ISSN 1807-1929
Revista Brasileira de Engenharia Agrícola e Ambiental
v.19, n.7, p.621–629, 2015
Campina Grande, PB, UAEA/UFCG – http://www.agriambi.com.br

DOI: http://dx.doi.org/10.1590/1807-1929/agriambi.v19n7p621-629

Water use optimization through alternative water depths in the Formoso Irrigation District

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Key words:
irrigation
planning
economic efficiency

A B S T R A C T

This study aimed to propose an optimal cultivation plan using a separable linear programming model, with alternative water depths, that allows maximizing the net revenue of the Formoso Irrigation District (FID), specifically with respect to the area of family plots. The model used in this study was based on data from the 2010 Annual Agricultural Report of the 2nd Regional Superintendency of CODEVASF (São Francisco and Parnaíba Valley Development Company), the 2011 Service and Extension Plan for the Formoso Irrigation District and further information provided by this government department. Based on the studied crops and their respective water response functions, on the constraints of cultivated area, prices and production costs, the maximization of the net revenue in the FID was equal to R$ 68,384,956.53, using the following cultivation pattern: 30 ha of pumpkin, 30 ha of *Phaseolus* beans, 977 ha of watermelon, 1868 ha of banana, 1200 ha of papaya and 300 ha of Tahiti lime. The optimal solution found by the model indicated that the monthly water availability in the FID did not constitute an effective restriction to crop production, since in all months the water volume needed was lower than the maximum volume that the FID can provide (10,833,500 m^3). For the monthly volumes used in the solution, the available annual volume will not be restrictive if the annual pumping capacity is higher than 79,649,000 m^3.

Palavras-chave:
irrigação
planejamento
eficiência econômica

Otimização do uso da água no Perímetro Irrigado Formoso aplicando lâminas alternativas de água

R E S U M O

O objetivo do presente trabalho consistiu em propor um plano ótimo de cultivo utilizando um modelo de programação linear separável, com lâminas alternativas de água, que proporcione a maximização do retorno líquido do Perímetro Irrigado Formoso (PIF), especificamente quanto a área de lotes familiares. O modelo utilizado neste estudo foi baseado em dados do Relatório Anual Agrícola da 2ª Superintendência Regional da CODEVASF em 2010, no Plano de Serviço e extensão para o PIF para 2011 e informações adicionais apresentadas por este departamento governamental. Com base nas culturas consideradas e suas respectivas funções de resposta à água, nas restrições de área cultivada, nos preços e nos custos de produção, a maximização da receita líquida no Perímetro Irrigado Formoso foi de R$ 68.384.956,53, utilizando o seguinte padrão de cultivo: 30 ha de abóbora, 30 ha de feijão Phaseolus, 977 ha de melancia, 1868 ha de banana, 1200 ha de mamão e 300 ha de limão Tahiti.A solução ótima encontrada pelo modelo indicou que a disponibilidade mensal de água no PIF não constituiu restrição efetiva à produção das culturas, uma vez que em todos os meses o volume de água necessário foi inferior ao volume máximo que o PIF pode disponibilizar (10.833.500 m^3). Para os volumes mensais utilizados na solução ótima, o volume anual disponível não será restritivo se a capacidade anual de bombeamento for superior a 79.649.000 m^3.

Protocolo 206-2014 – 20/06/2014 • Aprovado em 23/01/2015 • Publicado em 01/06/2015

Introduction

Water is the main limiting factor in the agricultural production of semiarid regions. When water resources are a limiting factor for the production, irrigation planning must be used in order to allow the maximization of the production per unit of irrigation water. Deficit irrigation is one option to maximize water use efficiency for high yields per unit of irrigation water applied (Bekele & Tilahun, 2007; Figueiredo et al., 2007; 2008; Detomini et al., 2009).

In the current situation of irrigation districts, irrigation planning is essential in order to make the various water uses compatible, making different production sectors viable, monitoring quantity and quality of water resources and improving global wateruse efficiency levels (Frizzone, 2007).

An efficient irrigation management requires information related to crop water demand and crop-water production functions. The use of response functions allows finding useful solutions for the optimization of the use of water and fertilizers, obtaining the maximum of the product with a certain production cost (Kim & Schaible, 2000; Soares et al., 2002; Huang et al., 2004).

In order to develop optimal irrigation strategies, it is necessary to use relations between the water applied and yield, which are known as crop-water production functions. In irrigated areas, where various crops in different irrigation regimes are competing for a limited amount of water, one way of selecting an economically viable water depth, among the different water depth options available, is the use of techniques that help the decision-making process, and linear programming (LP) is one of the tools used in the optimal allocation of these resources (Dantas Neto, 1997).

The linear programming was used in the studies to optimize agricultural production systems, considering single or multiple cropping systems, subjected or not to specific irrigation plans (Carvalho et al., 2000; Borges Júnior et al., 2008; Lima et al., 2008; Schonwald et al., 2008; Carvalho et al., 2009; Marques et al., 2009; Schardong et al., 2009; Santos et al., 2009; Cabral et al., 2011; Delgado et al., 2012).

Most problems involving water resources have non-linear functions. However, these functions can be linearized through the technique of piecewise linearization and the model can be treated as a separable linear programming problem (Frizzone et al., 1994).

This study aimed to propose an optimal cultivation plan, using a separable linear programming model, with alternative water depths (WD), which allows the maximization of the net revenue of the Formoso Irrigation District, especially with respect to the area of family plots, differently from the study conducted by Santos Júnior et al. (2014) for optimization, applying maximum water depths.

Material and Methods

The Formoso Irrigation District (FID) is located in the western part of Bahia state (BA), Brazil, approximately 30 km away from the city of Bom Jesus da Lapa – BA, in an area comprehended between the left bank of the São Francisco river, the right bank of the Corrente river and the Federal Highway BR – 349, connecting Bom Jesus da Lapa to Santa Maria da Vitória – BA.

The FID has a total area of 19,471.5 ha, with 12,100 ha of irrigable land, in the final stages of implementation and occupation, distributed as follows:

- Family plots: 913, totaling 4,700 ha; and
- Business plots: 249, totaling 7,400 ha.

In this study, it was considered an available area of 4,405 ha, representing the area for the family plots, already in operation in the Formoso Irrigation District.

According to Köppen's classification, the climate of the region is BSwh', semiarid hot, characterized by rains in the summer and a well-defined dry period during the winter, and absence of water surplus. The annual averages of temperature, rainfall and potential evapotranspiration are respectively 25.3 °C, 831 mm and 1,418 mm, and Latosols and Cambisols prevail in the region, with transitional vegetation between Caatinga and seasonal forest (Borges et al., 2010).

In this study, seven crops traditionally cultivated by the colonizers of the Formoso Irrigation District were considered: pumpkin, *Phaseolus* bean, *Vigna* bean, watermelon, maize, banana, papaya, besides Tahiti lime, which is under study. In the occasion, fruit cultivation represented 96.6% of the area, with banana representing 96.2% and the annual crops 3.4%, notably maize and bean (CODEVASF, 2010).

The general model is an adaptation proposed by Frizzone (1994), which aims to simulate an optimal cultivation pattern, in order to maximize the profit of the project, resulting from many crops (Table 1), considering the minimum and maximum water depths for each crop and their respective estimated yields.

The response functions of all studied crops were divided into linear segments. The criteria to select the zone for resource allocation for each crop were: water depth corresponding to maximum yield (w0); 2) minimum water depth below which

Table 1.Minimum and maximum water depths for each studied crop and their respective yields

Crops	Minimum value of W* (mm)	Minimum value of Y** (kg ha^{-1})	Maximum value of W (mm)	Maximum value of Y (kg ha^{-1})
Pumpkin	210	17317	384	22133
Phaseolus beans	380	1028	545	2669
*Vigna*beans	330	1178	500	1539
Watermelon	350	20482	887	33670
Maize	350	5828	775	7555
Banana	1520	18812	2900	37856
Papaya	800	33405	1400	94784
Tahiti lime	1400	10060	2100	31060

*W – Water depth; **Y – Yield

plants, in the studied area, suffer water stress and yield is greatly reduced (w9). Minimum and maximum amounts of water in order to obtain reasonable yields, as well as minimum and maximum yields for each crop are shown in Table 1.

The crop-water response functions that best fitted the available data are described in Table 2. Only the functions for the crops *Phaseolus* bean and maize were developed in the Formoso Irrigation District.

The objective function

The model used in this study was based on data from the 2010 Annual Agricultural Report of the 2nd Regional Superintendency of CODEVASF (São Francisco and Parnaíba Valley Development Company), the 2011 Technical Assistance and Rural Extension Plan for the Formoso Irrigation District (CODEVASF, 2011) and additional information provided by the mentioned government department.

The coefficients of the objective function represent the net profit per unit area for each crop (R\$ ha^{-1}). The obtention of the coefficients was based on product prices, yield levels and production costs throughout the period considered in the analysis. Therefore, the aim was to design the model, using the characteristics of the land use, which allowed determining the best plans, suiting the goals of the study, through a matrix layout specific to the model used.

Production cost data for 5-year and 3-year cultivations were used for the crops Tahiti lime and banana, respectively, corresponding to the periods in which these crops were in full production.

The objective function for the irrigated area was specified as the maximization of the annual net return, NR, subjected to constraints of water availability and other inputs. It can be schematically represented as follows:

$$\begin{aligned}\text{MAX NR} &= \left[\sum_{i=1}^{n} X_{i0} Y_{i0} - \sum_{i=1}^{n}\sum_{k=1}^{s} X_{ik}\Delta Y_{ik}\right] P_i - \\ &- \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{s} A_{ij}\left(X_{i0} - X_{ik}\right) C_j\end{aligned} \quad (1)$$

where:

NR - net return from the cultivation of n crops, in R\$;
Pi - unitary price of the product, in R\$ kg^{-1};
A_{ij} - amount of the input j demanded by the crop i;
Ci - unitary cost of the input j used by the crop i;
X_{ik} - area cultivated with the i-th crop, in ha; and
Yi - $f(w_i)$, yield obtained for the crop i, when applied w_i water unities, in kg ha^{-1}.

The model is subjected to the following constraints:

1. Area:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{s} A_{ij}\left(X_{i0} - X_{ik}\right) \le S_j \left(i = 1 \ldots n; k = 1 \ldots s; j = 1 \ldots m\right) \quad (2)$$

$$X_{ij} - X_{i0} \le 0 \left(i = 1 \ldots n; k = 1 \ldots s\right) \quad (3)$$

2.Water:

$$\begin{aligned}\sum_{i=1}^{n} X_{i0} W_{i0} \quad &- \quad \sum_{i=1}^{n}\sum_{k=1}^{s} X_{ik}\Delta Y_{ik} \le \\ &\le \quad WM_g \left(i = 1 \ldots n; k = 1 \ldots s; g = 1 \ldots 12\right)\end{aligned} \quad (4)$$

$$\begin{aligned}\sum_{i=1}^{n} X_{i0} W_{i0} \quad &- \quad \sum_{i=1}^{n}\sum_{k=1}^{s} X_{ik}\Delta Y_{ik} \le \\ &\le \quad WT_g \left(i = 1 \ldots n; k = 1 \ldots s\right)\end{aligned} \quad (5)$$

3. Market and industrial processing capacity:

$$X_i \le X_{max} \quad (6)$$

$$X_i \ge X_{min} \quad (7)$$

4. Non-negativity:

$$X_{ik} \ge 0 \left(i = 1 \ldots n; k = 1 \ldots s\right) \quad (8)$$

where:

WM - monthly amount of water available;
WT - total amount of water available;
S - maximum area available for cultivation in the month j, in ha;
X_{ij} - cultivated area in the month j, in ha;
X_{i0} - total cultivated area with the crop I (ha); and
X_{max}, X_{min} - constraint in the cultivated area of the crop i, in the period k.

The applied model

The objective function applied in the problem was:

$$\begin{aligned}\text{MAN NR} &= 2902.18X_{10} + 2902.18X_{20} + \\ &+ 2480.08X_{30} + \ldots - 2871.81X_{129}\end{aligned} \quad (9)$$

where:

Table 2.Crop-water response functions

Crop	Equations	Reference
1- Pumpkin	$Y = -1.269 + 0.122W - 0.159x10^{-3}W^2$	Marouelli (1999)
2- *Phaseolus* beans	$Y = 8.001813x10^{-2}EXP(3.8227x10^{-2}W - 3.507706x10^{-5}W^2)$	Gomes & Espinoza (1990a)
3- *Vigna* beans	$Y = 436.3158EXP(2.92035x10^{-3}W - 1.621882x10^{-6}W^2)$	Silva (1978)
4- Watermelon	$Y = -2301.15 + 81.0895W - 0.0457W^2$	EMBRAPA (1977)
5- Maize	$Y = -8481.9373 - 20.6859W + 1151.92245W^{0.5}$	Gomes & Espinoza (1990b)
6- Banana	$Y = 41.916 - 0.0304W + 1E^{-05}W^2$	Coelho et al.(2004)
7- Papaya	$Y = -89.1 + 9.4W 10^{-2}W - 1.7 x 10^{-5}W^2$	Almeida et al. (2004)
8- Tahiti lime	$Y = -31.94 + 0.030W$	Alves Júnior (2006)

Y: Yield ($kgha^{-1}$); W: Water depth (mm)

$X_{10} ... X_{19}$ - area (ha) planted with pumpkin in February;
$X_{20} ... X_{29}$ - area planted with pumpkinin June;
$X_{30} ... X_{39}$ - area planted with *Phaseolus* beans in March;
$X_{40} ... X_{49}$ - area planted with *Phaseolus*beans in June;
$X_{50} ... X_{59}$ - area planted with *Vigna*beans in April;
$X_{60} ... X_{69}$ - area planted with *Vigna* beans inJuly;
$X_{70} ... X_{79}$ - area planted with watermelon in April;
$X_{80} ... X_{89}$ - area planted with watermelon in July;
$X_{90} ... X_{99}$ - area planted with maize in November;
$X_{100} ... X_{109}$ - area planted with banana in the 2nd year of the crop;
$X_{110} ... X_{119}$ - area planted with papayain the 2nd year of the crop; and
$X_{120} ... X_{129}$ - area planted with Tahitilime in the 4th year of the crop.

The eight studied crops were represented by the indices (1, 2, ..., 12), where X_1 is the area planted with pumpkin in February, X_2 the area planted with pumpkin in June and X_{12} the area planted with Tahiti lime in its 5th year. In the model with alternative water depths, crops can be irrigated with different water depths, which are represented by the indices (9, 8, ..., 0). The highest numbers designate the lowest water depths and vice versa. Thus, X_{19} means area planted with pumpkin in February and irrigated with minimum water depth, that is, 210 mm, and X_{129} means area planted with Tahiti lime and irrigated with minimum water depth, that is, 1400 mm. On the other hand, X_{10} means area planted with pumpkin in February and irrigated with maximum water depth of 384 mm and X_{120} means area planted with Tahiti lime and irrigated with maximum water depth of 2100 mm.

Area constraints correspond to the combination of crops in the 12 months of the year and determine that the occupied area must be lower than or equal to the available area. An available area of 4405 ha was considered in this study, which represents the area of the family plots in the Formoso Irrigation District. The equations referring to area constraints(C.1) are numbered as Eq. 10 and 11.

1. Area Constraint:

$$AJAN = X_{90} + X_{100} + X_{110} + X_{120} \leq 4405 \tag{10}$$

$$ADEZ = X_{90} + X_{100} + X_{110} + X_{120} \leq 4405 \tag{11}$$

where:
AJAN - area irrigated in January; and
ADEZ - area irrigated in December.

Equations 12 and 13 ensure that, considering each crop separately, the area irrigated with certain irrigation depth must be less or equal to area irrigated with an immediately lower water depth.

$$A11 = X_{10} - X_{11} \leq 0 \tag{12}$$

$$A129 = X_{128} - X_{129} \leq 0 \tag{13}$$

Water constraints ensure that monthly crop water demand is not higher than the water volume available in the district during each month. The monthly volume used in the calculation of the available volume in the most critical month (October) was equal to 10,833,500 m^3, and the highest annual water volume offered to the users was equal to 79,649,300 m^3, in 2009. Water constraint equations are numbered as Eq. 14, 15 and 16.

2. Water Constraint:

$$\begin{aligned} WJAN &= 2.418X_{90} + 2.684X_{100} ... - \\ &- 0.1589X_{129} \leq 10.833.500 \text{ dm ha} \end{aligned} \tag{14}$$

$$\begin{aligned} WDEZ &= 2.148X_{90} + 2.622X_{100} ... - \\ &- 0.1940X_{129} \leq 10.833.500 \text{ dm ha} \end{aligned} \tag{15}$$

$$\begin{aligned} WTOTAL &= 3.84X_{10} + 3.84X_{20} ... - \\ &- 1.00X_{127} - 1.50X_{129} \leq 79.649.300 \text{ dm ha} \end{aligned} \tag{16}$$

where:
WJAN ... WDEZ - monthly water volume, in m^3, available from January to December; and
WTOTAL - total water volume available, in m^3.

Constraints of minimum and maximum areas to be cultivated were incorporated into the model, using market conditions, internal consumption, processing capacity or regional problems, based on market research. The values of each constraint were estimated as a function of the mean production expected per hectare, and per cultivated area, data provided by CODEVASF and PLENA Consultoria e Projetos, which is the company administering the Formoso Irrigation District.

3. Market Constraints

$$AREAMIN1 = X_{10} \geq 30 \tag{17}$$

$$AREAMIN3 = X_{30} \geq 30 \tag{18}$$

$$AREAMIN7 = X_{70} \geq 60 \tag{19}$$

$$AREAMAX7 = X_{70} \leq 800 \tag{20}$$

$$AREAMIN10 = X_{100} \geq 1500 \tag{21}$$

$$AREAMIN11 = X_{110} \geq 60 \tag{22}$$

$$AREAMAX11 = X_{110} \leq 1200 \tag{23}$$

$$AREAMAX12 = X_{120} \leq 300 \tag{24}$$

where:
AREAMIN1 - minimum area, in ha, to be planted with pumpkin in February;

AREAMIN3 - minimum area to be planted with *Phaseolus* beans in March;

AREAMIN7 - minimum area to be planted with watermelon in April;

AREAMAX7 - maximum area to be planted with watermelon in April;

AREAMIN10 - minimum area to be planted with banana in the 3rd year of the crop;

AREAMIN11 - minimum area to be planted with papaya in the 1st year of the crop;

AREAMAX11 - maximum area to be planted with papaya in the 1st year of the crop; and

AREAMAX12 - maximum area to be planted with Tahiti lime in the 5th year of the crop.

In the resolution of the linear programming model, the software LP - 88 (LINEAR PROGRAMMING – 88) was used, which solves linear equations using the revised simplex method, an iterative algorithm, for the optimization of net profit and for the study of sensitivity analysis.

Results and Discussion

In the Formoso Irrigation District (FID), the highest annual water volume offered to the users was equal to 79,649,300 m^3 and the highest monthly volume was 10,833,500 m^3, in 2009. Using these water constraints, the model found the optimal cultivation plan (Table 3), with the respective sensitivity analysis of the objective function.

Pumpkin and *Phaseolus* beans cultivations were only indicated for planting in February and March, respectively, with areas of 30 ha, defined by the production demand to meet internal consumption.

For watermelon, the financial return per produced unity of this crop can be altered to values ranging from R$ 4,103.33 to R$ 4,415.89 in the model, without changing the optimal solution (cultivation of 917 ha).

All crops in the optimal cultivation plan, except for papaya and Tahiti lime, used water depths lower than the maximum water depth, which shows that not always the highest water depth and, consequently, the highest yield leads to the highest financial return.

Santos Júnior et al. (2014) studied the allocation of water in the Formoso Irrigation District considering the irrigation water depths for maximum yield. In the study, they observed a financial return of R$ 67,643,676.78 for an optimal cultivation pattern that includes pumpkin 1, *Phaseolus* bean 1, watermelon 1, papaya and lemon, with cultivated areas equal to the ones obtained in this study. The crops watermelon 2 and banana had cultivated areas different from the ones obtained in this study: 1243 ha against 917 ha and 1542 ha against 1868 ha, respectively. This difference promoted a financial return 1.1% higher for the model with alternative water depths, representing a difference of R$ 741,279.75.

Marginal costs of crops not recommended for cultivation, called non-basic variables in the model, are shown in Table 4. Pumpkin and *Phaseolus* bean cultivations, with planting in June, were not recommended; in this case, there is a marginal cost associated with these activities, i.e., for each hectare cultivated with pumpkin during this period, there will be a reduction of R$ 531.52; for *Phaseolus* beans, the reduction will be equal to R$ 1,350.37.

The minimum value of the contribution to the profit that each crop, irrigated without deficit, must provide in order to be indicated for cultivation is shown in Table 4. Maize cultivation must not be recommended while its contribution to the profit is lower than R$ 3,820.27. *Phaseolus* beans, planted in June, will only participate in the optimal solution if its contribution to the profit is higher than R$ 3,830.37; this crop planted in March became a basic solution only because of the necessity to cultivate at least 30 ha to satisfy the internal demand.

Analysing Table 5, it is noticed that the resource land was not restrictive; in all months of the year, the occupied areas did not reach the limit of the available area, i.e., 4405 ha. The greatest occupation corresponds to the months of July, August and September, the period of the watermelon cycle (Table 5).

Table 4.Marginal costs associated with the non-basic activities and minimum value of the contribution to the profit for a volume

Crop	Planting period	Marginal cost	Minimum contribution to profit
		(R$ ha^{-1})	
Pumpkin	June	531.72	3,433.72
Phaseolus beans	June	1,350.37	3,830.37
Vigna beans	April / July	3,016.41	3,734.71
Maize	November	2,833.27	3,820.27

Table 3.Cultivated areas (ha), water depths with their respective yields and financial return for each studied crop, and sensitivity analysis of the model's objective function

Crop	Planting period/ stage	Optimal area (ha)	Water depth (mm)	Yield (kg ha^{-1})	Return per unit	Minimum	Maximum
						(R$ ha^{-1})	
Pumpkin 1	feb	30	364	22070	2,883.54	None	3,433.72
Pumpkin 2	jun	0	-	-	2,883.54	None	3,433.72
Phaseolus bean 1	mar	30	540	2667	2,478.29	None	3,830.37
Phaseolus bean 2	jun	0	-	-	2,480.00	None	3,830.37
Vigna bean 1	apr	0	-	-	718.30	None	3,734.71
Vigna bean 2	jul	0	-	-	718.30	None	3,734.71
Watermelon 1	apr	60	790	33238	4,415.89	4,415.89	None
Watermelon 2	jul	917	790	33238	4,415.89	4,103.33	4,415.89
Maize	nov	0	-	-	987.00	None	3,820.27
Banana	3° year	1868	1520	18812	9,011.00	7,948.71	9,188.56
Papaya	1° year	1200	1400	94789	36,270.00	5,848.72	None
Tahiti lime	5° year	300	2100	31060	13,990.00	7,492.95	None
Total Area (ha)		4405					
Financial Return (R$)						68,384,956.53	

Table 5.Sensitivity analysis of the resource land occupied per month

Month	Occupied area (ha)	Amount of slack (ha)	Shadow price (R$ ha^{-1})	Minimum area (ha)	Maximum area (ha)
January	3368	1037	0	3368	None
February	3398	1007	0	3398	None
March	3428	977	0	3428	None
April	3488	917	0	3488	None
May	3458	947	0	3458	None
June	3428	977	0	3428	None
July	4285	120	0	4285	None
August	4285	120	0	4285	None
September	4285	120	0	4285	None
October	3368	1037	0	3368	None
November	3368	1037	0	3368	None
December	3368	1037	0	3368	None

Similarly, land was not restrictive in several months of the year in a study conducted by Santos et al. (2009) in the Baixo Acaraú Irrigation Project - CE, also with analysis by linear programming tools. Andrade et al. (2008), seeking to establish an optimal cultivation pattern in the Gorutuba Irrigation District, MG, found that land-related constraints were not that pronounced in comparison to the effects of water and labor constraints.

The sensitivity analysis related to market and internal consumption constraints is shown in Table 6. The constraints of minimum area of pumpkin and *Phaseolus* beans have negative shadow prices, which means that the entry of these crops into the optimal solution, forced by the constraints, leads to reduction in the net return.

If the constraint AREAMIN3 (minimum area cultivated with *Phaseolus* beans planted in March), which has a minimum limit of 30 ha, had its value increased to 31 ha, the economic return of the objective function would be reduced in R$ 1,350.37. On the other hand, each hectare of land not cultivated with this crop would provide gain of R$ 1,350.37. The same reasoning applies to the constraint AREAMIN1.

The crops papaya and Tahiti lime, and the constraints AREAMAX11 and AREAMAX12 had positive shadow prices, which indicates that, if the maximum planting areas increase, the economic return will also increase. Thus, for each additional hectare of papaya, in the interval from 344 to 2291 ha, there would be a gain of R$ 9,042.13.

The constraint imposed to the banana crop AREAMIN10 has a shadow price equal to zero in the studied model. Thus, if the banana area (1868 ha) is increased or reduced, the basic variables composing the optimal solution will not be altered, but the values of the objective function will.

Table 6. Sensitivity analysis of market and internal consumption constraints

Constraints	Occupied area (ha)	Amount of slack (ha)	Shadow price (R$ ha^{-1})	Minimum area (ha)	Maximum area (ha)
AREAMIN1	30	0	-531.72	0	773
AREAMIN3	30	0	-1,350.37	0	836
AREAMIN7	60	0	0	0	800
AREAMAX7	60	740	0	60	None
AREAMIN10	1868	-368	0	None	1868
AREAMIN11	1200	-1140	0	None	1200
AREAMAX11	1200	0	9,042.13	344	2291
AREAMAX12	300	0	4,497.05	0	808

The sensitivity analysis for monthly and annual water volumes are shown in Table 7, considering a maximum availability of 79,649,300 m^3 per year and 10,833,500 m^3 per month.

The annual water availability in the Formoso Irrigation District presents itself as an effective constraint for the production system. The optimal cultivation pattern determined in the solution of the models resulted in the consumption of the entire volume available. Since the annual volume proved to be a scarce resource, a shadow price is associated to it (opportunity cost of using a certain water volume), which corresponds to the expected reduction in the value of the objective function if this volume becomes more restrictive in one unity. In this case, a unitary reduction in total water (1 m^3) from 79,649,300 to 72,997,000 m^3 would reduce the value of the objective function in R$ 234.89, without changing the basic variables of the optimal solution. On the other hand, each additional water unit from 79,649,300 m^3 to 96,255,000 m^3 would increase the value of the objective function in the same amount of monetary unit.

Monthly water availability was not limiting, since the volume of water used per month was always lower than the available value (10,833,500 m^3). Therefore, it has slack and its shadow price is equal to zero, indicating that the resource maximum monthly water volume that can be pumped to the cultivated area is not restrictive when the annual available volume is 79,649,300 m^3. The models only present minimum volume values that can be used in the constraints of water per month, without changing the basic variables (crops) of the optimal solution.

Santos et al. (2009), using linear programming for economic optimization in the Baixo Acaraú Irrigation Project – CE, corroborate with this study, as the water availability estimated for the Project was not limiting to the total use of land resources in six of the seven cultivation plans analysed.

Minimum and maximum water volume values represent the water availability limits for which the shadow price is valid and the current optimal solution is not altered; however, the values of cultivated areas can be changed. The annual water availability can range from 72,997,000 m^3 to 96,255,000 m^3 without changing the optimal solution and the shadow price.

The cultivated area of each crop, total area and the financial return for six water availability levels are shown in Table 8.

Table 7.Sensitivity analysis of the water volume in the project

Month	Volume used (1000 m^3)	Amount of slack (1000 m^3)	Shadow price (R$ 1000m^{-3})	Minimum area (1000 m^3)	Maximum area (1000 m^3)
January	6716	4117	0	6716	None
February	5856	4977	0	5856	None
March	5861	4973	0	5861	None
April	5371	5463	0	5371	None
May	5315	5518	0	5315	None
June	4687	6135	0	4699	None
July	6320	4514	0	6320	None
August	8364	2470	0	8364	None
September	9448	1386	0	9448	None
October	8108	2725	0	8108	None
November	6879	3954	0	6879	None
December	6724	4109	0	6724	None
Total Water	79649	0	234.89	72997	96255

Table 8.Cultivated area per crop (ha), total area (ha) and financial return (R$) for six water availability levels

Crops	Water availability (1000 m^3)					
	47790	61111	67198	72346	79649	97218
Pumpkin 1	30	1615	30	30	30	30
Pumpkin 2	0	0	1285	545	0	0
*Phaseolus*beans 1	30	30	30	30	30	30
*Phaseolus*beans 2	0	0	0	0	0	0
*Vigna*beans 1	0	0	0	0	0	0
*Vigna*beans 2	0	0	0	0	0	0
Watermelon 1	60	60	60	60	60	60
Watermelon 2	0	0	0	740	917	0
Maize	0	0	0	0	0	0
Banana	1500	1500	1500	1500	1868	2785
Papaya	1200	1200	1200	1200	1200	1200
Tahiti lime	0	0	300	300	300	300
Total Area	2820	4405	4405	4405	4405	4405
Return (R$)	52,207,842	59,263,243	64,674,113	65,870,267	68,384,957	72,394,587

The minimum value for the annual water volume available was equal to 45,273,000 m^3. Thus, for volumes lower than this one, the solution of the model was impossible, indicating that the constraints were not satisfied, specifically those limiting minimum cultivated area. The maximum value was determined when the annual water availability was not limiting, with shadow price equal to zero, and this volume was 97,218,000 m^3.

It is observed that the same crops (pumpkin 1, *Phaseolus* beans 1, watermelon 1, banana, papaya and Tahiti lime) are part of the cultivation pattern predicted by the model. Pumpkin 1, *Phaseolus* beans 1 and watermelon 1 remained with constant cultivated areas at all water availability levels, according to the imposed constraints.

Tahiti lime only appears in the solution from the water availability of 67,198,000 m^3. From this water availability on, the area cultivated with Tahiti lime was equal to 300 ha, maximum limit for the cultivation of this crop, which is being tested as an option in cultivation, substituting banana in areas affected by Panama disease.

Table 9 presents the water depths used in the crops that are part of the cultivation pattern predicted by the model, with their respective yields, at the six levels of available water volume.

Analyzing Table 9, it is observed that pumpkin 1, *Phaseolus* beans 1 and watermelon 1 had an increase in water depth when water availability increased from 47,790,000 m^3 to 79,649,300 m^3. At the level of 97,218,000 m^3, the highest one, there was reduction of water depths for *Phaseolus* beans 1 and watermelon 1, with yield reductions from 2,667 to 2,612 kg ha^{-1}and from 33,238 to 28,474 kg ha^{-1}, respectively. This water depth reduction provided an additional water volume to the model, which was used by the banana crop.

Papaya cultivation was not influenced by the increase in availability. For its high marginal revenue, the model attributes the highest water depth available to this crop (1400 mm) for a yield of 94,784 kg ha^{-1}.

Table 9.Water depth (W) in mm and yield (Y) in kg ha^{-1} for six water availability levels

Crop	Water availability (1000 m^3)					
	W	Y	W	Y	W	Y
	47790		61111		67198	
Pumpkin 1	310	21262	330	21669	350	21949
Pumpkin 2	-	-	-	-	350	21949
Phaseolus beans 1	520	2612	520	2612	530	2648
Watermelon 1	170	10190	650	31099	750	32810
Banana	1520	18812	1520	18812	1520	18812
Papaya	1400	94784	1400	94784	1400	94784
Tahiti lime			2100	31060	2100	31060
	72346		79649		97218	
Pumpkin 1	350	21949	364	22070	384	22134
Pumpkin 2	350	21949	-	-	-	-
Phaseolus beans 1	530	2648	540	2667	520	2612
Watermelon 1	750	32810	790	33238	550	28474
Watermelon 2	750	32810	790	33238	-	-
Banana	1520	18812	1520	18812	2900	37856
Papaya	1400	94784	1400	94784	1400	94784
Tahiti lime	2100	31060	2100	31060	2100	31060

Conclusion

1. The maximization of the net revenue in the Formoso Irrigation District obtained with the model was equal to R$ 68,384,956.53, using the following cultivation pattern: 30 ha of pumpkin, 30 ha of *Phaseolus* beans, 977 ha of watermelon, 1868 ha of banana, 1200 ha of papaya and 300 ha of Tahiti lime, for the annual water volume of 79,649,300 m^3.

2. The monthly water availability in the Formoso Irrigation District does not constitute effective restriction to the production of the crops if the monthly water volume provided is 10,833,500 m^3.

3. Considering the monthly water volumes used in the optimal solution, with the maximum volume of 9,449,000 m^3 in September and a minimum of 4,687,000 m^3 in July, the annual volume available will not be restrictive if the annual pumping capacity is higher than 79,649,000 m^3, which represents the highest water volume offered to the users in 2009.

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