A New Inflaton Model Beginning Near the Planck Epoch

Reuven Opher and Ana Pelinson
Departamento de Astronomia, IAG, Universidade de São Paulo, 05508-900, São Paulo, SP, Brazil

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The Starobinsky model predicts a primordial inflation period without the presence of an inflaton field. The modified version of this model predicts a simple time dependence for the Hubble parameter $H(t)$, which decreases slowly between the Planck epoch and the end of the inflation, $H(t) = M_{\rm Pl} - \beta M_{\rm Pl}^2 t$, where β is a dimensionless constant to be adjusted from observations. We investigate an inflaton model which has the same time dependence for $H(t)$. A reverse engineered inflaton potential for the time dependence of H is derived. Normalization of the derived inflaton potential is determined by the condition that the observed density fluctuations, $\delta\rho/\rho \approx 10^{-5}$, are created at $\sim 60\,e$-folds before the end of inflation. The derived potential indicates an energy (mass) scale, $M_{\rm end} \sim 10^{13}$ GeV, at the end of inflation. Using the slow roll parameters, which are obtained from this potential, we calculate the spectral index for the scalar modes n_S and the relative amplitude of the tensor to scalar modes r. A tensor contribution, $r \simeq 0.13$, and an approximately Harrison-Zeldovich density perturbation spectrum, $n_S \simeq 0.95$, are predicted.

Keywords: Inflation model; Planck epoch; Inflaton model

I. INTRODUCTION

The two major problems in cosmology are the origins of the primordial inflation period and the present "inflation" period of the universe. It is possible that both origins are linked. Primordial inflation could have been created by a non-zero vacuum energy. Subsequently, the vacuum energy could have decayed, creating the present period of acceleration. However, strong limits were recently placed on the possible decay of the vacuum energy into cold dark matter (CDM) or cosmic microwave background (CMB) photons [1].

The most popular model for the origin of primordial inflation remains the inflaton (scalar field) model. We investigate here an inflaton model based on the simple time dependence of the Hubble parameter, $H(t)$ [Eq.(3)], that was predicted by the modified Starobinsky model [2],[3]. (See [4] for the original Starobinsky model.) The Starobinsky model suggests that quantum fluctuations created a non-zero vacuum energy that induced the primordial inflation period.

Instead of assuming an ad hoc inflaton potential, as in the standard inflation model, we use the reverse engineering method of Ellis, Murugan and Tsagas [5] to derive the inflaton potential from the $H(t)$ of Eq.(3). The derived potential becomes negligible at the end of inflation, creating the observed density fluctuations, $\delta\rho/\rho \approx 10^{-5}$. These fluctuations are determined by the value of the potential and its first derivative at $60\,e$-folds before the end of inflation. This condition, together with the time dependence of the potential, determine a mass (energy) scale, $M_{\rm end} \simeq 10^{13}{\rm GeV} \sim 10^{-6} M_{\rm Pl}$, at the end of inflation. From the slow roll parameters obtained from the derived potential, we calculate the spectral index of the scalar modes n_S and the relative amplitude of the tensor to scalar modes r. The derived spectral index n_S is in agreement with the WMAP data [6, 7]. The ratio of tensor to scalar modes obtained, $r \sim 0.13$, is similar to that of most inflation models, which predict $r \sim 10-30\%$.

We can compare our scale $M_{\rm end}$ at the end of inflation with the results of Vilenkin [8] and Starobinsky [9]. Vilenkin noted that, in the Starobinsky model, the Hubble parameter defines a mass (energy) scale with a limiting value, $M_{\rm end} \lesssim 10^{16}$GeV, at the end of inflation. Starobinsky predicted that $M_{\rm end} \lesssim 10^{14}$GeV by requiring that the $\delta\rho/\rho$, resulting from inflation, is sufficiently small. Our derived value, $M_{\rm end} \sim 10^{13}$GeV, is consistent with the upper limits of both Vilenkin and Starobinsky for $M_{\rm end}$.

Although the potential that we obtain [Eq.(6)] is superficially similar to a standard inflation potential that depends on the square of the massive scalar field (see, for example, [10] for a recent review), our inflation model is very much different from the standard model for the following reasons:

1) The standard massive scalar inflation potential has two free parameters: the magnitude of the potential and its first derivative at $\sim 60\,e$-folds before the end of inflation. However, our potential in Eq.(6) is completely determined by a single parameter β, which is derived from the simple time dependence of the Hubble parameter in Eq.(3);

2) In the standard inflation model, there are many possible forms that the massive scalar potential can take. However, the form of our potential, a quadratic dependence on the field, is determined uniquely by Eq.(3);

3) The origin of the potential in the standard model is completely unknown. Moreover, there is no clear justification for its form; and

4) In the standard model, the inflation period begins when there is a displacement of the massive scalar field from the minimum of its potential. The origin of this displacement is left unexplained and the epoch in which it occurs is not specified. However, in our model, the beginning of inflation is specified to occur at the Planck epoch (i.e., at the beginning of the universe). The origin of the inflation is a direct result of the simple time dependence of the Hubble parameter in Eq.(3). Moreover, there is no initial displacement of the field that is left explained.

We present the algorithm for constructing the potential from the time dependence of the Hubble parameter in § 2. In § 3, we use this algorithm to obtain the effective potential from the Hubble parameter, $H(t) = M_{\rm Pl} - \beta M_{\rm Pl}^2 t$. From the potential, we calculate the spectral index of the $\delta\rho/\rho$ and the intensity of primordial gravitational waves. The mass (energy) scale at the end of inflation, $M_{\rm end} \sim 10^{13}$GeV, is determined from the requirement that the potential creates observed $\delta\rho/\rho \simeq 10^{-5}$ at $\sim 60\,e$-folds before the end of inflation. Finally, our conclusions are presented in § 4.

II. THE FRAMEWORK OF THE SINGLE SCALAR MODEL

Let us assume that there exists an inflaton field, $\phi = \phi(t)$, where t is the usual time function, in accordance with the Roberston-Walker symmetry [11]. The Lagrangian containing a minimally coupled scalar field is

$$L = \frac{1}{2}\,(\partial\phi)^2 - V\,(\phi) = \frac{1}{2}\,\dot{\phi}^2 - V\,(\phi)\,, \tag{1}$$

where $\dot{\phi} = d\phi/dt$. The scalar stress tensor takes the perfect fluid form,

$$T_{ab} = (p+\rho)\,u_a u_b + p\,g_{ab}\,, \tag{2}$$

with the following energy density and pressure of the scalar inflaton field:

$$\rho_\phi = \frac{1}{2}\,\dot{\phi}^2 + V\,(\phi)\,, \tag{3}$$

$$p_\phi = \frac{1}{2}\,\dot{\phi}^2 - V\,(\phi)\,. \tag{4}$$

The classical equation of motion for $\phi(t)$, which follows from the variation of the action $S = \int d^4x\,\sqrt{-g}\,L$, is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0\,, \tag{5}$$

where $H = \dot{\sigma}(t)$. The field equations for the Robertson Walker model, with $k = 0$, are

$$3\dot{H} + 3H^2 = (8\pi G)(V(\phi) - \dot{\phi}^2)\,, \tag{6}$$

$$3H^2 = (8\pi G)\left(V(\phi) + \frac{\dot{\phi}^2}{2}\right)\,. \tag{7}$$

Following Ellis, Murugan and Tsagas [5], we combine these two independent equations to obtain a more convenient set of equations,

$$V(\phi(t)) = \frac{1}{(8\pi G)}\left(\dot{H} + 3H^2\right)\,, \tag{8}$$

$$\dot{\phi}^2 = -\frac{1}{(4\pi G)}\,\dot{H}\,. \tag{9}$$

From $H(t)$, the above equations have been used to construct the effective potential in the following manner:
i) Eq.(9) is integrated to obtain $\phi(t)$;
ii) t as a function of ϕ is found;
iii) $t(\phi)$ is substituted in $H(t)$ to obtain $H(\phi)$; and
iv) the potential $V(\phi)$ is obtained, using Eq.(8).

III. THE EFFECTIVE INFLATON POTENTIAL

Assuming the simple Hubble parameter time dependence,

$$H(t) = M_{\rm Pl} - \beta M_{\rm Pl}^2 t\,, \tag{10}$$

we solved Eq.(9) for $\phi(t)$, obtaining t as a function of ϕ,

$$t(\phi) = \pm\frac{1}{M_{\rm Pl}^2\sqrt{2\beta}}\,(\phi(t) - \phi_0)\,, \tag{11}$$

where $|\phi_0| > |\phi|$. Choosing the positive sign in Eq.(11), we have $-\infty < \phi < 0$, as in [5]. From Eqs.(10) and (11),

$$H(\phi) = M_{\rm Pl} - \sqrt{\frac{\beta}{2}}\,(\phi(t) - \phi_0)\,. \tag{12}$$

Following the algorithm of the previous section to obtain $V(\phi)$, we substitute Eq.(12) into Eq.(8) to obtain

$$V(\phi) = M_{\rm Pl}^4\left\{-\beta + 3\left[1 - \frac{1}{M_{\rm Pl}}\sqrt{\frac{\beta}{2}}(\phi - \phi_0)\right]^2\right\} \tag{13}$$

or, in terms of the time,

$$V(t) = M_{\rm Pl}^2\left[-\beta M_{\rm Pl}^2 + 3\left(M_{\rm Pl} - \beta M_{\rm Pl}^2 t\right)^2\right]\,. \tag{14}$$

A realistic potential $V(\phi)$ describing inflation should:
1) become negligibly small at the end of the inflationary period, so that there is no important "cosmological constant" entering the FRW era; and
2) produce the density fluctuations at ~ 60 e-folds before the end of inflation (see e.g. [12]),

$$\frac{\delta\rho}{\rho} = \frac{1}{\sqrt{75}\,\pi M_{\rm Pl}^3}\,\frac{V^{3/2}(\phi)}{dV/d\phi}|_{N=60}\,, \tag{15}$$

which are observed to be $\sim 10^{-5}$.

For as long as the first term in Eq.(10) dominates, we have the inflationary expansion $a(t) = \exp M_{\rm Pl}t$. The second term in Eq.(10) decreases the expansion rate and is important near the maximum value of $\sigma(t) = \ln a(t)$. Following Vilenkin [8], we characterize the end of inflation by

$$H(t)|_{t=t_{\rm end}} = \mu M_{\rm Pl}\,, \tag{16}$$

where $H(t_{\rm end}) = \dot{\sigma}(t = t_{\rm end}) = M_{\rm end}$ and

$$\mu = \frac{M_{\rm end}}{M_{\rm Pl}} \tag{17}$$

is a dimensionless parameter (we should expect $M_{\rm end} < M_{\rm Pl} \sim 10^{19}$ GeV).

The time as a function of μ at the end of inflation is

$$t_{\rm end} = \frac{1}{\beta M_{\rm Pl}}\,(1-\mu)\,. \tag{18}$$

The number of e-folds of inflation before $t_{\rm end}$ is

$$N = \int_{t_{60}}^{t_{\rm end}} H(t)dt = \sigma(t_{\rm end}) - \sigma(t_{60})\,. \tag{19}$$

We are interested in $N \simeq 60$, the approximate time t_{60}, when the observed $\delta\rho/\rho$ (scalar) and the primordial gravitational (tensor) fluctuations were created. Substituting Eq.(18) into Eq.(10), we find

$$\sigma_{\rm end} = \sigma(t = t_{\rm end}) = \frac{1}{2\beta}\left(1-\mu^2\right), \qquad (20)$$

where we have used the customary normalization for $a(t = 0) = 1$. From Eq.(19), we have

$$\sigma_{60} = \sigma(t = t_{60}) = \frac{1}{2\beta}\left(1-\mu^2\right) - 60. \qquad (21)$$

Using this result to solve Eq.(10) for t_{60}, we obtain

$$t_{60} = \frac{1}{\beta M_{\rm Pl}}\left[1-\sqrt{1-2\beta\sigma_{60}}\right]. \qquad (22)$$

The slow roll parameters ε and η in terms of $H(\phi)$ are [12]

$$\varepsilon \equiv 2M_{\rm Pl}^2\left[\frac{H'(\phi)}{H(\phi)}\right]^2, \qquad (23)$$

$$\eta \equiv 2M_{\rm Pl}^2\left[\frac{H''(\phi)}{H(\phi)}\right]. \qquad (24)$$

To first order, the slow roll parameters are related to the ratio r of the tensor to scalar fluctuations, by the relation

$$r \sim 16\varepsilon \qquad (25)$$

and to the spectral index of the scalar $\delta\rho/\rho$ by

$$n_S - 1 \approx -\frac{3}{8}r + 2\eta \qquad (26)$$

[13]. The value for μ that characterizes the end of inflation, is constrained by the condition that $\varepsilon = 1$. From this condition and Eq.(12), we obtain

$$\mu^2 = \beta. \qquad (27)$$

Substituting the time at 60 e-folds before the end of inflation from Eq.(22) and β from Eq.(27) into Eq.(15), we obtain

$$\frac{\delta\rho}{\rho} = \frac{1}{\sqrt{75}\pi M_{\rm Pl}^3}\frac{V^{3/2}(t)}{V'(t)dt/d\phi}|_{t=t_{60}} \qquad (28)$$

$$\approx 5.42\mu. \qquad (29)$$

Using the above result, together with Eq.(17) and the observational evidence that the $\delta\rho/\rho$ produced at ~ 60 e-folds before the end of inflation is $\sim 10^{-5}$, we obtain the predicted value of $M_{\rm end}$, the mass (energy) scale at the end of inflation,

$$M_{\rm end} \approx 10^{13}\text{GeV}. \qquad (30)$$

This value is less than the GUT scale ($\sim 10^{14} - 10^{16}$GeV), but is consistent with the upper limits for the mass (energy) scale at the end of inflation given by Vilenkin [8] and Starobinsky [9].

Evaluating the spectral index of the scalar $\delta\rho/\rho$ from Eqs.(24) and (12), we observe that the parameter η is zero and that ε is very small, $\varepsilon \simeq 8.3\times10^{-3}$. From Eq.(26), we have $n_S \simeq 0.95$, an approximately Harrison-Zeldovich spectrum $n_S = 1$, in agreement with the WMAP data [6, 7]. These results do not depend on the exact value of ϕ_0.

From ε in Eq.(24) and Eq.(12), we obtain

$$r = 16\varepsilon \approx 0.13. \qquad (31)$$

This value is similar to those predicted by frequently discussed inflation models with $r \sim 10\% - 30\%$ (e.g., [13]).

IV. CONCLUSIONS

We investigated a model in which the Hubble parameter is decreasing slowly in time, as predicted by the modified Starobinsky model [2],[3], $H(t) = M_{\rm Pl} - \beta M_{\rm Pl}^2 t$, and constructed an inflaton potential for $H(t)$. The derived potential, normalized at ~ 60 e-folds before the end of inflation, creates the observed level of $\delta\rho/\rho \sim 10^{-5}$ and indicates an energy (mass) scale, $M_{\rm end} \sim 10^{13}$ GeV, at the end of inflation.

This energy scale at the end of inflation can be compared with those predicted by Vilenkin and Starobinsky. Vilenkin gave a limiting value $M_{\rm end} \lesssim 10^{16}$GeV for the scale at the end of inflation [8], while Starobinsky predicted $M_{\rm end} \lesssim 10^{14}$GeV [9].

From the inflaton potential, we calculated the spectral index of the scalar modes. The result, $n_S \simeq 0.95$, is compatible with the WMAP data. The potential also predicts a tensor contribution, $r \sim 0.13$, in accordance with most inflation models, which predict $r \sim 0.10 - 0.30$ and is in agreement with existing observational data.

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