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**EDITORIAL**

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## **Decolonizing Mathematics Instruction: Subordinating Teaching to Learning**

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Western European nations and the United States have significantly shaped the contemporary world through brutal and devious colonialist and imperialist ventures. Though those endeavors are different, they share common features. Those features include the economic, political, social, and cultural domination stretched over time of territories beyond the geographic boundaries of metropolitan centers to benefit the metropole's ruling classes and their populations. Vladimir I. Lenin (1939) views colonialism and imperialism hierarchically, whereby the latter is an advanced stage of the former and the final stage of capitalism. Contrastingly, the Palestinian intellectual, Edward Said (1993), distinguishes between imperialism and colonialism by stating, "imperialism involved 'the practice, the theory, and the attitudes of a dominating metropolitan center ruling a distant territory,' while colonialism refers to the 'implanting of settlements on a distant territory'" (SAID, 1993, p. 9). Gilmartin (2009) summarizes the British postcolonial theorist Robert J.C. Young's distinction as "imperialism is primarily a concept, and colonialism is primarily a practice" (GILMARTIN, 2009, p. 115). However, contrasted with imperialism, colonialism has been a project of economic and territorial control whereby, to effectuate dominance, the colonizer imposes its religion, language, and other cultural practices.

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In the second half of the last century, responding to the arduous and eventual victorious anticolonial struggles from Western colonial rule of peoples in Asia, Africa, Latin America, and the Caribbean, European and US imperialistic projects shifted from settler colonies to neocolonialism (KOTZ, 2002; QIAO, 2018). The anticolonial struggles were bloody as “colonialism only loosens its hold when the knife is at its throat” (FANON, 1961, p. 61). Nevertheless, neocolonialism advances the colonial goals. Under this indirect, without boots-on-the-ground control, former colonial powers accommodate the former subjugated masses’ illusion of self-determination and jurisdiction over their affairs, but maintain imperialistic hegemony through cunning economic and political mechanisms. Positing neocolonialism as the final state of imperialism, the Ghanaian pan-Africanist and political theorist Kwame Nkrumah (1965) argues that it constitutes a subtler form of control exercised through various economic means and, most importantly, through culture, politics, ideology, literature, and education. Though economic colonization may have ended, its social practices remain in how former colonial societies have established their educational systems and their reverence for the cultural and social traditions of former colonial masters. Consequently, beyond political decolonization, the work of social and cultural decolonization struggles remains, especially in the education realm.

For the education sphere, the neocolonial and neoliberal project manifests itself through the Programme for International Student Assessment (PISA), an arm of the Organization for Economic Co-operation and Development (OECD). PISA’s intentions coincide with OECD’s political and economic commitment to a global free-market capitalist economy. For two decades since its first round of assessments, OECD has successfully marketed PISA as a measure of educational quality, claiming to measure skills and knowledge essential for modern economies and the future. Nevertheless, the excellence defined by PISA is an illusion, a manufactured assertion without empirical evidence (ZHAO, 2020). Moreover, PISA imposes a monolithic, West-centric view of societies on the rest of the world and, as Zhao (2020) argues, distorts the purpose of education.

The skewed, West-centric educational view focuses on preparing citizens to function in a globalized, market-driven capitalistic world system. Though preparing children for economic participation is undoubtedly part of educational institutions’ responsibility, it is not schools’ only responsibility. For educators in many countries, the purpose of education includes considerably more than preparing economic beings. Among those purposes, mathematics educators have frequently mentioned these: citizenship, solidarity, equity, curiosity and engagement, compassion, empathy, cultural values, physical and mental health, and social

justice (JURDAK; VITHAL; FREITAS; GATES; KOLLOSCHE, 2016)<sup>1</sup>. However, these educational aims “are often forgotten or ignored when discussions about school quality rest on PISA scores and rankings” (SJØBERG, 2015, p. 113). Based on PISA results, the official OECD reports, reviews, and advice exert intense pressure toward global standardization of curricula, teaching, and learning (SJØBERG, 2015). Moreover, this monolithic, narrow view of education shows that OECD’s PISA is not mainly an educational venture but a social, political, and ideological project (SJØBERG, 2015). It fulfills a neocolonial, neoliberal project tied to imperialistic goals of international capital.

To escape PISA’s logic and project, education workers, particularly those in mathematics education, must struggle against the homogenization of education and celebration of authoritarian educational systems. This struggle calls for decolonizing standardized forms of instruction through an ongoing critique of Western worldviews and uplifting indigenous knowledge. Indigenous knowledge, as I understand it, means knowledge that originates and naturally occurs in a particular place. Furthermore, the idea of place can be considered geographical, socio-cultural, biological, or psychological. Therefore, mathematics education scholars must maintain a continuing and thorough analysis of dominant curricular models and pedagogical methods and engage in uplifting the indigenous knowledge of students. Decolonization of instruction employs instructional practices that recognize and support the mental brilliance of children and older students and promote students’ exercise of intellectual agency.

Mental brilliance with intellectual and emotional agency are naturally occurring in human beings. These claims coincide with results obtained by, among others, the developmental psychologist and mathematician, Caleb Gattegno<sup>2</sup> (1970, 1973, 1974/2010, 1987b, 1988). Gattegno (1973) studied the development of children from birth onward. From his observation of their acquisition of natural language, he theorized children’s indigenous mental powers to be among the following:

- extracting
- transforming
- visualizing
- evoking images
- abstracting
- noticing patterns
- interpreting
- analyzing
- synthesizing
- wondering

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<sup>1</sup> Also, for instance, consider the educational aims theorized and analyzed in the past 11 proceedings of the international grouping of progressive mathematics educators, Mathematics Education and Society, <<https://www.mescommunity.info>>. In addition, see issues of the Journal of Humanistic Mathematics <https://scholarship.claremont.edu/jhm/>.

<sup>2</sup> For further information about Gattegno and his contribution to mathematics education, see Arthur B. Powell (2007).

- recognizing
- anticipating
- stressing and ignoring

Recognizing these powers of the mind, Gattegno constructed a pedagogical approach he termed “the subordination of teaching to learning” (GATTEGNO, 1970). At its core, the approach invites learners to engage in tasks designed to allow them to continue as agentic beings employing their powers to build knowledge for themselves. Practitioners of this pedagogical approach to mathematics teaching have offered rich, challenging accounts of learners’ accomplishments in constructing powerful and personal mathematical ideas (for example, COLES, 2011; HAJAR, 1997; HEWITT, 1999, 2001a, 2001b; MADELL, 1985; POWELL, 1985/1986, 1993, 2019; POWELL; HOFFMAN, 1990).

Another underlying component of this perspective on mathematics teaching and learning is embodied in this statement: “we separate questions of language and notation from those of awareness of the relationships” (GATTEGNO, 2011, p. 15, original emphasis). The separation that Gattegno mentions identifies two categories of mathematical content: arbitrary and necessary (HEWITT, 1999, 2001a, 2001b). The first category refers to semiotic conventions such as names, labels, and notations. These cannot be constructed or appropriated through attentive noticing or awareness but instead must be given and retained through memorization and association. Unlike the first, the second category of mathematical ideas can be derived or built by attending to and noticing relations among objects. In this sense, these relations are (logically) necessary and, by inviting students to engage in appropriately designed tasks, can be left to students to discern. Discerning or awareness of mathematical relations among objects and producing or expressing those relations are two complementary processes involved in doing mathematics.

The subordination of mathematics teaching to its learning is not an instructional method. Instead, it is a pedagogical approach that supports a decolonial view of what it means to do mathematics. Specifically, from a discursive, participatory perspective, it builds on Gattegno’s (1987a, p. 13-14) view of mathematics:

No one doubts that mathematics stands by itself, is the clearest of the dialogues of the mind with itself. Mathematics is created by mathematicians conversing first with themselves and with one another. ... Based on the awareness that relations can be perceived as easily as objects, the dynamics linking different kinds of relationships were extracted by the minds of mathematicians and considered per se.

This discursive, participatory view of doing mathematics also implies learners’ agentic use of their will. Their will, a part of the active self, commits them to focus their attention so that the mind observes the content of their experience and, through dialogue with themselves

and others, they become aware of the particularities of their experience. In mathematics, the content of experiences, whether internal or external to the self, can be feelings, objects, relations among objects, and dynamics linking different relations.

To support subordinating mathematics teaching to its learning, a framework for working is practical. It consists of a four-phase sequence where an instructional unit is often longer than a single class meeting. The sequence consists of a coherent, flexible sequence of tasks intended to enable learners to work individually or collaboratively to educate their awareness about ideas of a mathematical topic. It attends to the arbitrary and necessary categories of mathematical content. The four phases—actual, virtual, written, and formalized actions—are described in Powell (2018). Furthermore, Amaral, Souza, and Powell (2021, 2022) provide an account of a recent implementation of the four phases for teaching fraction knowledge.

The pedagogical notion of subordinating teaching to learning has other manifestations in mathematics education. In several publications, the 1960s American civil rights fighter for Afro-descendants and founder of the Algebra Project, Bob Moses, and his colleagues (MOSES, 1994; MOSES; COBB, 2001; MOSES; KAMII; MCALLISTER SWAP; HOWARD, 1989; MOSES; WEST; DAVIS, 2009; SILVA, MOSES, RIVERS; JOHNSON, 1990) have argued strongly and convincingly that, in primary and secondary education, access to algebra by students Afro-descendants and other marginalized groups is part of the struggle for social justice. Bob designed the curricular process of the Algebra Project to transition between arithmetic and algebra and between experiential activities in society and abstract mathematical ideas. His pedagogical approach uses quotidian experiences culturally familiar to the students (MOSES; COBB, 2001)<sup>3,4</sup>.

The underlying pedagogical structure in the Algebra Project lessons involves a five-step process designed to move deliberately and carefully from the details of students' cultural experience to the symbolic representation of the mathematical features of that experience. First, the students start with an experience embedded in their cultural world, a physical event such as a trip on public transportation. This concrete activity forms the foundation from which the students proceed to an abstract mathematical concept, initially introducing “the abstract concept at a preconscious level” (SILVA; MOSES; RIVERS; JOHNSON, 1990, p. 380). Next, they reflect on the experience and represent it or model it, usually through a visual art such as

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<sup>3</sup> In an interview with mathematics education Professor Judith Green (2012), New York University, Bob Moses elaborates theoretically on foundations of the Algebra Project's curriculum and practice.

<sup>4</sup> For further information on Bob Moses' Algebra Project and its relation to critical mathematics (FRANKENSTEIN, 1983; POWELL, 2012), see Frankenstein and Powell (in press).

drawing. Then, they share and talk about their representations, using what the Algebra Project calls 'people talk,' intending to identify and isolate the mathematical features of their experience, eventually discussing it in more structured, regimented language, what the Algebra Project calls 'feature talk.' In the case of the public-transportation trip, the fundamental features are the start, the finish, the direction, and the trip's distance. Finally, students develop symbolic representations for their 'feature talk,' at first creatively from their ideas and subsequently learn the canonical academic mathematical symbols, a regimented abstract knowledge.

A strength of the Algebra Project's five-step model is that it engages learners to look at a phenomenon in depth and study it from the viewpoint of their cultural experience and then, from there, isolate the mathematics of the experiences. Grounded in students' lived experiences, the Algebra Project's approach takes time to detail and bridge those experiences to the symbolic representation of salient mathematical features. Further, as Moses and Cobb (2001, p. 120) explain,

students learn that math is the creation of people—people working together and depending on one another. Interaction, cooperation, and group communication are key components of this process. Students also help generate part of the content of instruction as well. They participate in the physical event that will generate data which becomes the vehicle for introducing mathematical concepts. Cooperation and participation in group activities, as well as personal responsibility for individual work, become important not only for the successful *functioning* of the learning group, but for the generation of instructional materials and various representations of the data as well.

The Bob Moses' Algebra Project assumes and builds on students' mental brilliance and intellectual and emotional agency. Like Gattegno's subordination of teaching to learning, Algebra Project's five-step model uses discursive and participatory actions to invite learners to develop mathematical ideas for themselves. As such, both pedagogical approaches decolonize mathematics education.

### **Concluding remarks**

Western-centric views of mathematics education as represented in curricular documents such as the US's Common Core State Standards (Common Core State Standards Initiative, 2010) and assessment instruments such as PISA are unwittingly or wittingly tools of neocolonial and neoliberal projects. The mathematics instruction they prompt dehumanizes learners as they present mathematics as something that emanates not from their minds but the minds of others, a somehow chosen few, represented and exulted by state and international educational authorities. Moreover, mathematics education curricula, as state education

authorities in the US have constructed them, view themselves as superior to students and further dehumanize them by ignoring students' intellectual assets and powers of the mind. Instead, it imposes mathematical ideas that students could develop independently.

Studies indicate that differences exist in how children of different cultural groups communicate, learn, and interact and that these differences reflect on mathematics learning in traditional classrooms in the United States and elsewhere (COBB; HODGE, 2002; DAMEROW; DUNKLEY; NEBRES; WERRY, 1984; GUTSTEIN; LIPMAN; HERNANDEZ; REVES, 1997; MOSCHKOVICH; NELSON-BARBER, 2009; MUKHOPADHYAY; POWELL; FRANKENSTEIN, 2009; NG; RAO, 2010; PRESMEG, 2007). Nevertheless, at the base of identified cultural differences are transcendent ways of learning based on mental powers such as noticing, comparing, discerning, and stressing and ignoring (GATTEGNO, 1970, 1973, 1974/2010). When mathematics teaching engages learners' mental powers, it provides learners access to constructing powerful and personal mathematical ideas for themselves (HAJAR, 1997; HEWITT, 1999, 2001a, 2001b), thereby, decolonizing mathematics instruction.

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