TECHNICAL PAPER

ECONOMIC OPTIMIZATION METHOD TO DESIGN TELESCOPE IRRIGATION OF MULTIPLES OUTLETS

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ABSTRACT: In this study is presented an economic optimization method to design telescope irrigation laterals (multidiameter) with regular spaced outlets. The proposed analytical hydraulic solution was validated by means of a pipeline composed of three different diameters. The minimum acquisition cost of the telescope pipeline was determined by an ideal arrangement of lengths and respective diameters for each one of the three segments. The mathematical optimization method based on the Lagrange multipliers provides a strategy for finding the maximum or minimum of a function subject to certain constraints. In this case, the objective function describes the acquisition cost of pipes, and the constraints are determined from hydraulic parameters as length of irrigation laterals and total head loss permitted. The developed analytical solution provides the ideal combination of each pipe segment length and respective diameter, resulting in a decreased of the acquisition cost.

KEYWORDS: conduits in series, minimum cost, Lagrange multipliers.

OTIMIZAÇÃO ECONÔMICA DE CONDUTOS TELESCÓPICOS COM MÚLTIPLAS SAÍDAS

RESUMO: O presente trabalho teve como objetivo desenvolver um procedimento de cálculo para otimização econômica, aplicado ao dimensionamento de linhas laterais de irrigação telescópicas com múltiplas saídas. A metodologia proposta pode ser empregada para a associação de condutos em série, sendo válida para o dimensionamento de trechos de tubulação com três diferentes diâmetros. Determinando-se a combinação ideal de comprimentos e respectivos diâmetros de cada trecho, obtém-se o resultado de mínimo custo na aquisição de tubulação. Para tal, utilizou-se a técnica dos multiplicadores de Lagrange, submetendo a função de custo às restrições do sistema, cujas variáveis de decisão são o comprimento da tubulação e a perda de carga total ao longo do percurso de escoamento. A técnica dos multiplicadores de Lagrange mostrou-se adequada para a otimização econômica do sistema em questão, quando comparada ao método-padrão de minimização de custos via função objetivo e respectivas restrições (solver), possibilitando o cálculo do comprimento de cada trecho da tubulação, bem como a redução nos custos de aquisição.

PALAVRAS-CHAVE: condutos em série, minimização de custo, multiplicadores de Lagrange.

INTRODUCTION

In general, the theoretical design of ducts refers to a diameter value commercially unavailable. In this case, there are three possible solutions: a) Adopting the commercial diameter immediately higher, a fact which provides lower load loss and higher fixed cost; b) Adopting the diameter immediately lower, resulting in higher load loss and lower cost of acquisition of the system; and c) Association of ducts in series, which is the most recommendable technical alternative in economic terms.

A duct is equivalent to another one or other number of ducts if pressure loss caused by the new system is equal to the original system, for the same flow rate (PORTO, 2006). Therefore, it is possible the dimensioning of systems in series and in parallel, when the pipeline is traversed by the

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same flow rate, or when there is a redistribution of flow between the ducts of the system, respectively.

When there is water intakes along the pipelines, they are treated as drilled ducts (AZEVEDO NETTO et al., 2005), distribution systems in motion (AZEVEDO NETTO et al., 2005) or, distribution systems in course (NEVES, 1974). An example of this condition is found in the lateral lines of irrigation, in which it is possible to make the combination of ducts in series in order to reduce costs with pipelines. The cost reduction with pipelines is more important in fixed irrigation systems, since in these systems the hydraulic network represents a large portion of the total cost of the system (SAAD et al., 1994). Therefore, reducing the costs of the hydraulic network (pipeline) without the increase of variable costs (energy) is an important point to be studied.

The calculation procedures for the association of ducts in lateral lines are usually presented for pipelines with only two sections (BERNARDO et al., 2006). For this situation, it is not possible a great economical solution, because there is only one condition that attends to the technical criteria for load loss. Similarly, when the number of sections is greater than two, there are innumerable combinations of length of pipeline of each diameter, for the same loss of load, which makes possible the adoption of the solution, which provides the minimization of the acquisition cost of the system.

In order to have a solution with technical viability, the optimization of the cost function should be submitted to the restrictions of the system (SAAD & MARCUSSI, 2006; BORGES JUNIOR et al., 2008; LUENBERGER, 2008; STEWART, 2010), since the minimum point of a linear function, with image being restricted to natural numbers, such as the cost function, results in pipeline length of a zero value, which does not refer to a solution with technical viability. The boundary conditions or the constraints are related to the entire length and the load loss of the lateral line.

There are several methodologies for the optimization of systems, among them it is highlighted the solution by using graphical processes, algorithms (simplex, the gradient method) and algebraic methods (Lagrange multipliers). The choice of the method to be employed depends on the particularities of each problem, that is, for those of greater complexity, it is required more sophisticated methods.

It may be used computational tools, such as "the Solver" from Microsoft Excel (obtaining the solution through attempts), in which is implemented the objective function, constraints and decision variables. This tool has the disadvantage of dependence on computational resources.

In view of the above mentioned, the aim of this study has been the development an algebraic calculation methodology for the dimensioning of a lateral line of irrigation, composed of three sections, using the optimization technique of Lagrange multipliers (STEWART, 2010) for minimizing the cost of acquisition of the system.

ISSUE DESCRIPTION

Hydraulic Modeling and Optimization

The dimensioning of perforated ducts, consisting of sections of different diameters may be done based on the concept of fictional flow, either to the association in series or in parallel, as it is shown in eqs.(1) and (2), respectively ((DENÍCULI et al., 2004). The chosen equation of load loss must attend particular technical requirements of each method, for all the sections of the duct, this is, an interesting alternative is the use of Darcy-Weisbach equation, which has no restrictions on the pipeline diameter.

$$\frac{{Q_1}^{m+1} - {Q_J}^{m+1}}{{D_e}^{n}} = \sum_{k=1}^{i} \frac{{Q_k}^{m+1} - {Q_{k+1}}^{m+1}}{{D_k}^{n}} \qquad (1)$$

$$\left(\frac{D_e{}^n}{L}\right)^{\frac{1}{m}} = \sum_{k=1}^{i}\left(\frac{D_k{}^n}{L_k}\right)^{\frac{1}{m}} \tag{2}$$

In which:

Q_k- flow rate of k passage;
D_e- calculated or equivalent diameter;
D_k- diameter of k passage;
L- entire length of the lateral line;
m e n- coefficients of the load loss equation;
Q_1- input flow rate of the lateral line;
Q_J- downstream flow rate of the lateral line;
$Q_{k+1} = Q_J$, when $k = i$, and
i- number of sections.

The equivalent diameter of the pipeline is obtained by applying an load loss equation, according to dimensioning criteria of lateral line of irrigation, which is the limitation of this parameter in 20% of the pressure of the transmitter, so that variation of flow does not exceed 10% along the lateral line, whether for irrigation by conventional or localized aspersion (BERNARDO et al., 2006).

In the case of the combination of three sections in series, as presented in Figure 1, eq.(1) may be transformed into eq.(6), considering the unitary flow (q_m), as presented in eqs.(3), (4) and (5). The commercial diameters D1 and D2 are, respectively, higher and lower than the calculated diameter.

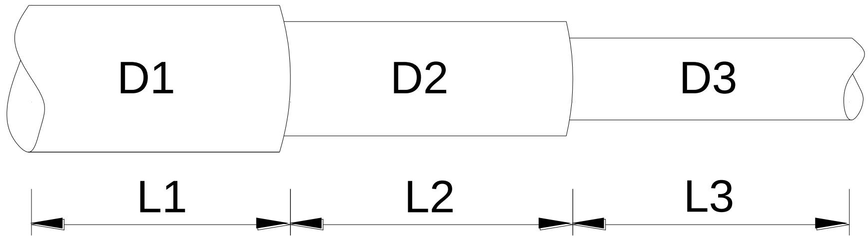


FIGURE 1. Schematic diagram of ducts in three sections.

The unitary flow rate is an assumption that the input flow of the system is continuously and constantly distributed along the pipe, i.e. the flow is distributed per unit of pipeline lengths, allowing calculating the loss of load along it.

In systems where the number of outputs of the lateral line is higher than 30 emitters, this approximation is quite accurate, differing in 5% of the load loss, calculated by means of the equation of Hazen-Williams in combination with the coefficient of Christiansen. However, the higher number of emitters and smaller spacing among them, the more accurate the calculation of load loss through this procedure (DENÍCULI et al., 2004).

$$Q_1 = q_m L \tag{3}$$

$$Q_2 = q_m(L_2 + L_3) \tag{4}$$

$$Q_3 = q_m L_3 \tag{5}$$

$$\frac{L^{m+1}}{D_e{}^n} = \frac{L^{m+1} - (L_2 + L_3)^{m+1}}{D_1{}^n} + \frac{(L_2 + L_3)^{m+1} - L_3{}^{m+1}}{D_2{}^n} + \frac{L_3{}^{m+1}}{D_3{}^n} \tag{6}$$

Equation (6) has no unique solution to satisfy the conditions for load loss established. However, the dimensioning may be done considering the minimization of the cost of the pipeline, i.e. the combination of the results, in order to provide lower cost of acquisition of the lateral line, reducing the total cost of the irrigation system.

Assuming that the price of the pipeline is proportional to the amount of required material for its manufacturing, its cost may be indirectly estimated, through the volume or weight of material that compose it. Therefore, the total cost of the pipeline of the lateral line, in proportional terms, may be calculated by using eq.(7):

$$C \approx \sum_{k=1}^{i} V_k L_k \tag{7}$$

In which:

C- yotal cost of the pipeline;

V_k- volume or unitary weight of material, which compose the pipeline, corresponding to the *k-th* section;

L_k - length of the corresponding section, and

i- number of sections.

The volume of material per unit of length may be determined by eq.(8):

$$V = \frac{\pi}{4} {D_{ex}}^2 - \frac{\pi}{4} D^2 \tag{8}$$

$$D_{ex} = \mathrm{D} + 2\mathrm{e} \tag{9}$$

In which:

D_{ex} - outside diameter of pipeline;

D - inner diameter of pipelines, and

e - thickness of the pipeline.

Substituting eq.(9) into (8), eq.(10) is obtained:

$$V = \pi e(D + e) \tag{10}$$

According to HIBBELER (2008), the thickness of the pipeline may be determined by eq.(11) (Mariotte Equation). This equation is based on the minimum thickness that pipeline should have to bear the nominal pressure, based on the circumferential tension and the resistance of the material, which the pipeline is formed. So that eq.(10) may be simplified in eq.(12):

$$e = \frac{PD}{2\sigma_c} \tag{11}$$

$$V = \pi \frac{P.D^2}{2\sigma_c}\left(1 + \frac{P}{2\sigma_c}\right) \tag{12}$$

In which:

P - internal pressure of the pipeline, and

σ_c - circumferential tension of the pipeline.

Assuming that all of the pipeline sections are composed of the same material, and that it is subject to the same pressure, i.e. it has the same nominal pressure value, eq.(12) results in eq.(13):

$$V \approx D^2 \tag{13}$$

Therefore, the minimum cost of the lateral line may be obtained by subjecting the eq.(7) to the system constraints (eqs.(14) and (15)), using the technique of Lagrange multipliers (λ), followed by the partial derivation. The point at which the system is optimized, is one in which the partial derivatives (eq.(17), (18), (19), (20) and (21)) of the resulting function (eq.(16)) have zero value:

$$\frac{L^{m+1}-(L_2+L_3)^{m+1}}{D_1{}^n}+\frac{(L_2+L_3)^{m+1}-L_3{}^{m+1}}{D_2{}^n}+\frac{L_3{}^{m+1}}{D_3{}^n}-\frac{L^{m+1}}{D_e{}^n}=0 \tag{14}$$

$$L_1+L_2+L_3-L=0 \tag{15}$$

$$\begin{aligned} F = V_1.L_1 + V_2.L_2 + V_3.L_3 \\ -\lambda_1\left(\frac{L^{m+1}-(L_2+L_3)^{m+1}}{D_1{}^n}+\frac{(L_2+L_3)^{m+1}-L_3{}^{m+1}}{D_2{}^n}+\frac{L_3{}^{m+1}}{D_3{}^n}-\frac{L^{m+1}}{D_e{}^n}\right) \\ -\lambda_2(L_1+L_2+L_3-L) \end{aligned} \tag{16}$$

$$\frac{\partial F}{\partial L_1}=V_1-\lambda_2=0 \rightarrow \lambda_2=V_1 \tag{17}$$

$$\frac{\partial F}{\partial L_2}=V_2-\lambda_1\left(\frac{-(m+1).(L_2+L_3)^m}{D_1{}^n}+\frac{(m+1).(L_2+L_3)^m}{D_2{}^n}\right)-\lambda_2=0 \tag{18}$$

$$\begin{aligned}\frac{\partial F}{\partial L_3}=V_3-\lambda_1\left(\frac{-(m+1).(L_2+L_3)^m}{D_1{}^n}+\frac{(m+1).(L_2+L_3)^m}{D_2{}^n}-\frac{(m+1)L_3{}^m}{D_2{}^n}+\frac{(m+1)L_3{}'}{D_3{}^n}\right. \\ -\lambda_2=0\end{aligned} \tag{19}$$

$$\frac{\partial F}{\partial \lambda_1}=\frac{L^{m+1}-(L_2+L_3)^{m+1}}{D_1{}^n}+\frac{(L_2+L_3)^{m+1}-L_3{}^{m+1}}{D_2{}^n}+\frac{L_3{}^{m+1}}{D_3{}^n}-\frac{L^{m+1}}{D_e{}^n}=0 \tag{20}$$

$$\frac{\partial F}{\partial \lambda_2}=L_1+L_2+L_3-L=0 \tag{21}$$

Substituting eq.(17) into (18), (17) and (22) into (19) results in eq.(22) and (23), respectively:

$$\lambda_1(m+1).(L_2+L_3)^m\left(\frac{1}{D_2{}^n}-\frac{1}{D_1{}^n}\right)=V_2-V_1 \tag{22}$$

$$\lambda_1(m+1)L_3{}^m\left(\frac{1}{D_3{}^n}-\frac{1}{D_2{}^n}\right)=V_3-V_2 \tag{23}$$

Dividing (23) by (22) and rearranging, results in eq.(24):

$$L_3=\left(\frac{(V_3-V_2)\left(\frac{1}{D_2{}^n}-\frac{1}{D_1{}^n}\right)}{(V_2-V_1)\left(\frac{1}{D_3{}^n}-\frac{1}{D_2{}^n}\right)}\right)^{\frac{1}{m}}(L_2+L_3) \tag{24}$$

As the linear volume and diameter of each section are constant, it is desirable that they are isolated from eq.(24), as it is shown in eq.(25). Thus, the same may be simplified in eq.(26):

$$k_1=\left(\frac{(V_3-V_2)\left(\frac{1}{D_2{}^n}-\frac{1}{D_1{}^n}\right)}{(V_2-V_1)\left(\frac{1}{D_3{}^n}-\frac{1}{D_2{}^n}\right)}\right)^{\frac{1}{m}} \tag{25}$$

$$L_3 = \frac{k_1\, L_2}{1 - k_1} \tag{26}$$

Rearranging eq.(20) results in eq.(27):

$$\frac{-(L_2 + L_3)^{m+1}}{D_1{}^{n}} + \frac{(L_2 + L_3)^{m+1} - L_3{}^{m+1}}{D_2{}^{n}} + \frac{L_3{}^{m+1}}{D_3{}^{n}} = \frac{L^{m+1}}{D_e{}^{n}} - \frac{L^{m+1}}{D_1{}^{n}} \tag{27}$$

Substituting eq.(26) in (27) results in eq.(28):

$$L_2 = \left(\frac{\dfrac{L^{m+1}}{D_e{}^{n}} - \dfrac{L^{m+1}}{D_1{}^{n}}}{\left[\left(1 + \dfrac{k_1}{1 - k_1}\right)^{m+1} \left(\dfrac{1}{D_2{}^{n}} - \dfrac{1}{D_1{}^{n}}\right) + \left(\dfrac{k_1}{1 - k_1}\right)^{m+1} \left(\dfrac{1}{D_3{}^{n}} - \dfrac{1}{D_2{}^{n}}\right) \right]} \right)^{\frac{1}{m+1}} \tag{28}$$

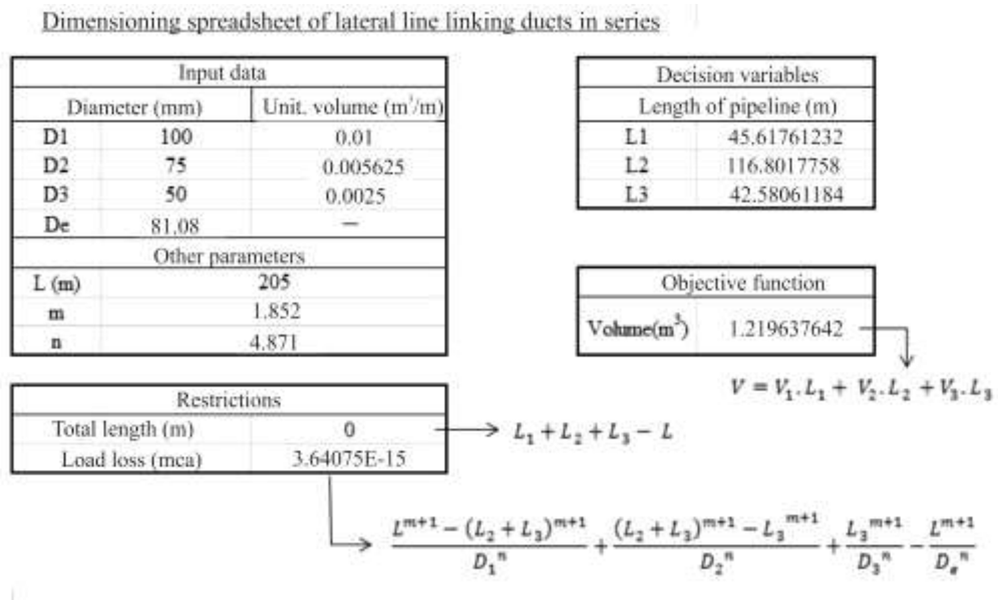
Rearranging eq.(21) results in eq.(29):

$$L_1 = L - (\, L_2 + L_3) \tag{29}$$

Thus, it is possible to dimension a perforated duct with minimal cost, by means of eq.(13), (25), (26), (28) and (29). The proposed model may be applied only in the case that the load loss equation used as a basis, not present restrictions at all sections of the lateral line.

Obtained model evaluation

To verify the hypothesis that the equations obtained algebraically through the optimization process (eqs.(26), (28) and (29)) provide the minimum cost of the lateral line, it was prepared a spreadsheet in Microsoft Excel software, with the SOLVER tool, using interative methods to find the optimal solution. The objective functions and restrictions were placed on the spreadsheet, as well as the decision variables, as it is shown in Figure 2.


Dimensioning spreadsheet of lateral line linking ducts in series

Input data		
Diameter (mm)		Unit. volume (m³/m)
D1	100	0.01
D2	75	0.005625
D3	50	0.0025
De	81.08	–
Other parameters		
L (m)	205	
m	1.852	
n	4.871	

Decision variables	
Length of pipeline (m)	
L1	45.61761232
L2	116.8017758
L3	42.58061184

Objective function	
Volume(m³)	1.219637642

$V = V_1.L_1 + V_2.L_2 + V_3.L_3$

Restrictions	
Total length (m)	0
Load loss (mca)	3.64075E-15

$L_1 + L_2 + L_3 - L$

$$\frac{L^{m+1} - (L_2+L_3)^{m+1}}{D_1{}^{n}} + \frac{(L_2+L_3)^{m+1} - L_3{}^{m+1}}{D_2{}^{n}} + \frac{L_3{}^{m+1}}{D_3{}^{n}} - \frac{L^{m+1}}{D_e{}^{n}}$$

FIGURE 2. Spreadsheet used to calculate the irrigation laterals arrangement for three diameters.

The implementation of SOLVER is shown in Figure 3, where restrictions were placed on the system, limiting the length of each section of pipeline in values greater than or equal to 0, so that negative values are not considered, since this situation has no physical meaning and provides an error in the equation that relates the load loss and length of each section. Other constraints were the same used in the development of optimization through the algebraic process.

As it may be seen in Figure 3, the target cell corresponds to that which it is located the objective function, or function that relates the length of each section of pipeline with the cost of the system (eq.(7)). In the variable cells are indicated the decision variables, i.e. the length of each section, which should be assigned any initial value, since it is a natural number, and attends the condition of total length of the lateral line.

Once entered restrictions, just click the button "*resolver/ solve*", that the lengths of each section of pipeline are shown in corresponding cells of the decision variables (Figure 2).

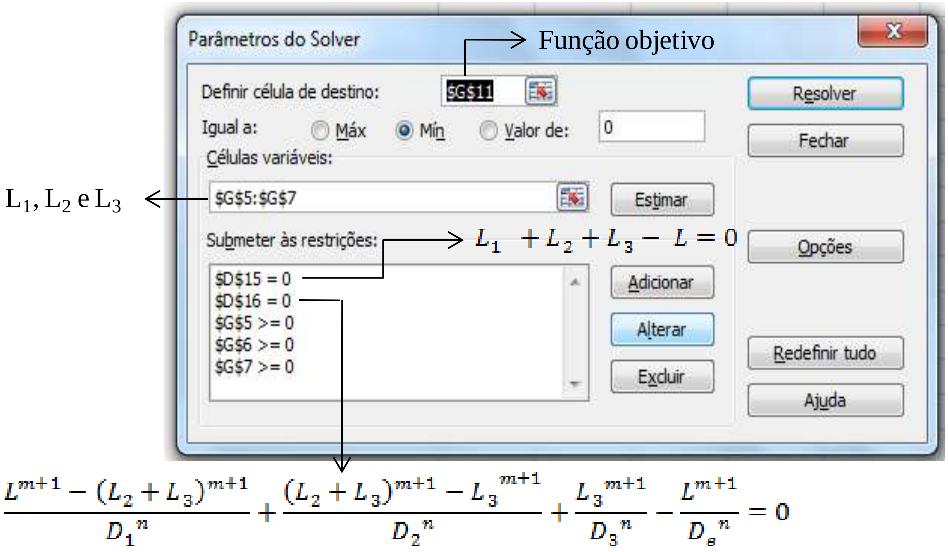


FIGURE 3. Excel SOLVER PARAMETERS screenshot, Brazilian Portuguese version.

For the example of dimensioning shown in Figure 2, it has as a result the total volume of 1.22 m^3 and lengths of 45.6, 116.8 and 42.6 m for the section 1, 2 and 3, respectively. The results obtained using the algebraic equations (eqs.(25), (27) and (28)) are practically identical to those obtained by the solution of the SOLVER, presenting differences only from the fourth decimal point. The accuracy of the Excel was 10^{-14}, tolerance of 5% and maximum number of interations equal to 100 (Figure 4).

FIGURE 4. SOLVER properties.

Cost analysis

Still regarding the example shown in Figure 2, an analysis was made which relates to the difference in the percent relative costs of the lateral line for different types of dimensioning (Figure 5). The analysis was performed for different equivalent diameters, limited in range of diameters D1 and D2 in the example shown in Figure 2. The difference in percent relative cost A1: the quotient of differences in cost between the duct with two sections and three sections, by the cost of the duct with three sections (eq.(30)); A2: the quotient of the difference in cost between the duct with diameters greater and three sections, by the cost of the duct with three sections (eq.(31)), and A3: the quotient of the difference in cost between the duct with diameter immediately higher and with two parts, by the cost of duct with two parts (eq.(32)).

$$A1 = \frac{\sum_{k=1}^{2} {D_k}^2 {L'}_k - \sum_{k=1}^{3} {D_k}^2 L_k}{\sum_{k=1}^{3} {D_k}^2 L_k} 100 \tag{30}$$

$$A2 = \frac{{D_1}^2 L - \sum_{k=1}^{3} {D_k}^2 L_k}{\sum_{k=1}^{3} {D_k}^2 L_k} 100 \tag{31}$$

$$A3 = \frac{{D_1}^2 L - \sum_{k=1}^{2} {D_k}^2 {L'}_k}{\sum_{k=1}^{2} {D_k}^2 {L'}_k} 100 \tag{32}$$

In which:

${L'}_1$ and ${L'}_2$ - length of section 1 and 2, respectively, calculated using eqs.(33) and (34) (DENÍCULI, et. al., 2004; BERNARDO et. al., 2006).

$${L'}_2 = L \left[\frac{\left(\frac{D_1}{D}\right)^n - 1}{\left(\frac{D_1}{D_2}\right)^n - 1} \right]^{\frac{1}{m+1}} \tag{33}$$

$${L'}_1 = L - {L'}_2 \tag{34}$$

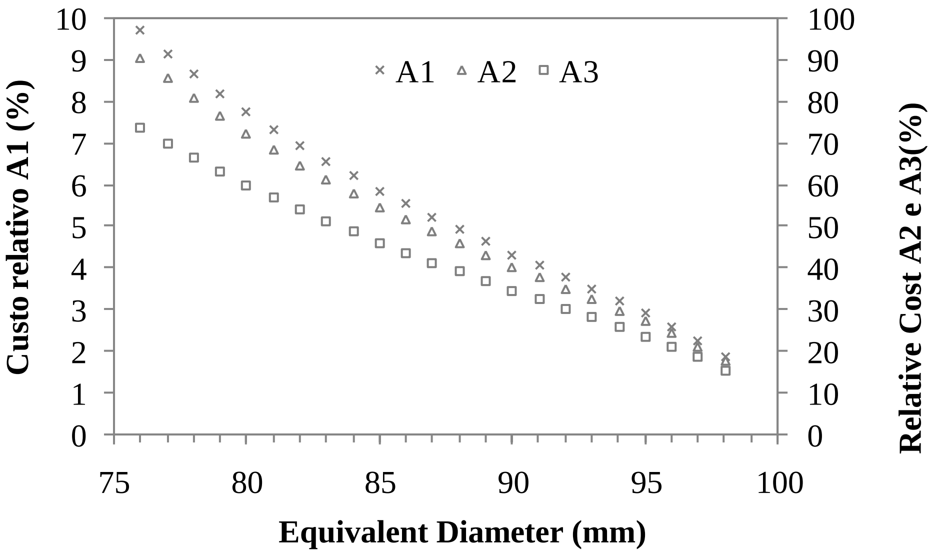


FIGURE 5. Lateral irrigation line cost comparing a situation of three diameters with another of equivalent diameter using data presented at Figure 2.

Analyzing Figure 5, there is evidence of association need of the ducts in the dimensioning of irrigation systems, whether the combination of two or three sections. Although not serving as a generic percentage of cost, in view that it is a particular case, it is clear the cost difference between the use of a diameter and three diameters / section (A2), ranging from 13.1 to 90.6%, with higher values corresponding to equivalent diameters close to the commercial diameter immediately below. According to BERNARDO et al. (2006), it may be adopted a smaller diameter, since the variation of pressure does not exceed 23.5% of the pressure of the transmitter.

Regarding the importance of optimizing the cost of the lateral line, it may be seen in Figure 5 (A1), that for combination with two sections the cost is associated with more than three sections, and this difference varies from 1.9 to 9.7%. It is noteworthy that the numerical results presented and discussed, refer only to the particular case of this example and should not be extrapolated to other cases.

CONCLUSIONS

The proposed calculating procedure allows the dimensioning of lateral lines with multiple outputs, so that the combination of lengths and respective diameters of each section of the pipeline provide the lowest acquisition cost.

The technique of Lagrange multipliers revealed adequate for the optimization of dimensioning of the irrigation lateral line.

REFERENCES

AZEVEDO NETTO, J.M.; FERNANDEZ, M.F.; ARAÚJO, R.; ITO, A.E. *Manual de hidráulica.* São Paulo: Edgard Blücher, 2005. 669 p.

BERNARDO, S.; SOARES, A.A.; MANTOVANI, E.C. *Manual de irrigação*. Viçosa: UFV, 2006. 625 p.

BORGES JÚNIOR, J.C.F.; FERREIRA, P.A.; ANDRADE, C.L.T.; HEDDEN-DUNKHORST, B. Computational modeling for irrigated agriculture planning. Part I: general description and linear programming. *Engenharia Agrícola*, Jaboticabal, v.28, n.3, p.471-482, 2008.

DENÍCULI, W.; SILVA, D.D. da; OLIVEIRA, R.A. de. *Hidráulica de condutos perfurados*. Viçosa: UFV, 2004. 93 p.

HIBBELER, R.C. *Resistência dos materiais.* São Paulo: Pearson Prentice Hall, 2008. 670 p.

LUENBERGER, D.G. *Linear and nonlinear programming*. New York : Springer, 2008. 546 p.

NEVES, E.T. *Curso de hidráulica.* Porto Alegre: Editora Globo, 1974. 577 p.

PORTO, R. de M. *Hidráulica Básica*. São Carlos: EESC/USP, 2006. 540 p.

SAAD, J.C.C.; MARCUSSI, F.F.N. Distribuição da carga hidráulica em linhas de derivação otimizadas por programação linear. *Engenharia Agrícola*, Jaboticabal, v.26, n.2, p.406-414, 2006.

SAAD, J.C.C.; TOMAZELA, C.; PERES, J.G.; PERES, F.C.; FRIZZONE, J.A. Otimização da rede hidráulica de um sistema de irrigação por gotejamento utilizando a programação linear. *Pesquisa Agropecuária Brasileira*, Brasília, v.29, n.5, p.797-805, 1994.

STEWART, J. *Cálculo*. São Paulo: Cengage Learning, 2010. 1077 p.