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TUNING OF MODEL PREDICTIVE CONTROL WITH MULTI-OBJECTIVE OPTIMIZATION

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Abstract - Two multi-objective optimization based tuning methods for model predictive control are proposed. Both methods consider the minimization of the error between the closed-loop response and an output reference trajectory as tuning goals. The first approach is based on the ranking of the outputs according to their importance to the plant operation and it is solved by a lexicographic optimization algorithm. The second method solves a compromise optimization problem. The former is designed for systems in which the number of inputs is equal to the number of outputs, while the latter can also be applied to non-square systems. The main contribution is an automated tuning framework based on a straightforward goal definition. The proposed methods are tested on a finite horizon model predictive controller in closed-loop with a 3x3 subsystem of the Shell Heavy Oil Fractionator benchmark system. The simulation results show that the methods proposed here can be a useful tool to reduce the commissioning time of the controller. The methods are compared to an existing multi-objective optimization based tuning approach. The computational time required to run the proposed tuning algorithms is considerably reduced when compared to the existing approach and, moreover, it does not need an *a posteriori* decision to select a solution from a set of Pareto optimal solutions.
Keywords: Model Predictive Control tuning; Lexicographic optimization tuning; Compromise tuning.

INTRODUCTION

Model Predictive Control (MPC) has been widely used in industry, especially in oil processing and petrochemical plants. It is a successful control strategy because it accounts for process constraints and can be easily extended to Multiple-Input Multiple-Output (MIMO) systems. The earliest reported MPC application in industry dates back to the 1970's. Motivated by industrial needs, the academic contributions started to improve the early MPC formulations, increasing robustness, enhancing performance and stability and reducing computational cost. The usual ingredients of MPC formulations are: (i) a process model to predict the behavior of the system and (ii) a rolling horizon strategy in which the optimum control moves result from the solution to a constrained optimization problem at each sampling time. The first control action is injected into the system and the procedure is repeated at the next sampling instant. In practical implementations, an incremental state-space model that avoids output offset is frequently adopted. The control cost function incorporates at least two weighted sum terms; the first one considers the deviations between the outputs and the output set points along a prediction horizon, weighted by a diagonal, positive matrix Q_y, and the second one considers the control moves along a control horizon,

*To whom correspondence should be addressed weighted by the diagonal, positive-definite matrix R. Closed-loop control performance is affected by a set of parameters, including the input and output horizons and the weighting matrices of the control cost function.

Depending to the approach that is followed to obtain the optimum tuning parameters, existing MPC tuning methods are usually divided into two major groups. The first one encompasses the methods based on analytical expressions obtained through some level of simplification, either in the process description or process model, and additional arbitrary selection of some of the parameters. The second group concerns the techniques based on multi-objective optimization, in which the goals regarding the performance metrics are put together into a single tuning objective function. In the latter approach, the techniques differ according to the goal definition and to which multi-objective optimization algorithm is used to solve the tuning problem. The methods show different tuning goal definitions, which may take into consideration time domain characteristics (e.g. settling time, rise time, overshoot); time domain mathematical metrics (e.g. Integral of Square Error (ISE), Integral of Absolute Error (IAE)); frequency domain sensitivity function norms; or a combination of the previously mentioned possibilities.

Garriga and Soroush (2010) presented a comprehensive review of the tuning techniques from the academic point of view, while Bauer and Craig (2008) assessed the MPC implementation and maintenance expectations from the industrial point of view. Both works pointed out the importance of properly tuning the MPC to ensure higher profit and smoother operation.

Shridhar and Cooper (1997, 1998) developed analytical tuning expressions for Single Input, Single Output (SISO) and MIMO Dynamic Matrix Control (DMC) applications. They set the conditioning number of the hessian of the DMC control problem equal to 500, which represents a good compromise between performance and robustness, and approximate the system model by first order plus dead time transfer functions, to develop analytical tuning expressions for the entries of matrix R. Liu and Wang (2000) considered the minimization of the sensitivity functions between the tuning parameters and the closed-loop performance as the goals of a mixed-integer nonlinear optimization problem.

Al-Ghazzawi *et al.* (2001) defined the tuning goals in terms of the closed-loop output constraining envelopes, and a linear approximation of the process dynamics allowed the authors to obtain analytical sensitivity functions for Q_y and R for constrained MPCs. When the constraints are active, sensitivity functions are calculated based on the Lagrange multipliers of the active constraints. Wojsznis *et al.* (2003) developed an expression for R based on experimental data collected from DMC applications. Adam and Marchetti (2004) and Vega *et al.* (2007) developed frequency domain tuning techniques based on the H_∞ -norm of a mixed sensitivity function for disturbance rejection scenarios. The model uncertainty is treated in Adam and Marchetti (2004) as minimum and maximum bounds on the parameters of the process transfer function of SISO structured controllers, while in Vega *et al*. (2007), uncertainty is considered in the sensitivity functions of input disturbances and output disturbances for DMC controllers.

Fuzzy, multi-objective goals based on the ISE considering the errors between outputs and reference trajectories, and time domain performance specifications are considered in Van der Lee *et al.* (2008) to define a tuning problem that can be solved through a genetic searching algorithm. Time domain goals are also considered in Susuki *et al.* (2008) to define a tuning method for unconstrained state-feedback controllers. The authors emphasized the role of the transient characteristics in process startups and they solved the tuning problem using the particle swarm optimization approach. Exadaktylos and Taylor (2010) developed a tuning technique for a state-space based MPC in which the cost function was defined in terms of the IAE considering the reference trajectories and the process outputs. The authors built the tuning algorithm as a goal attainment multi-objective optimization problem.

Other multi-objective optimization-based tuning methods, focusing on the characterization of the set of feasible optimum solutions for multi-objective optimization problems, known as Pareto set, are presented in Reynoso-Meza *et al.* (2013) and Vallerio *et al.* (2014). Reynoso-Meza *et al*. (2013) suggested several goal definitions: the IAE related with the controlled variables and their set points, the integral of the absolute values of input increments, the measurement of the coupling effects, the robust stability based on frequency domain requirements, and the control implementation objectives. However, a large number of objectives led to complex non-convex Pareto curves and the optimization problem needed to be solved using an evolutionary algorithm. Vallerio *et al.* (2014) considered two different multi-objective optimization approaches, the Normal Boundary Intersection (NBI) and extended normalized constraint method, to obtain a finite representation of the Pareto curve, from which the decision maker can arbitrarily choose a Pareto optimum solution that suits the process needs.

On a different perspective, Cairano and Bemporad (2009) used a linear controller gain, chosen by pole-placement as the tuning goal in a strategy developed for constrained state-feedback MPC. Matrices Q_y and R were chosen as the decision variables, optimized so that the MPC behaves as the target linear controller, in the absence of active constraints. The tuning problem was solved using Linear Matrix Inequalities; its cost function is the squared norm of the difference between the favorite controller gain and the unconstrained MPC gain. The same authors also proposed to consider, as the tuning goal, that the MPC behave as a LQR when the constraints are inactive, making the controller matching optimization problem independent of the MPC control horizon. Cairano and Bemporad (2010) extended the strategy to output-feedback MPC formulations.

Here we address the problem of how to optimally select the controller parameters corresponding to the weighting matrices Q_y and R to ensure that the given performance requirements or desirable control behavior are attained. The next section provides a brief explanation about the state-space based MPC controller and multi-objective optimization. The underlying framework that supports the tuning techniques proposed here is then presented. These methods are implemented in a 3x3 subsystem of the Heavy Oil Fractionator unit simulated in a case study and the paper closes with some conclusions.

PRELIMINARIES

The controller considered here is a state-space based finite horizon MPC. Differently from the DMC, its model representation is not as intuitive as a step response model, but on the other hand, allows for faster computations because it is more compact.

Control Algorithm

The controller assumes that the system is represented by the following state space model

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x \in \Re^{nx}, u \in \Re^{nu}, y \in \Re^{ny}$. Matrices A, B and C are the model matrices that carry all the system information required for future output predictions.

Then, for the system defined in (1), the MPC cost function includes the weighted sum of the squared deviation of the predicted outputs and the set point values over the prediction horizon, and the weighted sum of squared input increments over the control horizon. The control problem can be summarized as follows.

Problem 1

$$\begin{aligned} \min_{\Delta u_k} V_{1,k} = & \sum_{j=0}^{p} \left\| y(k+j \mid k) - y_{sp} \right\|_{Q_y}^2 \\ & + \sum_{j=0}^{m-1} \left\| \Delta u(k+j \mid k) \right\|_R^2 \end{aligned} \tag{2}$$

subject to (1) and

$$u_{\min} \le u(k+j) \le u_{\max}, \; j = 0,\ldots,m-1 \tag{3}$$

$$-\Delta u_{\max} \le \Delta u(k+j) \le \Delta u_{\max}, \; j = 0,\ldots,m-1 \tag{4}$$

where $y(k+j \mid k)$ is the output prediction calculated at time instant $k+j$ using the information available at time instant k, $y_{sp} \in \Re^{ny}$ is the vector of output set points, p is the output prediction horizon, $\Delta u(k+j \mid k) \in \Re^{nu}$ is the control move at time instant $k+j$ calculated using information available at time instant k, m is the control horizon, $Q_y \in \Re^{ny \times ny}$ is a positive semi-definite diagonal matrix, $R \in \Re^{nu \times nu}$ is a positive-definite diagonal matrix, $u_{\min}$, $u_{\max}$, and $\Delta u_{\max}$ are the lower and upper bounds of the system inputs and input increments, respectively.

The tuning algorithms developed here consider the unconstrained version of the MPC, disregarding the constraints defined in (3) and (4), so that the effects of the bounds on the system inputs and input increments do not affect the tuning results.

Multi-Objective Optimization

Multi-objective optimization searches for a single solution to problems with competing goals. There are two main alternatives to deal with the tradeoff between competing objectives: properly weighting objectives prior to the problem solution or choosing an optimum solution based on subjective criteria, after obtaining a set of optimum solutions.

A general multi-objective problem can be posed as follows:

Problem 2

$$\min_x F(x) = \left[F_1(x) \quad F_2(x) \quad \cdots \quad F_w(x) \right]^T \qquad (5)$$

subject to

$$g_j(x) \leq 0, \; j = 1, \ldots, z \; (6)$$

$$h_l(x) = 0, \; l = 1, \ldots, e \quad (7)$$

where $F(x)$ is a vector comprised of w objectives $F_i(x)$. Functions $g_j(x)$ and $h_l(x)$ are related with the inequality and equality constraints, respectively, $x \in \Re^{n_{dec}}$ is the vector of decision variables and n_{dec} is the number of decision variables. The feasible design space is defined as $\mathbf{X} = \{ x \in \Re^{n_{dec}} \mid g_j(x) \leq 0,$ $j = 1, \ldots, z$ and $h_l(x) = 0, l = 1, \ldots, e \}$, and the feasible criterion space is defined as $\mathbf{Z} = \{ z \in \Re^w \mid z = F(x),$ $x \in \mathbf{X} \}$. The objectives $F_i(x)$ are defined in terms of preferences, imposed by the decision-maker. The following statements characterize the optimum solutions in the multi-objective optimization problem.

Definition 1: A point $x^* \in \mathbf{X}$ is a Pareto optimum *iff* there does not exist another point $x \in \mathbf{X}$, such that $F(x) \leq F(x^*)$, and $F_i(x) < F_i(x^*)$ for at least one *i*.

Definition 2: A point $F^{\circ}(x) \in \mathbf{Z}$ is an Utopia point *iff* for each i=1,…,w, $F_i^{\circ}(x) = \min_x \{ F_i(x) \mid x \in \boldsymbol{X} \}$.

Lexicographic or Hierarchical Optimization

Lexicographic goals are useful when an optimization structure is properly represented by ranking the objective functions instead of by a single scalar-valued objective function (Luptáčik and Turnovec, 1991).

In many process systems where the MPC is usually implemented, the importance of the controlled outputs can be ranked and play the role of lexicographic goals. Consequently, the tuning of the controller can be built as a hierarchical optimization problem.

The lexicographic optimization framework used here assumes that the goals and their respective ranking are defined by the user. At each step, a single objective optimization problem is solved, where a single goal is addressed following the ordering of importance. In the subsequent steps, the previously obtained optimum cost function value is included as a constraint in the new optimization problem. The latter addresses a new, less important goal while preserving the performance of the previous, more important goals.

Compromise Optimization

The optimal compromise set has useful characteristics such as feasibility, Pareto optimality and independence of irrelevant alternatives (Ballestero and Romero, 1991). Moreover, a compromise solution method is attractive because, in real cases, it is virtually impossible to accurately translate the preferences of the decision maker into utility functions. Then, it is reasonable to suppose that, instead of selecting one of such functions, the decision maker would rather obtain the feasible point closest to the Utopia solution (Zelany, 1974). Compromise optimization has also been used as the framework of a nonlinear MPC, making the tuning step unnecessary altogether (Bemporad and Muñoz de la Peña, 2009; Zavala and Flores-Tlacuahuac, 2012).

The compromise optimization approach used here solves a multi-objective optimization problem finding the closest feasible solution, in terms of the Euclidian distance, to the Utopia point. Figure 1 shows the geometric representation of the compromise solution, considering a 2-objective problem.

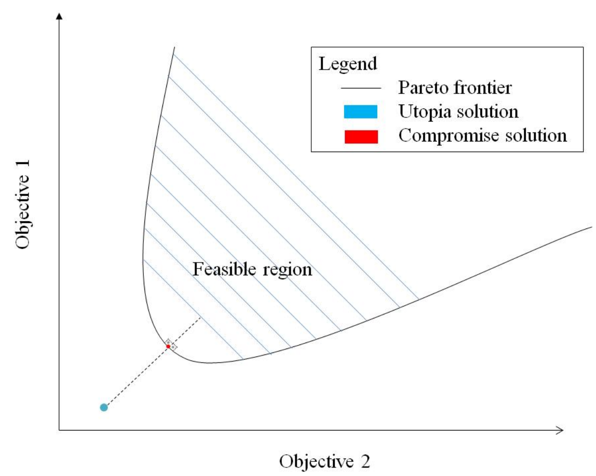


Figure 1: Geometric representation of the compromise optimization method considering two objectives.

TUNING APPROACHES

Parameters p, m, Q_y and R directly affect the performance of the MPC controller. The control horizon

m and the prediction horizon p are integer variables. For open stable systems, the effect of m on the controller performance seems to be as follows: small values of m usually lead to stable but conservative control actions while large values tend to reduce robustness and to increase aggressiveness. Choosing m between 3 and 5 is a recommended rule-of-thumb in the control literature (Garriga and Soroush, 2010). The MPC tuning literature recommends that the output prediction horizon p should be large enough to encompass the significant dynamics of the process. For instance, 80-90% of the open-loop step response rise time value is a reasonable value (Maurath *et al.*, 1988). Since, the rules to select the input and output horizons are already well established, in this work, we will focus only on obtaining the optimum values for parameters Q_y and R, to ensure that the controller fulfills one or more desired performance goals.

Lexicographic Tuning Technique (LTT)

A tuning technique based on lexicographic optimization, in which the objective functions are arranged in order of importance, is proposed here to address some of the shortcomings of the existing approaches. This technique is suitable for several formulations of MPC, including step response with finite output horizon and state-space based with infinite output horizon controllers. We consider a square system where the number of controlled outputs ny is the same as the number of inputs nu, represented by the linear model defined in (1). Steps 1 to 4 of Table 1 summarize the actions required to define the tuning goals. Once they are defined, a lexicographic optimum can be obtained by solving the resulting multi-objective optimization problem.

Table 1: LTT steps summary.

Step	Procedure
1	Define output importance
2	Specify input-output pairs
3	Normalize inputs, outputs and model gains
4	Specify tuning objectives
5	Lexicographic optimization

First, the user needs to assign the relative importance of the process outputs. Usually, economic, safety or environmental factors are used as guidelines. Second, an input-output pair is defined for each process output, following the order of importance of the outputs. Either the process knowledge or the Relative Gain Array (Bristol, 1966) and other similar techniques can be used to select the appropriate pairs. In the third step, numerical values of the inputs and outputs, as well as the gains of system model are normalized. This step aims for a better numerical conditioning of the tuning problem. Also, the values of the tuning cost function for different goals can be computed with orders of magnitude that are not too different.

In the fourth step, the tuning goals are defined as the minimization of the error between the output closed-loop responses and the pre-defined reference trajectories. This goal definition strategy has been adopted in several MPC tuning methods (Al-Ghazzawi *et al.*, 2001; Campi *et al.* 2002; Exadaktylos and Taylor, 2010; Shah and Engell, 2010), as it allows for a simple specification of both the desired control performance and time-domain objectives. In this work, the reference trajectory of a given output is defined as the step response of a target transfer function that is obtained from the approximation of the open-loop transfer function of the corresponding input-output pair defined in Step 2, by a first order plus dead time transfer function. Once this approximate transfer function is obtained, the user defines the time constant of the target function following the importance order of the outputs and the process operator specifications. Then, the multi-objective goals are defined as follows:

$$F_i(x) = \sum_{k=1}^{\theta_t} \left(y_i^{ref}(k) - y_i(k) \right)^2 \quad i = 1,\ldots,w \tag{8}$$

where θ_t is the tuning horizon, $y_i^{ref}(k)$ is the discretized reference trajectory of output i, $y_i(k)$ is the closed-loop trajectory of output i, $k=1,\ldots,\theta_t$, calculated using the unconstrained version of Problem 1; x is the vector of decision variables or tuning parameters, and w is the number of input-output pairs. Observe that if $Q_y = diag(q_1,...,q_{ny})$ and $R = diag(r_1,...,r_{nu})$, then $x = (q_1,...,q_{ny}, r_1,...,r_{nu})$. Observe also that since $y_i(k)$ is the response of the system in closed loop, it is a function of the tuning parameters that can be obtained by the minimization of the functions defined in (8). Moreover, even though the unconstrained MPC is considered, it is not possible to obtain an single analytical solution to the tuning problem because (8) is a discretized cost function, and the non-linearity of (2) with respect to the tuning parameters would lead very cumbersome expressions for the partial derivatives of the cost defined in (8).

The importance of the process outputs defined previously also characterizes the lexicographic optimization order, and the multi-step optimization problem is defined as follows.

Problem 3

$$\min_{x,\delta} V_3 = \sum_{i=1}^{w'} F_i\left(x\right) + \delta^T S_t \delta \tag{9}$$

subject to (8) and

$$F_i\left(x\right) - F_i^* - \delta_i \le 0, \quad i = 1,\ldots,w' - 1 \tag{10}$$

$$\delta_i \ge 0, \quad i = 1,\ldots,w' - 1 \tag{11}$$

$$LB \le x \le UB \tag{12}$$

where *w'* is the current tuning step and defines the number of the current output objectives, δ is a vector of slack variables, $S_t \in \Re^{(w'-1)\times(w'-1)}$ is a diagonal weighting matrix, *LB* and *UB* are the lower and upper bounds of the decision variables. The constraint defined in (10) tries to force that the optimum performance obtained for higher priority outputs does not deteriorate when lower priority output goals are addressed. The slack variable δ is included to ensure that Problem 3 is always feasible. F_i^* is the optimum value of the objective goal, defined in (8), for the variable y_i, obtained at the i-th lexicographic tuning step. Observe that once a F_i^* is obtained, it remains constant throughout the lexicographic method.

The proposed lexicographic approach is able to pursue as many objectives as the number of system inputs, because of the way the lexicographic optimization structure is constructed. Therefore, this method is appropriate mainly for square systems (*ny*=*nu*).

Compromise Tuning Technique (CTT)

In this method, we consider that the tuning goals are defined in the same way as in the previous section. Then, the Utopia solution is obtained by solving the optimization problems defined by (13) and (14).

$$F_i^\circ\left(x\right) = \min_x F_i\left(x\right), \; i = 1,...,w \tag{13}$$

subject to (8) and

$$LB \le x \le UB \tag{14}$$

where $x = \left(q_1,...,q_{ny},r_1,...,r_{nu}\right)$, $x \in \Re^{ny+nu}$, is the vector of decision variables. Observe that the Utopia point, $F^0(x)$, is unfeasible unless all the tuning goals share the same optimum solution. Once the Utopia solution is obtained, we try to find the closest feasible solution to it, in terms of the Euclidian distance. This procedure defines the CTT problem:

Problem 4

$$\min_x \left\|F^\circ - F\left(x\right)\right\|^2 \tag{15}$$

subject to (8) and

$$LB \le x \le UB \tag{16}$$

where $F^\circ \in \Re^w$ is the vector with components $F_i^\circ\left(x\right)$.

CASE STUDY

To illustrate the application of the tuning techniques proposed here, a MPC controller to be implemented in a subsystem of the Shell Heavy Oil Fractionator (HOF) benchmark system (Maciejowski, 2002) is tuned. The inputs of this subsystem are the top drawn flow rate (u_1), the side drawn flow rate (u_2) and the bottoms reflux heat duty (u_3). The controlled outputs are the top end point composition (y_1), the side end point composition (y_2) and the bottoms reflux temperature (y_3). Figure 2 shows a schematic representation of the process and Equation (17) defines the transfer functions that represent the HOF. The tuning strategies proposed above are applied to the MPC of the HOF and compared to the NBI tuning strategy proposed in Vallerio *et al.* (2014). The simulated scenarios involve the output tracking and the disturbance rejection.

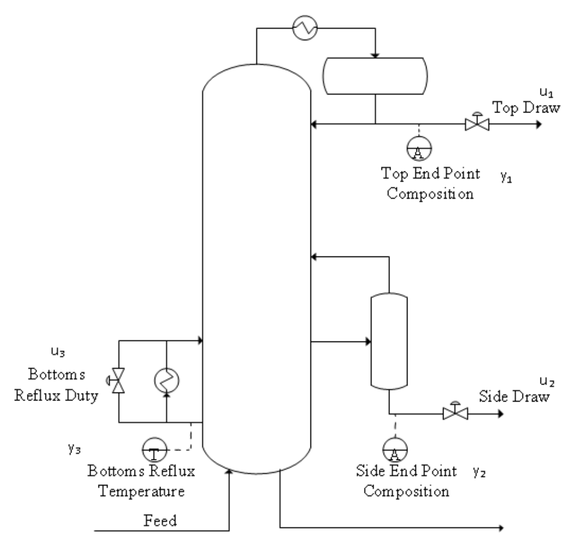


Figure 2: Shell Heavy Oil Fractionator 3x3 subsystem schematic representation.

$$G(s)=\begin{bmatrix} \dfrac{4.05e^{-27s}}{50s+1} & \dfrac{1.77e^{-28s}}{60s+1} & \dfrac{5.88e^{-27s}}{50s+1} \\ \dfrac{5.39e^{-18s}}{50s+1} & \dfrac{5.72e^{-14s}}{60s+1} & \dfrac{6.90e^{-15s}}{40s+1} \\ \dfrac{4.38e^{-20s}}{33s+1} & \dfrac{4.42e^{-22s}}{44s+1} & \dfrac{7.20}{19s+1} \end{bmatrix} \quad (17)$$

Tuning Goals

The reference trajectories corresponding to the tuning goals were defined according to the input-output pairing presented in Li *et al.* (2005). The open-loop transfer functions, $G_{i,j}(s)$ had their time constants multiplied by a response factor, $f_{res}(i)$, $i=1,\ldots,ny$, to obtain the reference transfer functions and reference trajectories for each output, $G_i^{REF}(s)$. The input-output pairs were selected as y_1-u_1, y_2-u_2 and y_3-u_3. Based on the process information available in Li *et al.* (2005), the vector of selected response factors was the following $f_{res}=[0.10\ 0.15\ 0.30]^T$. Response factors smaller than 1 indicate that the reference trajectory is faster than the corresponding open-loop response. Considering that the reference trajectory corresponds to a first order plus dead time transfer function, $G^{REF}(s)=\dfrac{K_i e^{(-\theta_i s)}}{\tau_i s+1}$, the resulting model parameters that define the reference trajectories for the tuning methods proposed here are given in Table 2.

Table 2: Parameters of the reference transfer function.

Output	K_i	τ_i	θ_i
y_1	1	5	27
y_2	1	9	14
y_3	1	5.7	0

For all the outputs, the tuning horizon, θ_t, is assumed to be equal to 450 min, which is large enough to include set point moves that drive the closed loop system to different directions. For the tuning procedure, the input and output initial values are $y_0=[0\quad 0\quad 0]^T$ and $u_0=[0\quad 0\quad 0]^T$. The output set points are changed to $y_{sp}=[0.2\quad 0.2\quad 0.2]^T$ at the initial time instant, to $y_{sp}=[0.0\quad 0.4\quad 0.1]^T$ at time instant 150 min and to $y_{sp}=[0.1\quad 0.3\quad 0.0]^T$ at time instant 300 min. Observe that this tuning scenario might be overly demanding in real applications, because only on rare occasions more than one output set point is driven towards new values at the same time.

In the closed loop simulation, the control problem (Problem 1) does not include the input constraints (3) and (4). Then, an optimum analytical solution could be obtained to Problem 1, which reduces the computational demand of the tuning problem. The control horizon is set equal to 5 and the prediction horizon is set equal to 70. All the problems pictured here were solved using an Intel® Core i5 320 GHz, 4 GB RAM computer. We assume that the process model considered in the controller is perfect and that the system states are fully measured.

Tuning the MPC of the HOF System with LTT

The LTT problem (Problem 3) was solved for the HOF system with the Matlab routine *fmincon*, which solves a NLP. At any step $w'=1,\ldots,nu$ of the tuning method, the vector of decision variables is $x=(q_1,\ldots,q_{w'},r_1,\ldots,r_{w'})$, and the initial guess is $x_0=\left[5\quad 1_{1,w'-1}\quad 1_{1,w'}\times 10^{-1}\right]$, with the lower and upper bounds of the decision variables equal to $LB=\left[5\quad 1_{1,w'-1}\times 10^{-2}\quad 1_{1,w'}\times 10^{-3}\right]$ and $UB=\left[5\quad 1_{1,w'-1}\times 10^{2}\quad 1_{1,w'}\times 10^{2}\right]$ respectively. Observe that the weight corresponding to output y_1 is kept at a fixed value ($q_1=5$). Otherwise the tuning problem shows multiple equivalent solutions. The same approach was adopted in the other tuning methods considered here.

The weighting matrix S_t of the slack variables was chosen considering the expression $s_{t,i}=10^{(w'-i)\times 2}$, $i=1,\ldots,w'-1$, where $s_{t,i}$ denotes the i-th diagonal element of matrix S_t. This approach guarantees that the slacks related to the more important goals will be more heavily weighted in the tuning cost function. Observe that, in the LTT approach, at any step the value of w' corresponds to the size of the subsystem that is considered.

The lexicographic optimization was performed as follows: at step 1, the goal defined by the reference trajectory of y_1 and its closed-loop response was minimized, using r_1 as a decision variable and a fixed q_1. At step 2, a cost function comprised of the sum of the goals defined by the reference trajectories of outputs y_1 and y_2 and their closed-loop responses was minimized. The decision variables of this problem were r_1, r_2, q_2; q_1 was fixed. A constraint was included in this optimization problem in order to ensure that the optimum cost function value obtained in the first step for the goal of y_1, which is a fraction of the cost function of the tuning problem defined in step 2, is kept smaller or equal to the value obtained at step 1. The constraint was softened by the inclusion of the slack variable δ_1. This slack was penalized in the cost function, with matrix S_t. At step 3, a similar problem was solved, observing that all three goals were included in the tuning function, as well as the two slack variables related with the cost function constraints corresponding to y_1 and y_2.

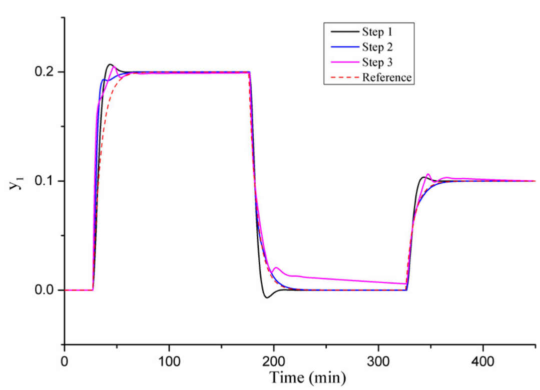
Tables 3 and 4 show the values of Q_y, R and δ resulting from the application of the LTT to the controller of the Heavy Oil Fractionator. The required computational time for this method was 4.27 hours. Observe that in Table 4 the values of δ at the third step are small, indicating that the LTT was able to properly adjust the closed-loop responses of outputs y_2 and y_3 to their reference trajectories without significantly degrading the response of y_1. This result is also observed in Figure 3 which shows the evolution of system outputs in closed-loop throughout the LTT method. The response related to output y_1 improves from step 1 to step 2, and degrades only a little from step 2 to step 3, while the response related to y_2 remains nearly the same from step 2 to step 3. The parameters shown in the last row of Table 3 are the optimum tuning parameters obtained by the LTT method.

Table 3: LTT optimum tuning parameters.

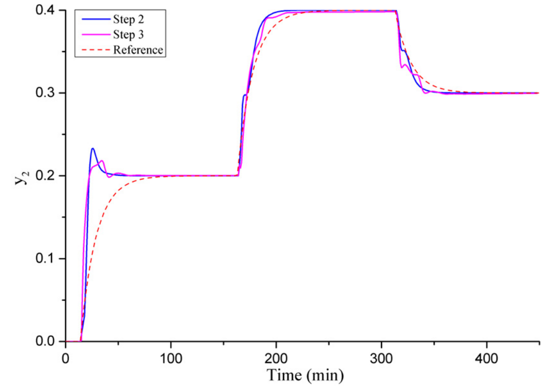
	Q_y			R		
Step	**1**	**2**	**3**	**1**	**2**	**3**
1	5			8.63		
2	5	1.32		2.62	6.49	
3	5	1.54	1.57	1.49	7.46	0.50

Table 4: LTT slack variables, δ.

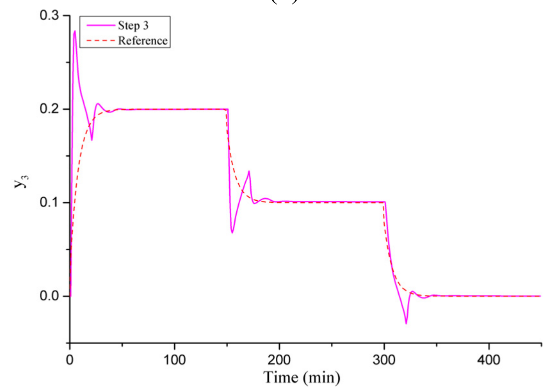
Step	1	2
1		
2	0	
3	0.013	0.010



(a)



(b)



(c)

Figure 3: Responses of (a) y_1, (b) y_2 and (c) y_3 with LTT.

Tuning the MPC of the HOF System with CTT

The same tuning goals as in the LTT method were defined, considering the input-output pairs adopted in the Tunning Goals section, as well as the transfer functions, and response factors are also assumed here. In the application of the CTT method to the Heavy Oil Fractionator, the individual objectives of the multi-objective optimization problem are built for each controlled output y_i, as follows

$$F_i(x) = \sum_{k=1}^{\theta_t} \left(y_i^{ref} - y_i(k) \right)^2 \quad i = 1, \ldots, ny \tag{18}$$

and this function is minimized with respect to all the tuning parameters, which means that $x = \left(q_1, \ldots, q_{ny}, r_1, \ldots, r_{nu} \right)$.

The CTT method (Problem 4) is solved for the tuning of the MPC of the HOF system using *fmincon* in MATLAB 2013®, with the following initial guess for the tuning parameters $x_0 = \left[5 \quad 1_{1,2} \quad 1_{1,3} \times 10^{-1} \right]$.

The lower and upper bounds of the decision variables are the same as in the LTT method.

The resulting Utopia vector is $F^0 = \begin{bmatrix} 1.893 & 0.465 & 0.004 \end{bmatrix}^T$, where the components of F^0 correspond to the solution of the problem defined by (13) and (14) for the objectives F_i, i=1,2,3. The optimum solution was obtained through the solution of Problem 4 that minimizes the Euclidian distance between the Utopia point and a feasible solution. The resulting computational time was 53 min. The optimum solution was

$Q_y^* = diag\left(\begin{bmatrix} 5 & 4.96 & 2.91 \end{bmatrix}\right)$ and $R^* = diag\left(\begin{bmatrix} 10^{-3} & 2.39 \times 10^{-2} & 0.98 \end{bmatrix}\right)$.

Differently from the LTT, the CTT does not depend on the number of inputs and outputs, because we can define as many objectives as the number of system outputs, and all of them are treated simultaneously, independently of the number of inputs. We also observe that the values of the parameters obtained with the CTT method are quite different from the values of the same parameters obtained with the LTT method. Then, the performances of the controllers with these two set of tuning parameters need to be compared through closed-loop simulations.

Simulation Results

Here, we analyze two different operating scenarios using the MPC controllers with the parameters obtained with the tuning techniques presented earlier, addressed as LTT and CTT. The first scenario corresponds to nearly the same conditions in which the MPC controlling the HOF system was tuned. Outputs are subject to set point changes one at a time, and finally driven back to the steady-state point, following the values defined in Table 5. The second simulation considers different set point changes that are given in Table 6, as well as unmeasured input disturbances. The different scenarios are used to validate the tuning results. Unmeasured disturbances are introduced through the system inputs u_2 and u_1, and affect the system from time instant 50 min to 55 min and from 220 min to 225 min, respectively. In both simulations, the constrained version of the MPC problem (Problem 1) is considered. A perfect MPC model and fully measured states are assumed, and the initial operating point of the system is $y_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and $u_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$; the input lower and upper bounds and maximum input increments are $u_{\min} = \begin{bmatrix} -0.5 & -0.5 & -0.5 \end{bmatrix}^T$, $u_{\max} = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}^T$, $\Delta u_{\max} = \begin{bmatrix} 0.05 & 0.05 & 0.05 \end{bmatrix}^T$.

The methods proposed here are also compared to a recent tuning method of the control literature: the NBI tuning method, developed by Vallerio *et al.* (2014). The method is based on an *a posteriori* choice of the optimum solution, from a set of Pareto solutions. The set is obtained through the Normal Boundary Intersection method, which performs a grid search over an evenly spaced parameterized segment for each objective. The points lying on the (quasi-) normal direction to a plane constituted of the individual optimum solutions (Utopia Point) and the feasible solution space should be Pareto optimum. The reader is referred to Das and Dennis (1998) and Vallerio *et al.* (2014) and the references therein for more detailed information. The method is implemented using 0.2 as the weight interval and the 3 goals defined previously, for the tuning techniques developed here. The final solution is chosen based on the minimum control cost to drive the system to the same targets as defined before and comparing the resulting 21 Pareto solutions. The approach leads to the following optimum tuning parameters: $Q_y^* = diag\left(\begin{bmatrix} 5 & 1.13 & 3.52 \end{bmatrix}\right)$ and $R^* = diag\left(\begin{bmatrix} 0.38 & 1.0 & 0.89 \end{bmatrix}\right)$. The NBI algorithm requires a computational time of 20.1 h to complete the tuning procedure, including the time required to obtain the Utopia solution. Table 7 summarizes the optimum parameters obtained by the tuning approaches used here and the required computational time.

Table 5: Simulation I set points.

Time (min)	y_1^{sp}	y_2^{sp}	y_3^{sp}
1 to 80	0.2	0.2	0.2
80 to 200	0.0	0.4	0.1
200 to 300	0.1	0.3	0.0
300 to 400	0	0	0

Table 6: Simulation II set points.

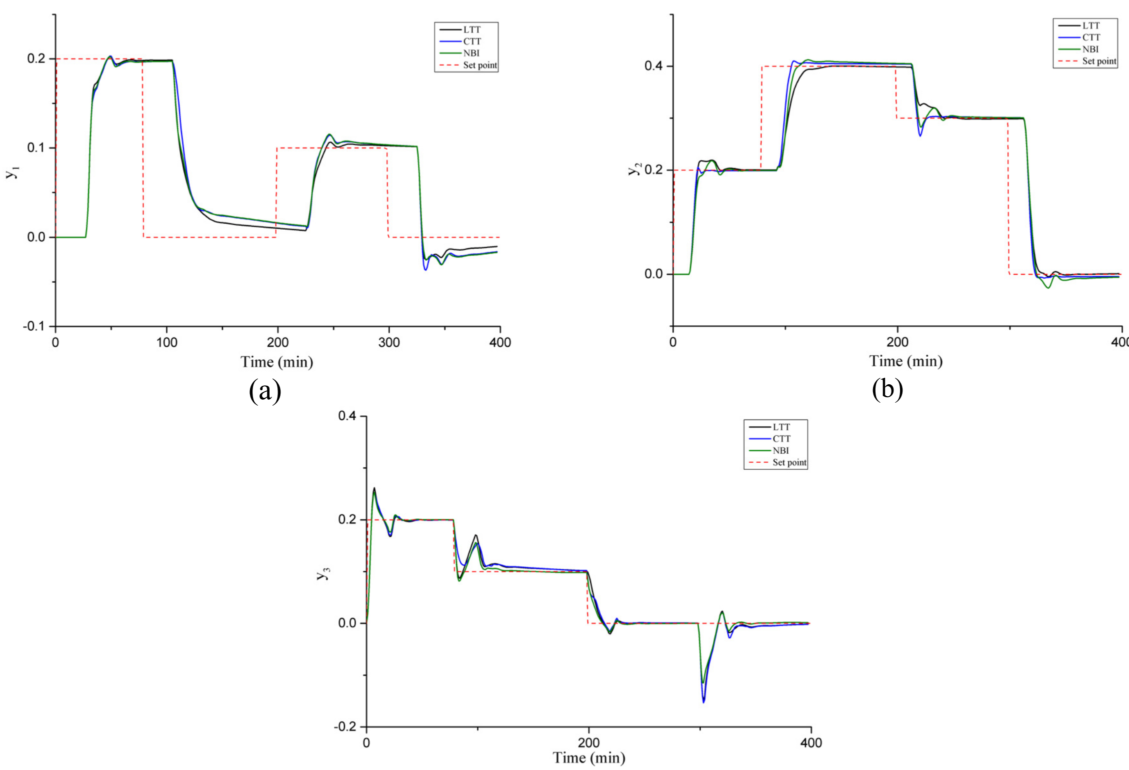
Time (min)	y_1^{sp}	y_2^{sp}	y_3^{sp}
1 to 120	-0.2	0	0
120 to 200	-0.2	0	0.3
200 to 300	0	-0.2	0.3
300 to 400	0.3	0	0.2

Table 7: Optimum tuning parameters summary.

Method	Q_y^*	R^*	Computational Time
LTT	$\begin{bmatrix}5 & 1.54 & 1.57\end{bmatrix}$	$\begin{bmatrix}1.49 & 7.46 & 0.50\end{bmatrix}$	4.27h
CTT	$\begin{bmatrix}5 & 4.96 & 2.91\end{bmatrix}$	$\begin{bmatrix}10^{-3} & 2.39\times10^{-2} & 0.98\end{bmatrix}$	53 min
NBI	$\begin{bmatrix}5 & 1.13 & 3.52\end{bmatrix}$	$\begin{bmatrix}0.38 & 1.00 & 0.89\end{bmatrix}$	20.1h

Figures 4 and 5 show the output and input responses corresponding to the scenario defined as Simulation I with the set-point changes represented in Table 5. These figures compare the behavior of the MPC tuned according to methods LTT, CTT and NBI. We observe that, although the numerical values of the tuning parameters provided by the three methods are quite different from each other, the responses of the closed loop system are not too different. Particularly the responses of y_1 and y_3 are quite the same for methods CTT and NBI. The responses corresponding to method LTT tend to be not the same as in the other methods but they are still close from a practical viewpoint. The same considerations can be given to the input responses corresponding to the three methods. Only the input responses corresponding to LTT seems to be slightly smoother than for the other methods.

The offsets observed in Figure 4a, after the second, third and fourth set-point changes are mostly due to the slow dynamics of the process. Observe that when the value of y_1 is driven back to 0 the responses are slower than when the variable y_1 is near 0 and its set-point is changed to a larger value. A similar trend is observed in Figure 4b, for the time period between 100 to 200 min and in Figure 4c, from 300 to 400 min.



(c)

Figure 4: Simulation I. Response of the HOF output (a) y_1, (b) y_2 and (c) y_3 to set point changes.

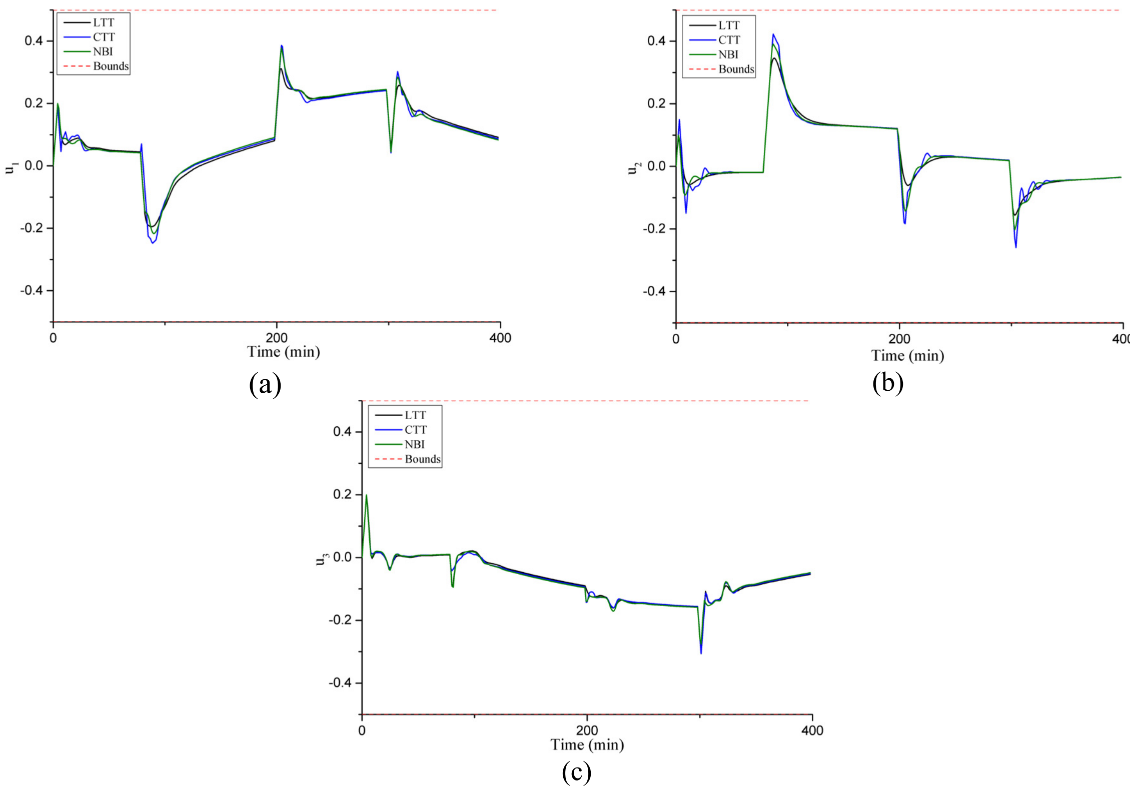


Figure 5: Simulation I. Response of the HOF input (a) u_1 (b) u_2 and (c) u_3 to set point changes.

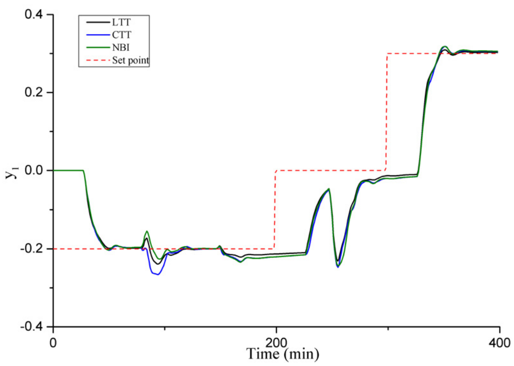
Figures 6 and 7 show the responses for Simulation II with the set-point changes defined in Table 6 and unknown pulse disturbances of intensity -0.8 on input u_1 from time 221 min to 225 min, and intensity 0.8 on input u_2 from time 51 min to 55 min. In this simulation, for the set-point changes, the responses of the three controllers seem to follow the same patterns as in the previous case. However, we can observe some differences in the responses for the disturbance rejection. The overshoots are different for the three methods, but the largest overshoot depends on the output and on the disturbance, and consequently, there is not a clear superiority of any of the methods. Concerning the inputs, the three methods seem to perform similarly, but LTT gives smoother responses. Tables 8 and 9 show the SSE between the system outputs and their set-points, calculated for Simulations I and II, respectively. The first table supports the same observation drawn from Figures 4 and 5. The total SSE obtained by the NBI and CTT methods are the same, up to the third decimal place, whereas the smooth LTT responses are reflected in the slightly higher values of the SSE, in both simulations.

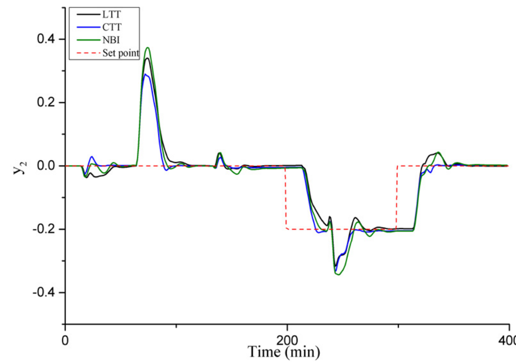
Table 8: Sum of Square Errors between outputs and their set-points, Simulation I.

Simulation I	y_1	y_2	y_3	Total
LTT	3.2331	3.6436	0.4259	7.3026
CTT	3.3161	3.5484	0.4155	7.2801
NBI	3.3126	3.549	0.4191	7.2806

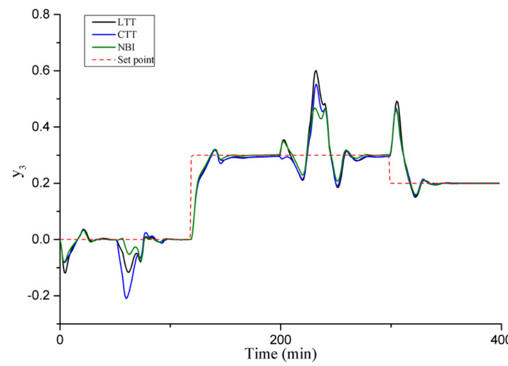
Table 9: Sum of Square Errors between outputs and their set-points, Simulation II.

Simulation II	y_1	y_2	y_3	Total
LTT	6.6925	3.1601	2.1992	12.0518
CTT	7.1799	2.6682	2.002	11.8501
NBI	7.1587	2.661	2.0217	11.8414



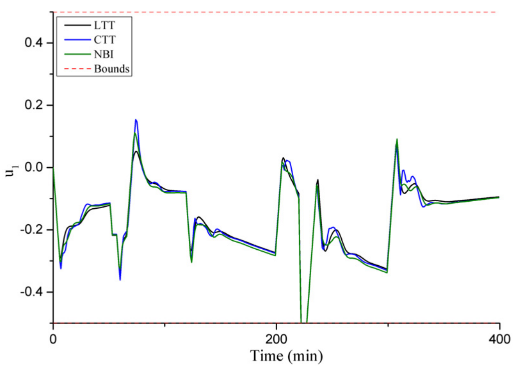


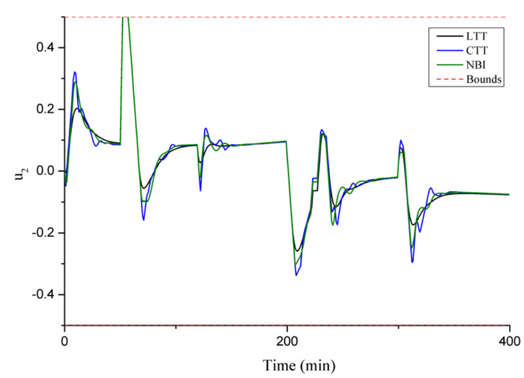
(a) (b)



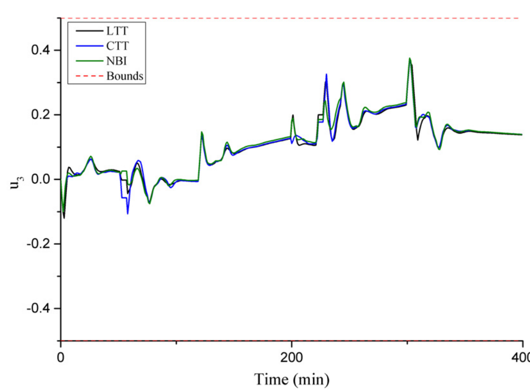
(c)

Figure 6: Simulation II. HOF output (a) y_1 (b) y_2 and (c) y_3 response to set point changes and unmeasured disturbances.





(a) (b)



(c)

Figure 7: Simulation II. HOF input (a) u_1 (b) u_2 and (c) u_3 response to set point changes and unmeasured disturbances.

CONCLUSIONS

Two tuning techniques for the conventional finite horizon MPC are presented here. In these methods the tuning goals are defined in terms of output reference trajectories, describing the desired time-domain characteristics. Two multi-objective optimization techniques are proposed to solve the MPC tuning problem, namely the lexicographic optimization (LTT) and the compromise optimization (CTT). These techniques lead to different sets of optimum parameters, as was observed in the tuning of the MPC implemented in the Shell Heavy Oil Fractionator benchmark subsystem. The LTT follows the usual tuning guidelines of industry, in which the goals are defined according to the number of system inputs available as degrees of freedom and is more suitable for systems in which the number of outputs is equal to the number of inputs, while the CTT can take into account as many objectives as necessary and is independent of size of the system. The LTT successfully prioritizes the more important objectives while the latter obtains the best attainable performance considering all objectives simultaneously. The tuning techniques developed here are compared to a multi-objective optimization based approach from the literature, which proved to be at least an order of magnitude more time consuming than any of the techniques developed here. Moreover, it requires the decision maker to pick out one of the Pareto optimum solution, whereas this selection is done automatically in both approaches developed here, once the tuning goals are specified. Regarding the simulation results, the existing technique yielded similar results compared to the compromise and lexicographic approaches. This observation emphasizes the complexity of the MPC tuning problem with its non-convexity and multiplicity of solutions. Besides, in this work, only the case of fixed output targets and unconstrained inputs is considered. In real industrial applications, to cope with real time optimization, the MPC controller works mainly with output control zones and input targets (González and Odloak, 2009), which implies additional tuning parameters.

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REFERENCES

Adam, E. J. and Marchetti, J. L., Designing and tuning robust feedforward controllers. Computers & Chemical Engineering, 28(9), 1899-1911 (2004).

Al-Ghazzawi, A., Ali, E., Nouh, A. and Zafiriou, E., On-line tuning strategy for model predictive controllers. Journal of Process Control, 11(3), 265-284 (2001).

Ballestero, E. and Romero, C., A theorem connecting utility function optimization and compromise programming. Operations Research Letters, 10(7), 421-427 (1991).

Bauer, M. and Craig, I. K., Economic assessment of advanced process control – A survey and framework. Journal of Process Control, 18(1), 2-18 (2008).

Bemporad, A. and Muñoz de la Peña, D., Multiobjective model predictive control. Automatica, 45(12), 2823-2830 (2009).

Bristol, E., On a new measure of interactions for multivariable process control. Automatic Control, IEEE Transactions on Automatic Control, 11(1), 133-134 (1966).

Cairano, S. Di, and Bemporad, A., Model predictive controller matching: Can MPC enjoy small signal properties of my favorite linear controller? In European Control Conference, Budapest, Hungary, 23th -26th (2009).

Cairano, S. Di, and Bemporad, A., Model predictive control tuning by controller matching. IEEE Transactions on Automatic Control, 55(1), 185-190 (2010).

Campi, M. C., Lecchini, A. and Savaresi, S. M., Virtual reference feedback tuning: A direct method for the design of feedback controllers. Automatica, 38(8), 1337-1346 (2002).

Das, I. and Dennis, J. E., Normal-boundary intersection: A new method for generating the pareto surface in nonlinear multicriteria optimization problems. SIAM Journal on Optimization, 8(3), 631-657 (1998).

Exadaktylos, V. and Taylor, C. J., Multi-objective performance optimisation for model predictive control by goal attainment. International Journal of Control, 83(7), 1374-1386 (2010).

Garriga, J. L. and Soroush, M., Model predictive control tuning methods: A review. Industrial & Engineering Chemistry Research, 49(8), 3505-3515 (2010).

González, A. H. and Odloak, D., A stable MPC with zone control. Journal of Process Control, 19(1), 110-122 (2009).

Li, S., Zhang, Y. and Zhu, Q., Nash-optimization enhanced distributed model predictive control applied to the Shell benchmark problem. Information Sciences, 170(2-4), 329-349 (2005).

Liu, W. and Wang, G., Auto-tuning procedure for model-based predictive controller. In SMC 2000 Conference Proceedings, 2000 IEEE International Conference on Systems, Man and Cybernetics. "Cybernetics Evolving to Systems, Humans, Organizations, and their Complex Interactions" (Cat. No.00CH37166) (Vol. 5, pp. 3421-3426), IEEE (2000).

Luptáčik, M. and Turnovec, F., Lexicographic geometric programming. European Journal of Operational Research, 51(2), 259-269 (1991).

Maciejowski, J. M., Predictive Control: with Constraints. (p. 331), Prentice Hall (2002).

Maurath, P. R., Laub, A. J., Seborg, D. E. and Mellichamp, D. A., Predictive controller design by principal components analysis. Industrial & Engineering Chemistry Research, 27(7), 1204-1212 (1988).

Reynoso-Meza, G., Garcia-Nieto, S., Sanchis, J. and Blasco, F. X., Controller tuning by means of multi-objective optimization algorithms: A global tuning framework. IEEE Transactions on Control Systems Technology, 21(2), 445-458 (2013).

Shah, G. and Engell, S., Tuning MPC for desired closed-loop performance for SISO systems. In 18th Mediterranean Conference on Control and Automation, MED'10 (pp. 628-633), IEEE (2010).

Shridhar, R. and Cooper, D. J., A Tuning strategy for unconstrained SISO model predictive control. Ind. Eng. Chem. Res., 36, 729-746 (1997).

Shridhar, R. and Cooper, D. J., A Tuning strategy for unconstrained multivariable model predictive control. Ind. Eng. Chem. Res., 5885(98), 4003-4016 (1998).

Susuki, R., Kawai, F., Nakazawa, C., Matsui, T. and Aiyoshi, E., Parameter optimization of model predictive control using PSO. In SICE Annual Conference, pp. 1981-1988 (2008).

Vallerio, M., Van Impe, J. and Logist, F., Tuning of NMPC controllers via multi-objective optimisation. Computers & Chemical Engineering, 61, 38-50 (2014).

Van der Lee, J. H., Svrcek, W. Y. and Young, B. R., A tuning algorithm for model predictive controllers based on genetic algorithms and fuzzy decision making. ISA Transactions, 47(1), 53-9 (2008).

Vega, P., Francisco, M. and Sanz, E., Norm based approaches for automatic tuning of Model Based Predictive Control. In Proceedings of European Congress of Chemical Engineering ECCE-6 (2007).

Wojsznis, W., Gudaz, J., Blevins, T. and Mehta, A., Practical approach to tuning MPC. ISA Transactions, 42(1), 149-162 (2003).

Zavala, V. M. and Flores-Tlacuahuac, A., Stability of multiobjective predictive control: A utopia-tracking approach. Automatica, 48(10), 2627-2632 (2012).

Zelany, M., A concept of compromise solutions and the method of the displaced ideal. Computers & Operations Research, 1(3-4), 479-496 (1974).