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ARTICLES

PRODUCTION INVENTORY MODELS WITH INTEGRATED STOCK AND PRICE DEPENDENT DEMANDS FOR DETERIORATIVE ITEMS

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ABSTRACT. This research takes into account production inventory models with price-dependent are developed: The first model uses integrated stock and price dependent demands, the second model uses stock dependent demand, and the third model uses price dependent demand. The models are built on the basis of a bipartition of the production cycle which in turn results in the holding cost. It was found that the holding cost is lower in integrated stock and price dependent demand compared to that of stock dependent demand and price dependent demand individually. Detailed mathematical models are presented for each model, as well as applicable illustrations are given for making the suggested technique clearer. In this scenario, the goal is to determine the order amounts and order intervals that will result in the lowest total cost. Each of the three models has its own individual sensitivity analysis offered. Visual Basic 6.0 was used in order to produce the required data.

Keywords: inventory, deterioration, stock dependent demand, price dependent demand, integrated and sensitivity analysis.

1 INTRODUCTION

Conventional models of production inventory control, including the EOQ (economic order quantity) model and EPQ (economic production quantity) models, are typically based on the idea of a constant and time-dependent (exponential, quadratic, linear, etc.) demand rate. However, in real practice, it was noticed that the rate of demand is typically determined by the price as well as the availability of the goods. The stock-dependent demand has an upwards sloping demand rate, in contrast to the price-dependent demand rate, which has a linear downward slope.

The realization that reduced selling prices often result in increased sales for a variety of items is the foundation of inventory models that include price-dependent demand rates. In addition, the demand for food products is often price-dependent. Many consumers will switch to a different brand or even a different food item when the price of these goods increases. However, in real life

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situations, most products will deteriorate during storage. Realistically, a company's resource is significantly associated with when it is employed owing to the return on investment. The time worth of money is taken into consideration when developing a degrading inventory model based on this interpretation.

The main purpose of the inventory model is to determine the optimum selling price, ordering frequency and preservation technology investment that maximizes the total profit. Ghare and Schrader (1963) were the first to investigate inventory issues based on the degradation of products and ongoing demand. In real-life inventory management challenges based on price-dependent and stock-dependent demand, a substantial number of authors have done comprehensive work. An inventory model for degrading products with a nonlinear price and a linear stock dependent market demand was explored by Mohammad Abdul Halim et al. (2021). In this case, language software may be used to address the associated optimization issue.

There are many other ways to express demand for a commodity. This article uses the idea of "price dependent demand," which is a combination of price-dependent and stock-dependent as well as other patterns such as $(a-\mathrm{bP})(x+\mathrm{yI}(t))$.With the purpose of constructing mathematical models for estimating the best quantity of inventory to be replenished in order to meet future demand, this work presents degrading-product demand models with price-dependent demand and exponential time-dependent demand. Three models are shown here: the first model uses integrated stock and price dependent demands, the second model uses stock dependent demand, and the third model uses price dependent demand. Each of the three models has its mathematical derivation, numerical demonstrations, and data given in Visual Basic 6.0, which is used in the presentation.

In addition to this, the following sections are included in the document: Section 2 provides an overview of important literature, while Section 3 provides the model's assumptions and notations. Section 4 introduces the inventory model and the best solution technique. Section 5 is devoted to a comparative examination. Sect. 6 concludes with a discussion of the findings and potential directions for further study.

2 LITERATURE REVIEW

The most recent and pertinent research is presented in this section. Since demand plays a significant role in the formation of deteriorating inventory, researchers have recognized and studied the variants of demand from the perspective of real-life situations. This is because demand plays a significant role in the formation of deteriorating inventory. For example, it's possible that demand fluctuates over time, is tied to supply, or is influenced by both.

Ghare and Schrader (1963) were the first researchers to study the effects of persistent demand and deterioration of things. Bernstein and Federgruen (2003) took into consideration a two-echelon distribution system in which a single supplier would distribute a product to N different retailers who would be in direct competition with one another. The demand rate at each store is dependent on the prices set by the other retailers; conversely, the price that each retailer is able to charge for its product is contingent on the sales volumes that are sought by the other shops. The provider maintains an adequate supply of goods by regularly placing orders (for purchases or manufacturing runs) with a third party that has an abundant supply. The items are then delivered to the various shops from that location. Carrying costs are incurred for all stocks, whereas fixed and variable costs are incurred for all supplier orders and transfers to the retailers from the suppliers. A model created by Teng and Chang (2005) referred to as EPQ explains circumstances where the demand rate is reliant on both existing stock levels and per-unit sales prices for degraded commodities.

Because there is a cap on the quantity of shelf and display space available, as well as an excessive amount of goods gives the customer a bad image, we restrict the number of items that may be shown at one time. The next step is to establish the essential circumstances to arrive at an ideal solution for the EPQ model that optimises earnings. Deng et al. (2008) enhanced this technique by introducing the inventory replenishment strategy across a limitless forecasting viewpoint with a negative exponential lead time crashing cost and taking time value into consideration. Panda and Maiti, (2009) constructed multi-item EPQ models with holding costs, stock dependent unit production, infinite production rate, and selling price dependent demand. These models also included an unlimited production rate. The manufacturing process has been updated to include flexibility and reliability considerations.

When the demand rate relies on the instantaneous inventory level, Toy and Chaudhuri (2009) developed two production inventory models for degrading products. These models are used in situations in which the inventory level is constantly changing. Not only does the amount of stock determine the production rate, but so does the amount of demand. The Weibull distribution degradation is employed in both of these models. Model I is designed and solved so that there are no shortages, whereas Model II is developed and solved so that there are shortages and backlogs.

Deteriorating products' optimum order quantity and selling price must be determined concurrently by Chun et al. (2010). On-display stock level, selling price per unit, and restricted shelf space are all thought to influence demand rate, as well as how quickly a product sells.

An inventory model for decaying products and selling price dependent demand was presented by Singh et al. (2011). Within this model, the inflationary environment and deterioration rate are addressed using a two-parameter Weibull distribution. Datta (2013) conducted research on an inventory management system for jointly determining product quality and selling price in a scenario in which some of the goods produced were flawed. It is presumable that just a portion of faulty products may be fixed or redone. The rate of demand is influenced not only by the product's quality but also by the price at which it is sold.

Alfares (2014) created solutions for gradual order replenishment using economic production quantity (EPQ). Stock-dependent demand models assume that demand is a linear function of the inventory level, which is the case in practice. It is assumed that the cost of storing each unit throughout the course of each period is a function of the entire duration in which the unit has been stored in models with variable holding costs.

Bhunia and Shaikh (2014) established two inventory models for degrading products with changing demand. This demand was based on the selling price of the item as well as the frequency with which it was advertised. In the first model, shortages are not permitted in any way, but in the second model, shortages are not only permitted but also partly backlogged at a variable rate that is depending on the amount of time that must pass until the arrival of the following lot. In each of these models, the rate of degradation is represented by a Weibull distribution with three parameters. An economic production quantity (EPQ) model with reworked faulty goods while a multi-shipment policy is in place was examined by Taleizadeh et al. (2015).

Kaliraman et al. (2017) created an inventory model that takes into account two warehouse models for degrading products with exponential demand rates. There is no reduction in the pace of degradation. This model takes into account both a leased and an owned warehouse. Mishra (2017) developed an EOQ inventory model that considers the demand rate as a function of stock and selling price. Shortages are permitted. Inventory models developed by Tripathi et al. (2017) allow for shortages while demand grows exponentially over time and degradation is also time-dependent. Unit cost and demand rate of production are thought to be correlated.

According to Shah et al. (2018), a price-sensitive stock demand scenario is a realistic circumstance. We have to deal with faulty products as the manufacturing process progresses. So, the production rate is an important factor in determining profit in the current model. Retail pricing and cycle time have a direct impact on profit. According to Singh and Rastogi (2018), in an inflationary climate, a production inventory model for degrading items may be used to predict demand and supply shortfalls. The pace at which a product is produced is determined by the amount of demand. The shortages experienced during the stock out period is presumed to be somewhat backlogged.

A production-inventory model created by Dharamender Singh (2019) has stock dependent and price dependent demand. Shortages are permitted and partly back-ordered at a rate that decreases with the length of time it takes for a new supply of an item to arrive. This study has been refined by Qi Feng (2019) to a class of intuitively attractive policies, through which the price decreases in the post-order stock level. In the context of stochastic functions and stochastic demand, the objective function is concave along such price routes, resulting in a simple base stock list pricing strategy. It also determines the top and lower bounds of a possible set of decreasing pricing pathways and demonstrates that any decreasing road outside of this set is always dominated by a path within the set in terms of profit performance.

There are several factors that affect the demand for a certain product, such as the amount of on-hand stock in a shop and its selling price, which were studied by Ruidas et al. (2020). Carbon tax policy and Remanufacturing subsidy policy were explored by Kaiying Cao et al. (2020) for two enterprises in a dual-channel supply chain selling both remanufactured and new items in a supply chain.

There are two categories of clients that a manufacturer serves: those who have a long-term commitment and are separated into many backorder classes, as well as those who do not have a long-term commitment and are grouped together as a single lost sales class. An inventory management and integrated production structure is used to address this issue. Wang and Zhang (2021) conducted research to determine how the selection of a distribution system's partners affected the value of the information that was shared between the system's one capacitated make-to-stock manufacturer along with its two retailers. A manufacturer may more precisely allocate inventory based on more predictable orders from the high-priority retailer whenever the store is the lone partner with a greater shortfall cost. Singh P et al. (2022) developed a production inventory model for deteriorating items with price-stock dependent demand rate under complete backlog. Mahata and Debnath (2022) addressed a single item two-level supply chain inventory model considering deterioration during carrying of deteriorating item from a supplier's warehouse to a retailer's warehouse as well as deterioration in the retailer's warehouse. Poswal P et al. (2022) studied demand function as price sensitive and stock dependent demand and a crisp model developed and then fuzzified the ordering cost, holding cost, deteriorative rate and shortage cost as a fuzzy trapezoidal number. Nita H. Shah et al. (2022) formulated an inventory model for perishable products for price and stock-dependent demand rate along with greening efforts. Qusay H. Al-Salami et al. (2022) developed an inventory control model-based Genetic Algorithm (GA) to minimize the total annual inventory cost function developed explicitly for the proposed model.

3 ASSUMPTIONS AND NOTATIONS

These assumptions and notations describe a single-item deterministic inventory model in order to degrade goods with stock and price dependent demand (PDD) rate.

3.1 Assumptions

(1) Because the replenishment rate is indefinite, so lead time is insignificant, and duration of all replenishment cycles is the same, only a typical planning cycle with a length of T is taken into consideration (which means that the planning horizon is [0, T]). (2) The planning horizon for the inventory is limitless, yet the inventory system only comprises a single stocking point and a single commodity. (3) In order to prevent revenue from being wasted, shortages are not permitted. (4) sales price is inversely proportionate to demand. i.e., D = $(a - \mathrm{bP})(x + \mathrm{yI}(t))$ which is the selling price function with stock-dependent demand (SDD) I(t) at time t; here 'a' and 'b' are positive constants in price dependent demand and x and y are positive constants in stock-dependent demand. (5) A constant deteriorating rate θ is only applied to on-hand inventory.

3.2 Notations

The following list of presumptions serves as the foundation for the model. (1) T - cycle time (decision variable), (2) Q - demanded quantity, that is order size, (3) C_S - setup cost per set, (4) C_P- unit purchase cost , (5) C_h - unit holding cost per unit time, (6) θ – rate of deteriorative items, (7) D_c - unit deteriorative cost, (8) $\mathrm{P_C}$ – rate of production during the first (upward) phase of the cycle ($P_C > D$), (9)T_1 – duration of first phase, (10) Q_1 – Stock at the end of the first phase, (11) $D = (a - \mathrm{bP})(x + \mathrm{yI}(t))$ demand rate.

4 MATHEMATICAL MODELS

4.1 Production inventory model for deteriorative items with integrated stock and price dependent demand

This model is developed for Price-dependent demand integrated with Stock dependent demand for deteriorating inventory model, that is $D = (a - \mathrm{bP})(x + \mathrm{yI}(t))$, where a denotes the constant demand in PDD and x the constant demand in SDD >0 and where b and y are coefficients of demand in PDD and SDD, respectively. The inventory level decreases due to demand and deteriorating till it becomes zero in the interval (0, T). The total process is repeated. The inventory level at different instants of time is shown in Figure 1.

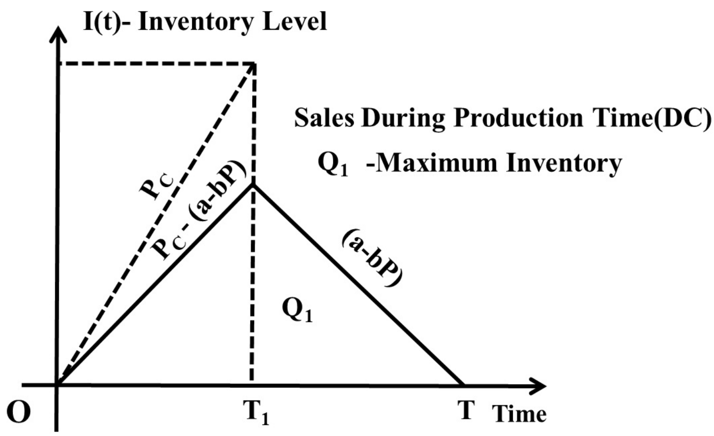

Figure 1 – Production Inventory Cycle

During the production stage, the inventory of good items increases due to production but decreases due to demand and loss of perishable items. Then, the inventory differential equation is

$$\frac{d}{dt}I(t) + \theta I(t) = P_c - (a - bP)(x + yI(t)), 0 \leq t \leq T_1. \tag{1}$$

The inventory differential equation during the consumption period with no production and subsequent reduction in the inventory level due to rate of perishable items is given by

$$\frac{d}{dt}I(t) + \theta I(t) = -(a - bP)(x + yI(t)), T_1 \leq t \leq T, \tag{2}$$

with the boundary conditions

$$I(0) = Q, I(T_1) = Q_1, I(T) = 0. \tag{3}$$

Solving the differential equation (1),

$$I(t) = \frac{(P_c - x(a-bP))}{\theta + y(a-bP)}\left(1 - e^{-(\theta + y(a-bP))t}\right). \tag{4}$$

Solving the differential equation (2),

$$I(t) = \frac{x(a-bP)}{\theta + y(a-bP)}\left(e^{-(\theta + y(a-bP))(T-t)} - 1\right). \tag{5}$$

To find the optimum quantity Q:

From equation (2) with the boundary conditions

$$I(0) = Q,$$

$$Q = \frac{x(a-bP)}{\theta + y(a-bP)}\left(e^{(\theta + y(a-bP))T} - 1\right). \tag{6}$$

To find T_1 and Q_1:

From equations (4) and (5) with the boundary conditions,

$$\frac{(P_c - x(a-bP))}{\theta + y(a-bP)}\left(1 - e^{-(\theta + y(a-bP))T_1}\right) = \frac{x(a-bP)}{\theta + y(a-bP)}\left(e^{-(\theta + y(a-bP))(T-T_1)} - 1\right).$$

On simplification,

$$T_1 = \frac{x(a-bP)T}{P_c}. \tag{7}$$

From equations (3) and (4) with the boundary conditions,

$$Q_1 = (P_c - x(a-bP))\,T_1. \tag{8}$$

Total Cost: Total cost comprises setup cost, holding cost, production cost, and deteriorative cost.

1. Production cost =

$$DC_P = (a-bP)(x + yI(t))C_p \tag{9}$$

2. Setup cost =

$$\frac{C_0}{T} \tag{10}$$

3. Holding cost

$$= \frac{C_h}{T}\left[\int_0^{T_1} \frac{(P_c - x(a-bP))}{\theta + y(a-bP)}\left(1 - e^{-(\theta + y(a-bP))t}\right)dt + \int_{T_1}^{T} \frac{x(a-bP)}{\theta + y(a-bP)}\left(e^{(\theta + y(a-bP))(T-t)} - 1\right)\right]$$

$$= \frac{C_h}{T}\left[\begin{array}{l} \dfrac{(P_c - x(a-bP))}{(\theta + y(a-bP))^2}\left[(\theta + y(a-bP))T_1 + e^{-(\theta + y(a-bP))T_1} - 1\right] \\ -\dfrac{x(a-bP)}{(\theta + y(a-bP))^2}\begin{pmatrix} 1 + (\theta + y(a-bP))T - e^{(\theta + y(a-bP))(T-T_1)} \\ -(\theta + y(a-bP))T_1 \end{pmatrix} \end{array}\right] \tag{11}$$

4. Deteriorative cost

$$=\frac{\theta C_d}{T}\left[\begin{array}{l}\frac{(P_c-x(a-bP))}{(\theta+y(a-bP))^2}\left[(\theta+y(a-bP))T_1+e^{-(\theta+y(a-bP))T_1}-1\right]\\ -\frac{x(a-bP)}{(\theta+y(a-bP))^2}\begin{pmatrix}1+(\theta+y(a-bP))T-e^{(\theta+y(a-bP))(T-T_1)}\\ -(\theta+y(a-bP))T_1\end{pmatrix}\end{array}\right] \tag{12}$$

Total cost (TC) = Production cost + Setup cost + Holding cost + Deteriorative cost

$$TC(T)=\frac{C_0}{T}+\frac{C_h+\theta C_d}{T}\left[\begin{array}{l}\frac{(P_c-x(a-bP))}{(\theta+y(a-bP))^2}\left[(\theta+y(a-bP))T_1+e^{-(\theta+y(a-bP))T_1}-1\right]\\ -\frac{x(a-bP)}{(\theta+y(a-bP))^2}\begin{pmatrix}1+(\theta+y(a-bP))T-e^{(\theta+y(a-bP))(T-T_1)}\\ -(\theta+y(a-bP))T_1\end{pmatrix}\end{array}\right] \tag{13}$$

Optimality:

$$\frac{\partial}{\partial T_1}[TC(T)]=0$$

and

$$\frac{\partial^2}{\partial {T_1}^2}[TC(T)]>0$$

and

$$\frac{\partial}{\partial T}[TC(T)]=0$$

and

$$\frac{\partial^2}{\partial T^2}[TC(T)]>0.$$

Partially differentiating equation (14) with respect to T_1,

$$\left[\begin{array}{l}\frac{(P_c-x(a-bP))}{(\theta+y(a-bP))^2}\left[(\theta+y(a-bP)-(\theta+y(a-bP))e^{-(\theta+y(a-bP))T_1}\right]\\ -\frac{x(a-bP)}{(\theta+y(a-bP))^2}\left[-(\theta+y(a-bP))+(\theta+y(a-bP))e^{(\theta+y(a-bP))(T-T_1)_1}\right]\end{array}\right]=0$$

On simplification, $T_1=\frac{x(a-bP)T}{P}$.

Partially differentiating (14) with respect to T,

$$\frac{\partial}{\partial T}TC(T)=-C_0+\frac{(C_h+\theta C_d)}{(\theta+y(a-bp))^2}\left[\begin{array}{l}-(P_c-x(a-bp))(e^{-(\theta+y(a-bp))T_1}+(\theta+y(a-bp))T_1-1)\\ +x(a-bp)\left[\begin{array}{l}T\{(\theta+y(a-bp))e^{(\theta+y(a-bp))(T-T_1)}-(\theta+y(a-bp))\}\\ -(e^{(\theta+y(a-bp))(T-T_1)}-(\theta+y(a-bp))(T-T_1)-1\end{array}\right]\end{array}\right]=0$$

$$\left[\begin{array}{l}\frac{-(P_c-x(a-bP))}{(\theta+y(a-bP))^2}\left[(\theta+y(a-bP))T_1+e^{-(\theta+y(a-bP))T_1}-1\right]\\ -\frac{x(a-bP)}{(\theta+y(a-bP))^2}\left[\begin{array}{l}(\theta+y(a-bP))T-(\theta+y(a-bP))Te^{(\theta+y(a-bP))(T-T_1)}\\ -1-(\theta+y(a-bP))(T-T_1)+e^{(\theta+y(a-bP))(T-T_1)}\end{array}\right]\end{array}\right]=\frac{C_0}{(C_h+\theta C_d)}$$

This is an optimum solution of T in a higher-order equation. This equation can be evaluated by using MATLAB. But for the reader's convenience the equation is reduced to the fourth-order equation and then the third order equation in T. On simplification,

$$\begin{bmatrix} \dfrac{-(P_c-x(a-bP))T_1^2}{2}+x(a-bP)T(T-T_1)-\dfrac{x(a-bP)(T-T_1)^2}{2} \\ +\dfrac{(P_c-x(a-bP))(\theta+y(a-bP))T_1^3}{6}+\dfrac{x(a-bP)(\theta+y(a-bP))T(T-T_1)^2}{2} \\ -\dfrac{x(a-bP)(\theta+y(a-bP)(T-T_1)^3}{6}-\dfrac{(P_c-x(a-bP))(\theta+y(a-bP)^2T_1^4}{24} \\ +\dfrac{x(a-bP)(\theta+y(a-bP))^2(T-T_1)^3T}{6}+\dfrac{x(a-bP)(\theta+y(a-bP)^2(T-T_1)^4}{24} \end{bmatrix}=\frac{C_0}{(C_h+\theta C_d)}$$

which is in fourth order equation. The reduced third order equation is

$$\begin{bmatrix} \dfrac{-(P_c-x(a-bP))T_1^2}{2}+x(a-bP)T(T-T_1)-\dfrac{x(a-bP)(T-T_1)^2}{2} \\ +\dfrac{(P_c-x(a-bP))(\theta+y(a-bP))T_1^3}{6}+\dfrac{x(a-bP)(\theta+y(a-bP))T(T-T_1)^2}{2} \\ -\dfrac{x(a-bP)(\theta+y(a-bP)(T-T_1)^3}{6} \end{bmatrix}=\frac{C_0}{(C_h+\theta C_d)}$$

Substituting the value of T_1 from equation (7) and simplifying,

$$\begin{bmatrix} \wedge\frac{(P_c-x(a-bP))(\theta+y(a-bP))x(a-bP)}{6P^2}\left[2P-x(a-bP)\right]T^3 \\ \wedge+\frac{(P_c-x(a-bP))x(a-bP)}{2P^2}T^2 \end{bmatrix}=\frac{C_0}{C_h+\theta C_d}$$

Further reduced,

$$(\theta+\mathbf{y}(\mathbf{a}-\mathbf{bP}))(\mathbf{2P_c}-\mathbf{x}(\mathbf{a}-\mathbf{bP}))\mathbf{T^3}+\mathbf{3PT^2}=\frac{\mathbf{6P_c{}^2C_0}}{(\mathbf{C_h}+\theta\mathbf{C_d})(\mathbf{P}-\mathbf{x}(\mathbf{a}-\mathbf{bP}))\mathbf{x}(\mathbf{a}-\mathbf{bP})} \tag{14}$$

which is the optimum solution for T in the third order equation.

Numerical example 1

The following values are given:

Production rate P_c = 500 units, Demand rate D = 450 units, Setup cost per set C_0= 130, Holding cost per unit per unit time C_h = 13, Production cost per unit C_P= 130, Deteriorative cost per unit C_d = 130, Rate of Deteriorative item θ = 0.01, Selling price per unit P = 150,

Constant demand rate in SDD x = 30, Coefficient of constant demand in SDD y = 0.1 Constant Demand rate in PDD a = 30, Coefficient of constant demand in PDD b = 0.1

Optimum solution

The solution is obtained as follows.

The third order equation $18686250T^3+33750000T^2-13636363.63=0$.

Optimum cycle time T= 0.5558, Optimum quantity Q = 250.12, Production time T_1 = 0.5002, Maximum inventory Q_1= 25.01, Production cost =58500, Setup cost =233.88, Holding cost = 162.58, Deteriorative cost = 16.25, Total cost = 58912.72, Total sales = 67500.00, Total profit = 8587.27.

Sensitivity analysis and discussion

In order to assess the relative impact of the different input parameters on the solution quantity, systematic sensitivity analysis was performed on the above example. The rate of the deterioration was given values from 0.01 to 0.1 and the other 11 values of Example 1 were kept constant.

Sensitivity Analysis with respect to Rate of Deteriorative items (θ)

Table 1 shows a study of the rate of the deteriorative items with cycle time, optimum quantity, production time, maximum inventory, setup cost, holding cost deteriorating cost, total cost and total profit. There is a positive relationship between the increase in the rate of deterioration for items θ with setup costs, deteriorating costs, and total costs, while there is a negative relationship between the increase in the rate of deterioration for items θ with cycle time, optimum quantity, production time, maximum inventory, holding cost and total profit.

Table 1 – Result of sensitivity analysis with respect to rate of deteriorative items.

θ	T	Q	T_1	Q_1	Setup cost	Holding cost	DC	Total cost	Total Profit
0.01	0.5558	250.12	0.5002	25.01	233.88	162.58	16.25	58912.72	8587.27
0.02	0.5342	240.40	0.4808	24.04	243.34	156.26	31.25	58930.85	8569.14
0.03	0.5150	231.75	0.4635	23.17	252.41	150.64	45.19	58948.25	8551.74
0.04	0.4977	224.00	0.4480	22.40	261.14	145.60	58.24	58964.99	8535.00
0.05	0.4822	217.00	0.4340	21.70	269.57	141.05	70.52	58981.15	8518.84
0.06	0.4680	210.64	0.4212	21.06	277.72	136.91	82.15	58996.71	8503.20
0.07	0.4551	204.81	0.4096	20.48	285.62	133.13	93.19	59011.94	8488.05
0.08	0.4432	199.45	0.3989	19.95	293.29	129.64	103.71	59026.66	8473.33
0.09	0.4322	194.51	0.3890	19.45	300.75	126.43	113.78	59040.97	8459.02
0.10	0.4220	189.91	0.3798	18.99	308.02	123.44	123.44	59054.92	8445.07

The graphical representation between rate of deteriorative items and total profit is given on Figure 2. It is observed that the total profit is in a downward curve.

Sensitivity Analysis with respect to constant demand in SDD (x):

Table 2 shows a study of the rate of the constant demand in SDD (x) with cycle time, optimum quantity, production time, maximum inventory, setup cost, holding cost deteriorating cost, total cost and total profit. There is a positive relationship between the increase in the rate of constant demand in SDD with optimum cycle time, optimum quantity, production time, total cost and total profit, while there is a negative relationship between the increase in the constant demand in SDD with maximum inventory, setup cost, holding cost, deteriorative cost.

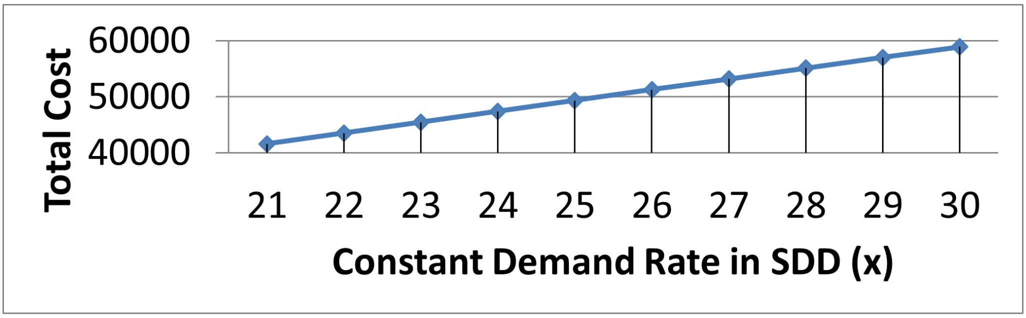

Figure 2 – Relationship between rate of deteriorative items and total profit.

Table 2 – Result of sensitivity analysis with respect to constant demand rate coefficient in stock dependent demand (x).

x	T	Q	T_1	Q_1	Setup cost	Holding cost	DC	Total cost	Total Profit
21	0.3540	111.54	0.2230	41.26	367.13	268.25	26.82	41612.21	5637.78
22	0.3609	119.12	0.2382	40.50	360.11	263.27	26.32	43549.71	5950.28
23	0.3697	127.54	0.2550	39.54	351.62	257.01	25.70	45484.34	6265.65
24	0.3805	137.01	0.2740	38.36	341.56	249.36	24.93	47415.87	6584.12
25	0.3942	147.82	0.2956	36.95	329.77	240.21	24.02	49344.01	6905.98
26	0.4113	160.41	0.3208	35.29	316.06	229.38	22.93	51268.38	7231.61
27	0.4331	175.42	0.3508	33.32	300.13	216.64	21.66	53188.44	7561.55
28	0.4616	193.89	0.3877	31.02	281.60	201.64	20.16	55103.41	7896.58
29	0.5002	217.62	0.4352	28.29	259.84	183.89	18.38	57012.13	8237.86
30	0.5558	250.12	0.5002	25.01	233.88	162.58	16.25	58912.72	8587.27

The graphical representation between constant demand rate (x) items and total profit is given below. It is observed that the total profit is in

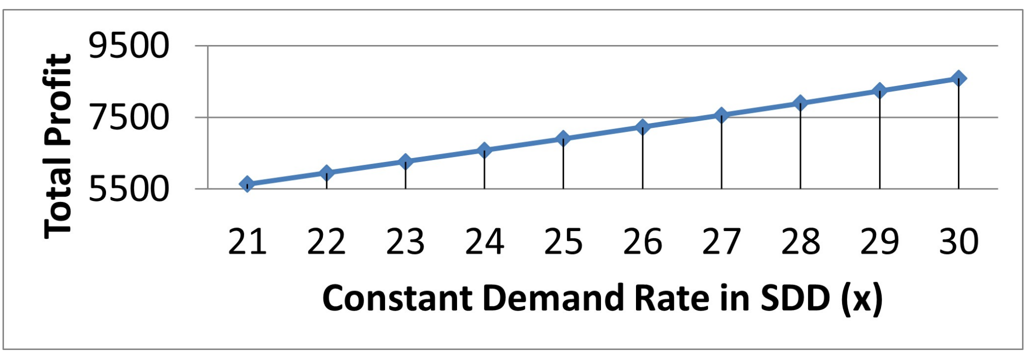

Figure 3 – Relationship between constant demand rate (x) and total profit.

Sensitivity Analysis with respect to constant rate (a) in PDD

Table 3 shows a study with constant rate of demand in SDD (a) with cycle time, optimum quantity, production time, maximum inventory, setup cost, holding cost deteriorating cost, total cost and total profit. There is a positive relationship between the increase in the rate of constant demand in SDD and optimum quantity, production time, maximum inventory, setup cost, holding cost, deteriorative cost, total cost and total profit, while there is a negative relationship between the increase in the constant demand in SDD and cycle time.

Table 3 – Result of sensitivity analysis with respect to constant demand rate coefficient in price dependent demand (a).

a	T	Q	T_1	Q_1	Setup cost	Holding cost	DC	Total cost	Total Profit
21	0.3745	67.42	0.1348	43.15	347.06	280.47	28.04	24055.59	2944.40
22	0.3625	76.13	0.1522	44.16	358.55	287.04	28.70	27974.30	3525.69
23	0.3565	85.56	0.1711	44.49	364.65	289.19	28.91	31882.76	4117.23
24	0.3556	96.02	0.1020	44.16	365.54	287.09	28.70	35781.35	4718.64
25	0.3599	107.97	0.2159	43.18	361.19	280.73	28.07	39670.00	5329.99
26	0.3700	122.11	0.2442	41.51	351.31	269.86	26.98	43548.17	5951.82
27	0.3876	139.56	0.2791	39.07	335.32	254.00	25.40	47414.77	6585.26
28	0.4164	162.41	0.3248	35.73	312.16	232.25	23.22	51267.64	7232.35
29	0.4646	195.14	0.3902	31.22	279.79	202.94	20.29	55103.03	7896.96
30	0.5558	250.12	0.5002	25.01	233.88	162.58	16.25	58912.72	8587.27

The graphical representation between constant demand rate (a) in PDD and total profit is given below. It is observed that the total profit is in an upward curve.

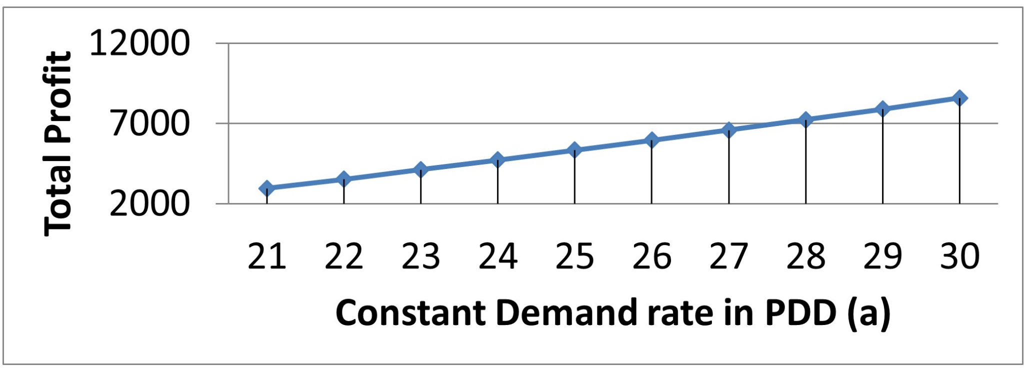

Figure 4 – Relationship between constant demand rate (a) and total profit.

Managerial insights

A sensitivity analysis is performed to study the effects of changes in the system parameters, ordering cost per order (C_0), holding cost per unit time (C_h), deteriorating cost per unit time (C_d), and coefficients in SDD on optimal values, that is, optimal cycle time (T), optimal quantity (Q), maximum inventory, production time, setup cost, holding cost, deteriorating cost, total cost

Table 4 – Sensitivity Analysis with respect to inventory cost parameters.

Cost Parameters		Optimum Solutions								
		T	Q	T_1	Q_1	Setup cost	Holding cost	DC	Total cost	Total Profit
C_0	110	0.5156	232.06	0.4641	23.20	213.30	150.84	15.08	58879.22	8620.77
	120	0.5362	241.31	0.4826	24.13	223.77	156.85	15.68	58896.31	8603.68
	130	0.5558	250.12	0.5002	25.01	233.88	162.58	16.25	58912.72	8587.27
	140	0.5745	258.54	0.5171	25.85	243.66	168.05	16.80	58928.53	8571.46
	150	0.5924	266.62	0.5332	26.66	253.16	173.30	17.33	58943.80	8556.19
C_h	11	0.5944	267.51	0.5350	26.75	218.67	147.13	17.38	58883.20	8616.80
	12	0.5741	258.36	0.5167	25.83	226.42	155.01	16.79	58898.23	8601.72
	13	0.5558	250.12	0.5002	25.01	233.88	162.58	16.25	58912.72	8587.27
	14	0.5392	242.66	0.4853	24.26	241.06	169.86	15.77	58926.71	8573.28
	15	0.5241	235.86	0.4717	23.58	248.01	176.90	15.33	58940.25	8559.74
C_d	110	0.5593	251.70	0.5034	25.17	232.41	163.61	13.84	58909.86	8590.13
	120	0.5575	250.91	0.5018	25.09	233.14	163.09	15.05	58911.29	8588.70
	130	0.5558	250.12	0.5002	25.01	233.88	162.58	16.25	58912.72	8587.27
	140	0.5541	249.34	0.4986	24.93	234.61	162.07	17.45	58914.14	8585.85
	150	0.5523	248.57	0.4971	24.85	235.33	161.57	18.64	58915.55	8584.44
y	0.06	0.5816	261.76	0.5235	26.17	223.48	170.14	17.01	58910.64	8589.35
	0.07	0.5746	258.61	0.5172	25.86	226.20	168.09	16.80	58911.11	8588.88
	0.08	0.5680	255.63	0.5112	25.58	228.84	166.16	16.61	58911.62	8588.37
	0.09	0.5618	252.81	0.5056	25.28	231.40	164.32	16.43	58912.15	8587.84
	0.10	0.5558	250.12	0.5002	25.01	233.88	162.58	16.25	58912.72	8587.27

and total profit. The sensitivity analysis is performed by changing (increasing or decreasing) the parameter taking at a time, keeping the remaining parameters at their original values. The following influences can be obtained from the sensitivity analysis based on Table 4.

1. There is a positive relationship between the increase in the setup costs per set (C_0) and the cycle time, the optimum quantity, maximum inventory, production time, setup costs, holding costs, deteriorating costs, total costs and total profit.

2. There is a positive relationship between the increase in the holding cost per unit time (C_h) and the setup cost, holding cost, and total cost, while there is a negative relationship between the increase in the holding cost per unit time (C_h) and the cycle time, optimal quantity, maximum inventory, production time, and deteriorating cost.

3. There is a positive relationship between the increase in the deteriorative cost per unit C_d) and the setup cost, deteriorative cost and total cost, while there is a negative relationship between the increase in the deteriorative cost per unit (C_d) and the cycle time, optimum quantity, production time, maximum inventory, holding cost, and total profit.

4. Similarly, other parameters, as the coefficient constant rate in SDD (y) can also be observed in Table 4.

4.2 Production inventory model for deteriorative items with stock dependent demand

This model is developed for deteriorating inventory model in which demand is Stock-dependent that is $D = (x + yI(t))$, where x (constant demand in SDD) > 0 at time t and y is coefficient of demand in SDD.

Thus, the inventory differential equation is

$$\mathbf{\frac{d}{dt}I(t)+\theta I(t) = P_c-(x+yI(t)), 0\leq t\leq T_1} \tag{15}$$

The inventory differential equation during the consumption period with no production and subsequent reduction in the inventory level due to the rate of perishable items is given by

$$\frac{d}{dt}I(t) + \theta I(t) = -(x + yI(t)), T_1 \leq t \leq T \tag{16}$$

with the boundary conditions

$$\mathbf{I(0) = 0, I(T_1) = Q_1, I(T) = 0.} \tag{17}$$

From the equation (17), the solution of the differential equation is

$$I(t) = \frac{P_c - x}{y + \theta}\left(1 - e^{-(y+\theta)t}\right). \tag{18}$$

Note: when y= 0 and x is replaced by D then $I(t) = \frac{P-D}{\theta}\left(1 - e^{-\theta t}\right)$ which is basic inventory model.

From the equation (18), the solution of the differential equation is

$$I(t)\frac{x}{y+\theta}\left(e^{(y+\theta)(T-t)} - 1\right) \tag{19}$$

Note: when y= 0 and x is replaced by D then $I(t)\frac{D}{\theta}\left(e^{(\theta(T-t)} - 1\right)$ which is basic inventory model.

To find Q: From the equations (19) and (21), $Q\frac{x}{y+\theta}\left(e^{(y+\theta)T} - 1\right)$

To find T_1 and Q_1: From the equations (20) and (21)

$$\frac{P_c - x}{y + \theta}\left(1 - e^{-(y+\theta)T_1}\right) = \frac{x}{y+\theta}\left(e^{(y+\theta)(T-t)} - 1\right) \text{ On simplification, } T_1 = \frac{xT}{P_c} \tag{20}$$

From the equations (19) and (20),

$$Q_1 = (P_c - x)\left(T_1 - \frac{(y+\theta)T_1^2}{2}\right) \tag{21}$$

Total cost TC(T)

Total cost is the sum of the production cost, setup cost, holding cost and deteriorating cost. They are grouped after evaluating each cost individually.

1. Production cost =

$$DC_P = (x + yI(t))C_p \tag{22}$$

2. Setup cost =

$$\frac{C_0}{T} \tag{23}$$

3. Holding cost

$$= \frac{C_h}{T}\left[\int_0^{T_1} \frac{P_c - x}{y+\theta}\left(1 - e^{-(y+\theta)t}\right) dt + \int_{T_1}^{T} \frac{x}{y+\theta}\left(e^{(y+\theta)(T-t)} - 1\right) dt\right]$$

$$= \frac{C_h}{T}\left[\begin{array}{l} \frac{P_c - x}{(y+\theta)^2}\left((y+\theta)T_1 + e^{-(y+\theta)T_1} - 1\right) \\ - \frac{x}{(y+\theta)^2}\left(1 + (y+\theta)T - e^{(y+\theta)(T-T_1)} - (y+\theta)T_1\right) \end{array}\right]$$

$$= \frac{C_h}{T}\left[\begin{array}{l} \frac{P_c - x}{(y+\theta)^2}\left(e^{-(y+\theta)T_1 - 1} + (y+\theta)T_1 - 1\right) \\ + \frac{x}{(y+\theta)^2}\left(e^{(y+\theta)(T-T_1)} - (y+\theta)(T - T_1) - 1\right) \end{array}\right] \tag{24}$$

4. Deteriorative cost =

$$\frac{\theta C_d}{T}\left[\begin{array}{l} \frac{P_c - x}{(y+\theta)^2}\left(e^{-(y+\theta)T_1 - 1} + (y+\theta)T_1 - 1\right) \\ + \frac{x}{(y+\theta)^2}\left(e^{(y+\theta)(T-T_1)} - (y+\theta)(T - T_1) - 1\right) \end{array}\right] \tag{25}$$

Total cost = Production cost + Setup cost + Holding cost + Deteriorative cost

$$TC(T) = DC_P + \frac{C_0}{T} + \frac{(C_h + \theta C_d)}{T}\left[\begin{array}{l} \frac{P_c - x}{(y+\theta)^2}\left(e^{-(y+\theta)T_1 - 1} + (y+\theta)T_1 - 1\right) \\ + \frac{x}{(y+\theta)^2}\left(e^{(y+\theta)(T-T_1)} - (y+\theta)(T - T_1) - 1\right) \end{array}\right] \tag{26}$$

Partially differentiating equation (28) with respect to T_1

$$\frac{\mathbf{P_c - x}}{(\mathbf{y}+\theta)^2}\left((\mathbf{y}+\theta) - (\mathbf{y}+\theta)\mathbf{e}^{-(\mathbf{y}+\theta)\mathbf{T_1}}\right) + \frac{\mathbf{x}}{(\mathbf{y}+\theta)^2}\left(-(\mathbf{y}+\theta)\mathbf{e}^{(\mathbf{y}+\theta)(\mathbf{T}-\mathbf{T_1})} + (\mathbf{y}+\theta)\right) = \mathbf{0}$$

On simplification,

Partially differentiating equation (28) with respect to T,

$$\left[\begin{array}{r} \&-\frac{\mathbf{P_c - x}}{(\mathbf{y}+\theta)^2}\left[\frac{(\mathbf{y}+\theta)^2\mathbf{T_1^2}}{\mathbf{2}} - \frac{(\mathbf{y}+\theta)^3\mathbf{T_1^3}}{\mathbf{6}} + \frac{(\mathbf{y}+\theta)^4\mathbf{T_1^4}}{\mathbf{24}}\right]\left(\mathbf{e}^{-(\mathbf{y}+\theta)\mathbf{T_1}} + (\mathbf{y}+\theta)\mathbf{T_1} - \mathbf{1}\right) \\ \&+\frac{\mathbf{x}}{(\mathbf{y}+\theta)^2}\left[(\mathbf{y}+\theta)\mathbf{T}\mathbf{e}^{(\mathbf{y}+\theta)(\mathbf{T}-\mathbf{T_1})} - (\mathbf{y}+\theta)\mathbf{T} - \mathbf{e}^{(\mathbf{y}+\theta)(\mathbf{T}-\mathbf{T_1})} + (\mathbf{y}+\theta)(\mathbf{T}-\mathbf{T_1}) + \mathbf{1}\right] \end{array}\right] = \frac{\mathbf{C_0}}{(\mathbf{C_h} + \theta\mathbf{C_d})}$$

This equation is reduced to the fourth-order equation and then the third order equation in T.

$$\begin{bmatrix} -(P_c-x)\left[\frac{T_1^2}{2}-\frac{(y+\theta)T_1^3}{6}+\frac{(y+\theta)^4T_1^4}{24}\right] \\ +\frac{x}{(y+\theta)^2}\begin{bmatrix} (y+\theta)^2T(T-T_1)+\frac{(y+\theta)^3T(T-T_1)^2}{2}+\frac{(y+\theta)^4T(T-T_1)^4}{6} \\ -\frac{(y+\theta)^2(T-T_1)^2}{2}-\frac{(y+\theta)^3(T-T_1)^3}{6}-\frac{(y+\theta)^4(T-T_1)^4}{24} \end{bmatrix} \end{bmatrix}=\frac{C_0}{C_h+\theta C_d}.$$

Substituting the value of T_1 and simplifying,

$$\begin{bmatrix} -\frac{(P_c-x)x^2T^2}{2P_c{}^2}+\frac{x(P_c-x)T^2}{P_c}-\frac{x(P_c-x)^2T^2}{2P_c{}^2} \\ +\frac{(P_c-x)(y+\theta)x^3T^3}{6P_c{}^3}+\frac{x(y+\theta)(P_c-x)^2T^3}{2P_c{}^2}-\frac{x(y+\theta)(P_c-x)^3T^3}{6P_c{}^3} \\ -\frac{(P_c-x)(y+\theta)^2x^4T^4}{24P_c{}^4}+\frac{x(y+\theta)^2(P_c-x)^3T^4}{6P_c{}^3}-\frac{x(y+\theta)^2(P_c-x)T^3}{24P_c{}^4}- \end{bmatrix}=\frac{C_0}{C_h+\theta C_d},$$

which is the fourth order equation.

This fourth order equation is further reduced to third order equation as follows:

$$\frac{x(P_c-x)T^2}{2P_c}+\frac{x(P_c-x)(y+\theta)(2P_c^2-2P_c)T^3}{6P_c^3}=\frac{C_0}{C_h+\theta C_d}$$

On simplification,

$$x(P_c-x)(y+\theta)(2P_c-x)T^3+3P_cx(P_c-x)T^2=\frac{6P_c{}^2C_0}{C_h+\theta C_d} \tag{27}$$

Note: when b = 0 and a is replaced by D then $T=\sqrt{\frac{2P_cC_0}{D(P_c-D)(C_h+\theta C_d)}}$.

Numerical example 2

The following values are given:

Production rate P_c = 500 units, Demand rate D = 450 units, Setup cost per set C_0= 130, Holding cost per unit per unit time C_h = 13, Production cost per unit C_P= 130, Deteriorative cost per unit C_d = 130, Rate of Deteriorative item θ = 0.01, Selling price per unit P = 150,

Constant demand rate in SDD x = 450, Coefficient of constant demand rate in SDD y = 0.1

Optimum solution

The solution is obtained as follows:

Optimum cycle time T= 0.6277, Optimum quantity Q = 282.48, Production time T_1 = 0.5649, Maximum inventory Q_1= 28.24, Production cost = 58500, Setup cost = 207.09, Holding cost = 183.61, Deteriorative cost = 18.36, Total cost = 58909.06, Total sales = 67500, Total profit = 8590.93

Sensitivity analysis and discussion

In order to assess the relative impact of the different input parameters on the solution quantity, a systematic sensitivity analysis was performed on the above example.

Sensitivity Analysis with respect to Rate of Deteriorative items (θ)

Table 5 shows a study of the relationship of the rate of deteriorative items with cycle time, optimum quantity, production time, maximum inventory, setup cost, holding cost deteriorating cost, total cost and total profit. There is a positive relationship between the increase in the rate of deterioration for items θ and setup costs, deteriorating costs, and total costs, while there is a negative relationship between the increase in the rate of deterioration for items θ and cycle time, optimum quantity, production time, maximum inventory, holding cost and total profit.

Table 5 – Result of sensitivity analysis with respect to deteriorative items.

θ	T	Q	T_1	Q_1	Setup cost	Holding cost	DC	Total cost	Profit
0.01	0.6277	282.48	0.5649	28.24	207.09	183.61	18.36	58909.06	8590.93
0.02	0.6006	270.31	0.5406	27.03	216.41	175.70	35.14	58927.25	8572.74
0.03	0.5768	259.57	0.5191	25.95	225.37	168.72	50.61	58944.70	8555.29
0.04	0.5555	250.00	0.5000	25.00	233.99	162.50	65.00	58961.49	8538.50
0.05	0.5364	241.41	0.4828	24.14	242.32	156.91	78.45	58977.70	8522.29
0.06	0.5191	233.63	0.4672	23.35	250.38	151.81	91.11	58993.37	8506.62
0.07	0.5034	226.56	0.4531	22.65	258.20	147.26	103.08	59008.55	8491.44
0.08	0.4890	220.08	0.4401	22.00	265.80	143.05	114.44	59023.30	8476.69
0.09	0.4758	214.12	0.4282	21.41	273.20	139.17	125.26	59037.64	8462.35
0.10	0.4635	208.61	0.4172	20.86	280.42	135.60	135.60	59051.62	8448.37

4.3 Production inventory model for deteriorative items with price dependent demand

This model is developed for deteriorating inventory model in which demand is Price-dependent demand that is $D=(a-bP)$, where a denotes a constant demand in PDD >0 and b is coefficient of demand in PDD >0.

During the production stage, the inventory of good items increases due to production but decreases due to demand and rate of perishable items. Thus, the inventory differential equation is

$$\mathbf{\frac{d}{dt}I(t)+\theta I(t)=P_c-(a-bP),0\leq t\leq T_1} \tag{28}$$

The inventory differential equation during the consumption period with no production and subsequent reduction in the inventory level due to the rate of perishable items is given by

$$\frac{d}{dt}I(t)+\theta I(t)=-(a-bP),\, T_1\leq t\leq T \tag{29}$$

With the boundary conditions,

$$\mathbf{I(0)=0,I(T_1)=Q_1,I(T)=0} \tag{30}$$

From the equation (17), the solution of the differential equation is

$$I(t)=\frac{P_c-(a-bP)}{\theta}\left(1-e^{-\theta t}\right) \tag{31}$$

Note: when b= 0 and a replaced by D then $I(t)=\frac{P_c-D}{\theta}\left(1-e^{-\theta t}\right)$ which is the basic inventory model.

From the equation (18), the solution of the differential equation is

$$I(t)\frac{a-bP}{\theta}\left(e^{\theta(T-t)}-1\right) \tag{32}$$

Note: when b= 0 and a replaced by D then $I(t)=\frac{P_c-D}{\theta}\left(1-e^{-\theta t}\right)$ which is the basic inventory model.

To find Q: From the equation (19) and (21),

$$I(t)\frac{a-bP}{\theta}\left(e^{\theta(T-t)}-1\right) \tag{33}$$

To find T_1 and Q_1: From the equation (20) and (21)

$$\frac{P_c-(a-bP)}{\theta}\left(1-e^{-\theta T_1}\right)=\frac{a-bP}{\theta}\left(e^{\theta(T-t)}-1\right) \tag{34}$$

On simplification, $T_1=\frac{(a-bP)T}{P_c}$

From the equations (19) and (20),

$$Q_1=(P_c-(a-bP))T_1 \tag{35}$$

Total cost TC (T)

Total cost is the sum of the production cost, setup cost, holding cost, and deteriorating cost. They are grouped after evaluating each cost individually.

1. Production cost =

$$DC_p=(a-bP)C_p \tag{36}$$

2. Setup cost =

$$\frac{C_0}{T} \tag{37}$$

3. Holding cost =

$$\frac{C_h}{T}\left[\int_0^{T_1}\frac{P_C-(a-bP)}{\theta}\left(1-e^{-\theta r}\right)dt+\int_{T_1}^{T}\frac{(a-bP)}{\theta}\left(e^{\theta(T-t)}-1\right)dt\right]$$

$$=\frac{C_h}{T}\left[\frac{P_C-(a-bP)}{\theta^2}\left(\theta T_1+e^{-\theta T_1}-1\right)-\frac{a-bP}{\theta^2}\left(1+\theta T-e^{\theta(T-T_1)}-\theta T_1\right)\right]$$

$$=\frac{C_h}{\theta^2 T}\left[(P_C-(a-bP))\left(\theta T_1+e^{-\theta T_1}-1\right)-(a-bP)\left(1+\theta T-e^{\theta(T-T_1)}-\theta T_1\right)\right] \tag{38}$$

4. Deteriorative cost

$$=\frac{\theta C_d}{\theta^2 T}\left[(P_C-(a-bP))\left(\theta T_1+e^{-\theta T_1}-1\right)-(a-bP)\left(1+\theta(T-T_1)-e^{\theta(T-T_1)}\right)\right] \tag{39}$$

Total cost = Production cost + Setup cost + Holding cost + Deteriorative cost

$$TC(T) = DC_p + \frac{C_0}{T} + \frac{C_h + C_d}{\theta^2 T}\begin{bmatrix} \wedge(P_C - (a-bP))\left(\theta T_1 + e^{-\theta T_1} - 1\right) \\ \wedge - (a-bP)\left(1 + \theta(T - T_1) - e^{\theta(T-T_1)}\right) \end{bmatrix} \tag{40}$$

Partially differenting equation (43) with respect to T_1

$$(P_C - (a-bP))(-\theta e^{-\theta T_1} + \theta) - (a-bP)(-\theta + \theta e^{\theta(T-T_1)}) = 0$$

On simplification.

Partially differentiating equation (43) with respect to T,

$$\begin{bmatrix} \wedge - (P_C - (a-bP))\left(e^{-\theta T_1} + \theta T_1 - 1\right) \\ \wedge - T\left\{(a-bP)(\theta - \theta e^{\theta(T-T_1)})\right\} + (a-bP)\left(1 + \theta(T - T_1) - e^{\theta(T-T_1)}\right) \end{bmatrix} = \frac{\theta^2 C_0}{C_h + \theta C_d}$$

On simplification

$$\begin{bmatrix} \wedge - (P_C - (a-bP))\left(e^{-\theta T_1} + \theta T_1 - 1\right) \\ \wedge + \theta(a-bP)T(e^{\theta(T-T_1)} - 1) + (a-bP)\left(1 + \theta(T - T_1) - e^{\theta(T-T_1)}\right) \end{bmatrix} = \frac{\theta^2 C_0}{C_h + \theta C_d}$$

This equation is now reduced to the fourth-order equation and then the third order equation in T.

$$\begin{bmatrix} -\frac{(P_C - (a-bP))T_1^2}{2} + \frac{\theta(P_C - (a-bP))T_1^3}{6} - \frac{\theta^2(P_C - (a-bP))T_1^4}{24} \\ +(a-bP)\begin{pmatrix} T(T-T_1) + \frac{\theta T(T-T_1)^2}{2} + \frac{\theta^2 T(T-T_1)^3}{6} \\ -\frac{(T-T_1)^2}{2} - \frac{\theta(T-T_1)^3}{6} - \frac{\theta^2(T-T_1)^4}{24} \end{pmatrix} \end{bmatrix} = \frac{C_0}{C_h + \theta C_d}$$

Substituting the value of T_1, and simplifying

$$\frac{(P_C - (a-bP))\theta(a-bP)(2P_C - (a-bP))T^3}{6P_C^2} + \frac{(P_C - (a-bP))(a-bP)T^2}{2P_C} = \frac{C_0}{C_h + \theta C_d}.$$

On simplification,

$$\theta(2P_C - (a-bP))T^3 + 3P_C T^2 = \frac{6P_C^2 C_0}{(C_h + \theta C_d)(P_C - (a-bP))(a-bP)},$$

which is the optimum solution of T in the third order equation.

Therefore,

$$T = \sqrt{\frac{2P_C C_0}{(a-bP)(C_h + \theta C_d)(P_C - (a-bP))}}. \tag{41}$$

Note: when b = 0 and a replaced by D, then $T = \sqrt{\frac{2P_C C_0}{D(P_C - D)(C_h + \theta C_d)}}$.

1. When x=1, y = 0, then (16) becomes
$\theta(2P_c-(a-bP))T^3+3P_cT^2=\frac{6{P_c}^2C_0}{(C_h+\theta C_d)((P_c-(a-bP))(a-bP)}$ which is the price dependent demand as per equation (45).

2. When a = 1 and b = 0 then (16) becomes
$(y+\theta)(2P_c-x)T^3+3P_cT^2=\frac{6P_c^2C_0}{(C_h+\theta C_d)x(P_c-x)}$ Which is stock dependent demand as per the equation (30).

Illustrative example 3

The following values are given:

Production rate P = 500 units, Demand rate D = 450 units, Setup cost per set C_0= 130, Holding cost per unit per unit time C_h = 13, Production cost per unit C_P= 130, Deteriorative cost per unit C_d = 130, Rate of Deteriorative item θ = 0.01, Selling price per unit P_S= 150, Constant demand rate in PDD a = 465, Coefficient of Constant demand rate in PDD = 0.1

Optimum solution: The solution is obtained as follows.

Third order equation is $2750T^3+750000T^2-303030.30=0$

Optimum cycle time T= 0.6349, Optimum quantity Q = 285.70, Production time T_1 = 0.5714, Maximum inventory Q_1= 28.57, Production cost = 58500 , Setup cost = 204.75, Holding cost = 185.71 , Deteriorative cost = 18.57, Total cost = 58909.03 , Total sales = 67500, Total profit = 8590.96.

Sensitivity analysis and discussion

In order to assess the relative impact of the different input parameters on the solution quantity, systematic sensitivity analysis was performed on the above example.

4.3.1. Sensitivity Analysis with respect to Rate of Deteriorative items (θ)

Table 6 shows a study of the rate of the deteriorative items with cycle time, optimum quantity, production time, maximum inventory, setup cost, and holding cost deteriorating cost, total cost and total profit. There is a positive relationship between the increase in the rate of deterioration for items θ with setup costs, deteriorating costs, and total costs, while there is a negative relationship between the increase in the rate of deterioration for items θ with cycle time, optimum quantity, production time, maximum inventory, holding cost and total profit.

Table 6 – Result of sensitivity analysis with respect to deteriorative items.

θ	T	Q	T_1	Q_1	Setup cost	Holding cost	DC	Total cost	Profit
0.01	0.6349	285.70	0.5714	28.57	204.75	185.70	18.57	58909.03	8590.96
0.02	0.6072	273.25	0.5465	27.32	214.08	177.61	35.52	58927.22	8572.77
0.03	0.5828	262.27	0.5245	26.22	223.04	170.48	51.14	58944.67	8555.32
0.04	0.5611	252.50	0.5050	25.25	231.67	164.13	65.65	58961.45	8538.54
0.05	0.5416	243.74	0.4874	24.37	240.00	158.43	79.21	58977.65	8522.34
0.06	0.5240	235.81	0.4716	23.58	248.07	153.28	91.96	58993.32	8506.67
0.07	0.5080	228.60	0.4572	22.86	255.90	148.59	104.01	59008.50	8491.49
0.08	0.4933	222.00	0.4440	22.20	263.50	144.30	115.44	59023.25	8476.74
0.09	0.4798	215.93	0.4318	21.59	270.90	140.36	126.32	59037.58	8462.40
0.10	0.4674	210.33	0.4206	21.03	278.12	136.71	136.71	59051.56	8448.43

5 COMPARATIVE STUDY

A comparative study was carried out between stock dependent demand, price dependent demand and integrated stock and price dependent demand, fixing the deterioration rate at a low value of 0.01. From the comparative study it is observed that the holding cost rate is very low in integrated Stock and Price dependent demand rather than in Stock dependent demand and Price dependent demand individually. Also it is observed that when demand value is kept equal in all the three models the values obtained for total sales are also equal in all the three models for comparative purpose.

Table 7 – Comparative Study.

S.No.	Inventory Parameters	Stock Dependent Demand	Price Dependent Demand	Stock and Price Dependent Demand
1.	Optimum cycle time	0.6277	0.6349	0.5558
2.	Optimum quantity	282.48	285.70	250.12
3.	Production time	0.5649	0.5714	0.5002
4.	Maximum inventory	28.24	28.57	25.01
5.	Demand	450	450	450
6.	Production cost	58500.00	58500.00	58500.00
7.	Setup cost	207.09	204.75	233.88
8.	Holding cost	183.61	185.71	162.58
9.	Deteriorative cost	18.36	18.57	16.25
10.	Total Cost	58909.06	58909.03	58912.72
11.	Total Sales	67500.00	67500.00	67500.00
12.	Total profit	8590.93	8590.96	8587.27

6 CONCLUSION AND FUTURE STUDIES

The price and stock dependent demand for deteriorating items was included in this study's inventory model. Instead of assuming constant values for variables such as supply and demand, the model makes use of actual assumptions about stock integration and price-dependent demand. For example, the model assumes that the demand rate is constant. It has been shown that the ideal cycle duration and the optimal lot size can be determined with the help of visual basic 6.0, which has led to the development of a mathematical model and the production of an effective solution.

Each model contains a numerical example that was analyzed as well as the sensitivity of the result examined. Sensitivity analysis shows that the two demand parameters a and b have the greatest impact on the choice variables and the total profit function. This implies that organizations must first focus on raising their income by improving client demand via well-designed marketing tactics. In order to boost profitability, organizations should focus on reducing buying and ordering expenses through negotiation with suppliers. To maximize profitability, the third stage might be to lower the selling price. Companies should lower the unit selling price for promoting demand and enhance revenues if ordering and buying expenses are decreased.

Future studies: The suggested model may be further developed in a variety of ways.

1. For example, supposing that the demand rate is a non-linear selling price function, stock level, or time, is one expansion that might be considered.

2. There are many other alternatives, such as taking into account scarcities, the rate at which objects degrade, and the worth of money in relation to the passage of time.

3. The influence that advertising has on the demand function may also be taken into account.

4. One further way is to model the demand as a time function.

5. Additionally, the time value concept might be introduced.

References

BERNSTEIN F & FEDERGRUEN A. 2003. Pricing and replenishment strategies in a distribution system with competing retailers. *Operations Research*, **51**(3): 409–426.

BHUNIA A & SHAIKH A. 2014. A deterministic inventory model for deteriorating items with selling price dependent demand and three-parameter Weibull distributed deterioration. *International Journal of Industrial Engineering Computations*, **5**(3), 497-510.

CHUN-TAO CHANG, YI-JU CHEN, TZONG-RU TSAI & SHUO-JYU WU. 2010. Production inventory models with stock-and-price dependent demand for deteriorating items based on limited shelf space. *Yugoslav Journal of Operation Research*, **20**(1): 55–69.

DEBDULAL PANDA & MANORANJAN MAITI. 2009. Multi-item inventory models with price dependent demand under flexibility and reliability consideration and imprecise space constraint:

A geometric programming approach. *Mathematical and Computer Modelling*, **49**(9-10): 1738 - 1749

DHARAMENDER SING. 2019. Production inventory model of deteriorating items with holding cost, stock and selling price with backlog. *International Journal of Mathematics in Operation Research*, **14** (2): 290-305.

GHARE P.M. & SCHRADER G.F. 1963. A model for exponentially decaying inventory, Journal of Industrial Engineering, Vol.14, pp.238-243.

HESHAM K. ALFARES. 2014. Production-inventory system with finite production rate, stock dependent demand, and variable holding cost. *RAIR Operational Research*, **48**: 135–150.

JINN-TSAIRTENG & CHANG TAO. 2005. Economic production quantity models for deteriorating items with price-and stock-dependent demand. *Computers & Operations Research*, **32**(2): 297–308.

KAIYING CAO, PING HE & ZHIXIN LIU. 2020. Production and Pricing decisions in a dual-channel supply chain under remanufacturing subsidy policy and carbon tax policy. *Journal of the Operational Research Society*, **71**(8): 1199–1215.

MOHAMMAD ABDUL HALIM, A. PAUL, MONA MAHMOUD, B.ALSHAHRANI, ATHEELAH Y.M.ALAZZAWI, GAMAL & M.ISMAIL . 2021. An Overtime production inventory model for deteriorating items with non-linear price and stock dependent demand. *Alexandria Engineering Journal*, **60**(3): 2779–2786.

MOHSEN E I HAFSI, JIANXIN FANG & ESSIA HAMOUDA. 2021. Optimal production and inventory control of multi-class mixed backorder and lost sales demand class models. *European Journal of Operational research*, **291**(1): 147–161.

NARESH KUMAR KALIRAMAN, RITU RAJ & SHALINI CHANDRA. 2017. Two Warehouse Inventory Model for Deteriorating item with Exponential Demand Rate and Permissible Delay in Payment. *Yugoslav Journal of Operations Research*, **27**(1), pp.109-124.

NITA H, SHAH, KAVITA RABARI & EKLA PATEL. 2022. Greening efforts and deteriorating inventory policies for price-sensitive stock dependent demand, *International Journal of Systems Science Operations and Logistics*. DOI: 10.1080/23302674.2021, 2022808.

NITA H. SHAH, DUSHYANTKUMAR G. PATEL & DIGESHKUMAR B. SHAH. 2018. EPQ model for returned/reworked inventories during imperfect production process under price-sensitive stock-dependent demand. *Operational Research*, **18**: 343–359.

PETER SHAOHUA DENG, ROBERT HUANG-JING LIN, CHIH -YOUND HUNG, JASON CHANG, HENRY CHAO, SUNG-TE JUNG, JENNIFER SHU-JEN LIN & PETERCHU . 2008. Inventory Models with a Negative Exponential Crashing Cost taking Time Value into Account. *Journal of the Operations Research Society of Japan*, **51**(3): 213-224.

PREETY POSWAL, ANANAN CHOUHAN, RAHUL BOADH & YOGENDRA KUMAR RAJORIA . 2022. *Materials Today: Proceedings*, **56**(6): 72–83.

PRIYANKA SINGH, UTTAM KUMAR KHEDLEKAR, R.P.S. CHAUDEL. 2022. Production inventory model with non-linear price-stock dependent demand Rate and Preservation Technology Investment under complete Backlog. *International Journal of Operation Research*, **19**(2): 41-50.

QI FENG, SIRONG LUO & J. GEORGE SHANTHIKUMAR. 2019. Integrating Dynamic Pricing with Inventory Decisions Under Lost Sales. *Management Science*, **66**(5): 1783–2290.

QUSAY H.AL-SALAMI, RABEEA KH. SALEH & AMMAR F.J. AL BAZI. 2022. An Efficient Inventory Model based on GA For Food Deterioration products in the Tourism Industry. *Pesquisa Operacional*, DOI: 10.1590/0101-7438.2022.042.00257447.

SINGH S, DUBE R & SINGH S.R.. 2011. Production model with selling price dependent demand and partially Backlogging under inflation. *International Journal of Mathematical Modelling & Computations*, **01**(01): 01-07.

SINGH S.R. & MOHIT RASTOGI. 2018. A production inventory model for deteriorating products with selling price dependent consumption rate and shortages under inflationary environment. *International Journal of Procurement Management*, **11**(1): 36–48.

SOURAN MAHATA & BIJOY KRISHNA DEBNATH. 2022. A profit maximization single item inventory problem considering deteriorating during carrying for price dependent demand and Preservation Technology Investment. *RAIRO Operational Research*, **56**: 1841 -1856.

SUBHENDU RUIDAS, MIJANUR RAHAMANSEIKH & PRASUN KUMAR NAYAK. 2020. An EPQ model with stock and selling price dependent demand and variable production rate in interval environment. *International Journal of System Assurance Engineering and Management*, **11**(2): 385–399.

T. TOY & K.S. CHAUDHURI. 2009. A production inventory model under stock dependent demand, weibull distribution deterioration and shortage. *International Transactions in Operational Research*, **16**(3): 325–346.

TALEIZADEH A.A., KALANTARI S.S. & CARDENAS-BARRON L.E .2015. Determining optimal price, replenishment lot size and number of shipments for an EPQ model with rework and multiple shipments. *Journal of Industrial Management Optimization*, **11**(4): 1059–1071.

TAPAN KUMAR DATTA. 2013. An Inventory Model with Price and Quality Dependent Demand Where Some items produced are Defective. *Advances in Operations Research*, **2013**, Article ID 795078.

UMAKANTA MISHRA, LEOPOLDO EDUARDO CARDENAS-BARRON, SUNIT TIWARI, ALIK AKHAR SHAIKH & GERARDO TREVINO-GARZA. 2017. An inventory model under price and stock dependent demand for controller able deterioration rate with shortages and preservation technology investment. *Annals of Operations Research*, **254**: 165-190.

YI WANG & SHENG HAO ZHANG. 2021. Optimal production and inventory rationing policies with selective-information sharing and two demand classes. *European Journal of Operational Research*, **288**(16), pp. 394–407.

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