Upper Bounds for Fusion Processes in Collisions of Weakly Bound Nuclei

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We obtain upper limits for the contributions of the incomplete fusion and sequential complete fusion processes to the total fusion cross section. Through those upper bounds we find that these processes are negligible in reactions induced by projectiles such as ^{6}He and ^{11}Li, which break up into neutrons and one fragment containing the full projectile charge.

The effects of channel coupling in fusion reactions induced by weakly bound projectiles have attracted great interest over the last decade [1]. Some theoretical studies predict strong influence of the breakup channel over the complete fusion (CF) cross section [2–7]. When one tries to compare these predictions with experimental data [8–14], one finds a serious problem. Sorting out complete and incomplete fusion (IF) events in an experiment may be a very difficult task, specially when uncharged fragments are produced in the breakup of the weakly bound collision partner. For this reason, most experiments measure the total fusion cross section, $\sigma_{TF} = \sigma_{CF} + \sigma_{IF}$. These results could not, in principle, be used to check theoretical predictions for σ_{CF}. However, the situation would be different when $\sigma_{IF} << \sigma_{CF}$. In this case one can approximate $\sigma_{TF} \simeq \sigma_{CF}$ and the measured cross section could be directly compared with theoretical predictions for σ_{CF}. In the present work, we present a method to find upper limits for σ_{IF} in collisions induced by weakly bound projectiles. With this method, we show that the incomplete fusion cross sections may be neglected when the projectile breakup produces uncharged fragments.

The appropriate theoretical tool to handle this problem is the coupled-channels method. However, its implementation becomes very complicated for the breakup channel, since it involves an infinite number of states in the continuum. For practical purposes, it is necessary to approximate the continuum by a finite set of states as in the Continuum Discretized Coupled-Channels method (CDCC) [15]. This procedure has been extended to the case of fusion reactions in refs. [5–7]. Recently, a semiclassical alternative based on the classical trajectory approximation of Alder and Winther (AW) [16] has been proposed [17]. This approximation was used to calculate breakup cross sections and the results were compared with those of the CDCC method. The agreement between these calculations was very good. Since this semiclassical version of the CDCC method is much simpler, it may be a very useful tool to calculate cross sections for other channels in reactions with weakly bound nuclei. Although the AW method has been extensively used for several nuclear reaction processes, only very recently it was applied to the estimate of the fusion cross section [18]. For this application it was considered a simplified two-channel problem for which the fusion cross section obtained with the AW method was compared with results of a full coupled-channels calculation. In spite of the large simplification in the calculation the agreement between these two calculations was again very good. Although such calculations may not be reliable for quantitative predictions, they lead to a very useful qualitative conclusion. At above-barrier energies, the fusion probability through channel-α at the partial-wave l can be written as a product of two factors. The first is the population of channel α, $\bar{P}_l^{(\alpha)}$, at the point of closest approach. The second is the tunneling probability, $T_l^{(\alpha)}$, through the effective (l-dependent) potential barrier. When dealing with the breakup channel, one should have in mind that different tunneling factors should be used for incomplete fusion of each breakup fragment. This point is not considered when one treats the breakup channel as a bound state. A quantitative semiclassical calculation of the fusion cross sections in reactions with weakly bound projectiles requires the inclusion of the continuum states associated with the breakup channel, as in ref. [17]. However, some simple upper bounds can be easily obtained.

As this work is devoted to reactions induced by weakly bound projectiles, the variables employed to describe the collision are the projectile-target separation vector, $\mathbf{r}$, and the relevant intrinsic degrees of freedom of the projectile, ξ. For simplicity, we neglect the internal structure of the target. The Hamiltonian is then given by

$$h = h_0(\xi) + V(\mathbf{r},\xi), \tag{1}$$

where $h_0(\xi)$ is the intrinsic Hamiltonian and $V(\mathbf{r},\xi)$ represents the projectile-target interaction. The eigenvectors of $h_0(\xi)$ are given by the equation

$$h_0\ |\phi_\alpha\rangle = \varepsilon_\alpha\ |\phi_\alpha\rangle\,. \tag{2}$$

The AW method [16] is implemented in two-steps. First, one employs classical mechanics for the time evolution of the variable $\mathbf{r}$. The ensuing trajectory depends on the collision energy, E, and the angular momentum, l. In its original version, an energy symmetrized Rutherford trajectory $\mathbf{r}_l(t)$ was used. In our case, the trajectory is the solution of the classical equations of motion with the potential $V(\mathbf{r}) = \langle\phi_0|V(\mathbf{r},\xi)|\phi_0\rangle$, where $|\phi_0\rangle$ is the ground state of the projectile. In this way, the coupling interaction becomes a time-dependent interaction in the ξ-space, $V_l(\xi,t) \equiv V(\mathbf{r}_l(t),\xi)$. The second step consists of treating the dynamics in the intrinsic space as a time-dependent quantum mechanics problem. Expanding the wave function in the basis of intrinsic eigenstates,

$$\psi(\xi,t) = \sum_{\alpha} a_{\alpha}(l,t)\ \phi_{\alpha}(\xi)\ e^{-i\varepsilon_{\alpha}t/\hbar}, \tag{3}$$

and inserting this expansion in the Schrödinger equation for $\psi(\xi,t)$, one obtains the AW's equations

$$i\hbar\ \dot{a}_{\alpha}(l,t) = \sum_{\beta}\ \langle\phi_{\alpha}|V_l(\xi,t)\,|\phi_{\beta}\rangle\ e^{i\left(\varepsilon_{\alpha}-\varepsilon_{\beta}\right)t/\hbar}\ a_{\beta}(l,t). \tag{4}$$

These equations are solved with the initial conditions $a_{\alpha}(l,t\to-\infty) = \delta_{\alpha 0}$, which means that before the collision $(t\to-\infty)$ the projectile was in its ground state. The final population of channel α in a collision with angular momentum l is $P_l^{(\alpha)} = |a_{\alpha}(l,t\to+\infty)|^2$ and the angle-integrated cross section is

$$\sigma_{\alpha} = \frac{\pi}{k^2}\ \sum_{l}(2l+1)\ P_l^{(\alpha)}. \tag{5}$$

To extend this method to fusion reactions, we start with the quantum mechanical calculation of the fusion cross section in a coupled channel problem. For simplicity, we assume that all channels are bound and have spin zero. The fusion cross section is a sum of contributions from each channel. Carrying out partial-wave expansions we get

$$\sigma_{TF} = \sum_{\alpha}\ \left[\frac{\pi}{k^2}\sum_{l}(2l+1)\ P_l^F(\alpha)\right], \tag{6}$$

with

$$P_l^F(\alpha) = \frac{4k}{E}\ \int dr\ |u_{\alpha l}(k_{\alpha},r)|^2\ W_{\alpha}^F(r). \tag{7}$$

Above, $u_{\alpha l}(k_{\alpha},r)$ represents the radial wave function for the l^{th}-partial-wave in channel α and W_{α}^F is the absolute value of the imaginary part of the optical potential associated to fusion.

To use the AW method to evaluate the complete fusion cross section, we make the approximation

$$P_l^F(\alpha) \simeq \bar{P}_l^{(\alpha)}\ T_l^{(\alpha)}(E_{\alpha}). \tag{8}$$

Above, $T_l^{(\alpha)}(E_{\alpha})$ is the probability that a particle with reduced mass $\mu_{\alpha} = m_0 A_P A_T/(A_P+A_T)$ and energy $E_{\alpha} = E-\varepsilon_{\alpha}$ tunnels through the potential barrier in channel α, and $\bar{P}_l^{(\alpha)}$ is the probability that the system is in channel-α at the point of closest approach on the classical trajectory.

We now proceed to study the complete and incomplete fusion cross sections in reactions induced by weakly bound projectiles. For simplicity, we assume that the GS is the only bound state of the projectile (as is the case of ^{11}Li projectiles) and that the breakup process produces only two projectile fragments, F_1 and F_2. In this way, the labels $\alpha = 0$ and $\alpha \neq 0$ correspond respectively to the GS and the breakup states represented by two unbound fragments. Neglecting any sequential contribution, the complete fusion can only arise from the elastic channel. In this way, the cross section σ_{CF} can be obtained from eq.(6), dropping the sum over channels and using in the single term

$$\bar{P}_l^{(0)} \equiv P_l^{Surv} = |a_0(l,t_{ca})|^2\,. \tag{9}$$

This probability is usually called survival (to breakup) probability. We get

$$\sigma_{CF} = \frac{\pi}{k^2}\sum_{l}(2l+1)\ P_l^{Surv}\ T_l^{(0)}(E). \tag{10}$$

The accuracy of the semiclassical fusion cross section has recently been checked in a preliminary two-channel calculation in the scattering of ^{6}He projectiles on a ^{238}U target, at near barrier energies [18]. The weakly bound ^{6}He nucleus dissociates into ^{4}He and two neutrons, with threshold energy $B = 0.975$ MeV. The elastic channel is strongly coupled to the breakup channel and the influence of this coupling on the fusion cross section is very important. In this model, the breakup channel is represented by a single effective state [19]. For simplicity, the effective channel is treated as a bound state but it is assumed to contribute only to incomplete fusion. The complete fusion cross section is therefore given by eq.(10) and the incomplete fusion cross section by considering only the $\alpha = 1$ term in eqs.(6) and (8). In [18] the threshold energy was neglected and the same potential barrier was used for both channels. That work showed that above the Coulomb barrier the semiclassical cross sections (both σ_{CF} and σ_{TF}) are in very good agreement with those calculated with the coupled-channels method. Further evidences of this fact will be presented in a forthcoming paper [20].

These calculations are rather schematic, since the continuum is represented by a single bound effective channel. In this way $T_l^{(1)}(E_1)$ is the tunneling probability of the projectile through the projectile-target potential barrier. However, incomplete fusion does not correspond to this process. It corresponds to the tunneling of a projectile's fragment through its barrier with respect to the target. In the particular collision studied in [18], that is ^{6}He - ^{238}U, incomplete fusion corresponds to the fusion of ^{4}He with ^{238}U. The ^{4}He fragment carries about 2/3 of the incident energy while the ^{4}He-^{238}U potential barrier is slightly higher then that for the entrance channel. Thus it is clear that the incomplete fusion cross section is overestimated in our previous work [18]. To illustrate this situation, in Fig. 1 we show the total fusion cross section (solid squares) of [18] where in σ_{TF} the incomplete fusion contribution was obtained from eq.(6) with $T_l^{(1)}(E_1)$ representing the projectile-target tunneling probability. We then re-calculate σ_{TF} modifying the contribution from incomplete fusion. We use the same $\bar{P}_l^{(1)}$ but replace the tunneling factor by that for the ^{4}He fragment. That is, we use the ^{4}He - ^{238}U potential barrier and the energy and angular momentum corresponding to the shares of ^{4}He in the ^{6}He projectile. For

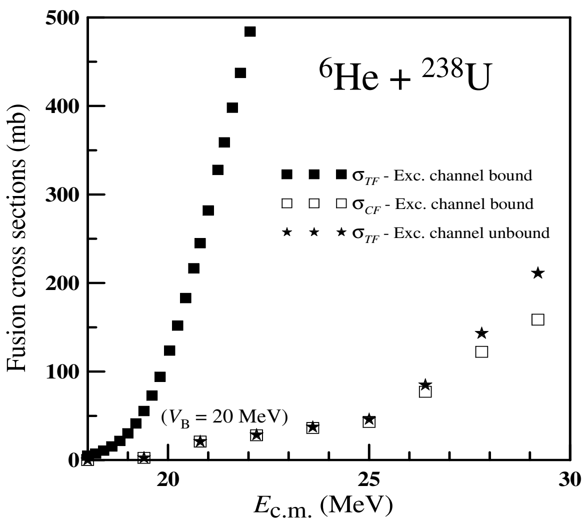

FIG. 1: Total fusion cross section of ref. [18] (solid squares) compared with that of the present work (stars). The present calculation uses the same potential, channel coupling and simplifying assumptions of [18]. The basic difference is that here the contribution from incomplete fusion uses the tunneling of the ^{4}He fragment, rather than the full ^{6}He projectile. For comparison, the complete fusion cross section of [18] is also shown (open squares).

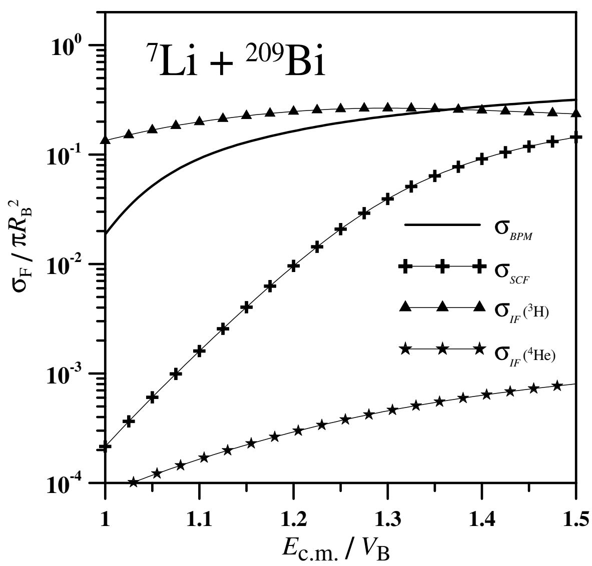

FIG. 2: Upper bounds of the contributions to the incomplete fusion cross section for the ^{7}Li + ^{209}Bi system, employing the Akyüz-Winther parametrization for the interaction potentials.

simplicity, we neglect the relative motion of the fragments of ^{6}He. The resulting σ_{TF} is shown in Fig. 1 as stars. It is clear that a proper treatment of the tunneling factor leads to a substantial reduction of σ_{TF}. The new cross section now is close to the complete fusion cross section σ_{CF} also obtained in [18] (open squares). This indicates that the incomplete fusion cross section σ_{IF} is very small.

As we mentioned before, the above results cannot be considered as a realistic prediction of the total fusion cross section, since the model does not use a realistic description of the continuum states corresponding to the breakup channel. Nevertheless we will show that such simple calculations are capable of yielding relevant information on the fusion process: more precisely, upper bounds for the incomplete fusion and the sequential complete fusion cross sections, σ_{IF} and σ_{SCF}, respectively, can be obtained from eq.(6) setting $\bar{P}_l^{(1)} = 1$ and evaluating the tunneling probability in a proper way, as discussed below.

To illustrate the application of this procedure, we show two examples. We employ the Akyüz-Winther parametrization for the interaction potentials for all the systems considered. Furthermore, the ingoing wave boundary condition is used in all these calculations. Note that in the schematic model of Fig. 1 we neglected the breakup threshold energy. However, in the following estimates of upper limits for the fusion cross sections, we do take it into account.

In the first case, shown in Fig. 2, we consider different fusion processes that appear for the case of a ^{7}Li projectile incident on a ^{209}Bi target, at energies just above the Coulomb barrier. Only energies above the barrier are shown, as this is the region of applicability of the present version of the method employed here [18]. The cross section for the incomplete fusion induced by the ^{3}H fragment is much larger than that for ^{4}He, which is negligible. This situation should be expected because of the lower Coulomb barrier energy for ^{3}H. Also shown is the single barrier penetration model cross section, σ_{BPM}, for ^{7}Li. We note that the upper bound for the incomplete fusion cross section induced by the ^{3}H fragment is large, exceeding σ_{BPM} in the low energy region. The experimental findings for this system [8] yield a value of the incomplete fusion cross section of about 30% of the total fusion cross section. Thus, although our upper bound is compatible with the data, not much is learnt in this case. Also shown in this figure is the upper bound for the cross section for sequential complete fusion, σ_{SCF}. Although negligible at low energies, it becomes appreciable for $E_{c.m.}/V_B \approx 1.5$. We should remark that to neglect the relative motion between the fragments tends to overestimate the sequential complete fusion cross section, and to decrease our estimate of the incomplete fusion cross sections. A quantitative investigation of these effects is under way [20].

In the case of ^{6}He incident on ^{238}U shown in Fig. 3, only the contribution from ^{4}He to the incomplete fusion cross section must be included, as the capture of one or both of the neutrons produced in the breakup of ^{6}He cannot be experimentally distinguished from the transfer process. In this case the upper bound for both the incomplete fusion cross section, and the sequential complete fusion cross sections are much smaller than the BPM estimate for the complete fusion cross section. This shows that, although it is difficult in this case to distinguish between the complete and total fusion cross sections, their difference is expected to be small, as the value of the incomplete fusion contributions to the total fusion cross

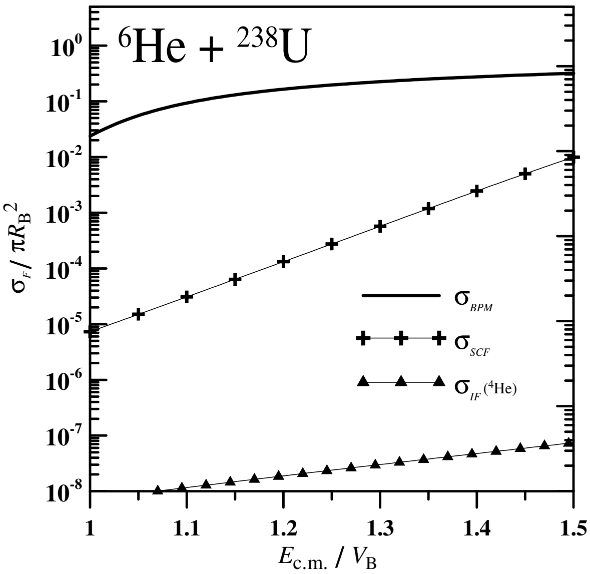

FIG. 3: Same as Fig. 2 for the ^{6}He + ^{209}Bi system. Note that only the ^{4}He contribution to the incomplete fusion has been shown. See text for details and further discussion.

section is not important.

In summary, we have illustrated how the application of the upper bounds to the incomplete fusion cross sections may be applied to the estimate of their contribution to the total fusion cross section. In cases where the unstable nucleus breaks into charged fragments, these upper bounds are consistent with the values measured. When one of the fragment posseses all of the charge of the unstable nucleus, we have shown that the complete fusion cross section, which is easy to evaluate theoretically, is a good estimate of the measured total fusion cross section. The calculations presented here are limited to energies above the Coulomb barrier. An extended version of the method exploring the classically forbidden region and including the relative motion between the fragments is presently being developed [20].

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