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Reliability-based analysis of seismic bearing capacity of shallow strip footings resting on soils with randomly varying geotechnical and earthquake parameters

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Article

Keywords
Randomness
Earthquake parameter
Shear strength
Probability density function
Failure probability

Abstract

Seismic bearing capacity of strip footings is a challenging task for geotechnical engineers due to its stochastic framework instigated by the natural uncertainties incorporated into geotechnical properties and earthquake parameters. Consequently, the introduction of the random field theory into reliability analysis may provide power tools to succor designers check how reliable their designs. This paper aims to assess the seismic bearing capacity of shallow strip footings resting on soils with randomly varying parameters. Bearing capacity formulas for purely cohesive and cohesive-frictional soils are considered. The influence of the type of the autocorrelation functions (ACFs), the scale of fluctuations (SOFs) and the coefficient of variation (COV) of the random parameters are investigated. Statistical moments, probability density function (PDF) and failure probability (P_f) of the seismic bearing capacity are computed. It is shown that the Single Exponential (SNE) ACF is the most appropriate function to characterize the spatial variability of the soil properties since it provides conservative results. On other hand, the results indicate that the increase in the coefficients of variation (COV) of the cohesion or the friction angle increases the variability of the seismic bearing capacity while this variability remains unaffected when the COV of the seismic coefficient increases. The results also highlight that the effect of the vertical SOF on the PDF and the failure probability is much more significant than that of the horizontal SOF. In addition, the mean seismic bearing capacity fluctuates slightly as the horizontal or vertical SOF increases so that the increment of variation is between 0.4% and 2% for the both two soil types.

1. Introduction

The seismic bearing capacity evaluation of strip footings is an essential issue for geotechnical engineers in a seismic zone. An earthquake loading may lead to a reduction of the bearing capacity and an increase in the settlement of shallow foundations. Several studies have been carried out by different researchers covering the seismic bearing capacity topic but they were based on the determination of the seismic bearing capacity factors following four main approaches: (i) the limit analysis (e.g. Richards Junior et al., 1993; Soubra, 1997; Ghosh, 2008; Yamamoto, 2010; Zhou et al., 2016; Conti, 2018; Rajaei et al., 2019; Qin & Chian, 2018), (ii) the limit equilibrium (e.g. Budhu & Al-Karni, 1993; Chen et al., 2007; Saha & Ghosh, 2015; Kurup & Kolathayar, 2018, Pakdel et al., 2021), (iii) the characteristic method (e.g. Kumar & Mohan Rao, 2002; Cascone & Casablanca, 2016) and (iv) the numerical methods (e.g. Pane et al., 2016; Saha et al., 2021; Boufarh et al., 2020). Moreover, the earthquake force within a soil mass was characterized primarily based on: (i) the pseudo static methods, (ii) the pseudo dynamic methods), and (3) the fully dynamic analyses. All these literature studies indicated that the seismic bearing capacity decreases significantly with increasing the horizontal seismic acceleration coefficient.

The seismic bearing capacity analysis is usually conducted for homogeneous soils and earthquake properties under the assumption of a deterministic set of parameters.

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Nevertheless, it is well known that the soil properties such as the shear strength parameters vary randomly, despite being in a single soil layer (Johari et al., 2017). Hence, the reliability analysis is an adequate way to consider the randomness of these properties and will provide a rational framework for adopting the appropriate bearing capacity that provides power tools to succor geotechnical designers in checking how reliable their designs.

Several studies have been carried out on the reliability analysis of shallow foundations under static loads, taking into account of the randomness of soil properties on the bearing capacity results in terms of the mean and standard deviation, i.e. the statistical moments, or the failure probability (e.g. Griffiths et al., 2002; Al-Bittar & Soubra, 2014; Puła & Chwała, 2015; Al-Bittar & Soubra, 2017; Jha, 2016; Al-Bittar et al., 2018; Brahmi et al., 2021; Puła & Chwała, 2018; Wu et al., 2019; Simões et al., 2020). In the dynamic bearing capacity context, a single available work in the literature has been conducted by Johari et al. (2017), to the authors' knowledge, where the spatial variability of the soil parameters was modelled using the random filed theory via the Cholesky decomposition approach. The authors showed that as the correlation length decreases, the mean value of the seismic bearing capacity increases while its standard deviation decreases. In addition, the mean seismic bearing capacity value increases and the standard deviation decreases when the correlation coefficient decreases.

It is aimed in this paper to conduct a reliability analysis of the seismic bearing capacity of shallow strip footings resting on soils with randomly varying properties (shear strength and unit weight) and earthquake parameters (horizontal seismic coefficients). The seismic bearing capacity formulas developed by Conti (2018) are considered for two kinds of soil supporting the shallow strip footing: a purely cohesive soil and a cohesive-frictional soil. The randomness of the soil parameters is captured by the Karhunen-Loève (KL) expansion method in the framework of the random field theory without considering variance reduction. The effects of the ACFs and the SOFs as well as the COV of the considered parameters on the probability density function (PDF), the probability of failure (P_f) and the statistical moments (mean, standard deviation and COV) of the seismic bearing capacity are investigated.

2. Basic equations for reliability analysis

The system reliability should be always described by a limit state function (or a performance function) "*Z (X)*", given as:

$$Z(X) = R(X) - S(X) \tag{1}$$

In Equation 1, "*R*" is the resistance, "*S*" the solicitation and "*X*" is the vector of the random input parameters. When $S(X) > R(X)$, which means that $Z(X) < 0$, the failure occurs (a failure domain), while when $S(X) < R(X)$, which means $Z(X) > 0$, the failure doesn't occur (a safe domain). In the case of $R(X) = S(X)$, which means $Z(X) = 0$, the system reliability is between the safe and unsafe domains. Therefore, this situation is called the limit state boundary.

On the other hand, the basic objective of the reliability analysis is to evaluate the probability of failure (P_f) for any chosen system. This objective can be achieved by the following equation:

$$P_f = P(Z < 0) = \int_{x \in F} \int \ldots \int I(x_1, x_2, \ldots, x_n) \, f_x(x_1, x_2, \ldots, x_n) \, dx_1 dx_2 \ldots dx_n \tag{2}$$

where I $(x_1, x_2, \ldots, x_n)$ is the indicator function with I$(x_1, x_2, \ldots, x_n)$ =1 if $x_1, x_2, \ldots, x_n$ are in the failure region and I$(x_1, x_2, \ldots, x_n)$ =0 if $x_1, x_2, \ldots, x_n$ are in the safe region.

In Equation 2, $f_x\ (x_1, x_2, \ldots, x_n)$ represents the joint probability density function for *X*. Mathematically, the integration of this equation is very difficult. For this reason, several methods are suggested in the literature to compute the probability of failure such as the FORM (First Order Reliability Method), SORM (Second Order Reliability Method), IS (Importance sampling), SS (Subset Simulations) and MCSs (Monte Carlo Simulations).

The spatial variability of the soil properties according to the random field theory are mainly described by the autocorrelation functions (ACFs), also called autocovariance functions. Five commonly used ACFs are reported in the literature as listed in Table 1. As shown in Table 1, different expressions of these functions exist resulting from diverse spatial correlations of the soil properties. In Table 1, ρ indicates the ACF, and T_x and T_y represent the absolute horizontal and vertical distances between two points within the soil unit, respectively. δ_h and δ_v indicate the horizontal and vertical SOF, respectively.

The spatial fluctuation of a soil property is most commonly and accurately modelled in the framework of the random field theory (Vanmarcke, 1977). Typically, it is described by a probability density function (PDF) and an autocorrelation function (ACF) (or covariance function). Most of the geotechnical issues require discrete fields for an accurate description of the required spatial variability. The commonly approaches adopted for the discretization of random field theory are the Karhunen–Loève expansion method, the Cholesky decomposition method and the local average subdivision method. In this paper, the Karhunen-Loève (KL) expansion method is followed to generate Gaussian random fields of the soil properties in one or two-dimensional space.

3. Numerical procedure

The numerical procedure followed in the present work consists in a probabilistic as well as reliability analysis taking into consideration of the spatial variability of the soil properties. The procedure combines the random field theory and simplified formulas of the seismic bearing capacity of strip footings resting on cohesive frictional soils or purely cohesive soils. The Karhunen–Loève (KL) expansion method for the generation of anisotropic Gaussian random fields of the soil properties in one or two-dimensional space is followed here (Constantine, 2022). The main steps followed to carry the seismic bearing capacity analysis are listed below:

1. Definition of the statistical inputs: the mean value, the variance (or COV), the number of simulation (Nsim) and the autocorrelation function. In the case of cohesive frictional soil, the cross-correlation coefficient ρ_{ij} between the cohesion and the frictional angle are defined. The horizontal and vertical scales of fluctuation (or autocorrelation lengths) are also defined here;
2. Discretization of the random fields: definition of the mesh around the edge of the footing (Figure 1);
3. Simulation of the Nsim realizations of the cross-correlated random field; an example of the generation of the shear strength parameters (cohesion c and friction angle φ) are displayed in Figure 2);
4. Introduction of the Nsim realizations of the random fields of the considered parameters into a simple Monte Carlo scheme to calculate the whole seismic bearing capacities using the formulas given in Table A1 in the Appendix 1;
5. Statistical response: outputs in terms of the mean, standard deviation, coefficient of variation of the bearing capacity as well as the probability density function are provided;
6. Obtaining the probability of failure: the bearing capacities resulted from the (Nsim) simulations of the random parameters is used in the Equation 1 of the limit state function where, at each time, the value of the applied load is changed to move from the safe domain ($p_f = 0$) to the failure domain ($p_f = 1$).

All these steps are coded in a Matlab program and the results are presented in tables and figures and then analyzed.

4. Validation examples

In this section, some validation examples are carried out in order to confirm the correctness of the obtained results, on one hand, and to compare the results provided by the simplified formulas with other numerical and rigorous methods, on the other hand.

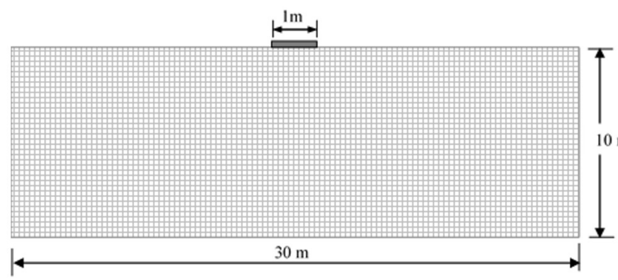

Figure 1. Mesh used for the discretization of random fields.

Table 1. Different types of autocorrelation functions (ACFs).

ACF type	Expression
Single exponential (SNE)	$\rho\left(\tau_x,\tau_y\right)=\exp\left[-2\left(\frac{\tau_x}{\delta_h}+\frac{\tau_y}{\delta_v}\right)\right]$
Cosine exponential (CE)	$\rho\left(\tau_x,\tau_y\right)=\exp\left[-\left(\frac{\tau_x}{\delta_h}+\frac{\tau_y}{\delta_v}\right)\right]\cos\left(\frac{\tau_x}{\delta_h}\right)\cos\left(\frac{\tau_y}{\delta_v}\right)$
Second-order Markov (SOM)	$\rho\left(\tau_x,\tau_y\right)=\exp\left[-4\left(\frac{\tau_x}{\delta_h}+\frac{\tau_y}{\delta_v}\right)\right]\left(1+\frac{4\tau_x}{\delta_h}\right)\left(1+\frac{4\tau_y}{\delta_v}\right)$
Squared exponential (SQE)	$\rho\left(\tau_x,\tau_y\right)=\exp\left[-\pi\left(\frac{\tau_x^2}{\delta_h^2}+\frac{\tau_y^2}{\delta_v^2}\right)\right]$
Binary noise (BN)	$\rho\left(\tau_x,\tau_y\right)=\begin{cases}\left(1-\frac{\tau_x}{\delta_h}\right)\left(1-\frac{\tau_y}{\delta_v}\right) & \text{for } \tau_x\le\delta_h \text{ and } \tau_y\le\delta_v\\ 0 & \text{otherwise}\end{cases}$

4.1 Random field realizations

First of all, an example of random fields of the cohesion and the friction angle discretized according to the normal distribution with statistical inputs as shown in Table 1 is carried out. A mesh of dimensions 128 x 64 is used for an element size 30 x 10 m. The attained values of the mean and the standard deviation using the Karhunen–Loève (KL) expansion method are compared in Table 2 to those attained by the local average subdivision method (LAS) (Alamanis & Dakoulas, 2021). As can be seen from Table 2, the KL method gives mean and standard deviation values of the cohesion and a standard deviation value of the friction angle closest to the exact values compared to the LAS method. The single realization of the random fields of the cohesion c and the friction angle φ are shown in Figure 2.

4.2 Verification of the statistical moments of the static bearing capacity

This example consists in the verification of the statistical moments of the bearing capacity of a shallow strip footing resting on soils with spatially and randomly varying properties with previous published results in the static case ($k_h = 0$). Due to the non-availability of the all statistical moments of the bearing capacity in a same work, different examples of the mean (μ) and standard deviation (σ) or coefficient of variation (COV) values of the strength parameters are considered.

Figure 3 confronts the variation of the mean normalized bearing capacity, versus the coefficient of variation of the undrained shear strength (cohesion c_u) (COV_{cu}), of a strip footing resting on a purely cohesive soil with mean value of c_u equal to 100 kPa to that published by Griffiths et al. (2002). These authors carried out the bearing capacity analyses with a conventional nonlinear finite element algorithm combined to the random field theory in conjunction with a Monte Carlo method for a strip footing of 1m width. As it is seen from Figure 3, the both results follow the same pattern with a maximum relative difference of about 28% for a $COV_{cu} = 50\%$.

The following example consists in the verification of the present results obtained by the simplified Conti (2018) formulas with those obtained by Luo & Bathurst (2017) when conducting a reliability bearing capacity analysis of a footing on cohesive soil slopes using the random finite element method (RFEM).

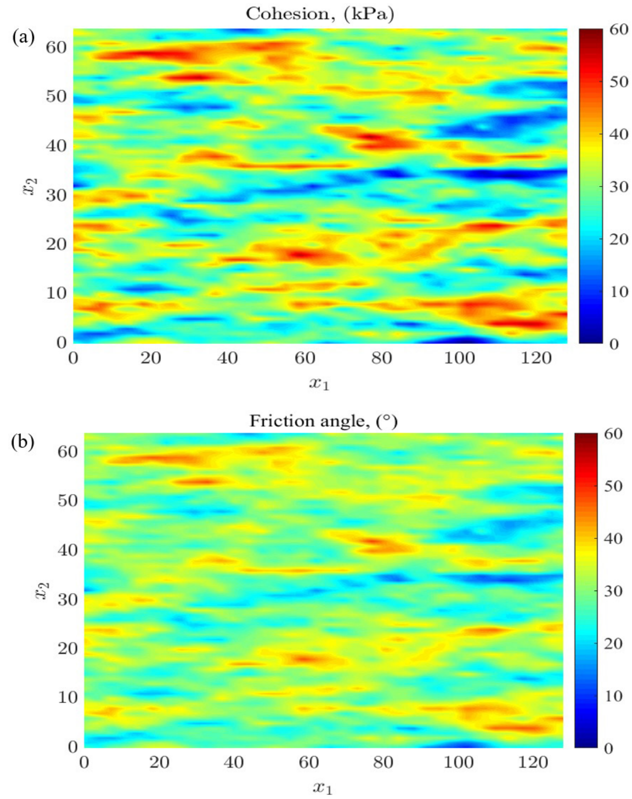

Figure 2. Simulation of Gaussian random field with δ_h= 20 m and δ_v= 2 m for: (a) soil cohesion with μ_c = 30 *kPa* and σ_c = 9 *kPa*; (b) soil friction angel with μ_ϕ = 30 *kPa* and σ_ϕ = 9.

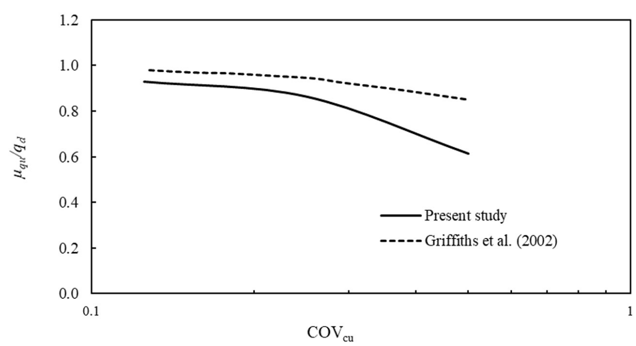

Figure 3. Comparison of the normalized bearing capacity of the present study with that of Griffiths et al. (2002) for a case of μ_{cu} = 100 kPa, $\delta_x = \delta_y$ = 2 m and k_h = 0.

Table 2. Comparison of exact and attained value of soil properties using KL method and LAS method.

Parameter	Exact mean (μ)	Attained μ by present KL	Attained μ by LAS	Exact standard deviation (σ)	Attained σ by present KL	Attained σ by LAS
Cohesion c (kPa)	30	30.0453	30.0746	9	8.7336	7.579
Friction angle φ (degree)	30	30.0508	30.0096	6	5.8752	5.111

Accordingly, Figure 4 shows the change of the COV of the bearing capacity factor (N_c) for a footing with 1m width resting of a purely cohesive soil with unit weight and mean cohesion equal to 20 kN/m^3 and 20 kPa, respectively. The showed results of Luo & Bathurst (2017) corresponds to the case of a strip footing on the level ground, i.e. without slope. As can be seen from this figure, the present results agree well with those of Luo & Bathurst (2017) and remain slightly lower as in the first example.

The following example consists in the verification of the statistical moments of the static bearing capacity obtained in the present study with those given by Cho & Park (2010). These authors studied the effect of the spatial variability of the cross-correlated strength parameters (c and φ) on the bearing capacity of a strip footing by mean of an approach integrating a commercial finite difference method and the random field theory. Cho & Park (2010) generated cross-correlated non-Gaussian random fields based on a Karhunen-Loève method. Not that in the deterministic analysis, Cho & Park (2010) estimated the bearing capacity to 1.01 MPa and jugged it in a good agreement with the value of 1.04 MPa obtained from the Terzaghi (1943) formula while in the present study it is estimated to 1.03 MPa. This results is evident since the used Conti (2018) formulas were based on the Terzaghi's equation for the vertical bearing capacity. Figure 5 and Figure 6 display the change of the mean value, the standard deviation and the COV of the bearing capacity versus the horizontal and vertical SOF, respectively. One may judge from these figures that the present results follow the patterns of the Cho & Park (2010) results but the present results are slightly higher than those of Cho & Park (2010). In other words, the present results based on the simplified Conti (2018) formulas are always conservative as far as cohesive-frictional soils are concerned due to the use of the all-minimum procedure as concluded by Conti (2018).

In addition, in the case of cohesive frictional soil, the mean static bearing capacity is almost unchanged as the horizontal SOF increases from 5 m to 30 m and this trend is comparable to that found by Cho & Park 2010 (Figure 5a). Dobrzański & Kawa (2021) found the same pattern for the case of purely cohesive soil for the same interval of the SOF. However, as shown in Figure 6a, the mean static bearing capacity fluctuates very slightly around a value of 1060 kPa as the vertical SOF increases from 1 m to 10 m comparable to the pattern found by Cho & Park (2010). Remember that this behavior was for COV$_c$ = 30% and COV$_\varphi$ = 20%. For purely cohesive soil however, Jha (2016) observed that, either for $\delta_h = \delta_v$ or δ_h different from δ_v, the mean normalized static bearing capacity decreases slightly as the horizontal SOF increases, reaches a minimum, and then increases also slightly. The maximum increment of variation is less that 1%. This trend was observed for two values of the COV$_{cu}$ (30% and 50%) but this reduction is less for 30% of COV$_{cu}$. A similar behavior was also observed in the study results carried by Puła & Chwała (2018).

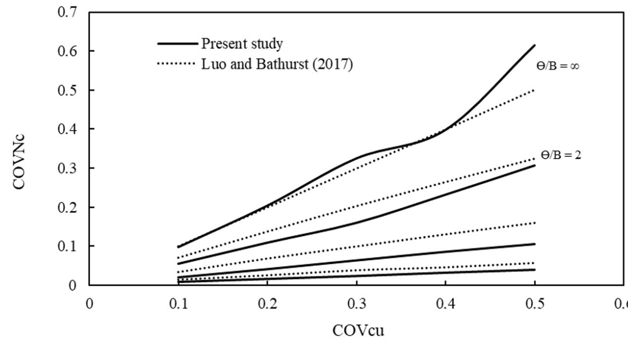

Figure 4. Comparison of the COV$_{Nc}$ of the present study with that of Luo & Bathurst (2017) for a case of μ_{cu} = 20 kPa, γ = 20 kN/m^3 and k_h = 0.

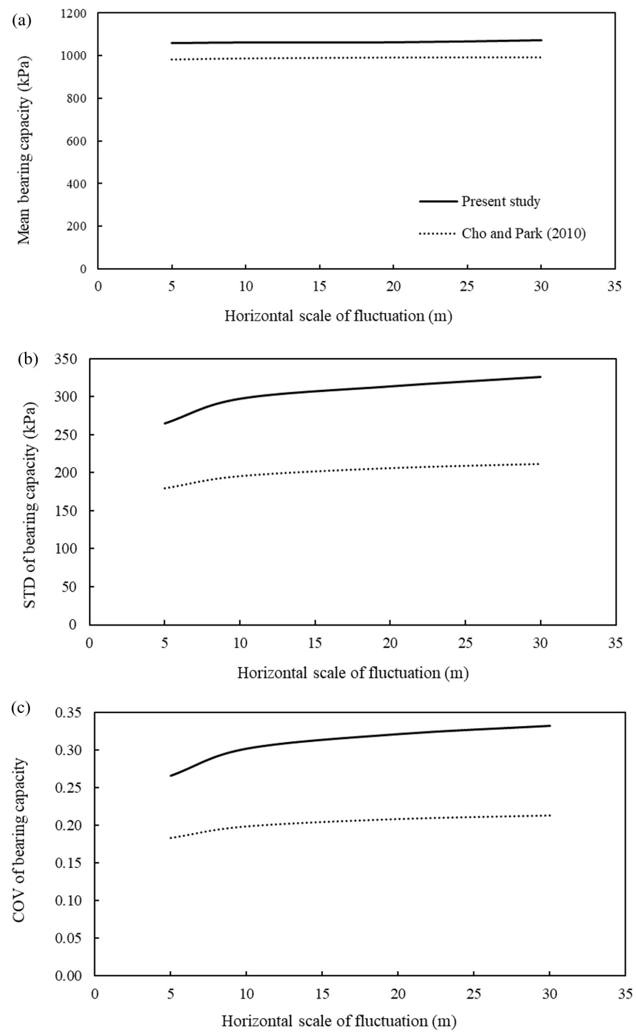

Figure 5. Verification of the statistical moments of the static bearing capacity of the present study with those of Cho & Park (2010) for a case of $r\ (c,\phi) = -0.5$, COV$_c$ = 30%, COV$_\phi$ = 20%, δ_v = 1 m and $k_h = 0$: (a) mean, (b) standard deviation and (c) coefficient of variation.

4.3 Verification of the failure probability of the static bearing capacity

This last validation example consists in the comparison of the failure probability of the static bearing capacity obtained from the present study with the results of Massih et al. (2008) and Krishnan & Chakraborty (2021). Note that the last authors explored the seismic bearing capacity of a strip footing over a *c-φ* soil using the finite element lower bound limit analysis formulation in conjunction with a modified pseudo-dynamic approach for the consideration of the seismic action. The soil properties (*c* and *φ*) are discretized spatially by mean of the Karhunen-Loève (KL) expansion method and the statistical responses are obtained via the Monte Carlo Simulation technique. However, Massih et al. (2008) investigated the ultimate bearing load of a *c-ϕ* soil in a reliability context using a pseudo-static approach with the help of the upper bound limit analysis. Note that in the present study, the random parameters are generated according to the normal distribution, as done in all the study, while for the others two papers for comparison they are obtained with the lognormal distribution. Figure 7 compares the failure probability (or CDF) plots of the ultimate bearing capacity for the static case of the three studies. Despite the normal distribution of the parameters in the present study in front of the lognormal one for the other two studies, it is clear from Figure 7 that the present results are the lowest while those of Massih et al. (2008) are the higher. In other words, the present results based on the use of the all-minimum procedure, are more conservative than those given by the lower bound method (Krishnan & Chakraborty, 2021) and consequently than those given by the upper bound method (Massih et al., 2008). Unfortunately, there are no results to compare in the seismic case.

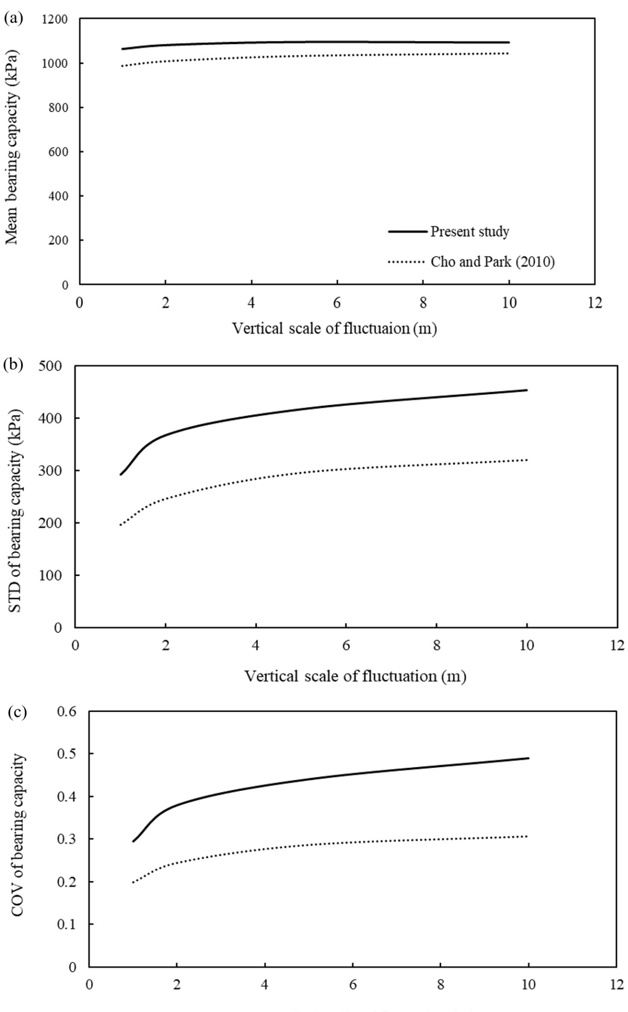

Figure 6. Verification of the statistical moment of the bearing capacity of the present study with those of Cho & Park (2010) for a case of *r (c,ϕ)* = − 0.5, COV_c = 30%, COV_ϕ = 20%, δ_h = 10 m and k_h = 0: (a) mean, (b) standard deviation and (c) coefficient of variation.

5. Results and discussions

The purpose of this section is to investigate the effect of the autocorrelation functions (ACFs), the scale of fluctuations (SOFs) and the coefficient of variation (COV) of the main parameters that govern the seismic bearing capacity on the probabilistic results for two different types of soil.

In order to achieve the objective, a shallow strip footing of 1 m width and subject to a seismic loading (q = 20 kPa) is considered. The shallow strip footing is assumed resting on two different kinds of soil. The first kind is a cohesive frictional soil ($c \neq 0$, $\varphi \neq 0$), while the second is a purely cohesive soil ($c = c_u$, $\varphi = 0$). Each soil is characterized by its statistical inputs as given in Tables 3 and 4.

5.1 Effect of Autocorrelation functions (ACFs) on seismic bearing capacity

In the case of a cohesive frictional soil, Figures 8a and 8b show the PDF and the failure probability, respectively, of the ultimate seismic bearing capacity for the five different types of autocorrelation functions (ACFs) given in Table 1.

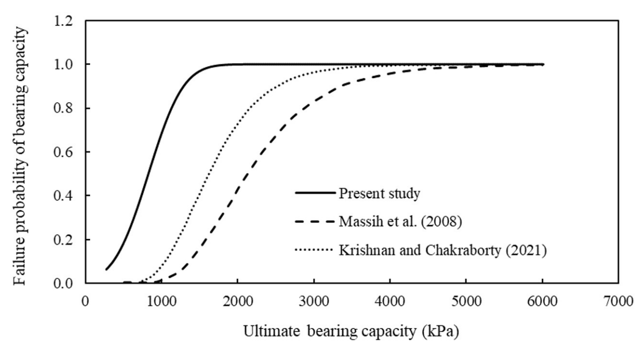

Figure 7. Verification of the failure probability of the bearing capacity of the present study with that of Massih et al. (2008) and Krishnan & Chakraborty (2021) for: *r (c, ϕ)* = −0.5, μ_ϕ = 30°, μ_c = *20 kPa*, COV_ϕ = 10%, COVc = 20% and k_h = 0.

Table 3. Statistical inputs of the cohesive frictional soil.

Parameter	Mean (μ)	Coefficient of variation (COV)	PDF
Cohesion c (kPa)	20	20%	Normal
Friction angle φ (degree)	30	10%	Normal
Horizontal seismic coefficient k_h	0.2	25%	Log-Normal

Table 4. Statistical inputs of the purely cohesive soil.

Parameter	Mean (μ)	Coefficient of variation (COV)	PDF
Undrained shear strength c_u (kPa)	20	20%	Normal
Horizontal seismic coefficient k_h	0.15	25%	Log-Normal

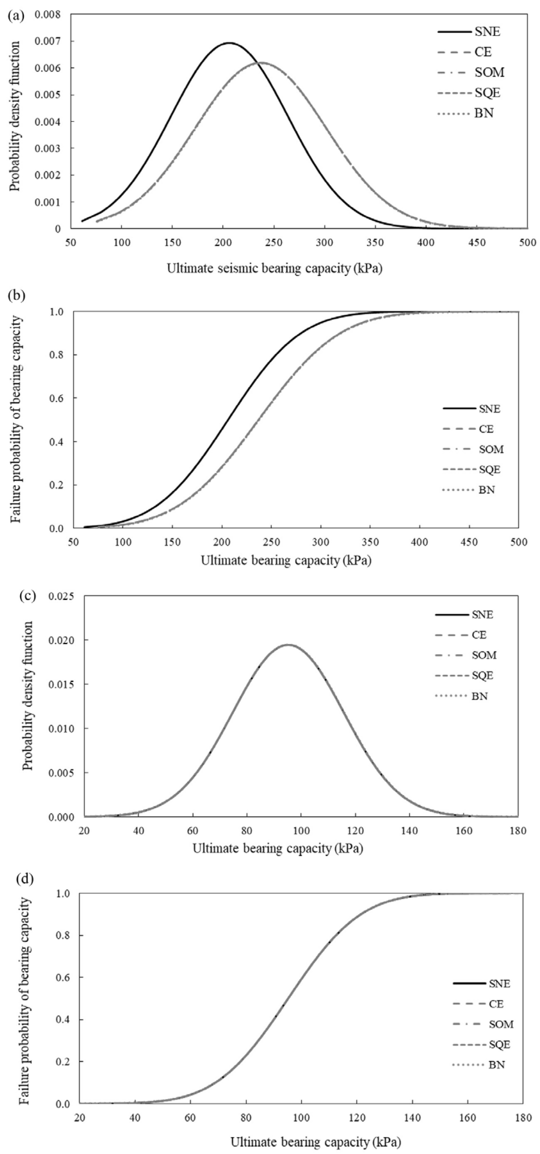

Figure 8. Effect of the type of the autocorrelation functions (ACFs), for δ_h = 20 m and δ_v = 2 m, on the probability density function (PDF) and the failure probability (P_f) of the seismic bearing capacity of a shallow strip footing resting on: (a) and (b) cohesive frictional soil, (c) and (d) purely cohesive soil.

One can note from these figures that-all the ACF types give the same variability (PDF) of the seismic bearing capacity and its corresponding probability of failure (P_f) for the purely cohesive soil and only the SNE ACF gives a PDF and a P_f different from the other four ACFs for the cohesive frictional soil.

Furthermore, the effect of the ACF type on the statistical moments of the seismic bearing capacity (mean value μ_{q_u}, standard deviation σ_{q_u}, coefficient of variation COV_{q_u}) is investigated as shown in Table 5. It is clear from Table 5, that only the SNE ACF provides statistical moments of the bearing capacity different from those provided by the others ACF types and smaller than them, for the cohesive frictional soil. This finding indicates that the commonly used SNE type of the ACFs provides conservative results. Note that only the SNE ACF will be used in the all subsequent applications.

5.2 Effect of the COVs of the seismic coefficients and the strength parameters on the seismic bearing capacity

In the case of a cohesive frictional soil, Figures 9a, 9b and 9c show the PDF of the seismic bearing capacity for various values of the COV of the seismic coefficient (COV_{kh}), the cohesion (COV_c) and the friction angle (COV_ϕ), respectively. For each one of the three Figures 9a, 9b and 9c, the COV of the concerned parameter is varied while the COV_s of the other two parameters are kept equal to the values given in Table 3. The results indicate that the increase in the COV of the cohesion or the friction angle increases the variability of the seismic bearing capacity while this variability remains unchanged when the COV of the seismic coefficient increases.

Moreover, it is found that the increment of variability is more significant for the friction angle. The statistical moments of the seismic bearing capacity are also sensitive to the randomness of the soil parameters as shown in Tables 6 and 7. For example, by increasing the COV_c from 10% up to 20% (with keeping the referred values of COV_φ and COV_{kh} equal to 10% and 25%, respectively (Table 6), the COV of the seismic bearing capacity (COV_{q_u})

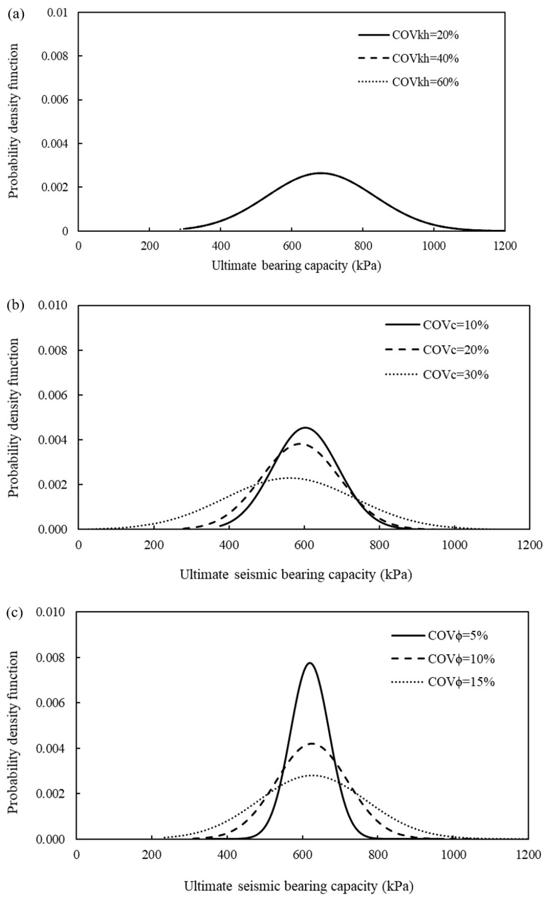

Figure 9. The PDF of the seismic bearing capacity of strip footing resting on cohesive frictional soil for δ_h = 20 m and δ_v = 2 m for various COVs of: (a) seismic coefficient, (b) cohesion, (c) frictional angle.

increases by about 22.49%. While, by increasing the COV_{φ} from 5% up to 10% (with keeping the referred values of COV_c and COV_{kh} equal to 20% and 25%, respectively), it is found that the COV_{qu} increases by about 32.80%. Otherwise, by increasing the COV_{kh} from 20% up to 40%, the COV_{qu} increases by only 0.56%. However, in the case of a purely cohesive soil, Figures 10a and 10b show the PDF of the undrained seismic bearing capacity for various values of the COV of the seismic coefficient (COV_{kh}) and the undrained shear strength (COV_{cu}), respectively. Each figure is drawn in the same way as in the previous case (cohesive frictional soil). The results show that the increase in the COV_{cu} increases the variability of the seismic bearing capacity (Figure 10b). Similarly, to the previous case of a cohesive frictional soil, by increasing the COV_{cu} from 15% up to 20% (with keeping the referred value of COV_{kh} equal to 25%), the COV_{qu} increases by about 35.22% (Table 7). While, the increment of the COV_{kh} from 20% up to 40% does not influence the COV_{qu}. (Table 7). As an explanation of this result, the dispersion of the ultimate seismic bearing capacity may depend on the choice of the probability distribution on the horizontal seismic coefficient (k_h). In fact, Massih et al. (2008) showed that the probability distribution of the punching safety factor for a shallow strip footing under a vertical load is significantly affected when an exponential distribution is chosen for the seismic coefficient. However, no significant effects were observed when different values of the coefficient of variation of the extreme value distribution for the seismic coefficient (20%, 40% and 60%) were used.

Table 5. Effect of the type of the autocorrelation functions (ACFs) on the statistical moments of the seismic bearing capacity of a shallow strip footing for δ_h = 20 m and δ_v = 2 m.

ACF type	Cohesive frictional soil			Purely cohesive soil		
	μ_{q_u} (kPa)	σ_{q_u} (kPa)	COV_{q_u} (%)	μ_{q_u} (kPa)	σ_{q_u} (kPa)	COV_{q_u} (%)
SNE	206.17	57.55	27.91	95.21	20.51	21.54
CE	237.08	64.47	27.19	95.21	20.51	21.54
SOM	237.07	64.47	27.19	95.21	20.51	21.54
SQE	237.07	64.47	27.19	95.21	20.51	21.54
BN	237.07	64.47	27.19	95.21	20.51	21.54

Table 6. Effect of the COV of the seismic coefficient (COV_{kh}), cohesion (COV_c) and frictional angle (COV_{ϕ}) on the statistical moments of the seismic bearing capacity of shallow strip footing resting on cohesive frictional soil for δ_h = 20 m and δ_v = 2 m.

COV_{k_h} (%)	COV_c (%)	COV_f (%)	μ_{q_u} (kPa)	σ_{q_u} (kPa)	COV_{q_u} (%)
20	20	10	678.96	155.03	22.83
40	20	10	677.77	155.65	22.96
60	20	10	675.52	156.72	23.20
25	10	10	584.59	97.77	16.72
25	20	10	563.50	115.40	20.48
25	30	10	509.64	153.44	30.11
25	20	5	571.70	89.92	15.73
25	20	10	563.56	117.72	20.89
25	20	15	549.17	155.85	28.38

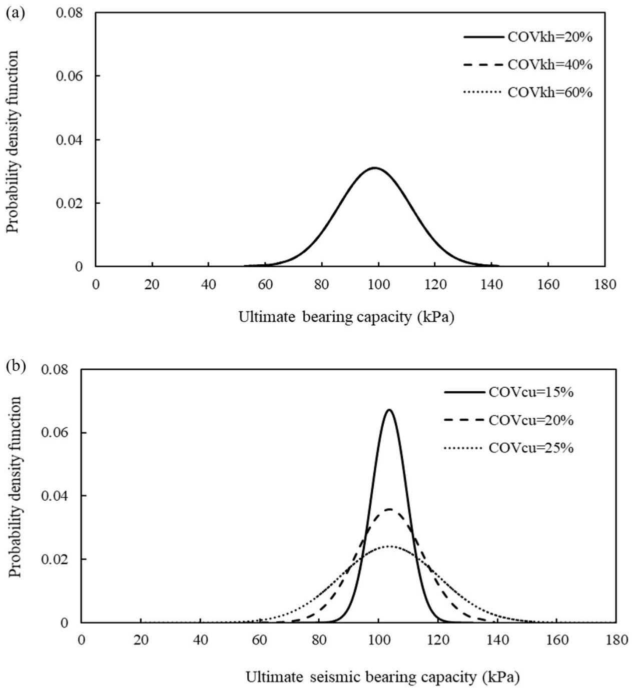

Figure 10. The PDF of seismic bearing capacity of strip footing resting on purely cohesive soil for δ_h = 20 m and δ_v = 2 m for various COV_s of: (a) seismic coefficient, (b) undrained shear strength.

5.3 Effect of scale of the fluctuations (SOFs) on the seismic bearing capacity

Figure 11 illustrates the effect of the variation of the horizontal and the vertical scale of fluctuations (SOFs) on the PDF and the failure probability of the seismic bearing capacity of a shallow strip footing resting on a cohesive frictional soil. The results exhibit that the PDF is more spread out as the vertical scale of fluctuations increases. In addition, the failure probability is more sensitive to the vertical SOF than it is to the horizontal SOF.

While in the case of a purely cohesive soil, Figure 12 shows the effect of the horizontal and vertical SOF on the PDF and the failure probability of the seismic bearing capacity. Similarly to the case of the cohesive frictional soil, it was found that the PDF is less spread out as the horizontal SOF increases and that the failure probability is more sensitive to the increase of the vertical SOF than it is for the horizontal SOF. It may be concluded from Figure 12 that, the effect of the vertical SOF on the PDF and the failure probability is much more significant than that of the horizontal SOF.

Tables 8 and 9 show the influence of the SOFs on the statistical moments (mean $_{q_u}$, standard deviation σ_{q_u} and coefficient of variation COV_{q_u}) of the seismic bearing capacity.

Table 7. Effect of the COV of the seismic coefficient (COV_{kh}) and the undrained shear strength (COV_{cu}) on the statistical moments of the seismic bearing capacity of a shallow strip footing resting on a purely cohesive soil for δ_h = 20 m and δ_v = 2 m.

COV_{k_h} (%)	COV_{cu} (%)	μ_{q_u} (kPa)	σ_{q_u} (kPa)	COV_{qu} (%)
20	20	98.77	12.87	13.36
40	20	98.77	12.87	13.36
60	20	98.77	12.87	13.36
25	15	100.49	10.12	10.08
25	20	99.09	13.50	13.63
25	25	94.20	16.92	17.97

Table 8. Effect of the horizontal SOF on the statistical moments of the seismic bearing capacity of a shallow strip footing for δ_v = 6 m.

Soil type	Cohesive frictional soil			Purely cohesive soil		
δ_h (m)	μ (kPa)	σ_{q_u} (kPa)	COV_{q_u} (%)	μ_{q_u} (kPa)	σ_{q_u} (kPa)	COV_{q_u} (%)
20	561.14	114.61	20.43	98.15	13.55	13.81
40	560.27	124.53	22.23	97.48	13.60	13.96
60	568.76	137.88	24.24	98.30	13.81	14.05
80	568.29	131.01	23.05	98.52	13.99	14.21
100	559.05	127.71	22.85	97.73	14.50	14.85

Table 9. Effect of the vertical SOF on the statistical moments of the seismic bearing capacity of a shallow strip footing for δ_h = 60 m.

Soil type	Cohesive frictional soil			Purely cohesive soil		
δ_v (m)	μ_{q_u} (kPa)	σ_{q_u} (kPa)	COV_{q_u} (%)	μ_{q_u} (kPa)	σ_{q_u} (kPa)	COV_{q_u} (%)
2	561.14	114.61	20.43	98.15	13.55	13.81
4	573.68	141.23	24.62	98.79	15.14	15.33
6	553.93	159.89	28.86	98.78	16.08	16.29
8	563.33	151.40	26.88	99.46	16.40	16.49
10	556.56	155.99	28.03	98.39	16.52	16.80

The results highlight that, the mean seismic bearing capacity fluctuates slightly so that it decreases, increases and then decreases for a variation of the horizontal SOF between 20 m and 100 m and that of the vertical SOF between 2 m and 10 m as may be observed from Table 8 and Table 9, respectively. The increment of variation is between 0.4% and 2% for the both two-soil types and for the both horizontal and vertical SOFs. A similar pattern was found by Chwała & Puła (2020) when evaluating the static baring capacity of a shallow foundation in the case of a two-layered soil where the spatial variability in the soil strength parameters was considered only for the bottom purely cohesive layer.

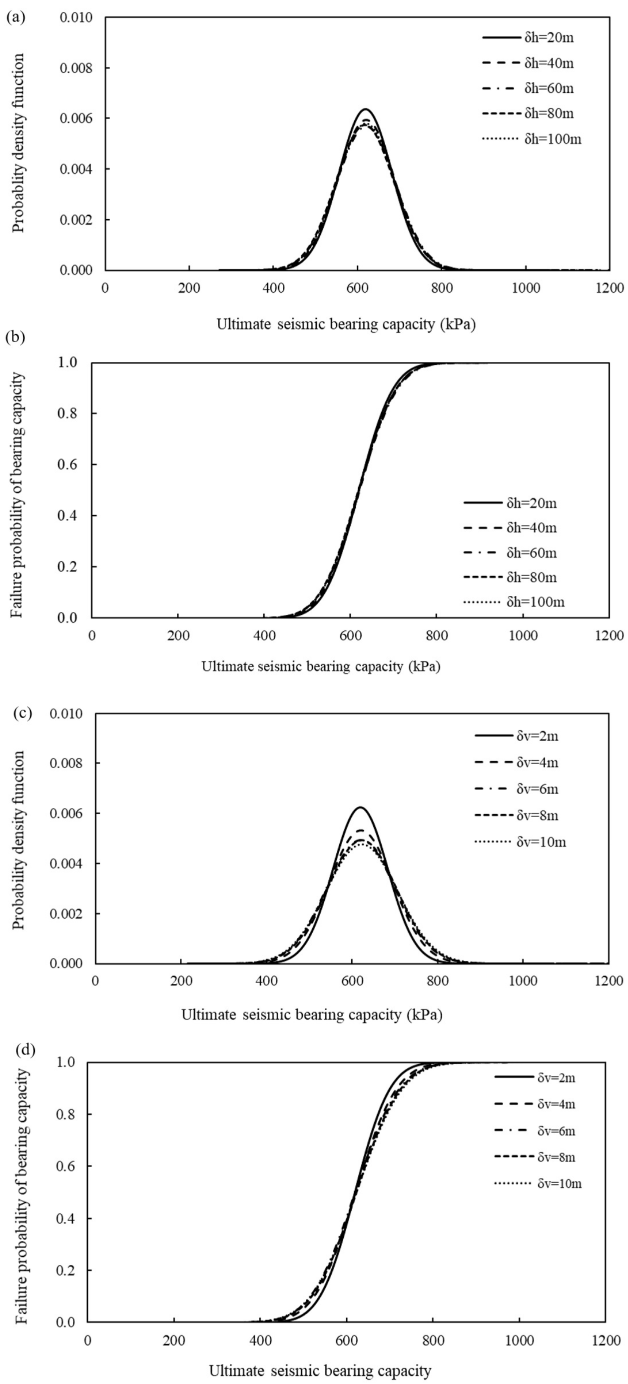

Figure 11. Probability density function and failure probability of the seismic bearing capacity of a strip footing resting on cohesive frictional soil for various values of: (a) and (b) horizontal SOF and $\delta_v = 2$ m, (c) and (d) vertical SOF and $\delta_h = 20$ m.

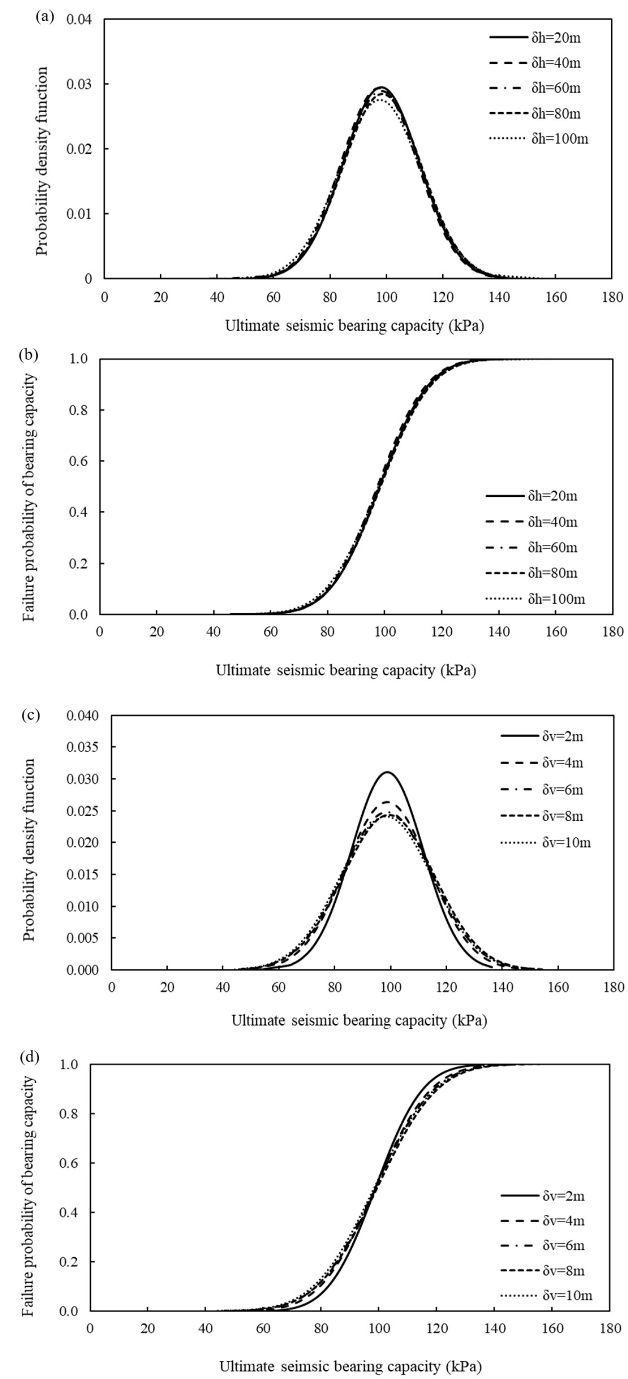

Figure 12. Probability density function and failure probability of the undrained seismic bearing capacity of a strip footing resting on a purely cohesive soil for various values of: (a) and (b) horizontal SOF and $\delta_v = 2$ m, (c) and (d) vertical SOF and $\delta_h = 20$ m.

On the other hand, the standard deviation of the seismic bearing capacity (σ_{qu}) increases as the horizontal or vertical SOF increases for the both kinds of soil. Nevertheless, the coefficient of variation (COV_{qu}) increases for the purely cohesive soil while it fluctuates for the cohesive frictional soil as the horizontal or vertical SOF increases.

6 Conclusions

This paper studied the seismic bearing capacity of a shallow strip footing by taking into account of the randomness of the shear strength properties and the seismic coefficient. The study is carried out in the framework of the random field theory through a reliability analysis of the seismic bearing capacity of a shallow strip footing assumed resting on two kinds of soils: a purely cohesive soil and a cohesive frictional soil. The Karhunen-Loève (KL) expansion method has been used to discretize the randomness of the soil parameters. The results have been obtained in terms of the statistical moments, the probability density function and the failure probability of the seismic bearing capacity, considering the effect of the ACFs, the SOFs and the coefficient of variation of the considered random parameters. The most important conclusions that can be drawn out from this study are as follows:

- Only the SNE ACF provides statistical moments of the bearing capacity that are different from those provided by the others used ACF types and are conservative for the cohesive frictional soil while for the purely cohesive soil all the ACF types give same results;
- The increase in the coefficients of variation of the cohesion or the friction angle increases the variability of the seismic bearing capacity while this variability remains unchanged when the COV of the seismic coefficient increases;
- The mean seismic bearing capacity fluctuates slightly as the horizontal SOF varies between 20 m and 100 m and the vertical SOF varies between 2 m and 10 m such that the increment of variation is less than 2% for the both two-soil types and for the both horizontal and vertical SOFs.

The present study served as a verification of the reliability of the used simplified formulas through comparisons with results of rigorous methods, which can make these them effective and suitable for the design practice.

Declaration of interest

The authors have no conflicts of interest to declare. All co-authors have observed and affirmed the contents of the paper and there is no financial interest to report.

Authors' contributions

Faiçal Bendriss: Writing original draft, Investigation, Discussion of results, Writing program of all calculations. Zamila Harichane: Methodology, Supervision, Discussion of results, Review & editing.

Data availability

Data availability is not applicable for this original research article.

List of symbols

c	Cohesion
c_u	Undrained shear strength
k_h	Horizontal seismic acceleration coefficient
ACF	Auto correlation function
BN	Binary noise
CE	Cosine exponential
COV	Coefficient of variation
COV_c	Coefficient of variation of the cohesion
COV_{cu}	Coefficient of variation of the undrained shear strength
COV_{kh}	Coefficient of variation of the horizontal seismic acceleration coefficient
COV_{Nc}	Coefficient variation of bearing capacity factor
COV_ϕ	Coefficient of variation of the friction angle
COV_{qu}	Coefficient of variation of the seismic bearing capacity
KL	Karhunen-Loève expansion method
LAS	Local average subdivision method
N_c	Bearing capacity factor
N_{sim}	Number of simulation
PDF	Probability density function
P_f	Probability of failure
RFEM	Random finite element method
SNE	Single exponential
SOF	Scale of fluctuation
SOM	Second-order Markov
SQE	Squared exponential
μ_ϕ	Friction angle
μ	Mean
μ_c	Cohesion mean value
μ_{qu}	Seismic bearing capacity mean value
μ_ϕ	Friction angle mean value
δ_h	Horizontal scale of fluctuation
δ_v	Vertical scale of fluctuation
σ	Standard deviation
σ_c	Standard deviation value of cohesion
σ_{qu}	Standard deviation value of seismic bearing capacity
σ_ϕ	Standard deviation value of friction angle
τ_x	Absolute horizontal distance between two points within the soil unit
τ_y	Absolute vertical distance between two points within the soil unit

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Appendix 1. Bearing capacity equations for shallow foundations.

Table A1. Seismic bearing capacity formulas for cohesive-frictional and purely cohesive soils.

	Cohesive-frictional soil	Purely cohesive soil
Ultimate seismic bearing capacity	$q_{uE} = \frac{1}{2}\gamma B N_{\gamma E} + cN_{cE} + qN_{qE}$	
Seismic bearing capacity factors	$N_{qE} = e_q^k N_{qS}$	$N_{qE} = e_q^k N_{qS}$
	$N_{cE} = e_c^k N_{cS}$	$N_{cE} = e_c^k N_{cS}$
	$N_{\gamma E} = e_\gamma^k N_{\gamma S}$	$N_{\gamma E} = e_\gamma^k$
Static bearing capacity factors	$N_{qS} = \left(\frac{1+sin\varphi}{1-sin\varphi}\right)e^{\pi tan\varphi}$	$N_{qS} = 1$
	$N_{cS} = \left(N_{qS} - \quad\right)cot\varphi$	$N_{cS} = 2+\pi$
	$N_{\gamma S} = 1.5\left(N_{qS}-1\right)tan\varphi$	$N_{\gamma S} = 0$
Soil inertia	$e_q^k = \left(1-\frac{k_h}{tan\varphi}\right)^{\left(0.37 tan\varphi^{0.5}\right)}$	$e_q^k = 1 - a_q\left(\frac{k_h}{k_{h,lim}}\right) - b_q\left(\frac{k_h}{k_{h,lim}}\right)^2$
		$e_c^k = 1$
		$e_\gamma^k = -a_\gamma\left(\frac{k_h}{k_{h,lim}}\right) - b_\gamma\left(\frac{k_h}{k_{h,lim}}\right)^2$
		Where:
		$a_q = 0.75k_{h,lim}$ $b_q = 1.4k_{h,lim}$
	$e_c^k = 1$	$a_\gamma = 1.75k_{h,lim}$ $b_\gamma = 1.4k_{h,lim}$
	$e_\gamma^k = \left(1-\frac{k_h}{tan\varphi}\right)^{0.47}$	$k_{h,lim} = \frac{c_u}{\gamma\left(D+\frac{B}{2}\right)}$