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A mathematical model for growth in weight of silver catfish (*Rhamdia quelen*) (Heptapteridae, Siluriformes, Teleostei)

Modelo matemático para crescimento em peso de jundiá (*Rhamdia quelen*) (Heptapteridae, Siluriformes, Teleostei)

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ABSTRACT

*The use of a mathematical model applied to biological science helps to predict the specific data. Based on biological data (weight and age) of silver catfish, **Rhamdia quelen**, a mathematical model was elaborated based on a nonlinear difference equation to demonstrate the relationship between age and growth in weight. Silver catfish growth was described following the Beverton-Holt model $P_{t+1} = (r\ P_t) / (1+ \alpha\ P_t)$, where $r > 0$ is the maximum growth rate and $\alpha > 0$ is a constant of growth inhibition. The solution of this equation is $P_t= 1 /\{[1/P_0 - \alpha / (r-1)]\ 1/r^t + \alpha/ (r-1)\}$, were P_0 is the initial weight of the fish. Through this model it was observed that the female reaches the theoretical maximum weight approximately at the age of 18 years and the male at the age of 12 years in a natural environment.*

Key words: *nonlinear difference equation, teleost, weight-at-age, growth.*

RESUMO

*A formulação de modelos matemáticos aplicado às ciências biológicas auxilia na previsão de dados específicos. Fundamentado em dados biológicos (peso e idade) de jundiá, **Rhamdia quelen**, elaborou-se um modelo matemático com base em equações a diferenças não lineares para demonstrar a relação entre idade e crescimento em peso. O crescimento do jundiá foi descrito segundo o modelo de Beverton-Holt $P_{t+1} = (r\ P_t) / (1+ \alpha\ P_t)$, onde $r > 0$ é a taxa de crescimento máxima e $\alpha > 0$ é uma constante de inibição do crescimento. A solução dessa equação é $P_t= 1 / \{[1/P_0 - \alpha / (r-1)]\ 1/r^t + \alpha/ (r-1)\}$, onde P0 é o peso inicial do peixe. Por esse modelo foi observado que fêmeas alcançam o peso máximo aproximadamente aos 18 anos e os machos aos 12 anos, em ambiente natural.*

Palavras-chave: *equação a diferenças não-lineares, esteleósteo, peso-idade, crescimento.*

INTRODUCTION

Studies dealing with the relationship between weight and age are of interest to fish biology because they can establish a simple method for evaluation of an individual weight related to its age. Computer modeling is a valuable tool for the analysis of complex systems, and is becoming an important component of research efforts to improve our understanding of aquaculture pond ecosystems and to develop management practices to optimize resource utilization (PIEDRAHITA, 1988). The use of retrospective data is recommended by RICKER (1969) because they can estimate true growth more accurately. Historically, retrospective data were widely used to estimate growth and projected yield in a fishery. The development of new software to properly analyze retrospective size-at-age data should encourage their use.

The models can be utilized to simulate better handling of fish stock or to increase our knowledge about biological interactions in the ecosystem. Complex models of simulation like the von Bertalanffy equation have been used by many fish biologists because of their virtue of including terms that represent metabolic properties of assimilation. However, varying environmental factors cause fish growth patterns to diverge more or less from idealized growth forms (WEATHERLEY & GILL, 1987). The von Bertalanffy model has also the disadvantage of not being appropriate for describing first year growth (GAMITO, 1998).

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When farmers invest on a specific fish species, they wish to know the time necessary for it to reach the slaughter stage and reproduction, so they may program their production and profit. Mathematical models can be used to estimate this desired data. Different equations have been developed to describe growth (JØRGENSEN, 1994). The choice of model depends on which species is being studied and also on the aims of the study or the research possibilities. The use of the exponential growth model for long periods of time is not recommended (WINBERG, 1971; CUENCO et al., 1985) because using this type of model, fish growth does not cease. Nevertheless, its use in aquaculture is relatively common (e.g. PORTER et al., 1986) because of its simplicity (BARNABE´, 1994). There are equations such as VON BERTALANFFY (1938), that describe fish growth but our objective was to develop a model which would describe this natural phenomenon based, simply, on difference equations. To demonstrate the efficiency of the model it was developed a research buy using data on the silver catfish, ***Rhamdia quelen*** (Heptapteridae).

Silver catfish lives in lakes and deep areas of rivers, mainly quiet waters with sand and mud bottoms. This species occurs from the central part of Argentina to the South of Mexico, showing adaptation to various environments and productive potential in fishponds, mainly in South Brazil (GOMES et al., 2000). Moreover, this species has excellent acceptance in consumer market and also in sport fishing (BALDISSEROTTO, 2003).

Based on the information given above and due to the fact that silver catfish is an important native fish species, it was developed a mathematical model from data obtained in the natural environment, relating weight and age distinguishing males and females.

MATERIAL AND METHODS

Size, weight and age empirical data

Size and age data of silver catfish (692 fish of both sexes) from the Santa Catarina swamps, Rio Grande do Sul, Southern Brazil were obtained from WEIS & CASTELLO (1983). These authors determined age through reading and counting of hyaline zones of otoliths and vertebrae because there is formation of one hyaline and one opaque zone per year. Growth curves for each sex were calculated through otoliths (Table 1) and vertebrae.

The relationships between total weight (Wt) and total length (Lt) were calculated by:

Males $W_t = 0.01083\, L_t^{2.970}$ ($r^2 = 0.958$)

Females $W_t = 0.00839\, L_t^{3.048}$ ($r^2 = 0.954$)

Mathematical Model

Difference equations were used in the construction of a mathematical model for silver catfish growth on weight. The main characteristic of these equations is that time is considered discrete in its formulation. In this case, the weight of silver catfish in the time t+1, denoted by P_{t+1}, is related to its weight in time t, P_t. This relationship is expressed as

$P_{t+1} = f(P_t)$, (1)

where f(P) is a nonlinear function of P.

Silver catfish growth was considered to be self-regulated to do an adequate choice for the function f(P). According to the previous expression, growth rate must decrease approximating zero as the weight of the fish increases to asymptotical value. This assures that the fish reaches its maximum theoretical size.

The growth of the fish was described following the Beverton-Holt model (KOT, 2001):

$P_{t+1} = (r\,P_t)/(1+ \alpha\,Pt)$, (2)

where r > 0 is the maximum growth rate and $\alpha > 0$ is denominated constant of growth inhibition.

A steady-state solution P* of difference equation is defined to be the value that satisfies the relations

$P_{t+1} = P_t = P^*$,

so that no change occurs from time step t to time step t+1. From equation (1) it follows that P* satisfies

$P^* = f(P^*)$. (3)

For the Beverton-Holt equation (2), steady states are computed by setting

$P^* = (r\,P^*)/(1+ \alpha\,P^*)$.

There is thus the trivial equilibrium at P1* = 0 and a nontrivial equilibrium at

$P_2^* = (r - 1)/\alpha$.

This steady state makes sense only if r >1, since a negative population P* would be biologically meaningless.

In the model P_2^* represents a threshold for the weight of the fish. It was verified if P_2^* was asymptotically stable, that is, given $0 < P_0 < (r - 1)/\ \alpha$, the sequence P_0, P_1 ... must monotocally approximates, as t increases, the non trivial equilibrium point $(r - 1)/\ \alpha$.

A steady state P* is asymptotically stable if $|f'(P^*)| < 1$ (see, eg., EDELSTEIN-KESHET, 1988). Thus, the non trivial steady state P_2^* is stable if, and only if, $1/r^2 < 1$, that is, if r > 1.

On the other hand, it was derived an exact, closed form solution for the equation (2) using the substitution $u_t = 1/P_t$ (KOT, 2001).

This solution was

$P_t = 1 / \{[1/P_0 - \alpha / (r-1)]\ 1/r^t + \alpha/ (r-1)\}$, (4)

where Pt is the fish initial weight.

Table 1 - Silver catfish lenght and respective age classes determined by otoliths reading. Data from WEIS & CASTELLO (1983).

Age (year)	1	2	3	4	5	6	7
Male length (cm)	17.4	26.6	31.3	34.8	38.3	41.6	-
Female length (cm)	19.1	26.8	31.6	35.1	38.6	41.8	45.0

Estimative of the model parameters

The parameters (and r were determined adjusting the data obtained in the literature (WEIS & CASTELLO, 1983): maximum weight for males (1.353kg) and females (3.018kg), in the solution (4), utilizing a least-square fit available in the software Mathematica version 4.0 (1999).

RESULTS AND DISCUSSION

The model developed here was built from nonlinear difference equations. Based on these equations, through qualitative methods, it is possible to obtain considerable information about its behavior. The simplicity in its formulation and computer implementation put the difference equations as a new tool for interdisciplinary works, once it facilitates the exchange of information between mathematicians and professionals of biological sciences. The simplicity of the development of this mathematical model was the main criteria to be considered.

Growth of silver catfish is pronounced in its initial years of life. Females and males of this species can reach 66.5cm and 52cm in length during the adult phase, respectively. The growth rate of males is higher than females up to the third of fourth year of life, when the situation changes because females start to grow faster. Females can grow older than males because life span is approximately 21 years for females and 11 years for males (WEIS & CASTELLO, 1983).

According to the proposed model silver catfish female reaches its maximum weight at approximately the age of 18 years, whereas the male of this species reaches its maximum weight at the approximate age of 12 years. Moreover, it is clearly shown that silver catfish male and female have a similar growth up to the sixth year of life. After the sixth year males continue to growth up but more slowly when compared to females. It is also clear that the maximum weight of silver catfish males is practically half of the maximum weight of females (Figure 1). The existence of sexual dimorphism was verified in relation to the weight-age for silver catfish.

Based on the results obtained, it was observed a significant difference between the sexes, showing that females reach larger size than males, observations also made for ***Pimelodus maculatus*** (NOMURA et al., 1972; BASILE-MARTINS, 1978). In relation to the size proportion between males and females, some authors obtained different results for other species. Due to the fact that weight is a biological variable which depends on biotic and abiotic factors, standardization does not occur between the sexes of the fish. HAGERMAN (1952) found females relatively heavier than males for ***Microstomus pacificus***. ANGELESCU et al. (1958) did not find differences in the relationship between males and females of the ***Argentinean merluza***, ***Merluccius merluccius hubbsi***. BAXTER (1960) did not observe significant differences on size between sexes in ***Seriola dorsalis,*** nor did YAMAGUTI & SANTOS (1966) for ***Macrodom ancylodon***. However, males from piava, ***Leporinus friderici,*** collected in Lobo reservoir and Moji-Guaçu river presented higher growth rate than females (BARBIERI & SANTOS, 1988). In addition various factors may be responsible for the differences in parameters of the weight-length relationships among seasons and years, such as temperature, salinity, food (quantity, quality and size), sex, time of year and stage of maturity (SHEPHERD & GRIMES, 1983; PAULY, 1984; WEATHERLEY & GILL, 1987)

The data used for the formulation of this mathematical model was obtained from the study of WEIS & CASTELO (1983) with silver catfish collected in their natural habitat from Rio Grande do Sul state, but it can also be applied to specimens of this species from other places because GOMES et al. (2000) demonstrated that the weight-length relationship for this species is similar in other Brazilian states (Paraná and São Paulo). BRANDER (1995) alluded to one of the key factors as being that the size at age of a fish represents the cumulative effect of its environmental experience from hatching. It could be suggested that, for example, in a fishpond in which the quantity and quality of the water and climate are more adequate, combined with a specific diet, silver catfish would reach more weight in less time. As a practical example, from the equation obtained in the present study, for one year old silver catfish the weight would be 52.5 g for males and 67.0g for females, while fish farmers claim

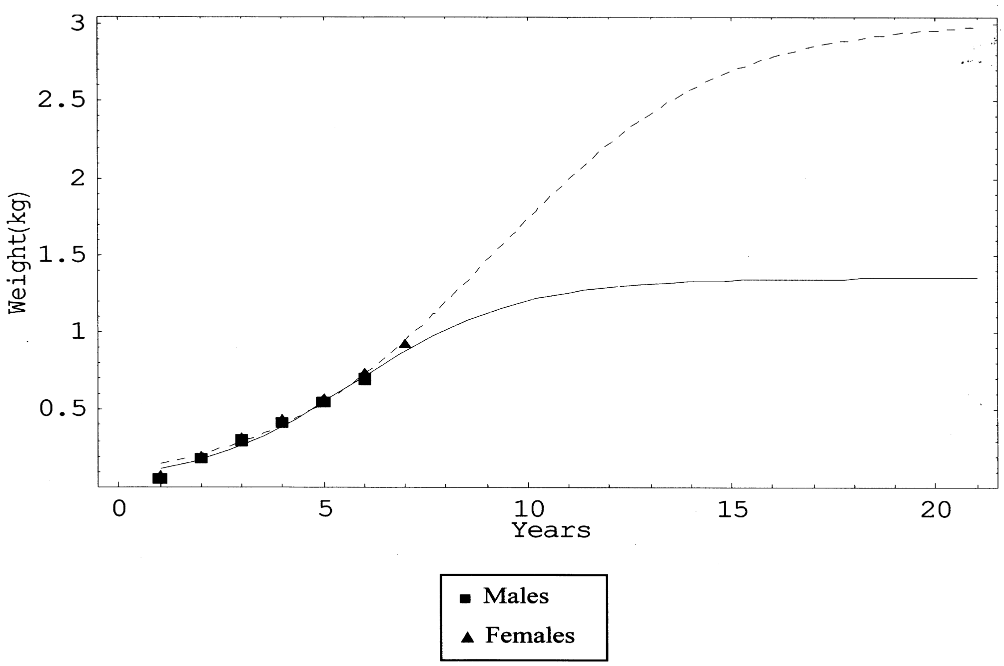

Figure 1 - Age-weight relationship for silver catfish males (full line) and females (dotted line). Empirical data from WEIS & CASTELLO (1983) for males (squares) and females (triangles).

that they can obtain specimens with 600 – 800 g for silver catfish of the same age raised in fish culture pond and fed with commercial food (BALDISSEROTTO, 2003).

Some modifications might be made to the model once the necessary information is available. Growth is much more sensitive to water quality parameters when they become limiting factors. Parameters related to food intake and net energy from feeding are also sensitive to the model output but are not well defined. Consequently, more basic research in aquaculture ecosystems is required to put aquaculture on a solid scientific base and make models become a useful design and management tool for commercial aquaculture.

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