SECURITY CONSTRAINED OPTIMAL ACTIVE POWER FLOW VIA NETWORK MODEL AND INTERIOR POINT METHOD

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ABSTRACT

This paper presents a new formulation for the security constrained optimal active power flow problem which enables the representation of three basic constraints: branch outage, generator outage and multiple equipment congestion. It consists of a network model with additional linear equality and inequality constraints and quadratic separable objective function, which is efficiently solved by a predictor-corrector interior point method. Sparsity techniques are used to exploit the matricial structure of the problem.Case studies with a 3,535-bus and a 4,238-branch Brazilian power system are presented and discussed, to demonstrate that the proposed model can be efficiently solved by an interior point method, providing security constrained solutions in a reasonable time.

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RESUMO

Este trabalho apresenta uma nova formulação do problema de fluxo de potência ótimo corrente contínua com restrições de segurança de três tipos: perda de ramo, perda de gerador e múltiplas perdas. A formulação emprega um modelo de fluxo em redes com restrições adicionais de igualdade e desigualdade e função objetivo quadrática separável que é eficientemente resolvido por um método de pontos interiores preditor-corretor. Técnicas de esparsidade são utilizadas para explorar a estrutura matricial do problema. Estudos de caso para o Sistema Interligado Nacional com 3535 barras e 4238 ramos são apresentados e discutidos, a fim de demonstrar que o modelo proposto pode ser resolvido através de um método de pontos interiores de maneira eficiente, fornecendo solu-

ções que respeitam as restrições de segurança em um tempo computacional razoável.

PALAVRAS-CHAVE: Segurança, despacho de potência ativa, fluxo de potência ótimo, fluxo em redes, controle de fluxo de potência, método de pontos interiores

1 INTRODUCTION

The optimal power flow (OPF) problem consists of obtaining the optimal settings for control variables in a power system so that certain operational goals can be achieved; these are represented by a predefined objective function f, which is subject to a set of constraints. The operating state of a power system provided by an OPF is one that guarantees affordability, reliability, security, and dependability (Momoh, 2001). Generally, the OPF problem can be expressed as

$$\begin{array}{rl} \text{Min} & f(x,u) \\ \text{s.t.} & g(x,u)=0 \\ & h(x,u)\le 0, \end{array} \qquad (1)$$

where x is the vector of dependent variables (bus voltage magnitudes and phase angles), u is a vector of control variables (as active power generation and active power flow), $g(x,u)$ is the set of nonlinear equality constraints (power flow equations), and $h(x,u)$ is the set of inequality constraints of the vector arguments x and u.

Minimization of active power losses, generation cost and reactive power generation of the system are possible objective functions. They may be achieved by setting control variables u, such as dispatching generating units, adjusting bus voltages and setting transformer taps. This set of constraints involves those conditions necessary to guarantee Kirchhoff's laws, bus voltage ranges, and the rated limits of equipment. The problem is usually solved for a normal operating condition of a power system ($n-0$ case).

A security constrained optimal power flow (SCOPF) is a special type of OPF where the optimum value of the objective function is computed while respecting the constraints, both under normal operating conditions and for specified disturbances, such as outages or equipment failures. These security constraints allow the OPF to determine the operation of the power system in a defensive manner (Wood e Wollenberg, 1996); i.e., the OPF will force the system to be operated in such a way that if a contingency is encountered, the resulting voltages and power flows will still be within the limits established. As for outages, SCOPF usually considers only single outages ($n-1$ case), although in some cases certain critical double outages ($n-2$ case) could be evolved.

As stated in (Biskas e Bakirtzis, 2004), a complete security analysis implies a specific constraint for each branch and unit outage for each monitored branch. The number of security constraints would thus be $m \times (m-1)$ for branches and $n \times (n-1)$ for generators (see Sec. 2 for notation). In order to constitute each constraint the computation of at least one load flow is necessary. Two mains approaches to contingency selection (Stott et al., 1987) are available: direct or indirect methods.

Examples of direct methods are those that use contingency filters (Harsan et al., 1997) and distribution factors (Wai, 1981). The Inverse Matrix Modification Lemma (IMML) is used either explicitly or implicitly for most contingency analysis studies. Specific versions of these approaches have been denominated compensation methods (Alsac et al., 1983). Indirect methods involve the consideration of contingency quantities without explicitly computing them.

Various optimization methods have been used to solve the OPF problem, including linear programming, nonlinear programming and integer programming (Dommel e Tinney, 1968; Happ, 1977; Huneault e Galiana, 1991; Momoh, 2001). Nonlinear programing, such as that used in this paper, can exploit a number of techniques, such as sequential quadratic programming, augmented Lagrangian methods, generalized reduced gradient methods, projected augmented Lagrangian functions, successive linear programming, and interior point methods. The choice of interior point methods (IPMs) was based on the robustness and efficiency reported for their use in OPF problems (Granville, 1994; Wu et al., 1994; Wei, 1996; Torres e Quintana, 1998; Quintana et al., 2000; Yan et al., 2006), specially those involving SCOPF problems (Lu e Unum, 1993; Vargas et al., 1993; Yan e Quintana, 1997; Jabr, 2002).

The main contribution of the present study is to develop an efficient model for a security constrained optimal active power flow (SCOAPF) that considers three types of contingency situations: branch outage, generator outage and multiple equipment congestion. The solution is obtained by using an IPM formulated as a network model with additional linear constraints and also considers a general quadratic separable objective function that may:

- Minimize the deviation from a specified generation obtained from a market pool or a dispatch model that does not consider transmission network constraints.
- Minimize the transmission losses.
- Realize both.

The outline of the paper is as follows. Nomenclature is presented in Sec. 2. The SCOAPF model is presented in detail in Sec. 3. An efficient interior point method for its solution is described in Sec. 4. In Sec. 5 some numerical results for the Brazilian power system are reported and commented on, and in Sec. 6 the conclusions are stated.

2 NOTATION

m number of branches.

n number of buses.

l number of independent circuit loops.

g number of generators.

c number of contingencies.

A network incidence matrix ($n \times m$).

L network loop matrix ($l \times m$).

X reactance diagonal matrix ($m \times m$).

R resistance diagonal matrix ($m \times m$).

E matrix ($n \times g$) formed by elements E_{ij} that are equal to one if generator j is connected to bus i, otherwise it is zero.

N active power flow contingency matrix ($c \times m$).

M active power generation contingency matrix ($c \times g$).

p active power generation vector ($g \times 1$).

d active power load vector ($n \times 1$).

f active power flow vector ($m \times 1$).

θ bus phase angle vector ($n \times 1$).

f_{min} lower bound for active power flow f.

f_{max} upper bound for active power flow f.

p_{min} lower bound for active power generation p.

p_{max} upper bound for active power generation p

α,β weights.

ϕ_1 function associated with power flow vector.

ϕ_2 function associated with power generation vector.

$*$ symbol for fixed or target value.

3 PROBLEM FORMULATION

The SCOAPF model is formulated as the following network model with additional linear equality and inequality constraints and quadratic separable objective function.

$$\min \alpha\, \phi_1\,(f) + \beta\, \phi_2\,(p)\,, \tag{2}$$

subject to

$$A\,f = E\,p - d \tag{3}$$
$$L\,X\,f = 0 \tag{4}$$
$$s_{min} \leq N\,f + M\,p \leq s_{max} \tag{5}$$
$$f_{min} \leq f \leq f_{max} \tag{6}$$
$$p_{min} \leq p \leq p_{max}\,. \tag{7}$$

The objective function (2) corresponds to the association of two different criteria, the first depending on power flow, $\phi_1(f)$, and the second on power generation, $\phi_2(p)$. Both criteria are represented by quadratic and separable functions, and can be combined using scalar weights α and β within a simple bi-objective optimization framework.

$\phi_1(f)$ is a quadratic separable function expressed by

$$\phi_1(f) = \frac{1}{2} f^t\, M_1\, f + m_2^t\, f + m_3, \tag{8}$$

where M_1, m_2, m_3 are a diagonal matrix, a vector and a scalar, respectively. By setting $M_1 = R$, $m_2 = 0$ and $m_3 = 0$, function $\phi_1(f)$ represents an approximation of the transmission power losses.

$\phi_2(p)$ is a quadratic separable function expressed by

$$\phi_2(p) = \frac{1}{2} p^t\, N_1\, p + n_2^t\, p + n_3, \tag{9}$$

where N_1, n_2 and n_3 are a diagonal matrix, a vector and a scalar, respectively. By setting adequate values for N_1, n_2 and n_3, function $\phi_2(p)$ will represent quadratic generation costs.

A quite useful objective function $\phi_2(p)$ is the quadratic deviation from a desirable generation dispatch. Such a dispatch can arise from a pool auction in an electricity market or from a dispatch model which does not take into consideration transmission network constraints. In such a case $\phi_2(p)$ can be represented by

$$\phi_2(p) = \frac{1}{2}(p - p^*)^t\, W\, (p - p^*), \quad (10)$$

where W is a diagonal matrix with the component w_i as the penalty term associated with deviation from the desired generation p_i^*. The equivalence between Eqs. (9) and (10) shows that $N_1 = W$, $n_2 = -p^{*t}\, W$ and $n_3 = \frac{1}{2}p^{*t}\, W\, p^*$.

Eq. (3) corresponds to the nodal balance according to the Kirchhoff's Current Law (KCL), while Eq. (4) represents the independent circuit loop equations, in accordance with Kirchhoff's Voltage Law (KVL). Efficient procedures for finding the loop matrix L from the incidence matrix A are discussed in (Oliveira et al., 2003; Expósito et al., 2006).

Eq. (5) represents a set of generic relationships between pieces of equipment in the power grid. This equation corresponds to the constraints applied to three basic situations:

- Branch outage: the most important branch outages have been selected, and for these, the line outage distribution factors (LODFs) are computed to formulate post-contingency constraints in the form of $f_{min} \leq N\, f \leq f_{max}$. For instance, for the outage of branch k-m, an overload on branch i-j is avoided by including a constraint of the type $(S_1 \leq P_{km} + \alpha\, P_{ij} \leq S_2)$.
- Generator outage: for certain selected generator outages, the generalized generation distribution factors (GGDFs) are computed, leading to post-contingency constraints having the form of $f_{min} \leq N\, f + M\, p \leq f_{max}$. For instance, for the outage of generator k, an overload on branch i-j is avoided by including a constraint of the type $(S_3 \leq P_k + \beta\, P_{ij} \leq S_4)$.
- Multiple equipment congestion: this involves limits on the interchange between areas and bottlenecks already identified by experience of the grid operator. These constraints have the form of $f_{min} \leq N\, f + M\, p \leq f_{max}$. For instance, the power flow between two areas can be limited by adding the constraint $(S_5 \leq P_k + \gamma\, P_{ij} + \eta\, P_{lm} \leq S_6)$.

The first two sets of constraints can be seen as to represent security constraints generated after the analysis of contingency cases, as discussed in (Stott, 1974; Stott e Hobson, 1978; Stott e Marinho, 1979). The automatic generation of sets of security constraints is not the main focus of this paper and the theme will not be discussed further. Although the definition and how to calculate LODFS and GGDFS using contingency cases can be seen in (Sauer, 1981) and (Ng, 1981), respectively. The contribution of this work is the use of LODFS and GGDFS to form a set that considers important contingency constraints and then efficiently solve the resulting problem. Then, the focus is how to efficiently find an OPF solution for which the three situations mentioned above can be handled by the inclusion of Eq. (5).

Eqs. (6) and (7) represent the bounds for active power flow and generation, respectively. Note that transmission limits are imposed directly on the power flow variables, which constitutes one of the main advantages of approaches based on a network model.

The SCOAPF model (2)-(7) corresponds to a DC model where transmission losses are not considered in the active power balance equations. In order to compute more realistic solutions, the following procedure $P1$ can be adopted for the computation of transmission loss:

- Solve the SCOAPF model for the original load vector d. Let (p^0, f^0) be the optimal solution.
- For the solution (p^0, f^0) calculate the power loss for each branch using the equation $f_{km}^{loss} = (\frac{r_{km}}{r_{km}^2 + x_{km}^2})(x_{km} f_{km})^2$.
- Compute a new load vector $\tilde{d}$ by including the branch power loss as an incremental load equally distributed between the terminal buses k and m.
- Solve the SCOAPF model for the new load vector $\tilde{d}$. Let (p^1, f^1) be the optimal solution.
- Verify if the relative difference between f_{km}^0 and f_{km}^1 is less then a specified tolerance. If so, the procedure is finished. Otherwise recalculate the branch power loss for the new solution (p^1, f^1), and repeat the procedure.

In general, this procedure requires only two iterations of OAPF or SCOAPF to achieve convergence within a tolerance of 10^{-2}.

4 SOLUTION TECHNIQUE

For the sake of simplicity, assume that the lower bounds in Eqs. (6) and (7) are all zero and that $\alpha = \beta = 1$ in Eq. (2). The dual problem for the security constrained optimal active power flow model (2)-(7) is given by

$$\begin{array}{rc} rl\min & \hat{d}^t y - f_{max}^t w_f - \phi_1(f) - (p_{max}^t w_p) - \phi_2(p) \\ s.t. & B^t y + z_f - w_f - M_1 f + N^t y_2 = m_2 \\ & -\hat{E}^t y + z_p - w_p - N_1 p + M^t y_2 = n_2 \\ & -y_2 - w_s + z_s = 0 \\ & (z_p, w_p) \geq 0, (z_f, w_f) \geq 0, (z_s, w_s) \geq 0, \end{array}$$

where z_f, z_p, and z_s are slack variables, $B = \begin{pmatrix} A \\ LX \end{pmatrix}$, $\hat{d} = \begin{pmatrix} -d \\ 0 \end{pmatrix}$ and $\hat{E} = \begin{pmatrix} E \\ 0 \end{pmatrix}$, with 0 being a $(l \times g)$ zero matrix.

The optimality conditions for the primal and dual problems are given by primal and dual feasibility and complementarity conditions

$$\begin{cases} F\, z_f = 0, & W_f\, s_f = 0, \\ P\, z_p = 0, & W_p\, s_p = 0, \quad and \\ S\, z_s = 0, & W_s\, s_s = 0, \end{cases}$$

where s_p, s_f, and s_s are slack variables for the bound constraints on active power generation, active power flow and security constraints, respectively. Moreover, the notation $F = \text{diag}(f)$ for diagonal matrices formed by vectors is introduced.

4.1 Primal-Dual Interior Point Methods

Most primal-dual interior point methods can be seen as variants of the application of Newton's method to the first order optimality conditions. The following outlines a framework for such methods, where $x = (f, p, s_f, s_p)$ and $t = (z_f, z_p, w_f, w_p)$ are used.

Assume y^0 and $(x^0, t^0) > 0$. For $k = 0, 1, 2, \cdots$, do

1. Choose $\sigma^k \in [0, 1)$ and set $\mu^k = \sigma^k \left(\gamma^k / n\right)$, where n is the dimension of x and $\gamma^k = (x^k)' t^k$.

2. Compute Newton search directions $\left(\Delta x^k, \Delta y^k, \Delta t^k\right)$.

3. Choose an appropriate step size so that the point remains interior: $\alpha^k = \min(1, \tau^k \rho_p^k, \tau^k \rho_d^k)$, where $\tau^k \in (0, 1)$, $\rho_p^k = (-1/min_i(\Delta x_i^k / x_i^k))$, and $\rho_d^k = (-1/min_i(\Delta t_i^k / t_i^k))$.

4. Compute $\left(x^{k+1}, y^{k+1}, t^{k+1}\right) = \left(x^k, y^k, t^k\right) + \alpha^k \left(\Delta x^k, \Delta y^k, \Delta t^k\right)$.

The step size for both primal and dual variables is the same, since for quadratic problems, primal variables appear in the dual problem constraint set. Parameters σ and τ and the starting point will be discussed later. Newton search directions are defined by the following linear system [1].

[1] From this point on, superscript k will be omitted for a cleaner notation.

$$\begin{cases} A\Delta f - E\Delta p = -d - Af + p & \equiv r_i \\ X\Delta f = -Xf & \equiv r_v \\ \Delta f + \Delta s_f = f_{max} - f - s_f & \equiv r_f \\ \Delta p + \Delta s_p = p_{max} - p - s_p & \equiv r_p \\ \Delta s + \Delta s_s = s_{max} - s - s_s & \equiv r_s \\ B^t\Delta y + \Delta z_f - \Delta w_f - M_1\Delta f + N^t\Delta y_2 & = r_y \\ -\hat{E}^t\Delta y + \Delta z_p - \Delta w_p - N_1\Delta p + M^t\Delta y_2 & = r_g \\ -\Delta y_2 - \Delta w_s + \Delta z_s & = r_{y2} \\ N\Delta f + M\Delta p - \Delta s & = r_{ss} \end{cases} \quad (11)$$

$$\begin{cases} Z_f\Delta f + F\Delta z_f & = \mu e - FZ_f e \equiv r_{zf} \\ Z_p\Delta p + P\Delta z_p & = \mu e - PZ_p e \equiv r_{zp} \\ Z_s\Delta s + S\Delta z_s & = \mu e - SZ_s e \equiv r_{zs} \\ W_f\Delta s_f + S_f\Delta w_f & = \mu e - S_f W_f e \equiv r_{wf} \\ W_p\Delta s_p + S_p\Delta w_p & = \mu e - S_p W_p e \equiv r_{wp} \\ W_s\Delta s_s + S_s\Delta w_s & = \mu e - S_s W_s e \equiv r_{ws} \end{cases} \quad (12)$$

where e is the column vector consisting exclusively of ones, $r_y \equiv m_2 - B^t y - z_f + w_f + M_1 f - N^t y_2$, $r_g \equiv n_2 + y(p) - z_p + w_p + N_1 p - M^t y_2$, $r_{y2} = y_2 + w_s - z_s$, and $r_{ss} \equiv -Nf - Mp + \underline{s} + s$.

4.2 The Predictor-Corrector Method

For the predictor-corrector (PC) approach (Mehrotra, 1992), two linear systems must be solved. First, *affine directions* $(\Delta\tilde{x}, \Delta\tilde{y}, \Delta\tilde{t})$ are computed by solving Eqs.(11) and (12) for $\mu = 0$. The search directions are then given by solving Eq. (11) and

$$\begin{cases} Z_f\Delta f + F\Delta z_f & = \mu e - FZ_f e \equiv \tilde{r}_{zf} \\ Z_p\Delta p + P\Delta z_p & = \mu e - PZ_p e \equiv \tilde{r}_{zp} \\ Z_s\Delta s + S\Delta z_s & = \mu e - SZ_s e \equiv \tilde{r}_{zs} \\ W_f\Delta s_f + S_f\Delta w_f & = \mu e - S_f W_f e \equiv \tilde{r}_{wf} \\ W_p\Delta s_p + S_p\Delta w_p & = \mu e - S_p W_p e \equiv \tilde{r}_{wp} \\ W_s\Delta s_s + S_s\Delta w_s & = \mu e - S_s W_s e \equiv \tilde{r}_{ws}. \end{cases}$$

4.3 Implementation Issues

Parameters $\tau = 0.9995$ and $\sigma = n^{-\frac{1}{2}}$ are fixed. For the predictor-corrector approach, the barrier parameter is given by $\mu = (\tilde{\gamma}/\gamma)^2(\tilde{\gamma}/n^2)$, where $\tilde{\gamma} = (x + \Delta\tilde{x})'(t + \Delta\tilde{t})$. In both versions, however, if $\gamma < 1$ then $\mu = (\gamma/n)^2$. The following starting point is suggested: $y^0 = 0$, $f^0 = s_f^0 = f_{max}/2$, $p^0 = s_p^0 = p_{max}/2$, $z_f^0 = w_f^0 = (R + I)e$, $z_p^0 = w_p^0 = e$, $w_s^0 = z_s^0 = Ie$, $su = s_{max} - s_{min}$, $s^0 = 2su/3$, $s_s^0 = su/3$.

5 LINEAR SYSTEM SOLUTION

Since the two linear systems introduced in Sec. 4.2 share the same matrix, the following discussion will consider only the system involving Eqs. (11) and (12). The dimension of this linear system can be reduced by substitutions involving various sets of variables without changing the sparse pattern of the matrix. First, slack variables are eliminated:

$$\begin{array}{l}\Delta z_f = F^{-1}(r_{zf} - Z_f \Delta f)\\ \Delta z_p = P^{-1}(r_{zp} - Z_p \Delta p)\\ \Delta z_s = S^{-1}(r_{zs} - Z_s \Delta s)\\ \Delta w_f = S_f^{-1}(r_{wf} - W_f \Delta s_f)\\ \Delta w_p = S_p^{-1}(r_{wp} - W_p \Delta s_p)\\ \Delta w_s = S_s^{-1}(r_{ws} - W_s \Delta s_s)\\ \Delta s_f = r_f - \Delta f; \Delta s_p = r_p - \Delta p; \Delta s_s = r_s - \Delta s,\end{array}$$

reducing Eq. (11) to

$$\left\{\begin{array}{ll} A\Delta f - E\Delta p = -d - Af + p & \equiv r_i\\ X\Delta f = -Xf & \equiv r_v\\ B^t \Delta y - D_f \Delta f + N^t \Delta y_2 & = r_a\\ -\hat{E}^t \Delta y - D_p \Delta p + M^t \Delta y_2 & = r_b\\ -D_s \Delta s - \Delta y_2 & = r_{sy2}\\ N\Delta f + M\Delta p - \Delta s & = r_{ss}, \end{array}\right. \tag{13}$$

where $D_f = F^{-1}Z_f + S_f^{-1}W_f + M_1$, $D_p = P^{-1}Z_p + S_p^{-1}W_p + N_1$, $D_s = S^{-1}Z_s + S_s^{-1}W_s$, $r_a = r_y - F^{-1}r_{zf} + S_f^{-1}(r_{wf} - W_f r_f)$, $r_b = r_g - P^{-1}r_{zp} + S_p^{-1}(r_{wp} - W_p r_p)$, and $r_{sy2} = r_{y2} + S_s^{-1}(r_{ws} - W_s r_s) - S^{-1}r_{zs}$. Note that only the inverse of diagonal matrices are involved. Now the active power generation and transmission variables in (13) can be eliminated with $\Delta f = -D_f^{-1}(r_a - B^t\Delta y - N^t \Delta y_2)$, $\Delta p = -D_p^{-1}(r_b + \hat{E}^t \Delta y - M^t \Delta y_2)$, and $\Delta s = -D_s^{-1}(r_{sy2} + \Delta y_2)$, resulting in

$$D_{y2}\Delta y_2 = (r_{ys} - D_s^{-1} r_{sy2} + B_s^t \Delta y) \tag{14}$$

$$D_y \Delta y = r, \tag{15}$$

where $D_y = BD_f^{-1}B^t + \hat{E}D_p^{-1}\hat{E}^t - B_s D_{y2}^{-1} B_s^t$, $D_{y2} = ND_f^{-1}N^t + MD_p^{-1}M^t + D_s^{-1}$, $B_s = -BD_f^{-1}N^t + \hat{E}D_p^{-1}M^t$, $r_{ys} = r_{ss} + ND_f^{-1}r_a + MD_p^{-1}r_b - D_s^{-1}r_{sy2}$, and $r = \begin{bmatrix} r_i \\ r_v \end{bmatrix} + BD_f^{-1}r_a - \hat{E}D_p^{-1}r_b + B_s D_{y2}^{-1}(r_{ys} - D_s^{-1}r_{sy2})$.

In order to solve Eq. (14), it is necessary to solve a system with dimension constituted exclusively by the number of security constraints, which in practice is much smaller than that of branches or even buses. The most intensive computational work is involved in solving Eq. (15).

6 NUMERICAL RESULTS

The proposed SCOAPF model was implemented in Matlab 7.0, running on an Intel Pentium 2.0 GHz personal computer with 2 GB of RAM in a Windows XP Professional environment. The predictor-corrector IPM approach was tested for the Brazilian Power System (BPS), a predominantly hydro system (90%) with 3,535 buses, 4,238 branches, 300 generators and 157 security constraints involving all three of the types described in Sec. 3. Three load levels (light, medium, and heavy) involving slightly different configurations were considered, as shown in Table 1. Table 2 describe the number of contingencies of each type (Branch, Generator or Multiple). All data were provided by the Brazilian Independent System Operator (ISO).

Table 1: BPS configuration for each load level

Load Level	Branches	Buses	Load [MW]
Light	4238	3535	36,249
Medium	4228	3531	40,239
Heavy	4237	3533	53,467

Table 2: Number of BPS security constraints by type

Type	Number
Branch(B)	122
Generation(G)	5
Multiple(M)	30
Total	157

Two case studies with different objective functions were considered:

- CS_1: Minimization of transmission power losses, effected by setting $\alpha = 1$, $\beta = 0$ and $\phi_1(f) = \frac{1}{2} f^t\, R\, f$.
- CS_2: Minimization of quadratic deviation from a predefined generation dispatch p^*. This involves setting $\alpha = 0$, $\beta = 1$ and $\phi_2(p) = \frac{1}{2}(p - p^*)^t\, I\,(p - p^*)$, where I is the identity matrix. Generation dispatch p^* corresponds, in this case, to the economic dispatch that minimizes thermal fuel cost in the BPS and is calculated by the ISO without considering transmission network constraints.

6.1 OAPF model

An initial set of results is presented in which the proposed model is evaluated without consideration of security constraints, but considering transmission losses by the procedure $P1$ described in Section 4 which demands the use of only two successive OAPFs. Table 3 summarizes the performance of the OAPF model in three load scenarios by exposing the total time spent and the number of iterations for each OAPF.

Table 3: Performance of the OAPF model

	Total Time[s](Iter. OPF 1/Iter. OPF2)	
Load Level	CS_1	CS_2
Light	14.53 (7/7)	12.54 (10/6)
Medium	16.23 (8/8)	14.76 (14/6)
Heavy	16.79 (8/8)	23.31 (21/8)

The proposed predictor-corrector IPM approach presented an effective performance in terms of number of iterations and CPU time, that decreases slightly with the increase in load. Table 4 shows the power generation for each case study and load level.

Table 4: Power generation on the OAPF solution

	Generation [MW]	
Load Level	CS_1	CS_2
Light	36,613	37,633
Medium	40,883	42,171
Heavy	53,995	56,602

As expected, the solution for CS_1 provides lower transmission losses. The transmission power losses for light, medium and heavy loads were reduced from 3.7%, 4.6% and 5.5% for CS_2 to 1.0%, 1.6% and 0.98% for CS_1, respectively.

Table 5 shows the number of security constraints violated by the optimal solutions of both case studies. It is interesting to observe that a smaller number of violations occurred for all load levels in CS_1. This can be explained by the network topology of the BPS, as shown in Figure 1. Most of the load is located along the east coast (Atlantic ocean), and the most important hydro plants are located in rural areas, such as the Itaipu and Tucuruí hydro plants. When economic dispatch assigns more generation to these distant hydro plants, (CS_2), the long transmission lines that bring this generation to the load centers operate closer to their capacities and more security constraints become active. When dispatch is designed to minimize transmission loss, (CS_1), more generation is assigned to conventional and nuclear thermal plants which are closer to the load centers, such as the Angra, Fluminense and Fortaleza thermal plants, thus reducing the power flow along the long transmission lines, resulting in less active security constraints. These shifts in generation are presented in Table 6 related to the generating units presented in Figure 1. These two case studies illustrate the trade-off in the BPS between optimal dispatch from an "economic" point of view (which maximizes hydro generation and minimizes thermal fuel costs) and optimal dispatch from an "electric" point of view (which minimizes transmission loss).

Table 5: Number of constraints violated by type on the OAPF solution

	Number of violations							
Load Level	CS_1				CS_2			
	B	G	M	Total	B	G	M	Total
Light	0	0	0	0	2	0	1	3
Medium	0	0	0	0	5	0	1	6
Heavy	0	0	5	5	16	2	4	22

Table 6: Generation of most important plants for CS_1 and CS_2

	Capacity [MW]	Generation [MW]	
Plant		CS_1	CS_2
Itaipu	12600	3522	12412
Tucuruí	5625	1859	5510
Angra	2229	2229	1243
Fluminense	1011	1011	104
Fortaleza	391	391	0

Table 7 shows the values for the seven largest of the 22 security constraints violated in case study CS_2 (heavy load).

Table 7: Most important violations for CS_2 (heavy load)

		Violation[MW]		
Constraint	Type	Light	Medium	Heavy
79	B	0	0	241
84	M	0	0	760
86	G	0	0	714
111	M	0	0	408
135	B	149	534	640
155	B	46	179	274
157	M	0	0	910

Constraint 79 (Region 5 in Figure 1) is violated in Heavy level; it consists of maintaining the transformer shown in Fig. 2 in secure operation. This constraint is responsible for preventing the contingency that one of the three lines will cause a fault in the transformer that connects the 500 kV area and 230 kV area. This situation corresponds to the branch outage case mentioned in Section 3; it can be stated mathematically by the following expression $-3100 \leq 1.00(f_{235-92} + f_{235-93} + f_{235-94}) + 1.00f_{3965-230} + 1.00f_{235-230} \leq 3100$. (matricial form $s_{min} \leq Nf \leq s_{max}$). Constraints 135 and 155 represent similar situations.

Constraint 84 (Region 3 in Figure 1) is necessary to prevent a power flow inversion from the 500 kV area to the 440 kV area, as illustrated in Fig. 3. This situation corresponds to the

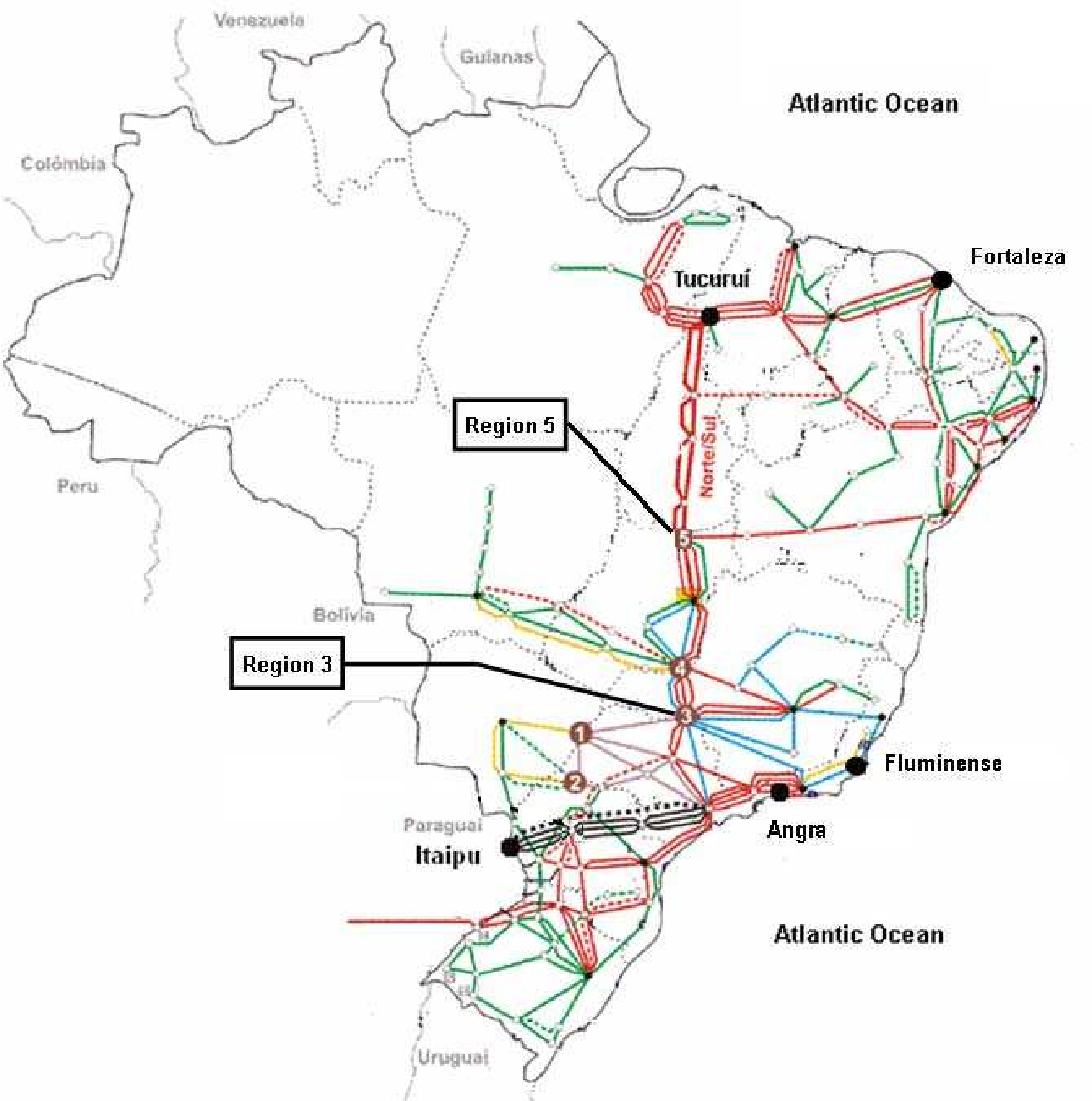

Figure 1: Overview of the BPS grid.

multiple equipment congestion case mentioned in Section 3; it can be stated mathematically as $-8050 \leq f_{535-536} + p_{500} + p_{501} + p_{502} + p_{503} + p_{507} + p_{510} + p_{513} + p_{520} \leq 8050$. (matricial form $s_{min} \leq Nf + Mp \leq s_{max}$). Similar situations are represented by constraints 111 and 157.

Constraint 86 (Region 3 in Figure 1) consists of the prevention of congestion by limiting the maximum power output of the group of generators shown in Fig. 3. This constraint can be treated as a special case of multiple equipment congestion; it can be stated mathematically as $-5550 \leq p_{501} + p_{502} + p_{503} + p_{510} + p_{513} + p_{520} \leq 5550$. (matricial form $s_{min} \leq Mp \leq s_{max}$).

6.2 SCOAPF model

A second set of numerical results is now presented to evaluate the proposed model when security constraints are considered. Table 8 summarizes the performance of the proposed approach, which corresponds in this case to the SCOAPF model, for a heavy load. The inclusion of security constraints influenced the performance of the model in relation to the situation in which these constraints were not considered (Table 3).

The difference in performance between the OAPF and SCOAPF models can be explained as follows: when no security constraints are considered, Eq. (14) is no longer necessary, and an IPM is produced in which each itera-

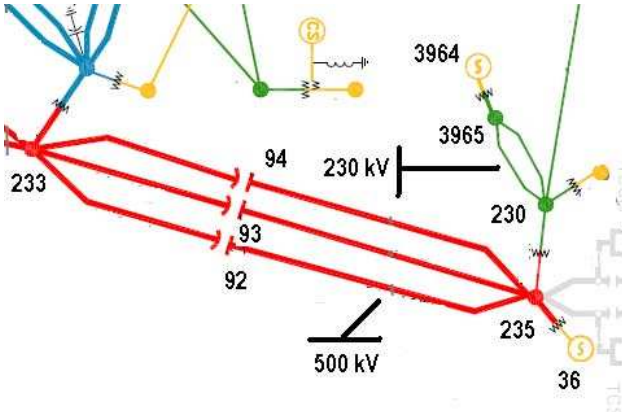

Figure 2: Illustration of Constraint 79 of Region 5.

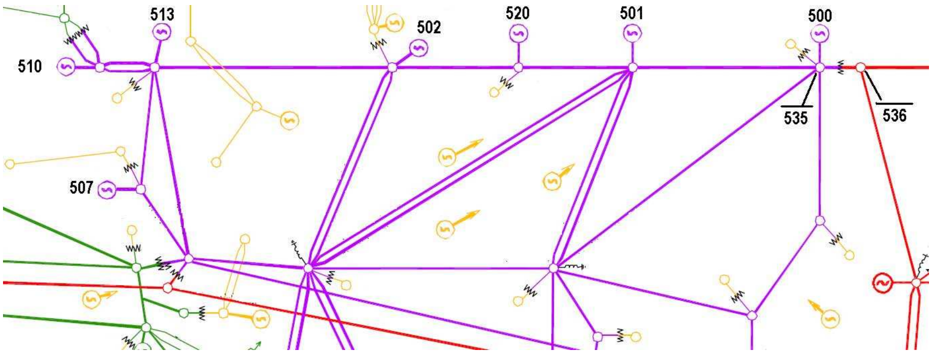

Figure 3: Illustration of constraints 84 and 86 from Region 3.

Table 8: Performance of the SCOAPF model

	Time[s](Iter. OPF1/ Iter. OPF2)	
Load Level	CS_1	CS_2
Heavy	280(9/9)	998(41/24)

tion is computationally less costly. Moreover, Matrix D_y presents 0.2320% of non-zero elements without security constraints, whereas the same matrix with security constraints has 0.3122% of non-zero elements (an increase of 34.57% in the number of non-zero elements). Furthermore, the inclusion of constraints results in an increase in the number of iterations.

Since only a few of the security constraints are active in the optimal solution, one alternative to reduce CPU time would be to adopt a scheme similar to that found in (Stott e Hobson, 1978), in which only the security constraints actually violated, identified after running the OAPF, are included in the SCOAPF model. Note that these constraints are here simultaneously included in the SCOAPF, although in (Stott e Hobson, 1978) they were included one at a time. The application of this procedure for CS_2 and heavy load, where only 22 of the 157 security constraints are violated, reduced the CPU time of about 30%.

7 CONCLUSION

This paper has presented a new security constrained optimal active power flow formulation which enables the representation of three basic constraints: branch outage, generator outage and multiple equipment congestion. Quadratic separable objective functions of active power generation and flows can be considered, thus allowing the minimization of transmission losses, generation cost or quadratic deviation from a desired dispatch, or a combination of both. The model was formulated as a network flow problem with additional linear constraints and enables the efficient solution using an interior point method exploiting the specific matricial structure by sparsity techniques. The model was tested on the Brazilian power system with 3,535 buses, 4,238 branches, and 157 security constraints. The results showed that the technique is flexible, robust and efficient.

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