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EFFECTS OF TEMPERATURE-DEPENDENT VISCOSITY ON FLUID FLOW AND HEAT TRANSFER IN A HELICAL RECTANGULAR DUCT WITH A FINITE PITCH

Cuihua Wang[1,2], Shengju Liu[1], Jianhua Wu[1,2*] and Zhang Li[3]

[1]School of Energy and Power Engineering, Shenyang University of Chemical Technology,
Shenyang, Liaoning 110142, China.
E-mail: lsj890511@126.com
[2]School of Chemical Engineering and Technology, Tianjin University, Tianjin 300072, China.
[3]School of Chemical Engineering, Shenyang University of Chemical Technology,
Shenyang, Liaoning 110142, China.

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Abstract - An incompressible fully developed laminar flow in a helical rectangular duct having finite pitch and curvature with temperature-dependent viscosity under heating condition is studied in this work. Both the cases of one wall heated and four walls heated are studied. The cross-sectional dimensions of the rectangular duct are 2a and 2b. The aspect ratio $\eta = 2b/2a$ is 0.5. Water is used as the fluid and Reynolds number (Re) is varied in the range of 100 to 400. The secondary flow with temperature-dependent viscosity is enhanced markedly as compared to constant viscosity. An additional pair of vortices is obtained near the center of the outer wall at Re=400 for the model of four walls heated with temperature-dependent viscosity, y, while for constant viscosity, the appearance of two additional vortices near the outer wall cannot be found. Besides, the axial velocity decreases and the temperature increases at the central region of the rectangular duct when the temperature-dependent viscosity is considered. Due to the decrease of the viscosity near the walls, the friction factor obtained with temperature-dependent viscosity is lower than that of constant viscosity, while the convective heat transfer for temperature-dependent viscosity is significantly enhanced owing to the strengthened secondary flow. Especially for four heated walls, the effects of viscosity variation on the flow resistance and heat transfer are more significant.
Keywords: Temperature-Dependent; Helical Rectangular Duct; Flow resistance; Heat transfer.

INTRODUCTION

The problem of temperature-varying properties of the fluid is more complex than that of constant properties. The different property ratio correlations of different fluids increase the complexity of the variable-temperature properties problem. It is also difficult to give a correction for the temperature-dependent behavior of all viscous liquids in all tubes. The viscosity varies more markedly than the other thermo-physical properties for most liquids, so plenty of researcher on the variation of viscosity has been performed in the literature. A few studies on fluid flow and heat transfer for temperature-dependent thermo-physical properties in straight tubes with circular cross section have been carried out, e.g., by Shah and London (1978), Harms *et al*. (1998) and Kakac (1987). Shah and London reported the laminar

*To whom correspondence should be addressed

flow and heat transfer of gases and liquids and Harms *et al*. gave reasonable predictions for many fluids with corrections based on different models for temperature-dependent behavior. However, both the range of temperature and the forms of geometries considered were limited. Kakac developed a correction for the temperature-dependent viscosity effect on Nusselt number and the friction factor of laminar flow.

There are also many studies of fluid flow and heat transfer in straight rectangular ducts, but only several of them considered the variation of the fluid properties dependent on temperature. Sehyun Shin (1993, 1994) and Chang-Hyun Sohn (2006) investigated the influence of variable viscosity of temperature-dependent fluids on the laminar heat transfer and friction factor in straight rectangular ducts. They reported that the effect of temperature-dependent viscosity on the laminar heat transfer is more significant in a non-circular duct than in a circular duct because, in the latter, a symmetric thermal boundary condition must be used and the effect of variable viscosity on heat transfer cannot be capitalized. Mohamed Eissa (2000) studied the fully developed MHD laminar flow and heat transfer through a rectangular duct of a viscous incompressible electrically-conducting fluid.

Since enhanced heat transfer in curved tubes due to the secondary flow was first reported by Dean (1927), more and more temperature-dependent properties problems in curved tubes have been studied. Bergles *et al.* (1983) investigated a curved circular tube neglecting the curvature effects. Kumar *et al.*(2007) and Andrade and Zaparoli (2001) reported the flow and heat transfer for both heating and cooling conditions in a curved circular tube with temperature -dependent properties. The former took into account four thermo-physical properties variations of water with increasing temperature and their results showed that the specific heat and thermal conductivity have negligible effects on the velocity and thermal profiles and density has a small effect, while viscosity plays the dominant role. The latter study focused on the effects of varying viscosity on the fully developed laminar flow and heat transfer for the case of water, and indicated that the frictional factor and the Nusselt number for water are markedly dependent on the viscosity.

Helical rectangular ducts have an extensive application in industrial processes, such as in heat exchangers, ventilators, gas turbines, aircraft intakes and centrifugal pumps (2005). It is clear that all the literature mentioned above considered the variation of the fluid properties dependent on the temperature in straight rectangular ducts or circular curved ducts and none considered helical rectangular ducts. The purpose of this paper is therefore to investigate the effect of temperature-dependent viscosity on the laminar flow and heat transfer in helical rectangular ducts having finite curvature and pitch. The flow is considered to be steady, and hydrodynamically and thermally fully developed. Both the cases of constant temperature at one heated wall and four heated walls are simulated. The hydrodynamics and thermal profiles for varied viscosity flow are presented and compared with those for constant viscosity flow. The friction factor and Nusselt number results are predicted considering the temperature-dependent viscosity.

GOVERNING EQUATIONS

Figure 1 illustrates the geometry and systems of coordinates for the helical rectangular duct, where, R_c is the curvature radius of the helical duct; H is the so-called pitch; 2a and 2b are the width and height of the cross section, respectively. (r, θ, z) are the cylindrical coordinate system and v_r, v_θ, v_z are the corresponding velocity components. Neglecting viscous dissipation and axial conduction, the governing equations for a steady, incompressible, fully developed laminar flow can be written in the cylindrical coordinate system as (Rennie and Raghavan, 2006).

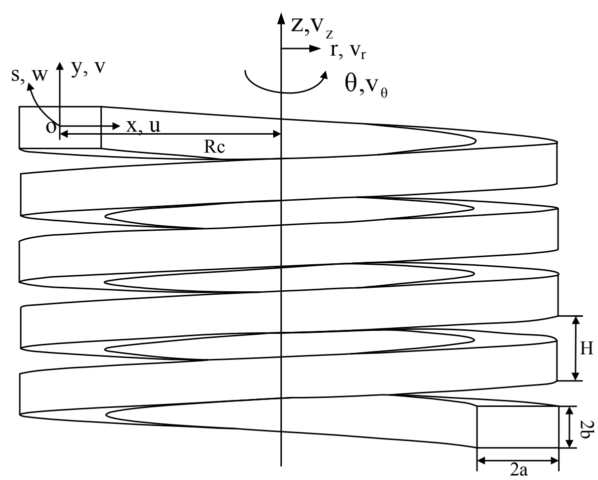


Figure 1: Schematic geometry and coordinates of the helical rectangular duct model.

The continuity equation is:

$$\frac{\partial v_r}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}\frac{\partial v_z}{\partial z}+\frac{v_r}{r}=0 \tag{1}$$

The momentum equations are:

$$v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial r}-\frac{v_\theta^2}{r}$$
$$=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}\right. \tag{2}$$
$$\left.+\frac{\partial^2 v_r}{\partial z^2}-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right)$$

$$v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial r}-\frac{v_r v_\theta}{r}$$
$$=-\frac{1}{\rho}\frac{\partial p}{\partial \theta}+\frac{\mu}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}\right. \tag{3}$$
$$\left.+\frac{\partial^2 v_\theta}{\partial z^2}-\frac{v_\theta}{r^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right)$$

$$v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial r}$$
$$=-\frac{1}{\rho}\frac{\partial p}{\partial \theta}+\frac{\mu}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right) \tag{4}$$

The energy equation is:

$$v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}$$
$$=\frac{\lambda}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

where ρ is the density; p is the pressure; T is the temperature; λ is the thermal conductivity; c_p is the specific heat at constant pressure; μ is the viscosity and is dependent on the temperature of the flow. For water, the temperature-dependent viscosity μ can be calculated using a polynomial function of the form:

$$\mu=A_0+A_1T+A_2T^2+A_3T^3+A_4T^4+A_5T^5 \tag{6}$$

where A_i (i=1, 2, 3, 4, and 5) are coefficients determined by a polynomial fitting of the viscosity data of water with MATLAB. The maximum deviation of Eq. (6) from the viscosity data is less than 0.5% and the fitting values of A_i are shown in Table 1.

Table 1: Coefficients of Temperature-Dependent Viscosity for Water.

Coefficient	A_0	A_1	A_2	A_3	A_4	A_5
Value	1.9216	-2.8296 E10-02	1.6721 E10-04	-4.9509 E10-07	7.3389 4E10-10	-4.3550 E10-13

The no-slip boundary condition at the walls of the duct gives $v_r = v_\theta = v_z = 0$, and constant temperature (T_w) is imposed on the heated wall. For the model of one heated wall, the inner wall is the heated wall and the others are adiabatic walls.

VELOCITY TRANSFORMATION AND ANALYSES

In Figure 1, (x, y, s) is an orthogonal helical coordinate system and u, v, w are the velocity components in the x, y, s directions; w is the axial velocity of the spiral orthogonal coordinate system and u, v constitute the secondary flow. According to Germano (1989), u, v, w can be derived from v_r, v_θ, v_z as:

$$w=-\sin\varphi\cos v_r+\cos\varphi\cos\alpha v_\theta+\sin\alpha v_z \tag{7}$$

$$u=-\cos\varphi v_r-\sin\varphi v_\theta \tag{8}$$

$$v=\sin\varphi\sin v_r-\cos\varphi\sin\alpha v_\theta+\cos\alpha v_z \tag{9}$$

$$\sin\varphi=\frac{K(z\text{-}z_o)}{K^2+R_c r},\quad \sin\varphi=K\Big/\sqrt{R_c^2+K^2}, \quad \cos\alpha=R_c\Big/\sqrt{R_c^2+K^2} \tag{10}$$

where $K=H/2\pi$; α is the slope of the centerline relative to the plane z =constant; φ is the angle along the centerline; z_o is the ordinate value of the midpoint in the cross section.

Dimensionless parameters are defined as follows:

$$x'=x/d_h,\ y'=y/d_h,\ w'=w/w_m,\quad (u',v')=\rho(u,v)d_h/\mu \tag{11}$$

$$\kappa=R_c d_h/(R_c^2+K^2),\ \tau=Kd_h/(R_c^2+K^2) \tag{12}$$

Here $d_h=4ab/(a+b)$ is the hydraulic diameter for a rectangular cross-section, w_m is the mean value of the axial velocity, and κ and τ are the dimensionless curvature and torsion ratio, respectively.

The vorticity ω is defined as:

$$\omega=\partial v'/\partial x'-\partial u'/\partial y' \tag{13}$$

Here $Re=d_h w_m\rho/\mu$.

The Nusselt numbers are obtained as:

$$Nu_l = \frac{h_l d_H}{\lambda}, \quad Nu_m = \frac{\int_0^A Nu_l dA_w}{w_m A_w} \tag{14}$$

where q_w denotes the heat flux on the heated walls, Nu_l is the local Nusselt number along the heated walls, Nu_m is the average Nusselt number, T_b is the bulk temperature and is defined as:

$$T_b = \frac{\int_0^A w_m T dA}{w_m A} \tag{15}$$

where A is the area of the duct cross-section.

The friction factors are:

$$fRe = \left(\frac{-d_h (dp/ds)}{2\rho w_m^2} \right) Re \tag{16}$$

SOLUTION METHOD

In the present work, the governing equations are solved by the computational fluid dynamics software FLUENT6.3 based on the finite volume method. The velocity and pressure coupling is solved by using the SIMPLEC algorithm and the second-order upwind scheme is employed for momentum and energy discretization. Using structured hexahedron grids, the face grid spacing of 0.7 mm and the volume grid spacing of 1.2mm are adopted for the final results. The numerical computation is considered to be converged when the maximum errors of all variables are $\leq 10^{-6}$.

The Nusselt number and friction factor obtained in square helical ducts with four heated walls for constant properties are compared with the predictions made by Borlinder and Sunden (1996) and Zhang *et al*. (2011) in Table 2.

Table 2: Verification of the simulation method.

Re	fRe			Nu_m		
	Emp. (Sunden, 1996)	Sim.	Difference (%)	Emp. (Zhang, 2011)	Sim.	Difference (%)
225	18.47	18.01	2.49	9.82	9.87	0.51
250	18.91	18.51	2.12	10.23	10.25	0.20
275	19.35	18.99	1.86	10.62	10.64	0.19
300	19.8	19.48	1.62	10.99	11.04	0.45
325	20.25	19.95	1.48	11.35	11.45	0.88

Apparently, the simulated results are in good agreement with the empirical results with absolute maximum deviations of 2.49% for the friction factor and 0.88% for the Nusselt number, respectively. The comparative results indicate that this simulation method is accurate.

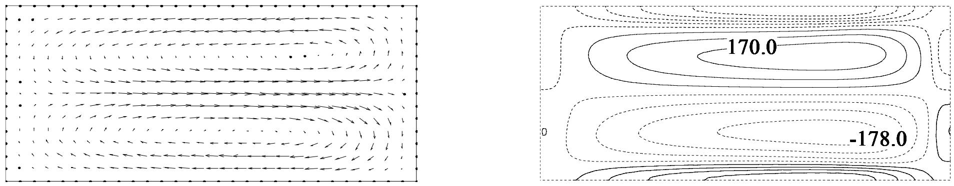
RESULTS AND DISCUSSION

Analyses were carried out for Reynolds number varying in the range of 100 to 400 and the uniform temperature of the heated wall (or walls) was 350 K. For the model of one heated wall, the inner wall was set as the heated wall and the others were considered to be adiabatic. For the model of four heated walls, all the walls are set at 350 K. Both the constant viscosity and the temperature-dependent viscosity are calculated at the mean temperature of the duct inlet. Bara (1992) and Borlinder and Sunden (1995) indicated that 2.5 turns should suffice to establish a fully developed flow with constant viscosity in helical triangular ducts and Kumar *et al*. reported that the flow in curved tubes with circular cross section is fully developed at $\theta=1.5\pi$ in the case of temperature-dependent viscosity; therefore we took the cross section of $\theta=6\pi$ into consideration.

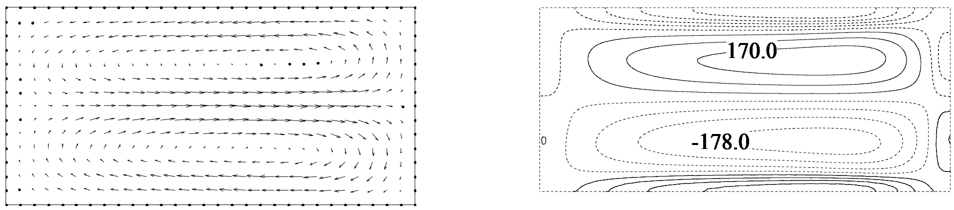
Fig. 3 shows the effect of temperature-dependent viscosity on the secondary flow for both the case of one heated wall and four heated walls. For constant viscosity, where the parameters of the fluid flow remain constant with the change of temperature, the heating condition has no effect on the laminar flow fields. The secondary flow for constant viscosity with one heated wall is the same as that of four heated walls (see part (a) and part (b) of Fig. 2). The simulation results for one heated wall (or four heated walls) representing the flow fields for constant viscosity are presented in Fig. 3. As seen in Fig. 3, the absolute values of the vorticity in the case of temperature-dependent viscosity are higher than the corresponding results of constant viscosity at Re= 200, 300, and 400. Furthermore, the centers of the upper and lower vortices are shifted toward the top wall and bottom wall, respectively. This means that the secondary flow is enhanced for constant Re when the viscosity variation with temperature is considered, and the action of the viscosity variation is more marked in the case of four heated walls.

With the increase of Reynolds number, the secondary flow is strengthened markedly for both temperature-dependent viscosity and constant viscosity in terms of the change of the vorticity in Fig. 3. It is noted that an additional pair of vortices was obtained near the center of the outer wall for the temperature-dependent viscosity with four heated walls at Reynolds number 400 (shown in the part (iii) of Fig. 3), while for constant viscosity at Re=400, there are no additional vortices near the outer wall.

secondary flow vector vorticity



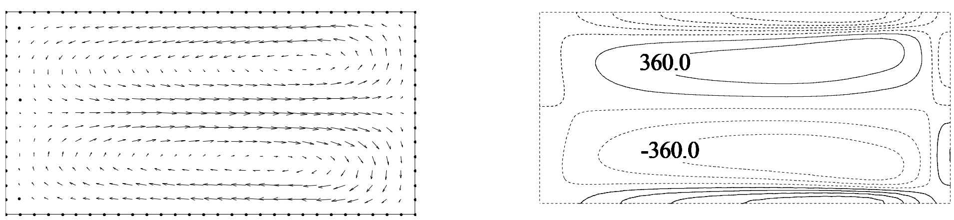
(a) One heated wall



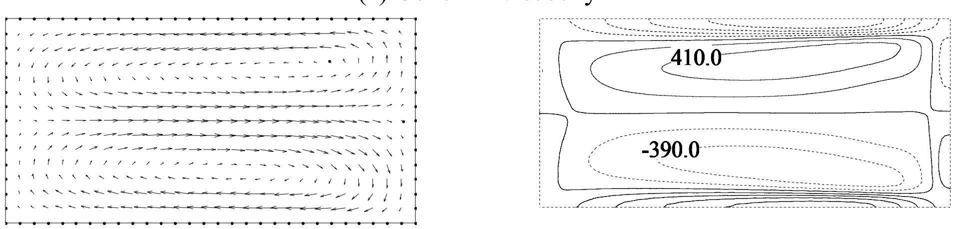
(b) Four heated walls

Figure 2: Flow fields of the helical duct at Reynolds number 100 for constant viscosity (left is the inner wall, κ=0.222, τ=0.0118).

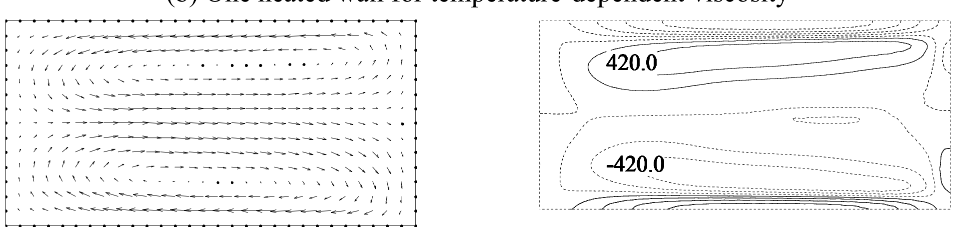
secondary flow vector vorticity



(a) Constant viscosity

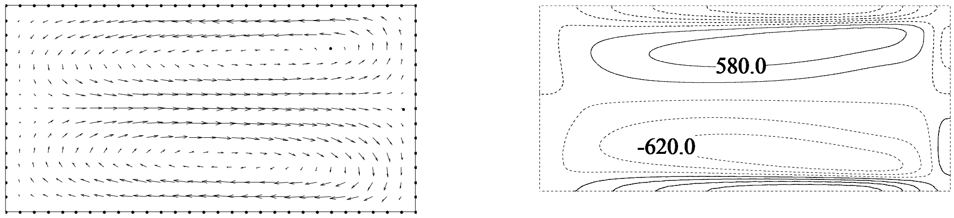


(b) One heated wall for temperature-dependent viscosity

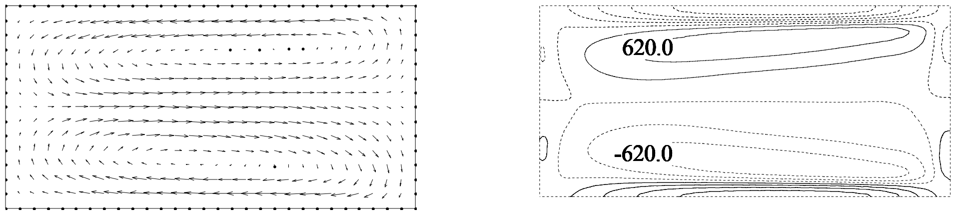


(c) Four heated walls for temperature-dependent viscosity

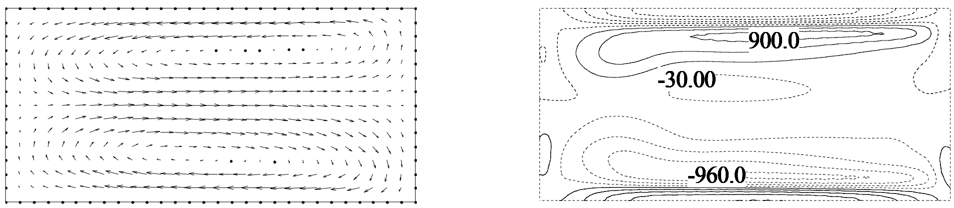
(i)



(a) Constant viscosity

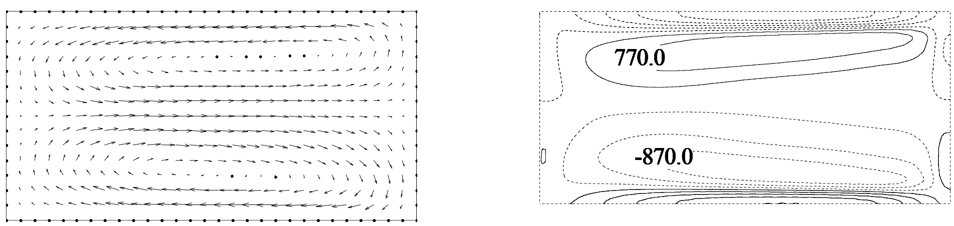


(b) One heated wall for temperature-dependent viscosity

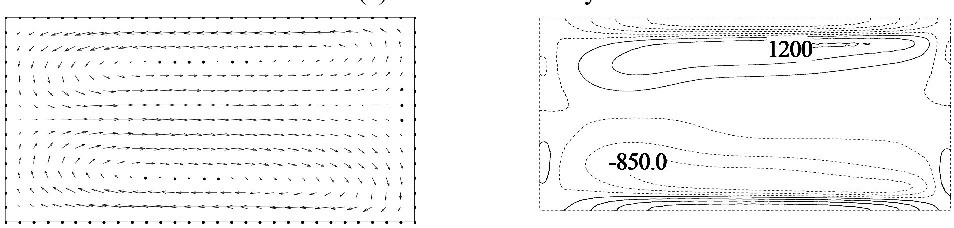


(c) Four heated walls for temperature-dependent viscosity

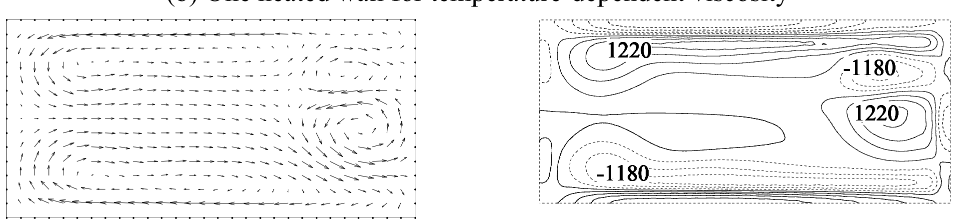
(ii)



(a) Constant viscosity



(b) One heated wall for temperature-dependent viscosity



(c) Four heated walls for temperature-dependent viscosity

(iii)

Figure 3: Flow fields of the helical duct at (i) Re=200 (ii) Re=300, (iii) Re=400 (left is the inner wall, κ=0.222, τ=0.0118).

When the outer wall is flat, centrifugal instability gives rise to the formation of two small additional vortices (1975, 1987). Nandakumar *et al.* (1993) indicated that smaller viscosity makes it easier to form four vortices. In this work, the decrease of viscosity with increasing temperature for four heated walls probably results in the appearance of the additional vortices.

Figures 4 and 5 show the velocity profiles at the horizontal and vertical centerlines for temperature-dependent viscosity and constant viscosity, respectively. Due to the centrifugal force, the maximum axial velocities for both the cases of constant viscosity and varied viscosity move toward the outer wall. However, the dimensionless axial velocities for temperature-dependent viscosity at the horizontal and vertical centerline decrease at the central region of the rectangular duct due to the decreasing viscosity as the fluid temperature increases, especially for the models with four heated walls.

Figures 6 to 9 give the temperature and viscosity distributions at the horizontal and vertical centerlines for temperature-dependent viscosity at Reynolds number 300. It can be seen from Fig. 6, for four heated walls, the minimum temperature values for both constant viscosity and varied viscosity shift to the outer wall of the cross section due to the influence of centrifugal force. At the horizontal and vertical centerlines the temperature of fluid for varied viscosity with one heated wall (or four heated walls) is higher than the corresponding value of the constant viscosity model with one heated wall (or four heated walls) at the central region of the rectangular duct. In Figures 8 and 9, μ_l and μ_w represents the local viscosity and the viscosity at the wall, respectively. Because the viscosity varies with temperature, the viscosity profiles along the horizontal and vertical centerlines are also affected by the matching temperature. It is evident in Figures 8 and 9 that the location of the minimum temperature for the one heated wall (or four heated walls) case is associated with the maximum of the viscosity. Consequently, the viscosity profiles for the four heated walls case is smoother due to the smaller variation of temperature compared with those of the one heated wall case.

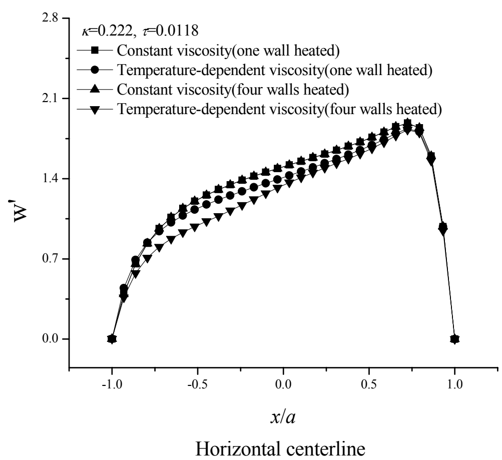


Figure 4: Axial velocities at the horizontal centerline for Reynolds number of 300.

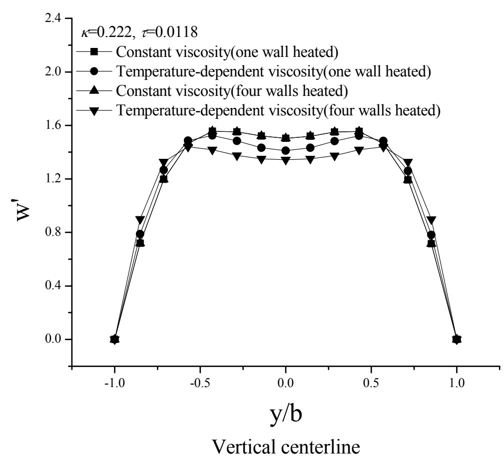


Figure 5: Axial velocities at the vertical centerline for Reynolds number of 300.

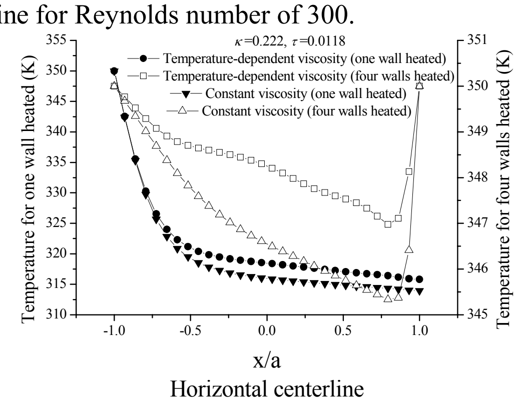


Figure 6: Temperature distributions at the horizontal centerline.

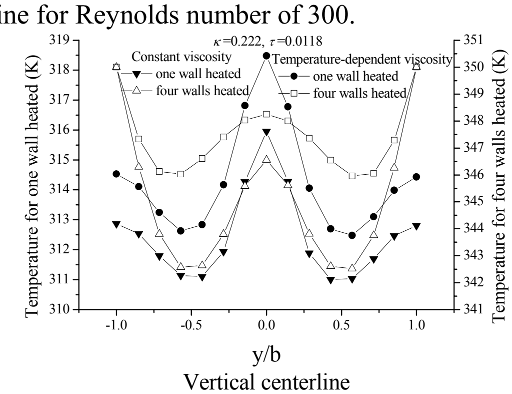


Figure 7: Temperature distributions at the vertical centerline.

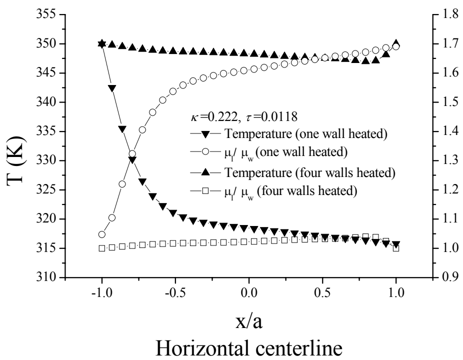


Figure 8: Temperature and viscosity distributions at the horizontal centerline for temperature-dependent viscosity.

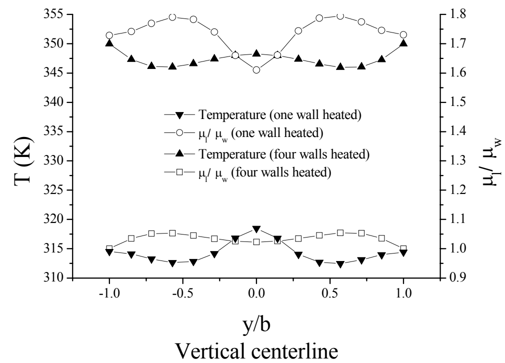


Figure 9: Temperature and viscosity distributions at the vertical centerline for temperature-dependent viscosity.

In order to demonstrate the heat transfer enhancement due to the temperature-dependent viscosity models on the laminar heat transfer, Figure 10 shows the local Nusselt number on the inner heated wall for both constant viscosity and temperature-dependent viscosity models with Reynolds number ranging from 100 to 400. Figure 11 gives the difference between the local Nusselt number of the temperature-dependent viscosity models and that of constant viscosity models (Nu_{lvp} - Nu_{lcp}). That the secondary flow can contributes to enhancement of heat transfer in curved or helical ducts is very well known. At two sides near the secondary vortices for all the models, the fast moving secondary flow enhances heat transfer to a great extent, and two peaks of the local Nusselt number are located there, while, the lowest local Nusselt number is located at the center of the heated wall. In addition, it is verified that the larger local Nusselt number is observed (see Figure 10) when the viscosity variation with temperature is taken into account. Then it is unquestionable that the mean Nusselt number for varying viscosity is higher than that of constant viscosity (see Figure 13). Furthermore, the difference in Figure 11 is enlarged with increasing Reynolds number, which means that the temperature dependent viscosity has a greater effect on heat transfer for larger Reynolds number.

Figures 12 and 13 show the variation of the friction factor and the mean Nusselt number with the increase of the Reynolds number, respectively. The friction factor and the mean Nusselt number all increase with Reynolds number for the models of constant viscosity and temperature-dependent viscosity (Figures 12 and 13), which is in good agreement with results on helical ducts reported in the literature. It is evident from Figures 12 and 13 that the friction factors for the temperature-dependent viscosity with one heated wall and four heated walls are lower than those of the constant viscosity when the Reynolds number is constant, while the mean Nusselt numbers for the temperature-dependent viscosity are higher than the corresponding values for constant viscosity.

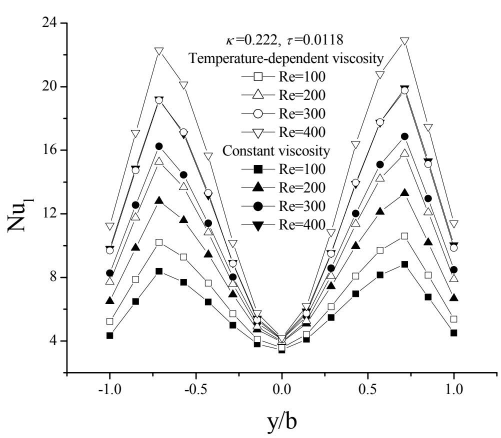


Figure 10: Distribution of local Nusselt number along the inner heated wall for different Reynolds number.

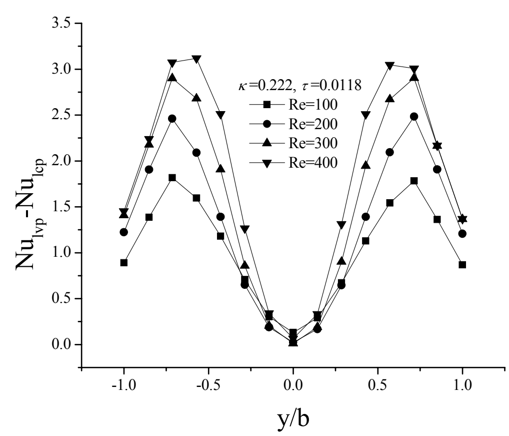


Figure 11: Distribution of Nu_{lvp} - Nu_{lcp} along the inner heated wall for different Reynolds number

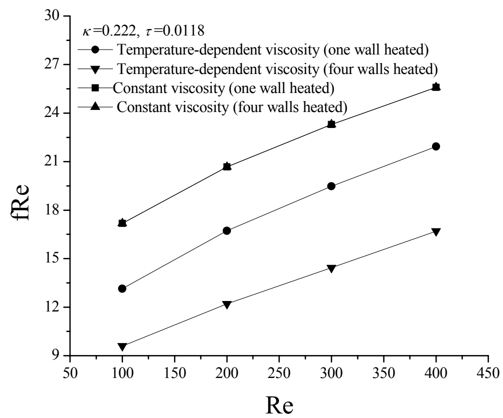


Figure 12: Variation of fRe with Reynolds number.

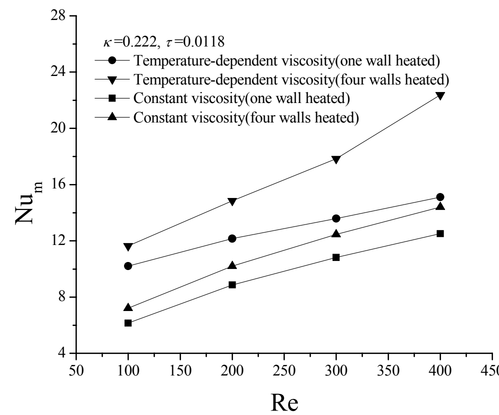


Figure 13: Variation of mean Nusselt number with Reynolds number.

The results may be attributed to the increase of the mean temperature in the rectangular duct for temperature-dependent viscosity compared to constant viscosity. Due to the higher mean temperature, the temperature-dependent viscosity is lower compared to the case of constant viscosity, which reduces the flow resistance and strengthens the effect of the secondary flow on heat transfer enhancement. Especially for four heated walls, the effects of varied viscosity on the laminar flow resistance and convective heat transfer are more significant.

The effects of dimensionless curvature and torsion ratio on the friction factor and mean Nusselt number in helical rectangular ducts are also obtained considering temperature -dependent properties. The results are shown in Table 3. For both the cases of one heated wall and four heated walls, the Nusselt number and friction factor increase with increasing dimensionless curvature ratio ($0.17 \le \kappa \le 0.33$), and the smaller dimensionless torsion ratio ($0.0088 \le \tau \le 0.0264$) has a weaker effect on heat transfer and flow resistance, which are similar to the results reported in the literature for constant viscosity. Like Kumar *et al.* (2007), we do not go into details here as well.

Table 3: Effects of κ and τ on fRe and Nu_m at Reynolds number 300.

κ	τ	one heated wall		four heated walls	
		fRe	Nu_m	fRe	Nu_m
0.22	0.0118	19.64	13.90	14.34	17.82
0.22	0.0175	19.67	12.60	14.34	17.41
0.22	0.0088	19.64	12.91	14.34	17.83
0.17	0.0066	17.94	11.64	13.24	16.86
0.33	0.0264	22.33	15.16	16.22	18.63

CONCLUSIONS

Fully developed laminar flow and heat transfer in a helical rectangular duct with temperature-dependent viscosity were investigated under both one heated wall and four heated walls conditions. The results show that the secondary flow is enhanced markedly when the temperature-dependent viscosity is considered. Compared to the flow field for constant viscosity, there is an additional pair of vortices near the center of the outer wall at Re=400 for the model of four heated walls with temperature-dependent viscosity. Moreover, the profiles of the velocity and temperature along the horizontal and vertical centerline tend to flatten. Because the viscosity decreases with increasing temperature, the viscosity distributions at the horizontal and vertical centerlines also are affected by temperature. Due to the lower viscosity at the higher mean temperature for the temperature-dependent viscosity, the frictional factor decreases and the Nusselt number increases in comparison with constant viscosity. Especially for four heated walls in larger Reynolds number, the effects of temperature-dependent viscosity on the laminar flow and heat transfer are more significant.

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NOMENCLATURE

A_i	coefficient for thermo-physical properties (i=1, 2, 3, 4, 5, and 6)	(-)

η	aspect ratio	η=b/a
a	width of the cross section	m
b	height of the cross section	m
A	area	m^2
c_p	specific heat	J/kg K
Dn	Dean number	$=Re\kappa^{0.5}$
d_h	hydraulic diameter	m
f	frictional factor	(-)
Gn	Germano number	$=Re\tau$
H	pitch of the curved duct	m
Nu	Nusselt number,	$=hd_h/\lambda$
n, m	exponents in Eqs. (17) and (18)	(-)
Pr	Prandtl number	$=\mu c_p/\lambda$
q_w	heat flux	$W\cdot m^{-2}$
p	pressure	Pa
R_c	radius of the curved duct	m
Re	Reynolds number	$=w_m d_h/\upsilon$
T	temperature	K
u′, v′, w′	dimensionless velocity in x′, y′, s′ direction	(-)
u, v, w	velocity in x, y, s direction	$m\cdot s^{-1}$
x′, y′, s′	dimensionless coordinates	(-)
x, y, s	orthogonal helical coordinates	m
r, θ, z	cylindrical coordinates	m
v_r, v_θ, v_z	velocity in r, θ, z direction	m/s

Greek Letters

κ	dimensionless curvature ratio	(-)
λ	thermal conductivity	W/m K
μ	dynamic viscosity	Pa s
ν	kinematic viscosity	m^2/s
ρ	density	kg/m^{-3}
τ	dimensionless torsion ratio	(-)
ω	dimensionless axial vorticity	(-)

Subscripts

cp	value with constant property	(-)
l	local value	(-)
m	mean value	(-)
vp	value with variable property	(-)
w	value at the wall	(-)

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