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Multifractal and joint multifractal analysis of soil invertebrate fauna, altitude, and organic carbon[1]

Análise multifractal e joint multifractal da fauna invertebrada do solo, altitude e carbono orgânico

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HIGHLIGHTS:

Joint multifractal analysis revealed multiple scales between altitude, organic carbon, and soil fauna.
The altitude, organic carbon, and soil fauna correspond to complex systems with scale variability.
Altitude and organic carbon were correlated with soil fauna diversity at multiple scales.

ABSTRACT: The objectives of this study were to evaluate the degree of multifractality of the spatial distribution of altitude, organic carbon concentration, and invertebrate fauna diversity, and to characterize the degree of joint multifractal association among these variables. Soil sampling was performed every 20 m across a 2,540 m transect, with a total of 128 sampling points in a sugarcane area in Goiana municipality, Pernambuco State. For each sampling point, the altitude, organic carbon concentration, and macrofauna diversity (diversity indices and functional groups) were evaluated. Spatial distributions of altitude, organic carbon concentration, and macrofauna diversity were characterized by the generalized dimension spectrum (Dq) and singularity spectrums [f(α) versus α], which presented multifractal behavior with different degrees of heterogeneity in scales. Joint multifractal analysis was useful for revealing the relationships at multiple scales between the studied variables, as demonstrated by the non-detected associations using traditional statistical methods. To quantify the spatial variability of edaphic fauna based on the multiple scales and association sets in the joint dimension, the impact of agricultural production systems on biological diversity can be described. All of the studied variables displayed a multifractal behavior with greater or lower heterogeneity degree depending on the variable, with altitude and organic carbon being the most homogeneous attributes.

Key words: scale heterogeneity, singularity spectrum, soil fauna diversity, soil attributes

RESUMO: Os objetivos deste estudo foram avaliar o grau de multifractalidade da distribuição espacial da altitude, concentração de carbono orgânico e diversidade da fauna de invertebrados, e caracterizar o grau de associação joint multifractal dessas variáveis. A amostragem do solo foi realizada a cada 20 m em um transecto de 2.540 m, totalizando 128 pontos de amostragem em uma área de cana-de-açúcar localizada no município de Goiana, Estado de Pernambuco. Para cada ponto de amostragem foram avaliados altitude, concentração de carbono orgânico e diversidade da macrofauna (índices de diversidade e grupos funcionais). A distribuição espacial da altitude, carbono orgânico e a diversidade da macrofauna foram caracterizados por meio do espectro de dimensão generalizada (Dq) e espectros de singularidade [f(α) versus α] que apresentaram comportamento multifractal com diferentes graus de heterogeneidade nas escalas. A análise joint multifractal foi útil para mostrar as relações em múltiplas escalas entre as variáveis estudadas, e evidenciaram associações não detectadas pelos métodos estatísticos tradicionais. Quantificar a variabilidade espacial da fauna edáfica considerando as múltiplas escalas e conjuntos de associações em dimensão conjunta permite descrever o impacto do sistema de produção agrícola na diversidade biológica. Todas as variáveis estudadas apresentaram um comportamento multifractal com maior ou menor grau de heterogeneidade dependendo da variável, sendo a altitude e o carbono orgânico os atributos mais homogêneos.

Palavras-chave: heterogeneidade de escala, espectro de singularidade, diversidade da fauna do solo, atributos do solo

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Introduction

The invertebrate fauna is responsible for several dynamic processes within the soil ecosystem (Wagg et al., 2014) and organic matter decomposition (Catterall et al., 2001; Gholami et al., 2016). In fact, the invertebrate fauna participates in nutrient cycling (Silva et al., 2018) and improves soil structure (Bernardes et al., 2020). Therefore, it is necessary to understand the factors that act on soil fauna along the landscape, which can be achieved using mathematical models that allow us to comprehend the variability at different scales.

Among these mathematical models, multifractal analysis permits the study of the spatial variability of soil fauna based on different variability scales. Multifractal analysis allows the description of a system through a continuum spectrum of scale exponents (Evertsz & Mandelbrot, 1992). According to Banerjee et al. (2011) and Leiva et al. (2019), multifractal analysis presents several relevant information regarding system heterogeneity. Multifractal analysis focuses on evaluating the distribution of one variable along a geometric support while joint multifractal analysis refers to the joint distribution of two or more variables on a common support or temporal space (Zeleke & Si, 2006; Banerjee et al., 2011; Biswas et al., 2012).

Soils are highly variable and heterogeneous within a landscape, and across fields or soil profiles (Wilson et al., 2016; Leiva et al., 2019). This variability directly affects the diversity and distribution of invertebrate fauna. The diversity of invertebrate fauna is also affected by altitude (Catterall et al., 2001; Wang et al., 2009; Begum et al., 2010), and its distribution by soil organic carbon (OC) (Gholami et al., 2016; Silva et al., 2018; Bernardes et al., 2020). Therefore, it is important to assess the variability of these soil properties along the landscape to enable specific management and achieve sustainable development.

In this context, the objectives of the current study were to evaluate the degree of multifractality of the spatial distribution of altitude, OC concentration, and invertebrate fauna diversity, and to characterize the degree of joint multifractal association among these variables.

Material and Methods

The study was carried out in the Goiana municipality (State of Pernambuco, Brazil) at the coordinates 07° 34' 25" S and 34° 55' 39" W. The regional relief is wavy smooth with a mean altitude of 46 m. The climate in the region is tropical humid, with a mean annual temperature of 24 °C and annual mean precipitation of 1,654 mm. The soil in this region is classified as Ultisol. The selected soil physical and chemical characteristics at 0 to 0.20 m soil depth are shown in Table 1. The study area was used to cultivate sugarcane in 1910, with manual harvesting. Due to cane field renovation, the soil area was plowed, harrowed, subsoiled, and furrowed for sugarcane planting. After planting, the area was irrigated in the dry season and an annual dose of 60 mm of sugarcane stillage has been applied since 2003.

Soil sampling was performed on 11/10/2015 every 20 m across a 2,540 m transect, and a total of 128 sampling points were established in a sugarcane area. For each sampling point, altitude (m), organic carbon (OC) concentration (g kg^{-1}), and macrofauna diversity (diversity indices and functional groups) were evaluated. Altitude was determined using a static global positioning system with differential correction post-processed (static DGPS). Disturbed soil samples were collected at 0 to 0.20 m soil depth to determine OC concentration according to Raij et al. (2001). Invertebrate soil faunas were sampled using one pitfall trap at each sampling point (128 points) containing 200 mL of 4% formalin (Siqueira et al., 2016). The traps remained in the field for a period of seven days, and the collected organisms were identified at the level of class, order, and family (Lawrence, 1994). A total of 823 individuals were collected from the following groups: Acari (197), Araneae (18), Collembola (92), Coleoptera (16), Diptera (19), Formicidae (443), Isoptera (3), and Scorpianida (29). After identification, the number of individuals per trap per day and total richness were determined, and the following biological diversity indices were estimated: Shannon diversity (H') and Pielou uniformity (J'). Thereafter, the invertebrate macrofauna individuals were grouped based on their functionality, relationships with the environment, and food habits, as described by Maggiotto et al. (2019). The identified organisms were grouped into detritivores (Acari and Collembola), predators (Araneae, Coleoptera, Diptera, and Scorpianida), and socials (Formicidae and Isoptera).

The following parameters of descriptive statistics were calculated: mean, standard deviation, coefficient of variation (CV), asymmetry, kurtosis, and D - maximum deviation from related to the normal frequency distribution by the Kolmogorov-Smirnov test ($p \leq 0.01$). Linear correlations among variables were determined using Pearson's correlation coefficient ($p \leq 0.01$). Joint multifractal analysis was used to assess the joint correlation ($p \leq 0.05$) among variables using the singularity indices [$\alpha(q,t)$ and $\beta(q,t)$].

Multifractal analysis was carried out according to the moment method (Halsey et al., 1986), generating successive partitions to k (k = 1, 2, 3…), where at each scale δ, a number of segments, $N(\delta) = 2^k$ with characteristic size length, $\delta = L \times 2^{-k}$, were obtained, covering the entire extent of the support, L (Evertsz & Mandelbrot, 1992; Vidal-Vázquez et al., 2013). In this study, the transect total length was employed as the support, L.

The attributes were converted into a mass distribution along the support, and the probability mass function (p_i) for each segment was estimated as a portion of the total mass:

Table 1. Chemical and physical characterization at 0 to 0.20 m soil depth of the study area cultivated with sugarcane

Sand	Silt	Clay	OC	pH	P	K^+	Ca^{2+}	Mg^{2+}	H + Al	SB	CEC	V
(g kg^{-1})				($CaCl_2$)	(mg dm^{-3})	(mmol$_c$ dm^{-3})						(%)
841.9	56	102.1	13.7	5.5	39.4	1.3	26.7	7.2	26.4	48.2	74	55.6

OC - Organic carbon; P - Phosphorus; K^+ - Potassium; Ca^{2+} - Calcium; Mg^{2+} - Magnesium; H + Al - Potential acidity; SB - Sum of bases; CEC - Cationic exchange capacity; V - Base saturation

$$p_i(\delta) = \frac{N_i(\delta)}{N_t} \quad (1)$$

where:

$N_i(\delta)$ - value of the measure in the i^{th} segment; and

N_t - total mass of entire transect.

The partition function χ(q,δ) of order q was calculated from the probability mass function $p_i(\delta)$:

$$\chi(q,\delta) = \sum_{i=1}^{n(\delta)} p_i^q \quad (2)$$

where:

q - statistical moments, defined for -∞ < q < ∞; and

n(δ) - number of segments with size δ.

The function τ(q) was obtained from the slopes of a log–log plot of the quantity χ(q,δ) versus δ for different q values:

$$\chi(q,\delta) \propto \delta^{-\tau(q)} \quad (3)$$

where:

χ(q,δ) - partition function in moments (q) and generated segments of order (δ); and

τ(q) - nonlinear function of q, referred to as the mass exponent function. When the measurements behave as multifractals, a non-linear function(q) is adjusted; however, for monofractals, a linear functionτ(q) is adjusted (Halsey et al., 1986).

The generalized dimension is used to characterize multifractal measurements of order q, where Dq is directly obtained from the relation with the mass exponent, τq (Hentschel & Procaccia, 1983). The generalized dimension was estimated using the moment method (Evertsz & Mandelbrot, 1992) (Eq. 4), when q ≠ 1. However, when q = 1, D_1 becomes indeterminate because the value of the denominator is zero. Thus, when q = 1, D_q is obtained using l'Hôpital's rule according to Eq. 5.

$$D_q = \frac{1}{q-1}\lim_{\delta\to 0}\frac{\log\left[\chi(q,\delta)\right]}{\log(\delta)} = \frac{\tau(q)}{q-1}, \text{ for } q \neq 1 \quad (4)$$

$$D_1 = \lim_{\delta\to 0}\frac{\sum_{i=1}^{n(\delta)}\mu_i(\delta)\log\chi(q,\delta)}{\log(\delta)}, \text{ for } q \neq 1 \quad (5)$$

For multifractal measures, $N_\delta(\alpha)$ of segments of size δ, with a singularity or Hölder's exponent equal to α increases with the decrease in δ and obeys the power law, $N(\alpha) \propto \delta^{-f(\alpha)}$, where the exponent f(α) is a continuous function of α. The singularity spectrum, a plot of f(α) versus α, is usually a concave parabola with a downward shape, where the range of α values increases with the heterogeneity of the measure.

The scaling functions, f(α) and α_q, were obtained using the direct method of Chhabra & Jensen (1989). This procedure relies on the scaling properties of a modified partition function χ(q,δ,), based on the contributions of individual segments. Once the generating function has been obtained, the normalized variable, $\mu_i(q, \delta)$, is defined as

$$\mu_i(q,\delta) = \frac{\mu_i^q(\delta)}{\sum_{i=1}^{n(\varepsilon)}\mu_i^q(\delta)}.$$

Thereafter, using a set of real numbers, q, the relationships applied to compute $f(\alpha_{(q)})$ and $\alpha_{(q)}$ are:

$$\alpha(q) \propto \frac{\sum_{i=1}^{n(\delta)}\mu_i(q,\delta)\log\left[\mu_i(\delta)\right]}{\log(\delta)} \quad (6)$$

$$f\left[\alpha(q)\right] \propto \frac{\sum_{i=1}^{n(\delta)}\mu_i(q,\delta)\log\left[\mu_i(q,\delta)\right]}{\log(\delta)} \quad (7)$$

The generalized dimension spectra, D_q, were calculated in the range of statistical moments -10 ≤ q ≤ 10 at 1 lag increments. Values, α and f(α), were accepted in the singularity spectrum only if the numerators of Eqs. 6 and 7 varied linearly with the logarithm of δ (in the denominator), with coefficients of determination (R^2 = 0.90).

The approach of the joint partition function (Zeleke & Si, 2006; Banerjee et al., 2011; Biswas et al., 2012; Siqueira et al., 2018) can be considered as an extension of multifractal analysis, allowing for a description of the variability of two or more datasets along a transect. The first step involves box counting, where the total length of the transect (L) is partitioned into boxes of size δ. The probability of the measures of these two coexisting variables in the n segment is defined as $p_i(\delta)$ and $r_i(\delta)$. The Holder exponents of these two variables, which are called α and β, present the following relationships: $p_i(\delta) \propto (\delta)^\alpha$ and $r_i(\delta) \propto (\delta)^\beta$, respectively. Similar to the multifractal analysis of one variable, the normalized joint partition function, $\mu_i(q,t,\delta)$, for the joint distribution of $p_i(\delta)$ and $r_i(\delta)$ was calculated as follows:

$$\mu_i(q,t,\delta) = \frac{\left[p_i(\delta)\right]^q\left[r_i(\delta)\right]^t}{\sum_{i=1}^{n(\varepsilon)}\left[p_i(\delta)\right]^q\left[r_i(\delta)\right]^t} \quad (8)$$

where:

q and t - represent the moment orders; and

δ - scale.

The singularity indices, α(q,t) and β(q,t), were calculated as the average values of α and β according to the μ measurements, defined by Eqs. 9 and 10 (Zeleke & Si, 2006; Banerjee et al., 2011; Siqueira et al., 2018):

$$\alpha(q,t) = \lim_{\varepsilon \to 0} \frac{\sum_{i=1}^{n(\varepsilon)} \left[\mu_i(q,t,\delta) \log p_i(\delta)\right]}{\log(\delta)} \quad (9)$$

$$\beta(q,t) = \lim_{\varepsilon \to 0} \frac{\sum_{i=1}^{n(\varepsilon)} \left[\mu_i(q,t,\delta) \log r_i(\delta)\right]}{\log(\delta)} \quad (10)$$

The joint multifractal dimension, f(α,β), of the set on which α(q,t) and β(q,t) - are the mean of the local exponent singularity of both measures, is given by

$$f(\alpha,\beta) = \lim_{\varepsilon \to 0} \frac{\sum_{i=1}^{n(\varepsilon)} \left[\mu_i(q,t,\varepsilon) \log(q,t,\varepsilon)\right]}{\log(\varepsilon)} \quad (11)$$

Joint multifractal spectra were obtained by plotting the joint dimension, f(α,β), versus the singularity indices, α(t,q) and β(t,q).

Results and Discussion

The average concentration of OC along the transect was 13.72 $g \cdot kg^{-1}$ (Table 2), with higher values related to lower altitudes (Figures 1A and B). In the same area, Siqueira et al. (2018) found OC mean values of 42 $g \cdot kg^{-1}$ in the lower parts of the terrain, corroborating the highest OC values observed in the current study in the final part of the terrain.

On average, the diversity index was 0.95 individuals per trap per day, the richness was 2.04, and Pielou equitability, and Shannon diversity indices were 2.04, 0.55, and 0.23, respectively (Figure 1 and Table 2). The number of individuals per trap per day displayed a similar behavior to that of altitude and OC, increasing by approximately 500% in the final part

Table 2. Descriptive statistics of altitude, organic carbon concentration, diversity indices, and functional groups along the transect with 128 points in a sugarcane area

Variable	Unit	Mean	SD	CV (%)	Skewness	Kurtosis	D*
Altitude	m	50.66	20.15	39.78	0.031	-1.661	0.175Ln
Organic carbon	g kg^{-1}	13.72	3.50	25.51	1.452	4.809	0.155Ln
Individuals per trap per day	individuals	0.95	1.03	108.07	2.204	5.984	0.179Ln
Richness	-	2.04	1.35	65.99	0.245	-0.248	0.166Ln
Pielou	-	0.55	0.40	72.61	-0.501	-1.502	0.236Ln
Shannon	-	0.23	0.19	82.09	0.188	-0.986	0.209Ln
Detritivores	individuals	4.25	2.88	67.80	1.868	4.304	0.217Ln
Predators	individuals	2.64	1.64	62.27	6.011	47.066	0.348Ln
Socials	individuals	5.48	5.61	102.30	2.975	10.253	0.267Ln

SD - Standard deviation; CV - Coefficient of variation; D* - Kolmogorov-Smirnov test of normality (p < 0.01); Ln - Lognormal

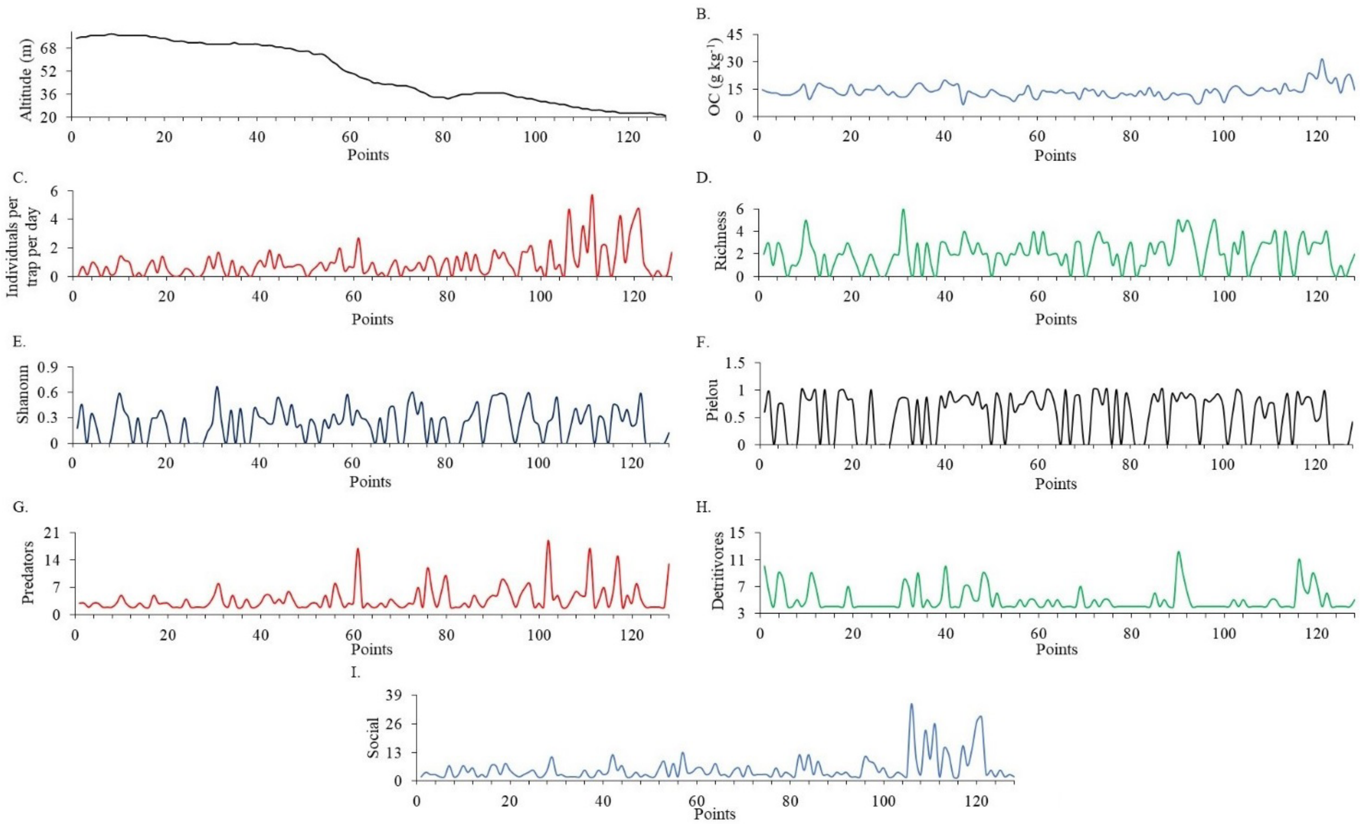


Figure 1. Spatial distribution of variables along the transect: altitude (A); organic carbon concentration - OC (B); Individuals per trap per day (C); Richness (D); Shannon (E); Pielou (F); Detritivores (G); Predators (H); and Socials (I)

of the transect (Figure 1C). According to Silva et al. (2018), individuals per trap per day represent the organism abundance and its increase is related to an increase in soil quality; however, it does not represent an increase in diversity: richness, J', and H' (Figures 1D, E, and F).

Social insects were more abundant along the transect, with an average occurrence of 5.48 individuals per sample along the transect (Table 2). Detritivore and predator organisms had averages of 4.25 and 2.64 individuals per sample, respectively, along the transect. The abundance of social insects, represented in this study, mainly by the Formicidae family, is justified by their high colonization capacity (Holldobler & Wilson, 1990), aggregated social behavior (Holldobler & Wilson, 1990), and diversified food habits (Siqueira et al., 2016). The spatial distribution of social insects is related to individuals per trap per day, OC, and altitude (Figures 1A, B, and C), which corroborates the findings of Catterall et al. (2001), Wang et al. (2009), Begum et al. (2010), and Wagg et al. (2014). The spatial distribution of detritivores and predator organisms (Figures 1G and H) did not show a consistent relationship with altitude and OC (Figures 1A and B). According to Bernardes et al. (2020), soil OC quality and water content influence the spatial distribution of these organisms more than the OC content in the soil.

The CV values (Table 2) varied from 25.51% for OC to 108.07% for the number of individuals per trap per day. The evaluated variables presented a lognormal frequency distribution according to the Kolmogorov-Smirnov test ($p \leq 0.01$), which can be observed based on the values of mean and standard deviation (Table 2).

The highest values of D_0 were found for altitude, OC, and functional groups ($D_0 = 0.999$) while the lowest values were found for Shannon and Pielou ($D_0 = 0.935$) (Table 3). D_1 values varied from 0.914 (Shannon index) to 0.992 (OC), while D_2 ranged from 0.915 (Pielou index) to 0.985 (OC). The biggest difference in D_{-10}-D_{10} was observed for individuals per trap per day (D_{-10}-D_{10} = 0.645) while the smallest difference was found for OC (D_{-10}-D_{10} = 0.152) (Table 3). The spectrum of the generalized dimension (Dq, Figure 2) describes a typical sigma-shaped curve and provides indicator parameters of properties of multifractal dimension (Vidal-Vázquez et al., 2013; Leiva et al., 2019). The left sides of the spectra represent the q negative moments, which correspond to higher measured concentrations, while the right side represents the q-positive moments, which correspond to the lower measured concentrations (Vidal-Vázquez et al., 2013). The presence of low values of D_0 for diversity indices (H' and J') was found because such indices tended to zero when a unique group was found in a given sample (Table 3). At many sampling points, the value of the indices was equal to zero (Figure 1), which resulted in a partition with null values, as described by Zeleke & Si (2006) and Banerjee et al. (2011).

The dimension of information (D_1) represents the entropy (i.e., the degree of system disorder), and its value ranges from 0 to 1. When D_1 is close to 1, system values are uniformly distributed in all scales, whereas D_1 values close to zero represent a subset of scales where the irregularities are concentrated (Zeleke & Si, 2006; Banerjee et al., 2011; Biswas et al., 2012; Leiva et al., 2019). Accordingly, we may deduce that D_1 values indicate a uniform distribution of our data along the transect. D_2 values are mathematically associated with the correlation function and computing the correlation of the measures contained between intervals. Therefore, the differences in these three moments of the generalized dimension (Dq) were used to evaluate the heterogeneity of scale properties (Vidal-Vázquez et al., 2013; Paz-Ferreiro et al., 2018). When $D_0 = D_1 = D_2$, the distribution of the data series is characterized as monofractal; however, if $D_0 > D_1 > D_2$, the measure distribution is considered to be multifractal (Zeleke & Si, 2006; Marinho et al., 2017; Paz-Ferreiro et al., 2018). The heterogeneity of altitude was greater than that of OC (Figure 2A), according to the spectra of the generalized

Table 3. Multifractal parameters obtained from the generalized dimension (D_{-10}-D_{10}, D_{10}, D_{-10}, D_0, D_1 and D_2) and the singularity spectrum (q_+, q_- α_0, α_{-10} and α_{10}) for altitude, OC concentration, diversity indices, and functional groups along the transect with 128 points during sugarcane cultivation

Variable	Generalized dimension spectra					
	D_{-10}-D_{10}	D_{10}	D_{-10}	D_0	D_1	D_2
Altitude	0.130	0.954 ± 0.018	1.084 ± 0.028	0.999 ± 0.000	0.987 ± 0.004	0.978 ± 0.008
Organic carbon	0.152	0.915 ± 0.006	1.067 ± 0.010	0.999 ± 0.001	0.992 ± 0.000	0.985 ± 0.001
Individuals per trap per day	0.645	0.719 ± 0.021	1.364 ± 0.068	0.977 ± 0.013	0.915 ± 0.015	0.861 ± 0.018
Richness	0.350	0.846 ± 0.022	1.196 ± 0.068	0.977 ± 0.013	0.950 ± 0.012	0.929 ± 0.014
Shannon	0.336	0.827 ± 0.020	1.163 ± 0.106	0.935 ± 0.022	0.914 ± 0.019	0.898 ± 0.018
Pielou	0.212	0.888 ± 0.009	1.100 ± 0.097	0.935 ± 0.022	0.923 ± 0.017	0.915 ± 0.014
Detritívores	0.187	0.857 ± 0.026	1.044 ± 0.003	0.999 ± 0.007	0.982 ± 0.003	0.961 ± 0.008
Predators	0.329	0.787 ± 0.044	1.116 ± 0.006	0.999 ± 0.006	0.960 ± 0.008	0.918 ± 0.018
Socials	0.455	0.706 ± 0.024	1.161 ± 0.010	0.999 ± 0.005	0.935 ± 0.008	0.867 ± 0.016
	Singularity spectra					
	q_-	q_+	α_0	α_{10}	α_{-10}	
Altitude	10	-10	1.011 ± 0.010	1.125 ± 0.076	0.943 ± 0.042	
Organic carbon	8	-7	1.005 ± 0.000	1.106 ± 0.036	0.868 ± 0.019	
Individuals per trap per day	6	-2	1.043 ± 0.023	1.337 ± 0.116	0.687 ± 0.043	
Richness	7	-3	1.008 ± 0.033	1.216 ± 0.148	0.816 ± 0.048	
Shannon	7	-2	0.959 ± 0.055	1.098 ± 0.209	0.802 ± 0.044	
Pielou	10	-2	0.950 ± 0.057	1.046 ± 0.177	0.875 ± 0.019	
Detritívores	5	-9	1.013 ± 0.004	1.054 ± 0.012	0.843 ± 0.061	
Predators	3	-8	1.033 ± 0.010	1.150 ± 0.031	0.815 ± 0.085	
Socials	6	-10	1.055 ± 0.011	1.206 ± 0.027	0.671 ± 0.050	

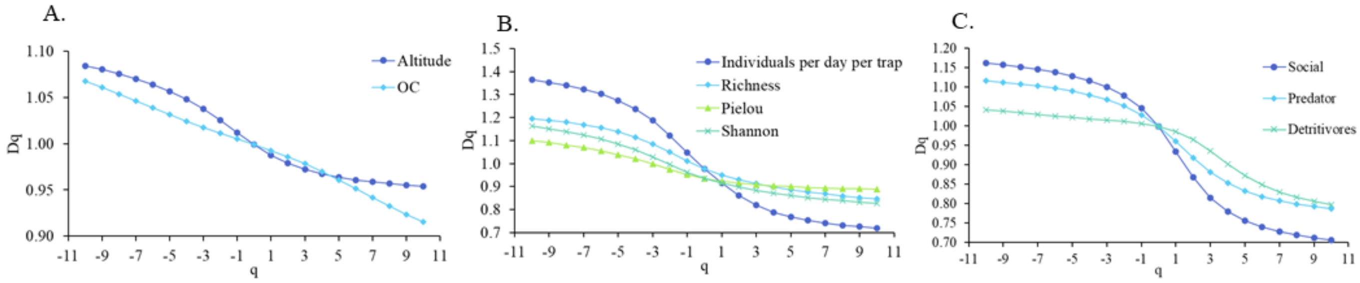


Figure 2. Generalized dimension spectra (Dq versus q) of altitude and organic carbon - OC (A); biological diversity indices (individuals per trap per day, Richness, Shannon and Pielou) (B); and functional groups of edaphic fauna (Detritivores, Predators and Socials) (C)

dimension (Dq). Diversity indices had a well-defined spectrum of generalized dimension with decreasing values and the highest heterogeneity for individuals per trap per day (Figure 2B). For the functional groups, the highest heterogeneity was found for social organisms (Figure 2C).

The curve of the generalized dimension spectrum of OC (Dq - Figure 2) did not show a side with high curvature, suggesting a quasi-monofractal behavior. By assessing the OC multifractality in two transects, Vidal-Vázquez et al. (2013) found a quasi-monofractal tendency of OC for one transect and multifractal scales for the other transect. These differences can be attributed to relief alteration, soil parental material, and landscape vegetal covering. Siqueira et al. (2018) evaluated OC multifractality in a sugarcane area but did not observe differences in the Dq curvature. According to these researchers, the low variability of OC is associated with soil management in sugarcane production.

According to Biswas et al. (2012) and Siqueira et al. (2018), variables with multifractal behavior present curves of singularity spectrums with concave shapes. The asymmetry and amplitude shown in the curves of the singularity spectrum indicate the degree of data heterogeneity (Paz-Ferreiro et al., 2018). The degree of heterogeneity of a measure can be evaluated by considering the difference between D_{-10}-D_{10} (Martínez et al., 2013; Vidal-Vázquez et al., 2013). All variables in this study presented different degrees of heterogeneity; the highest value was found for individuals per trap per day (D_{-10}-D_{10} = 0.645), while the lowest value was observed for OC (D_{-10}-D_{10} = 0.152). Siqueira et al. (2018) and Vidal-Vázquez et al. (2013) found higher multifractality for OC data series. Marinho et al. (2017) reported that the heterogeneity of OC concentration is related to crop history. Further, Liu et al. (2011) revealed that the heterogeneity of OC followed alterations in landscape relief. In the current study, the graph of singularity spectra for altitude had asymmetry to the left, indicating the dominance of high measurement values; however, OC had asymmetry to the right, indicating the dominance of low measurement values in the transect (Figure 3A). The right side of the singularity spectra was wider in the case of altitude, richness, and J' indices, but was less elongated than the left side, except for altitude. A wider right side indicates a greater variety of high singularity exponents that correspond to measures with low concentrations. However, the left side was longer, indicating high concentrations of these variables (Zeleke & Si, 2006; Martínez et al., 2013). For OC, individuals per trap per day, H', and all functional groups, the left side of the curve f(α) - α was asymmetric and longer, which suggests a high frequency of low values for these variables along the transect (Martínez et al., 2013).

The horizontal axis [α(q,t)] in the contour graphs corresponds to altitude (Figure 4) and OC content (Figure 5), which were used as predictor variables for other variables (individuals per trap per day, richness, Shannon, Pielou, Detritivores, Predators, and Socials) presented on the vertical axis [β(q, t)].

The highest linear correlation coefficient, with altitude as a predictor variable, was observed for the pair altitude and individuals per trap per day ($r = 0.342$, $p \leq 0.05$) (Figure 4). In the case of joint correlation, the highest value was described for the pair altitude and social organisms ($r = 0.634$, $p \leq 0.05$) while the lowest value was described for the pair altitude and J' ($r = 0.086$, $p \leq 0.05$). When OC is considered as a predictor

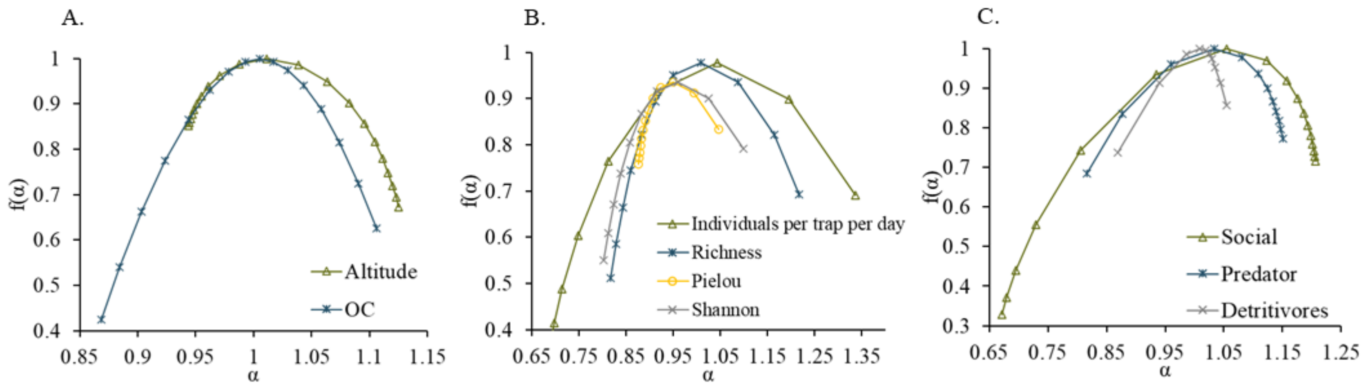
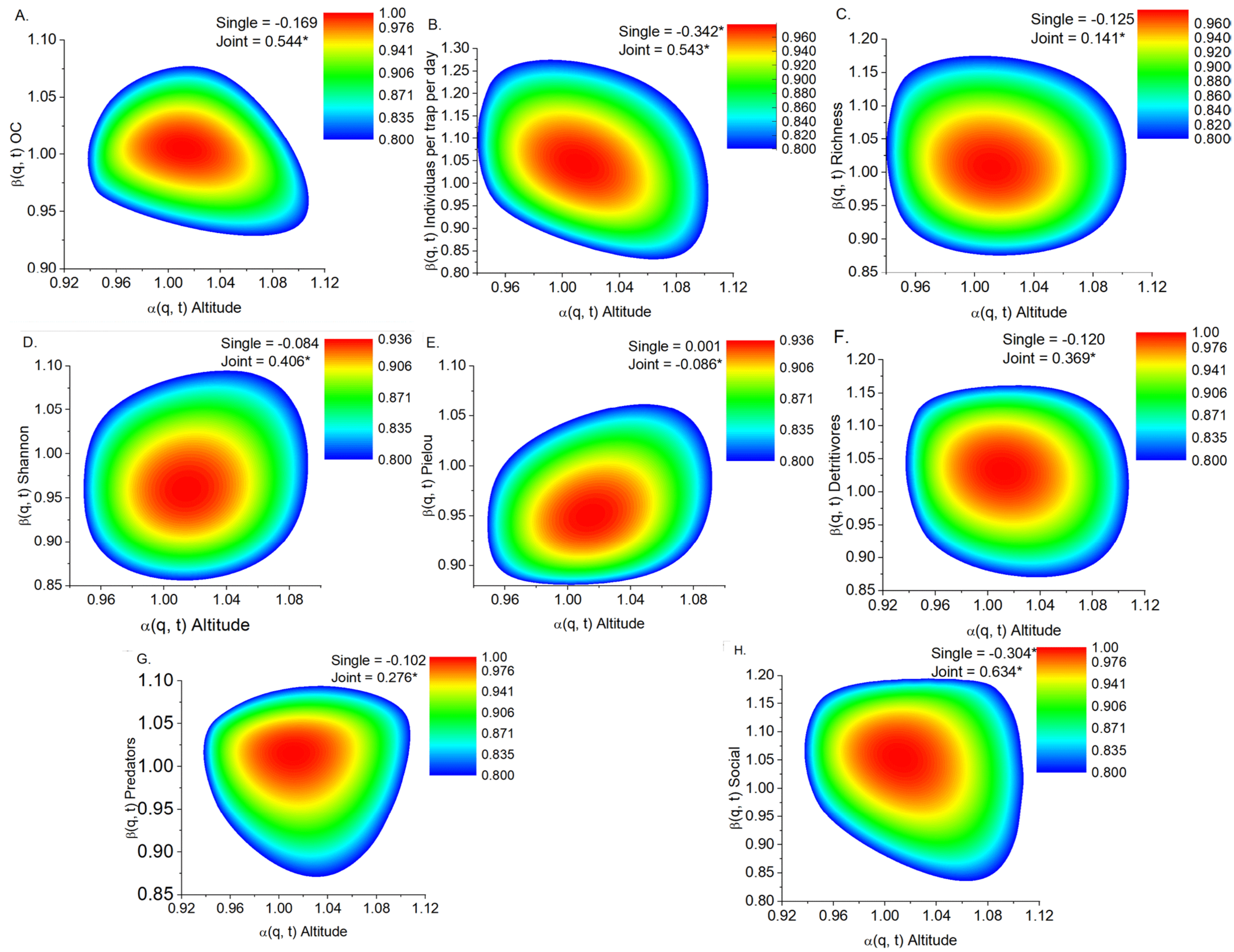


Figure 3. Multifractal spectra calculated for statistical moments of order q (-10 < q < 10) for the following variables: altitude and organic carbon - OC (A); biological diversity indices (individuals per trap per day, Richness, Shannon, and Pielou) (B); and functional groups of edaphic fauna (Detritivores, Predators, and Socials) (C)



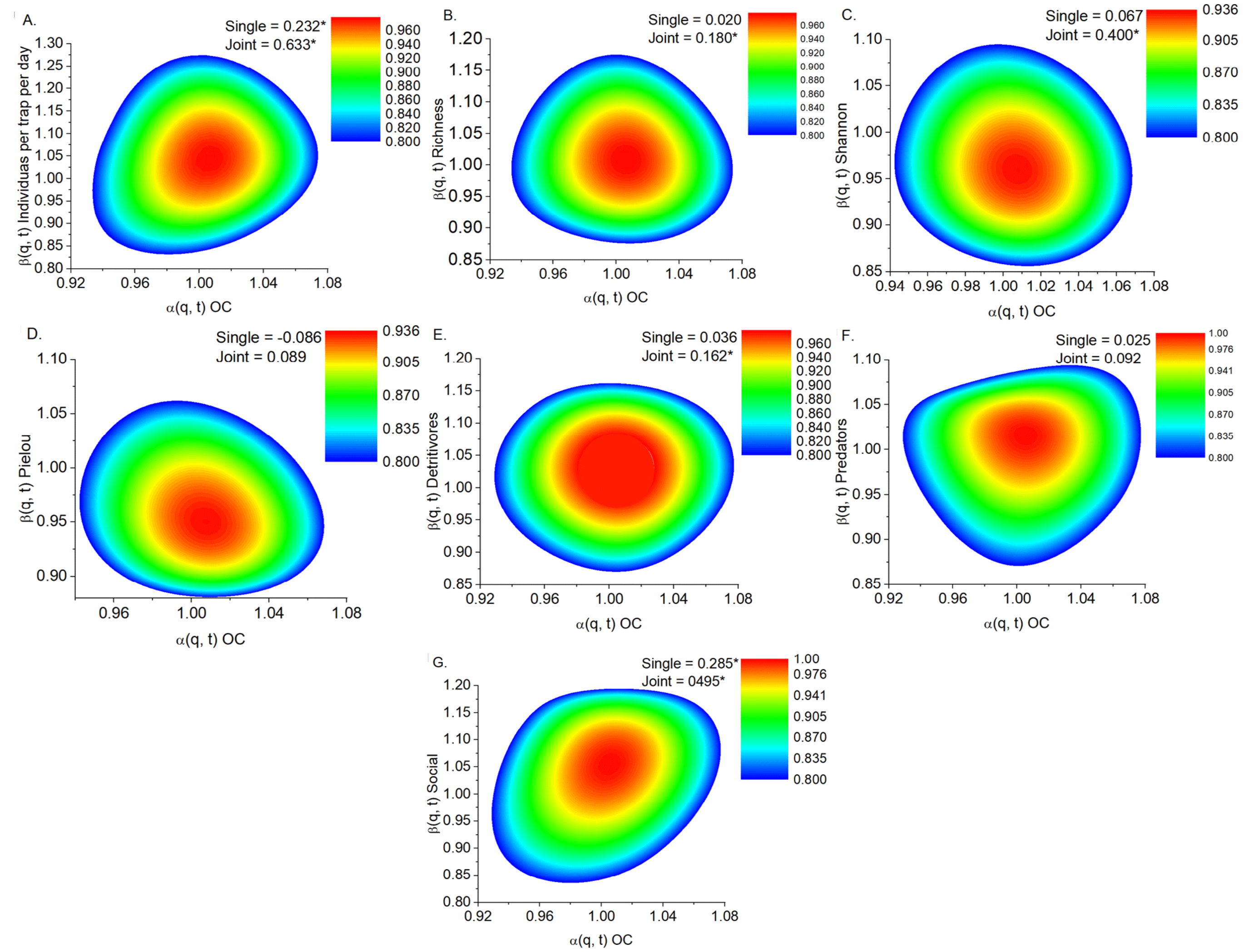
The different colors indicate the joint dimensions of the two scale indices. * p ≤ 0.05

Figure 4. Multifractal spectra of joint distribution of altitude (horizontal axis) and biological attributes (vertical axis). Single - Pearson's correlation in the scale of observation; and Joint - between scales [α(q,t) and β(q,t)] obtained in multiple spatial scales: OC x Altitude (A), individuals per trap per day x Altitude (B), Richness x Altitude (C), Shannon x Altitude (D), Pielou x Altitude (E), Detritivores x Altitude (F), Predators x Altitude (G), and Social x Altitude (H)

variable, the highest correlation coefficient was found for OC and social organisms (r = 0.285, p ≤ 0.05) while the lowest correlation coefficient was found for OC and richness (r = -0.020, not significant) (Figure 5). In the case of joint correlation with OC as a predictor variable, the highest value was observed for OC versus individuals per trap per day (r = 0.633, p ≤ 0.05), while the lowest value was observed for OC versus J' (r = 0.089, not significant).

According to the graphs of the joint multifractal spectrum, the use of altitude (Figure 4) as a predictor variable showed better Pearson's correlation coefficients (single and joint) than the use of OC as a predictor variable (Figure 5). The lower-left side exhibited a joint dimension of high values for two variables, while the upper-right side showed the lowest values of the data series assessed (Zeleke & Si, 2006). Graphs with diagonal orientation, narrow in relation to contour lines and lines in an ellipse, are strongly correlated with the variable pairs (Zeleke & Si, 2006; Banerjee et al., 2011; Biswas et al., 2012).

A greater correlation between scale dimensions [α(q,t) e β(q,t)] was obtained for the pair altitude versus socials (r = 0.634, p ≤ 0.05); however, the graph of contour lines did not present a diagonal orientation with narrow and elliptic lines. This result indicates that even though Pearson's correlation was significant, the association degree of joint multifractals was discrete (Figure 4H). An association was found in most of the spatial scales, and the heterogeneity of the spatial distribution of social arthropods can be described by the relief changes. During their study on invertebrate soil fauna in eucalyptus forests, Catterall et al. (2001) found a positive relationship between social organisms (Formicidae family) and altitude. The correlation of scale dimensions [α(q,t) e β(q,t)] between altitude and individuals per trap per day (r = 0.543, p ≤ 0.05; Figure 4B) was lower than that found for social organisms (Figure 4H). However, there was a higher elongation on the diagonal of the contours of the spectrum (Figure 4B). Low correlations were found between altitude and Shannon (r = 0.406, p ≤ 0.05), altitude versus detritivores (r = 0.369, p ≤ 0.05), altitude versus predators (r = 0.276, p ≤ 0.05), and altitude versus richness (r = 0.141, p ≤ 0.05), with irregular contour lines. However, for altitude versus Pielou, the correlation was negative and weak (r = -0.086, p ≤ 0.05), suggesting that on a minor scale, the community uniformity of soil invertebrates was negatively associated with altitude.



The different colors indicate the joint dimensions of the two scale indices. * p ≤ 0.05

Figure 5. Multifractal spectra of joint distribution of OC (horizontal axis) and biological attributes (vertical axis). Single - Pearson's correlation in the scale of observation; and Joint - between scales [α(q,t) and β(q,t)] obtained in multiple spatial scales: individuals per trap per day x OC (A), Richness x OC (B), Shannon x OC (C), Pielou x OC (D), Detritivores x OC (E), Predators x OC (F), and Social x OC (G)

Scale indices α(q,t) to OC and β(q,t) for all diversity indices and functional groups had joint positive correlations (Figure 5). A higher joint correlation or a stronger correlation with OC as a predictor variable was found versus individuals per trap per day ($r = 0.633$, $p \leq 0.05$), and versus socials ($r = 0.495$, $p \leq 0.05$), thereby presenting distribution contours to joint dimensions narrower and a diagonal tendency relative to other variables (Figure 5). However, these relationships are disclosed when the correlation of scale indices is considered at multiple scales (joint correlation). Therefore, more studies are needed to understand the magnitude of the differences between scales.

Overall, the two coexistent variables can be simultaneously characterized as having a strong joint multifractal [f(α,β)] correlation and a weak Pearson's correlation; this is because traditional statistics can only explain the relations between variables on a fixed scale. Joint multifractal analysis provides a complex analysis of scale indices [α(q,t) and β(q,t)]. Consequently, the joint multifractal and linear correlation focuses on different aspects of the dataset (Banerjee et al., 2011), but are complementary methods for understanding the association between the scales of two variables. Our findings revealed the potential use of altitude and OC as predictor variables for soil invertebrate diversity.

Conclusions

1. Joint multifractal analysis was useful for highlighting the relationships at multiple scales between the studied variables, revealing associations that could not be detected using the traditional statistical methods.

2. The quantification of the spatial variability of edaphic fauna based on multiple scales and association sets in a joint dimension allowed us to describe the impact of agricultural production systems on biological diversity.

3. All of the variables evaluated herein showed a multifractal behavior with greater or lower heterogeneity degree, with altitude and organic carbon concentration being the most homogeneous variables.

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