristics of the jump, through the establishment of the water surface profile and through parameters such as the jump length and the sequent depths.

The main studies on the internal characteristics of the sloping jump are the ones performed by Pinto et al. (1988), Marques et al. (1999) and Wiest et al. (2005).

A good performance of the structure can be assured if the effects of the jump over the basin and its extent are known. This is why so many researches investigate the jump length and the roller length, such as: Rajaratnam (1967), Marques et al. (1997), Lopardo et al. (2004). The latter suggests a novel expression for the determination of the length of submerged jumps. Teixeira (2003) and Trierweiler (2006) present and extensive list of different methods for the determination of the length of the jump.

Hager (1988) and Ohtsu & Yasuda (1991) performed experimental studies aiming to determine the ratio of the sequent depths in a hydraulic jump. The relevance of their studies is given by the fact that their experiments include both A and B-jumps. The equations are presented in Table 1. The relationship derived by Hager (1988) is valid for B-jumps at an upstream bottom slope of 45º and for Froude numbers between 3 and 11, while the one derived by Ohtsu & Yasuda (1991) is valid for $0° \le \alpha \le 60°$ and $4 \le Fr_1 \le 14$.

Carollo et al. (2011) also developed an equation for the estimation of the conjugate depths, valid for A and B jumps on smooth channels. The authors compared data collected in physical models with previously existent methodologies for the determination of the sequent depths for this type of jump (including the ones developed by the previously mentioned authors), and verified that none of the analyzed equations yielded good estimates of the ratio Y_2 / Y_1. From the dimensional analysis of the phenomenon and using the collected data, an equation with a mean error of 4.9% was adjusted. This equation can be applied to determine the sequent depth ratio of type B hydraulic jumps on smooth channels for $8.5° \le \alpha \le 30°$. It is worth mentioning that this expression, presented in Table 1, depends on the knowledge of the parameter Z, that defines the start position of the jump over the spillway profile. Thus, the practical application of the equation is limited by the absence of an expression capable of estimating this parameter.

In a similar manner, Bejestan & Shokrian (2015) developed an expression for the determination of sequent depth ratio. The authors argued that this relation, shown in Table 1, is more precise that the one developed by Carollo et al. (2011). In addition, it has the advantage of not being dependent on the slope of the channel upstream of the apron. Their experimental tests comprised slopes between 14.5° and 27.5°.

Table 1. Expressions for the determination of conjugate depths ratio of B-jumps.

Equation	Authors (year)
$\frac{T_w}{Y_1 \cdot \cos(\alpha)} = 23.5 \cdot \left(1+\frac{3}{2}E\right)^{-2} + \frac{\sqrt{2}\left(Fr_1 - 3\right)}{\tanh\left(\frac{5}{2}E\right)}$	Hager (1988)
$\frac{x_{sj}}{Y_2} = \left[\frac{2.3}{\left(\tan\alpha\right)^{0.73}} - 0.8\right]\left(\frac{T_w}{Y_2} - 1\right)^{0.75}$	Ohtsu & Yasuda (1991)
$\frac{T_w}{Y_1 \cdot \cos(\alpha)} = \sqrt{2}\exp\left(-\frac{\tan\alpha}{8.42}\right)E^{-\left[0.181/\left(\tan\alpha\right)^{0.663}\right]}\left(Fr_1 - 1\right)^{0.963} + \frac{1}{E}$	Carollo et al. (2011)
$\frac{T_w}{Y_1 \cdot \cos(\alpha)} = \left[7.9 - 10.82\frac{k_s}{h_c}\right] \cdot M^{\left[0.73+1.86\frac{k_s}{h_c}\right]} \cdot N^{2.37} + M$	Bejestan & Shokrian (2015)

In which: α is the slope of the spillway (radians); T_w is the tail water depth downstream of the sloping jump; $E = \left(T_w - Z\right)/T_w$, with Z being the elevation of the spillway at the start position of the jump, with respect to the basin elevation; k_s is absolute roughness; h_c is the critical depth; $M = \frac{T_w - k}{Y_1 \cdot \cos(\alpha)}$ is a dimensionless parameter; $N = 1 - \frac{Z + Y_1 \cdot \cos(\alpha)}{T_w}$ is a dimensionless parameter; k is the vertical distance between the water levels upstream and downstream of the sloping jump.

The purpose of these equations is to determine the conjugate depths of sloping jumps. Except for the one proposed by Ohtsu & Yasuda (1991), they all depend on the parameter Z, which is the difference between the basin elevation and the elevation of the spillway at the start position of the jump. However, these equations are implicit with respect to Z, while very often this parameter is the one that is being searched for.

Carollo et al. (2012) developed an equation capable of determining the roller length of A-jumps over smooth and rough channels, applicable also to B-jumps over rough channels. The expression proved to be efficient, except for jumps close to the undular condition.

The knowledge of the jump length, or even the roller length, is important to predict which part of the structure must be reinforced in order to receive the forces coming from the turbulence and the sudden pressure and velocity oscillations intrinsic to the phenomenon. In sloping jumps, however, the mere determination of the length is not enough. The determination of the start position of the jump, which will occur over both the basin and the spillway, is also necessary. After an extensive review of the literature, it was verified that there are no recent studies that cover this subject. Thus, the goal of this paper is to establish a methodology for the determination of the start position of the sloping jump over a Creager spillway.

MATERIALS AND METHODS

Experimental facility

The experimental work of this study was carried out at the Laboratório de Hidráulica Experimental – LAHE, at Furnas Centrais Hidrelétricas, Rio de Janeiro. A two-dimensional model of the Porto Colômbia HPP was used, in a 1:50 scale. The spillway and the stilling basin of the model are made of concrete and were installed on a 1.0 m large channel, with approximately 17 m of length. The stretch upstream of the spillway is 8.7 m, and 6.5 m of the channel are downstream of the basin. Figure 3 presents an overall view of the test channel, while Figure 4 shows the spillway and the stilling basin in detail. In Figure 5, the end part of the spillway, the stilling basin and the end-sill are shown, with an equivalent prototype discharge of 4,000 m^3/s and submergence factor of $S = 1.61$. The cross section of the model, with the equation of the Creager profile, the radius of the toe curve and the spillway crest

Figure 3. Overall view of the test channel (Wiest, 2008).

Figure 4. Spillway, stilling basin and end-sill (Wiest, 2008).

Figure 5. Hydraulic jump formed for Q_p = 4,000 m³/s and S = 1.61 (Wiest, 2008).

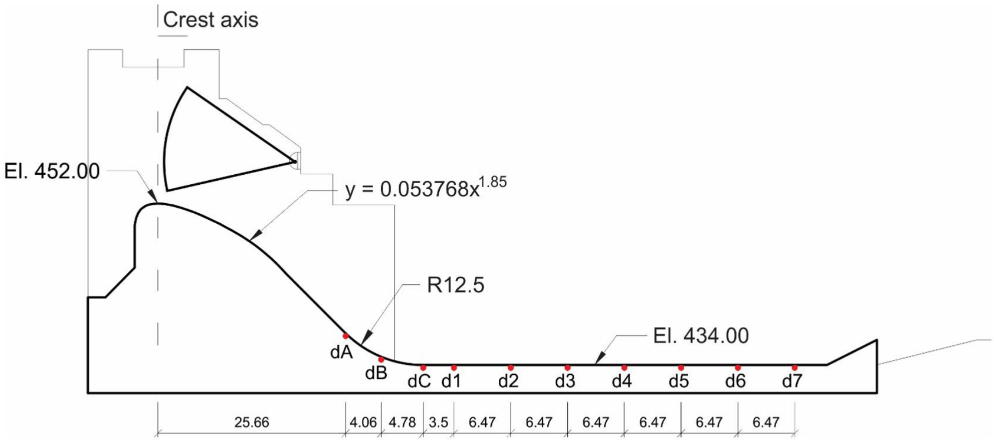

Figure 6. Cross section of the model (prototype scale in meters).

and apron elevations are shown in Figure 6. It is worth mentioning that the dimensions shown refer to the real structure (prototype).

The hydraulic circuit of the model consists of an inferior tank, from where the water is pumped to a superior tank with constant level. From this tank, the water flows by gravity to the upstream part of the channel, through cast iron pipes.

The tail water was measured using a point gauge placed 4.0 m from the end of the stilling basin. The jump is positioned using a gate placed approximately 6.5 m downstream of the end of the basin. Then, water is led to a Bazin weir, in which the discharge is read.

Experimental methodology

Besides water level and velocity measurements, the flow was also recorded. Given the goal of quantifying the effects of the submergence on the jump, a series of experiments was conducted, comprising different submergence factors and different flow conditions over the stilling basin. An attempt was made to study, at least, three degrees of submergence for each discharge. 20 tests, distributed in six different discharges, were performed. The analyzed discharges, in prototype scale, were 1,000 m³/s, 2,000 m³/s, 3,000 m³/s, 4,000 m³/s, 6,000 m³/s and 8,000 m³/s.

The upstream conjugate depth and the start position of the jump were measured using point gauges. Due to limitations in the assembled structure, it was not possible to perform these measurements in some flow conditions. From the 20 tests under different hydraulic conditions, six of them presented some kind of difficulty regarding the determination of the start position of the jump and its respective water depth. This limitation occurred because the longitudinal displacement of the point gauge was restricted, sometimes not reaching the desired position. Therefore, an expression describing the effect of the submergence on the start position of the jump along the spillway was adjusted.

RESULTS AND DISCUSSION

The definition of the start position of the jump is important mainly because of the analysis of the longitudinal distribution of pressures and its fluctuations along the stilling basin. As the submergence factor of the jump increases, the start position of the jump is dislocated upstream. The spillway, however, exerts resistance to this movement, what induces the jump to find a balance situation.

From the data collected on the model, a theoretical analysis was performed aiming to determine the characteristic parameters of the jump, such as the conjugate depths (Y_1 and Y_2) and the Froude number (Fr_1) of the jump, allowing for the determination of the submergence factor (S) imposed to the phenomenon. Figure 1 presents the mentioned parameters, while Table 2 contains the performed tests with the respective start position of the jump, in which:

EL_{sj} is the elevation of the start position of the sloping jump (m);

x_{sj} is the distance to the start position of the sloping jump, taken from the joint of the toe curve and the horizontal apron (m);

Z is the difference between the bottom elevation at the start position of the jump (on the structure) and the elevation of the basin (m);

α is the slope of the spillway at the start position of the jump (m);

Y_1 is the vertical component of the water depth at the start position of the jump ($Y_1 = N_1 \cdot \cos\alpha$, m);

N_1 is the water depth perpendicular to the spillway at the start position of the jump (m);

Y_2 is the downstream conjugate depth, calculated with the Bélanger's equation (Equation 1, m);

Fr_1 is the Froude number at the start position of the sloping jump, determined on the horizontal direction, i. e., using Y_1 (dimensionless);

S is the submergence factor of the jump (dimensionless).

From the analysis of the various parameters established as a function of the theoretical analysis, it was possible to determine an adjustment that best represents the behavior of the start position of the sloping jump. Figure 7 presents the data collected on the model compared with the adjustment that best described the points, correlating the vertical position of the start of the jump (Z_{calc}), the critical depth (h_c), the Froude number (Fr_1) and the submergence factor of the phenomenon (S). It can be verified that the proposed adjustment fitted well to the collected data. The relationship that describes the proposed curve is expressed as in Equation 4.

$$\frac{Z_{calc}}{h_c} = \left(Fr_1 \cdot (S-1)\right)^{0.61} \quad (4)$$

In which,

Z_{calc} is the theoretical vertical position of the start of the jump (m);

h_c is the critical depth (m);

S is the submergence factor of the jump (dimensionless).

A comparison of the data generated from the adjusted equation and the data collected during the tests reveals that the equation is well fitted. Table 3 shows the relative error between the theoretical results obtained using Equation 4 and the data

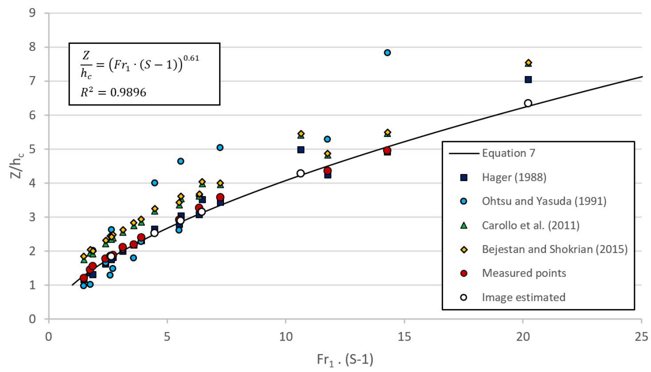

Figure 7. Start position of sloping jump as a function of Z, correspondent to the elevation of the spillway at the beginning of the jump.

Table 2. Parameters relative to the start position of the sloping jump (data in prototype scale).

Q	EL_{us}	EL_{ds}	EL_{sj}	x_{sj}	Z	α	Y_1	Fr_1	Y_2	S
(m³/s)	(m)	(m)	(m)	(m)	(m)	(º)	(m)	(m)	-	-
997.64	465.78	442.92	441.94	12.91	7.73	45.00	0.29	10.50	4.21	2.12
1004.17	465.84	444.08	443.01	13.99	8.82	45.00	0.30	9.99	4.15	2.43
1005.80	465.80	447.79	**445.97**	**16.57**	**11.28**	**41.61**	**0.35**	**8.43**	**4.05**	**3.40**
2000.00	465.67	444.26	442.68	13.38	8.20	45.00	0.60	7.25	5.83	1.76
1995.57	465.65	444.96	443.66	14.37	9.19	45.00	0.62	6.91	5.72	1.92
1995.57	465.69	445.92	444.60	15.26	10.07	43.93	0.65	6.52	5.66	2.11
1995.57	465.70	450.09	**446.75**	**17.41**	**12.00**	**40.11**	**0.75**	**5.57**	**5.54**	**2.91**
3000.00	465.74	445.36	442.83	13.25	8.08	45.00	0.90	5.89	7.05	1.61
3005.05	465.71	445.77	443.92	14.05	8.87	45.00	0.93	5.58	6.91	1.70
2995.44	465.77	448.33	**445.66**	**15.86**	**10.63**	**42.80**	**1.03**	**4.98**	**6.75**	**2.12**
3005.05	465.76	450.06	**446.74**	**16.97**	**11.63**	**40.91**	**1.11**	**4.59**	**6.68**	**2.41**
4001.03	465.75	446.16	443.06	13.12	7.94	45.00	1.20	5.11	8.07	1.51
4006.36	465.77	446.34	443.76	13.47	8.29	45.00	1.23	4.94	7.97	1.55
4011.68	465.78	447.03	444.55	14.61	9.43	45.00	1.26	4.74	7.85	1.66
4019.68	465.77	450.05	**446.72**	**16.57**	**11.28**	**41.61**	**1.45**	**4.07**	**7.66**	**2.10**
5995.32	465.71	447.82	444.02	13.62	8.45	45.00	1.82	4.08	9.64	1.43
6007.67	465.72	449.67	446.78	16.26	11.00	42.41	2.10	3.44	9.23	1.70
6020.03	465.72	450.32	**446.81**	**15.95**	**10.72**	**42.66**	**2.10**	**3.44**	**9.22**	**1.77**
7998.56	465.73	449.55	444.68	13.68	8.50	45.00	2.43	3.53	10.97	1.42
8005.44	465.72	451.10	447.47	16.32	11.05	42.18	2.82	2.96	10.48	1.63

Obs.: Values in bold are the ones estimated using the proposed methodology.

Table 3. Relative error between the experimental data, the proposed adjustment and the equations derived by other authors.

Q	Fr_1	S	Fr_1 . (S-1)	Equation 4	Hager (1988)	Ohtsu & Yasuda (1991)	Carollo et al. (2011)	Bejestan & Shokrian (2015)
(m^3/s)	-	-	-	(%)	(%)	(%)	(%)	(%)
997.64	10.50	2.12	11.76	3.79	-2.89	21.25	10.58	11.64
1004.17	9.99	2.43	14.29	2.99	-0.75	58.02	10.27	10.88
2000.00	7.25	1.76	5.51	-1.86	-4.36	-10.34	15.18	17.78
1995.57	6.91	1.92	6.36	-4.91	-5.90	11.08	10.63	12.49
1995.57	6.52	2.11	7.24	-6.16	-4.16	40.82	10.32	11.90
3000.00	5.89	1.61	3.59	0.77	-0.73	-18.29	25.31	29.38
3005.05	5.58	1.70	3.91	-3.34	-3.79	-5.50	19.01	22.14
4001.03	5.11	1.51	2.61	1.44	-0.53	-27.18	32.20	37.89
4006.36	4.94	1.55	2.72	0.02	-2.23	-20.24	28.81	33.87
4011.68	4.74	1.66	3.13	-3.95	-5.17	-3.16	21.10	24.69
5995.32	4.08	1.43	1.75	-0.95	-5.69	-30.15	34.66	41.67
6007.67	3.44	1.70	2.41	-9.10	-13.66	-12.34	18.28	23.12
7998.56	3.53	1.42	1.48	6.01	-2.34	-18.44	45.97	53.72
8005.44	2.96	1.63	1.86	-5.45	-16.18	29.28	23.06	28.77

collected on the model. The mean relative error and the maximum relative error resulted in 3.6%, and 9.1%, respectively.

The empty points in Figure 7 represent the data collected during the tests in which it was not possible to measure the start position of the jump, such that an estimate was made using the registered pictures of the flow. It is observed that the behavior of these points is very similar to the established adjustment. The measurements indicate that the adjusted curve adequately describes the points not measured directly on the model. It is worth mentioning that the points estimated using the images were not used to derive Equation 4.

From the adjustment proposed and with the jump initial data (upstream and downstream levels, friction loss at the gates, critical depth and unit discharge), it is possible to define, through an iterative process, the incident Froude number (Fr_1) and the submergence factor (S). Therefore, it is possible to calculate the theoretical vertical position of the jump (Z, as in Figure 1) and the horizontal start position of the jump along the spillway for different conditions of flow. The process presented a quick convergence, with up to four iterations being enough. Table 4 presents the obtained results for the six tests in which it was not possible to carry out the measurements.

Table 4. Parameters calculated through the proposed theoretical analysis.

Q	EL_{us}	EL_{ds}	EL_{sj}	x_{sj}	Z	S
(m^3/s)	(m)	(m)	(m)	(m)	(m)	-
1005.80	465.80	447.79	445.63	16.57	11.28	3.40
1995.57	465.70	450.09	446.75	17.41	12.00	2.91
2995.44	465.77	448.33	445.66	15.86	10.63	2.12
3005.05	465.76	450.06	446.74	16.97	11.63	2.41
4019.68	465.77	450.05	446.72	16.57	11.28	2.10
6020.03	465.72	450.32	446.81	15.95	10.72	1.77

The points measured on the experimental setup, shown in Figure 7, represented through the dimensionless relation Z/h_c, contain uncertainties due to many parameters, such as the test flowrate (Q) and the water depth measurements upstream (EL_{us}), downstream (EL_{ds}) and at the start position of the jump (EL_{sj}). In addition, the estimated points, which are determined through instantaneous pictures of each test, also contain, apart from the previously mentioned uncertainties, the subjectivity of the determination of the graphic reading of the parameters (it is known that the hydraulic jump is a macroturbulent phenomenon and that there is oscillation of position and levels every moment). It is very difficult to quantitatively estimate the uncertainties of these metrics, especially in a combined way. It is even possible that these errors counterbalance each other. Therefore, it is understood that the differences on Z/h_c generated by the presented adjustment (Figure 7) are satisfactory to determine the start position of the sloping hydraulic jump, given the possible sources of error in the process.

The data presented in Table 2 was applied to the equations shown in Table 1 in order to implicitly find the calculated value of the parameter Z. The exception was the equation proposed by Ohtsu & Yasuda (1991), for which the value of x_{sj} was calculated, and used to find the correspondent value of Z on the spillway.

The comparison of the values found through the other author's equations with the measured values of Z and the values calculated through Equation 4 is shown in Figure 7. The equation that best fitted the measured points was the one derived by Hager (1988), especially for the lowest values of $Fr_1 \cdot (S-1)$. For this range, the equation derived by Ohtsu & Yasuda (1991) also generated satisfactory results. The good agreement with Hager (1988) is due to his value of α being the same as in the experimental setup of the present paper. In addition, his experimental setup was the only one with a toe curve between the spillway and the apron, in a similar manner of the channel of this paper. The slope range tested by Carollo et al. (2011) and Bejestan & Shokrian (2015) do not comprise the inclination of the channel of this paper, which explains the discrepancies of their points. The relative errors for the author's equations with respect to the measured points are shown in Table 3.

CONCLUSIONS AND RECOMMENDATIONS

Knowing where the hydraulic jump will occur as a function of the downstream conditions, whether partly over the spillway or completely over the stilling basin, is essential to the design of these structures. Despite its relevance, there is not a significant amount of literature studies on this specific matter, especially when compared with the classical hydraulic jump. The results herein presented indicate that the start position of a sloping jump along the spillway are a function of both the Froude number Fr_1 and the submergence factor S. A simpler and novel explicit equation has shown to yield results as good as those obtained using previously existing implicit equations that depend on intricate parameters. The proposed curve fitted well to the experimental data, with errors of up to 9.10%. It is recommended that future studies investigate data collected on prototypes, as well as data collected on models of different settings and geometries. In addition, it is suggested that future researches study the length of this type of jump and its variation as a function of the submergence factor.

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Rafael André Wiest: Performed the methodology and obtained the results.

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Mauricio Dai Prá: Contributed with technical notes and revised the text.

Aloysio Portugal Maia Saliba: Contributed with technical notes and revised the text.

Marcelo Giulian Marques: Defined the objectives, contributed with technical notes and revised the text.