

The influence of the 2008 financial crisis on the predictiveness of risky asset pricing models in Brazil*

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ABSTRACT

This article examines three models for pricing risky assets, the capital asset pricing model (CAPM) from Sharpe and Lintner, the three factor model from Fama and French, and the four factor model from Carhart, in the Brazilian market for the period from 2002 to 2013. The data is composed of shares traded on the São Paulo Stock, Commodities, and Futures Exchange (BM&FBOVESPA) on a monthly basis, excluding financial sector shares, those with negative net equity, and those without consecutive monthly quotations. The proxy for market return is the Brazil Index (IBrX) and for riskless assets savings accounts are used. The 2008 crisis, an event of immense proportions and market losses, may have caused alterations in the relationship structure of risky assets, causing changes in pricing model results. Division of the total period into pre-crisis and post-crisis sub-periods is the strategy used in order to achieve the main objective: to analyze the effects of the crisis on asset pricing model results and their predictive power. It is verified that the factors considered are relevant in the Brazilian market in both periods, but between the periods, changes occur in the statistical relevance of sensitivities to the market premium and to the value factor. Moreover, the predictive ability of the pricing models is greater in the post-crisis period, especially for the multifactor models, with the four factor model able to improve predictions of portfolio returns in this period by up to 80%, when compared to the CAPM.

Keywords: CAPM, market anomalies, multifactor model, prediction, 2008 financial crisis.

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1 INTRODUCTION

The capital asset pricing model (CAPM), developed by Sharpe (1964) and Lintner (1965) has been the model that is most widely used by the market for calculating expected rates of return on risky assets. Although the CAPM calculates expected excess return on risky assets as the only function of their systematic risk, subsequent empirical studies indicate the existence of other factors that influence achieved historic returns. Banz (1981) found evidence of greater historic records for small stock companies. Stattman (1980) showed that the average return on US shares is greater for value companies, or rather, those with a high book value of net equity to market value ratio (high book-to-market index [BM]). Based on these market anomalies, Fama and French (1993) proposed an empirical model in which they identified three risk factors that would determine expected return on shares: the market factor, the factor related to the size of the firm, and another to the BM index. Subsequently, Carhart (1997) added a fourth factor, related to momentum, based on evidence from Jegadeesh and Titman (1993) regarding significantly positive historic returns from adopting a strategy of buying winning shares funded by the sale of losing shares.

Garcia and Ghysels (1998) documented the importance of studying structural changes in emerging markets, and recently, articles that address estimating systematic risk in a dynamic way have gained more ground in the literature, since the structure of correlation between the factors involved in the models suffers from an alteration over time, especially when structural breaks exist in the time series, arising for example from a crisis period. Silva, Pinto, Melo, and Camargos (2009), Garcia and Bonomo (2001), Machado, Bortoluzzo, Martins, and Sanvicente (2013), and Tambosi Filho, da Costa, and Rossetto (2006) published papers that evaluated the efficiency of the conditional CAPM model (C-CAPM) proposed by Bodurtha and Mark (1991) in the Brazilian market. On the other hand, Lewellen and Nagel (2006) found empirical evidence that the results from the C-CAPM do not differ significantly from the results from the non-conditional CAPM, since the relationship between the betas and the market risk premium varies very smoothly

over time. This leads to the belief that in order for there to be significant changes in the asset pricing model coefficients, an event with greater intensity needs to occur in order to provoke an alteration in the dependency structure of the variables, such as a crisis.

The main aim of this article is to verify whether there was any alteration in the correlation structure of the single and multifactor model variables due to the occurrence of the 2008 crisis, that is, to examine the behavior of risk premiums in the Brazilian market in the periods before and after the 2008 financial crisis. To do so, the period from 2002 to 2013 was divided into three (pre-crisis, crisis, and post-crisis), and for the first and last of these sub-periods, non-conditional single and multifactor models were estimated in order to compare the results in a descriptive way and using the Chow parameter stability test. The results from this test indicate the existence of significant differences before and after the crisis, both in the relationship between the factors and in the values and significance of the coefficients of the CAPM and of the three factor model from Fama and French and the four factor model from Carhart, which justifies the division of the sample into the sub-periods.

The paper also analyzes the predictive ability of each one of the models evaluated for the Brazilian market based on calculating the root mean square error (RMSE) and the mean absolute percentage error (MAPE), which are measures that compare observed return with the return estimated for the portfolios by a particular model. These measures indicate the supremacy of the multifactor models in comparison with the CAPM in the two sub-periods analyzed, with the four factor model presenting slightly better performance than the three factor and all of the models presenting better predictions in the period following the 2008 crisis.

The next section describes the single and multifactor models, their variables, and expected results, and then an empirical literature review of studies of the Brazilian market is carried out. Finally, the results from the models are presented, analyzed, and interpreted.

2 SINGLE AND MULTIFACTOR RISK MODELS

The CAPM, from Sharpe and Lintner, is a theoretical model that, according to Black, Jensen, and Scholes (1972), depends on the following hypotheses: (a) all investors are

wealth maximizers and have a utility function that is adverse to a single risk horizon and choose their portfolios based only on average and variance in returns; (b) no transaction

costs exist; (c) all investors have information symmetry regarding the joint probability density function of returns on assets; (d) all investors can lend and borrow at a risk-

free rate. Under these hypotheses, the main result from the model indicates that:

$$E(r_i) - r_f = \beta_i (E(r_m) - r_f) \quad 1$$

in which $E(r_i)$ represents the expected return on a risky asset i , r_f is the return on a risk-free asset, $E(r_m)$ is the expected return on a portfolio representative of the market, and β_i is the measure of sensitivity of the return from asset i to market return, representing the asset's systematic risk coefficient.

Although Roll (1977) considers it to be impossible to test the CAPM empirically, alleging that no adequate proxy would exist for a market portfolio as defined by the model,

various authors (Banz, 1981; Carhart, 1997; Fama & French, 1993; Jegadeesh & Titman, 1993; Stattman, 1980) have found empirical evidence that the return on a risky asset depends on other risk factors that are not captured by the β of the asset.

Unlike the CAPM, which is a theoretical model, the empirical three factor model from Fama and French (1993) aims to capture the sensitivity of return to other risk factors, and is represented by equation 2:

$$E(r_i) - r_f = b_i(r_m - r_f) + s_i(SMB) + h_i(HML) \quad 2$$

in which $E(r_i) - r_f$ is the expected excess return on portfolio i , $r_m - r_f$ is the excess return on the market portfolio, *SMB*, meaning small minus big, or size factor, is the risk premium for maintaining a portfolio bought in shares in small companies and sold in shares in big companies, *HML*, meaning high minus low, or value factor, is the risk premium for maintaining a portfolio bought in shares in companies with a high BM index (value shares) and sold in shares with a low BM index (growth shares), and b_i , s_i , and h_i are the sensitivities related to the respective factors. Fama and

French (1993) considered the factors as proxies for risk factors. According to them, there is empirical evidence that the two factors are related to return; in other words, large companies or growth ones would have lower returns on assets when compared to small companies or with value companies, respectively. Thus, it is expected that positive risk premiums will be found for each one of the three factors.

Carhart (1997) adds the momentum factor (*WML*, or winners minus losers) to the three factor model, resulting in the four factor model:

$$E(r_i) - r_f = b_i(r_m - r_f) + s_i(SMB) + h_i(HML) + w_i(WML) \quad 3$$

in which *WML* is the risk premium for maintaining a portfolio bought in winning company shares (which obtained above average returns in the previous year) and sold in losing shares (which obtained below average returns in the previous year), and w_i is the sensitivity of the portfolio with relation to the *WML*.

Just like the size and value factors, the addition of *WML* is empirically justified. Vayanos and Woolley (2013) explain *WML* as a result of the gradual removal of funds invested in a discredited company. In a second phase, the capital flow could be large enough for its market value to diverge too much from its fair value, at which point a reversal would occur. Jegadeesh and Titman (2001), however, verified that the factor is neither consistent with the random walk hypotheses, nor with the financial behavior hypotheses, since reversal only occurs in some of the sub-periods analyzed, with some anomaly thus being involved. The anomaly is characterized by a positive above-market return in the year

following the year in which the share was winning (even if there is a reversal, this takes place after this period). Thus, we expect a positive premium for *WML*.

To evaluate the predictive power of models (1), (2), and (3), time series regressions are carried out with the inclusion of the α_i intercept, which must have its significance tested. As set out by Black et al. (1972), the α_i term, also known as the Jensen alpha in its CAPM version, if statistically different from zero, indicates a violation of hypothesis (a) of the CAPM. As the other models also propose to explain the totality of excess return on well diversified portfolios, the α_i coefficient should be statistically equal to zero for the three models analyzed. If the model presents an intercept that is statistically above (below) zero for some portfolio, it means that that portfolio had a positive (negative) excess return after controlling all of the risk factors in the model, and there is therefore evidence that, for that portfolio, some risk factor was not captured by the model.

3 MAIN EMPIRICAL EVIDENCE IN THE BRAZILIAN MARKET

Málaga and Securato (2004) obtained results that confirmed the superiority of the three factor model in relation to the CAPM in explaining returns on Brazilian shares. Mussa, Rogers, and Securato (2009) produced a paper in which they tested the predictive ability of the CAPM and the three factor model, carrying out the two stage procedure in accordance with Fama and MacBeth (1973) and concluded that neither of the two models is efficient in predicting returns on Brazilian shares, given the significance observed in the regression intercepts.

In another paper, Rogers and Securato (2009) carried out a study of the three factor model from Fama and French in the Brazilian market, comparing with the traditional single factor CAPM and the reward beta approach, and concluded that the multiple factor model was superior, however excluding accounting HML over market value, which was not shown to be statistically significant. The period covered was 1995 to 2006, subdivided into two sub-periods: one to estimate risk premiums and the other to test the models.

Mussa, Famá, and Santos (2012) carried out a study of the Brazilian equities market in which they tested the three factor model from Fama and French, as well as the four factor model proposed by Carhart, incorporating WML. The results from the study indicated the validity of including the four factors in the broadened version of the original CAPM, in which despite SMB and WML presenting a negative premium, contradicting the original studies from Fama and French (1993) in the case of SMB, and from Carhart (1997) in the case of WML, other results obtained for the Brazilian context were confirmed, such as that from Málaga and Securato (2004).

Argolo, Leal, and Almeida (2012) carried out a study regarding the three factor model from Fama and French with data from Brazil and covering the period from 1995 to 2007, that is, subsequent to the Brazilian currency stabilization plan and prior to the period that succeeded the subprime financial crisis. The authors concluded that using the three factor model was statistically valid, incorporating a premium for

SMB and another for the so-called value stocks factor. They verified, therefore, that the three factor model has greater explanatory power compared to the single factor model, corresponding to the CAPM from Sharpe and Lintner. However, it was verified that the historical averages of the HML and SMB premiums are very high and they did not present better estimates than the one factor traditional model for estimating the cost of equity capital in the Brazilian context.

Rayes, Araújo, and Barbedo (2012) tested the three factor model from Fama and French, considering in particular the structural break occurring in the Brazilian equities market as a result of the sharp increase in liquidity in mid-2006. They selected 40 shares with higher liquidity in 2004 and traded on the stock exchange in the period from July 2000 to June 2008. Considering the shares both individually and combined in portfolios, they found that the SMB and HML factors do not explain market returns more, due to the structural break mentioned above.

In a more recent paper, Noda, Martelanc, and Kayo (2015) included the risk factor Price-to-Earnings Ratio in the traditional CAPM, verifying the validity of using this factor in explaining returns on Brazilian shares, covering the period from 1995 to 2014. The results from the study indicated that the greater the Price-to-Earnings Ratio, the greater the return on shares tends to be, complementary to the CAPM beta effect. This factor was shown to be significant even after controlling for the other components of the three factor model from Fama and French.

In light of this, we can infer that there is not yet any clear evidence of the superiority of the three or four factor models, as well as that little evidence exists regarding the predictive ability of these models, at least for the context of the Brazilian equities market. The most recent articles do not address the crisis period specifically, which makes it impossible to verify the adjustment of the models in this period, in which a modification occurs in the dependency structure of the data involved in the pricing models.

4 METHODOLOGY

The population analyzed comprises all non-financial company shares listed on the São Paulo Stock, Commodities, and Futures Exchange (BM&FBOVESPA) between January 2002 and December 2013, with all of the secondary data collected from the Economatica information system. Shares in financial sector companies were ignored, due to them presenting high debt ratios that influence the BM multiple. In

composing the database, shares that had not been traded in any session in the analysis period were excluded. Companies that had more than one type of share had the least liquid ones excluded from the base. These exclusions alone restricted the database to 253 shares.

For the portfolio formation, which was carried out in December of each year, the following additional exclusions

were necessary: shares that did not present consecutive monthly quotations for the 12 month period after the portfolio formation so that return on the shares can be calculated, shares that did not present consecutive monthly quotations for the 12 month period before the portfolio formation so that classification between winners and losers is possible, and shares without a market value on December 31 of each year, and companies that do not have positive liquid equity on December 31 of each year.

Thus, for each year, the sample had a different number of shares available for forming part of the portfolios. For example, for 2002, our sample presented 59 shares with available data, while in 2013 there were 207 shares with available data. On average, the sample had 127 shares with data available for composing the portfolios.

It is verified that the use of the methodology proposed by Fama and French (1993) for forming portfolios results in portfolios with only one or two shares in some years, which violates the hypothesis that portfolios are diversified, implying a high specific risk component, which could undermine the statistical tests regarding the coefficients of the models analyzed. In order to eliminate the problem of lack of diversification, common in less developed markets in which the number of companies listed is very small,

such as that of Brazil, this article uses the same alternative methodology for forming portfolios proposed in Rogers and Securato (2009) and described below. First, the shares are ordered by the BM index and divided into three groups with approximately the same number of shares in accordance with the percentages 30% and 70%; second, for each one of the three groups separately, the shares are ordered by their market value and each group is subdivided between small and large companies using the median of each group, resulting in six groups with approximately the same number of shares; in the third and last step, each one of the six groups is ordered separately in accordance with the previous year's returns and subdivided into two more, between winning and losing shares, in accordance with the median of each subgroup. The logic of the method is illustrated in Figure 1. The difference in the method proposed is that the annual divisions of the groups are carried out using the percentages from the subgroups formed in the previous stage, whereas the Fama and French method uses the percentages from the total sample in each stage. Using the method proposed, it is guaranteed that each one of the portfolios has approximately the same number of shares in its composition and that all the portfolios are relatively well diversified.

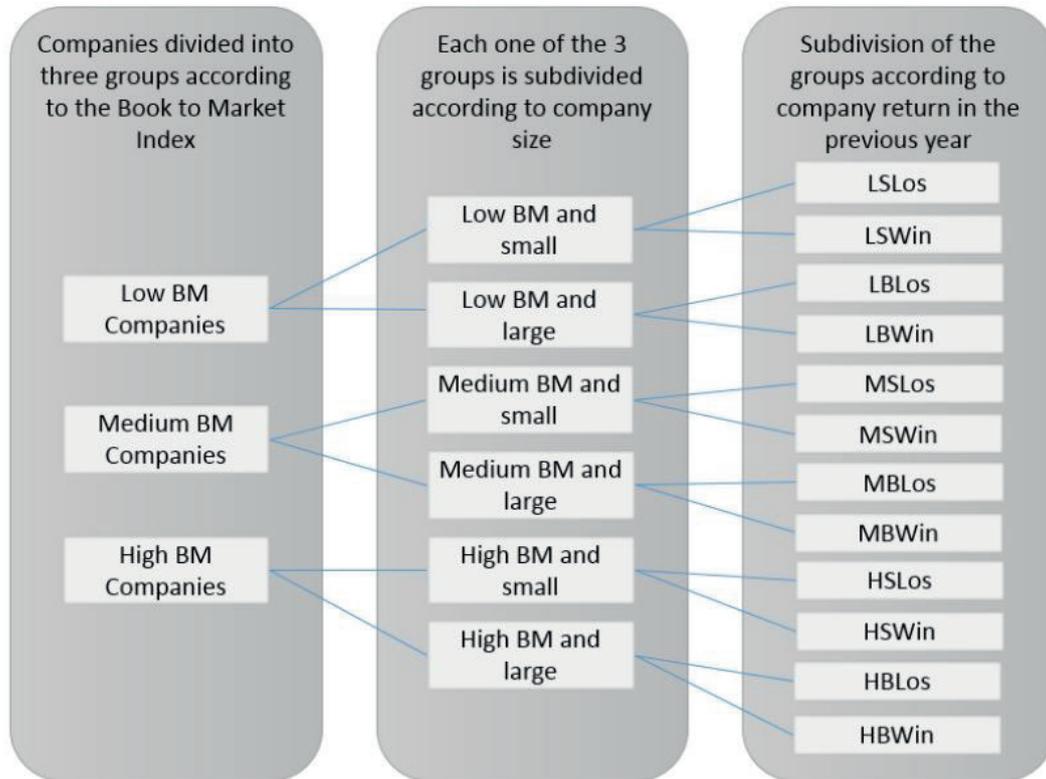


Figure 1 Methodology for separating portfolios for emerging markets

BM: book-to-market index; L = companies with low BM; M = companies with medium BM; H = companies with high BM; B = big companies; S = small companies; Los = losers; Win = winners.

Source: Elaborated by the authors.

Despite the benefits of the methodology proposed, it only makes sense to use this alternative if the original variables BM, size, and return are weakly correlated so that it is guaranteed that, for example, a portfolio with small company has a lower market value than a portfolio with big company, as occurs in the proposal from Fama and French. In the period evaluated, the correlations were very weak in magnitude, lower than 0.09 for BM and return vs. size and lower than 0.3 in the case of return vs. BM, which makes the use of this type of modification possible in compiling portfolios for the Brazilian market without causing bias in the portfolios formed. Moreover, we validated the construction

of portfolios using the modified methodology observing average and standard deviation (SD) values from each portfolio for each one of the years from 2002 to 2013 and verifying the same ordering of portfolios by size and return, as in the Fama and French method. Table 1 shows the results obtained for the portfolios in 2013 in which it is observed, in the second column, that for the method proposed, as well as for the Fama and French method, all of the small company portfolios have a lower market value when compared with big company portfolios. It is also possible to observe that average size and average return have very close values between the two methodologies.

Table 1 Quantity of shares, size, and return of portfolios formed in 2013 using the Fama and French (FF) method and with the modification proposed in this paper (New)

Assets (R\$ thousand)			Size (R\$ thousand)			Return (%)		
Method			Method			Method		
Portfolio	FF	New	Portfolio	FF	New	Portfolio	FF	New
LSLos	8	16	LSLos	500,866	1,817,066	LSLos	-28.43	-0.55
LSWin	9	15	LSWin	1,299,633	2,823,315	LBLos	5.11	17.40
LBLos	12	16	MSLos	710,643	788,818	MSLos	-14.84	-5.12
LBWin	33	15	MSWin	1,048,125	1,217,058	MBLos	-0.35	0.85
MSLos	16	21	HSLos	330,613	97,088	HSLos	-22.47	-43.08
MSWin	23	21	HSWin	921,305	185,708	HBLos	-18.39	-28.21
			Average	801,864	1,154,842	Average	-13.23	-9.79
MBLos	20	21	LBLos	13,473,852	18,873,244	LSWin	89.68	89.50
MBWin	24	20	LBWin	20,453,979	31,931,765	LBWin	60.77	68.96
HSLos	38	16	MBLos	22,117,773	21,190,639	MSWin	50.20	58.72
HSWin	10	15	MBWin	8,285,358	9,477,719	MBWin	63.43	64.46
HBLos	10	16	HBLos	27,060,877	16,772,027	HSWin	36.65	9.70
HBWin	4	15	HBWin	8,422,178	3,558,824	HBWin	32.71	30.30
			Average	16,635,669	16,967,370	Average	55.57	53.61

Note. *HBLos* = high book-to-market (BM) index, big, and loser companies; *HBWin* = high BM index, big, and winner companies; *HSLos* = high BM index, small, and loser companies; *HSWin* = high BM index, small, and winner companies; *LBLos* = low BM index, big, and loser companies; *LBWin* = low BM index, big, and winner companies; *LSLos* = low BM index, small, and loser companies; *LSWin* = low BM index, small, and winner companies; *MBLos* = medium BM index, big, and loser companies; *MBWin* = medium BM index, big, and winner companies; *MSLos* = medium BM index, small, and loser companies; *MSWin* = medium BM index, small, and winner companies.

Source: Elaborated by the authors.

After compiling the portfolios, for each one of the methodologies time series regressions are carried out for models (1), (2), and (3), for the pre-crisis, post-crisis, and total periods. The financial crisis from 2007 to 2009 was subdivided by Phillips and Yu (2011) into three burst bubbles, namely: the subprime crisis, from August to December 2007, the commodities crisis, from March to July 2008, and the bonds crisis, from September 2008 to April 2009. The subprime crisis was triggered in the United States of America and did not significantly affect the Brazilian market. By means of a graphic analysis of the Brazilian stock exchange index

it is verified that it was only affected after the commodities bubble burst so that the period between March 2008 and April 2009 was used to define the crisis period. Thus, the period before the crisis was defined as that between January 2002 and February 2008, and the post-crisis period as that between May 2009 and December 2013.

Despite the study from Araújo, Oliveira, and Silva (2012) indicating the prevalence in Brazilian studies of the Ibovespa Index (Ibovespa) and of the Special System for Settlement and Custody (SELIC) as proxies for the market portfolio and the risk-free asset, respectively, we adopted the Brazil

Index (IBrX) and savings accounts due to them being more consistent with the CAPM theory. The Ibovespa was an equities index weighted only by liquidity and came to be by the market value with a limit on participation based on liquidity, while the IBrX is an index of shares weighted only by the market value of shares and contains the 100 most liquid assets on the Brazilian exchange. It is worth

highlighting that the change in weighting criterion for Ibovespa shares only occurred after the period covered in our sample. Savings accounts present a low DP and are accessible to any investor. Their use is justified by Silva, Pinto, Melo, and Camargos (2009) and they are used in a recent study from Sanvicente (2014), among other papers.

5 RESULTS

Tables 2 presents the risk premiums calculated for the period from 2002 to 2013 (total) and for the pre-crisis, crisis, and post-crisis sub-periods, as well as the correlation between the market factors, SMB, HML, and WML. It is verified that the market risk premium for the whole sample was positive, as expected, and equal to 0.89% a month and statistically significant to 10%. This value is lower than those found by Sanvicente (2014), Machado and Medeiros (2011), Málaga and Securato (2004), and Mussa, Famá, and Santos (2012), which were 1.65%, 3.09%, 1.09%, and 1.56%, respectively, this being expected due to the long history of low market returns for the sample used in this paper, with the crisis and post-crisis periods.

Analyzing the sub-periods, it is important to highlight that the market risk premium is positive and significant only for the pre-crisis period, being negative in the crisis period and close to zero in the post-crisis period, but without statistical relevance in these last two sub-periods. It can also be noted that, in using the total sample in the time regression, the pre-crisis period predominates due to it having more observations, influencing the regression coefficients. However, as suggested in Bortoluzzo, Minardi, and Passos (2014), and Sandoval Jr., Bortoluzzo, and Venezuela (2014), the short timeframe can more efficiently explain the relationship between risk and return. Alterations of this nature can be observed for the other factors, especially for SMB and WML.

SMB was statistically significant in the crisis period and its negative sign indicates that in this period the returns of large companies were greater than those of small companies, probably due to the liquidity problem of low market value companies. Although there is the expectation of higher returns for these companies in the long run, the reason for this premium would precisely be the difficulty in selling these shares at times of crisis or selling them with large negative variations in price, given the pressure to sell. Comparing the values for the pre-crisis and post-crisis periods, it is perceived that before the crisis this premium was positive, and after it became negative, although not statistically significant.

HLM behaves in a more stable way over time; negative and statistically significant for all of the sub-periods, however a little lower in the post-crisis period, compared to the pre-crisis one. It is worth highlighting that the negative sign result

for Brazil is different from the result obtained by Fama and French (1993), indicating that the factor captures a different anomaly for Brazil in comparison to the US market. One possible explanation for this difference is that in the United States of America this factor is based on the existence of growth companies, in which the book value of assets is very small when compared with market value, such as in technology companies, which are intensive in intangible non-accounted assets. In Brazil, these companies would be those that have a consistent historic increase in valuation over the years, so that market value ends up exceeding book value; that is, in Brazil we do not have practically any technology companies listed on the stock exchange and companies classified as growth companies are those that presented great increase in valuation in the study period, which results in high market value compared to net book value of equity. In our study, the companies that fit this description were large consolidated groups. For example, *Companhia de Bebidas das Américas* (AMBEV), a company that in the US market would be considered as value due to a recent history of share appreciation, ended up being classified as growth, and there are various other similar cases.

WML presented the expected sign, however with statistical significance in the pre-crisis sub-period only. In the post-crisis sub-period there was practically no difference between return on the assets of winning and losing companies, with a value close to zero that was not relevant. This fact reveals indications that the crisis may have caused some regime change not absorbed by the traditional pricing models.

Table 2 illustrates the differentiated behavior of risk premiums in the sub-periods evaluated, which could suggest the existence of a structural break in the time series and indicate the need to work with a pricing model only with the most recent past. In order to evaluate the differences in the dependency structure of the factors over time, the Box M test (Silva, 2016) was used, which indicated the existence of an alteration in the behavior of the correlations between the pre-crisis and post-crisis sub-periods (p value < 0.001). Some of the relationships that suffered alterations are: dependency between the market and value factors, the relationship between which was inversely proportional before the crisis and became directly proportional in the crisis and

remained this way after the crisis; the relationship between the market and WML factors, which was negative and weak before the crisis and became moderate during and after the

crisis; and the relationship between WML and SMB, which presented an alteration from a positive sign before the crisis to a negative sign in the post-crisis period.

Table 2 Risk premiums in the period from 2002 to 2013 and in the pre-crisis, crisis, and post-crisis sub-periods

	Risk premium	Monthly average (%)	SD (%)	t	p	$r_m - r_f$	SMB	HML
Total	$r_m - r_f$	0.89	6.44	1.6583	0.0995	1	-	-
	SMB	-0.04	4.23	-0.1244	0.9012	-0.2424	1	-
	HML	-8.99	17.29	-6.2381	0.0000	-0.0369	0.3703	1
	WML	0.49	4.23	1.3959	0.1649	-0.1726	0.1789	0.0483
Pre-crisis	$r_m - r_f$	2.07	6.49	2.6441	0.0100	1	-	-
	SMB	0.70	5.05	1.2824	0.2038	-0.4685	1	-
	HML	-7.80	21.36	-3.1313	0.0025	-0.1506	0.3772	1
	WML	1.17	4.36	2.4408	0.0171	-0.1112	0.3600	0.0848
Crisis	$r_m - r_f$	-2.22	10.43	-0.7950	0.4409	1	-	-
	SMB	-1.84	3.68	-1.8640	0.0851	0.4360	1	-
	HML	-9.31	15.56	-2.2393	0.0433	0.1462	0.5126	1
	WML	-1.37	6.20	-0.8269	0.4232	-0.3440	-0.3019	0.3880
Post-crisis	$r_m - r_f$	0.11	4.65	0.1771	0.8601	1	-	-
	SMB	-0.58	2.80	-1.5617	0.1241	-0.3009	1	-
	HML	-10.48	10.88	-7.2052	0.0000	0.1538	0.2575	1
	WML	0.06	3.21	0.1354	0.8928	-0.3805	-0.2625	-0.3443

Note. On the right the correlation matrix of the explanatory variables of the time series is presented. In bold are the premium estimates that presented statistical significance to 10%. Pre-crisis: January 2002 to February 2008; Post-crisis: May 2009 to December 2013.

SD = Standard Deviation; HML = high minus low, or value factor; $r_m - r_f$ = excess return on market portfolio; SMB = small minus big, or size factor; WML = winners minus losers, or momentum factor.

Source: Elaborated by the authors.

Tables 3 and 4 present the results of the time series regressions for models (1), (2), and (3), in the pre-crisis and post-crisis periods, using the modified method for portfolio formation. All of the analyses were carried out using the SELIC rate as a risk-free asset and the results obtained were similar. The analyses were also made compiling the portfolios using the method proposed by Fama and French, however there was a gain in the predictive power of the models using the method proposed in this article, which varied from 3% for the CAPM to 61% in the case of the four factor model for the post-crisis period. All of the results are available upon requesting them from the authors.

Analyzing the CAPM intercept, it is verified that for three out of 12 portfolios before the crisis, and five out of 12 portfolios after the crisis, this coefficient is different from zero, with 95% confidence, which would contradict the CAPM hypotheses, as previously discussed. This indicates that the market risk factor is not enough to capture all of the risk premiums in the Brazilian market. Thus, the multifactor models would be better, since for only one portfolio the intercept was statistically relevant to 5% significance before

the crisis, and this was the case for none of them after the crisis, which leads to the conclusion that the multifactor models used managed to capture the anomalies existing in the market. Also according to Table 4, in considering only the market risk factor, the four low BM index portfolios present positive excess returns (α), always to 5% significance. This result differs from that indicated by Stattman (1980) and Fama and French (1993), as was mentioned.

In accordance with what was expected, the β coefficient, which measures sensitivity to the market risk factor, presented a positive sign and statistical significance to 5% for all of the portfolios and models, that is, even after considering the other factors. Despite the importance of the market risk factor, it was not enough to capture all of the risk premiums in the Brazilian market, as was mentioned in the previous paragraph.

The four factor model is the one that presents the highest adjusted R^2 for practically all of the portfolios in both the pre-crisis and post-crisis periods. For some portfolios, the quality of the adjustment of the three and four factors models was similar, such as for the portfolios of low BM index, small,

and winning companies, low BM index, big, and winning companies, medium BM index, big, and winning companies, and high BM index, big and winning companies.

Based on the results from the four factor model, it is important to note that 75% of the portfolios (9 out of 12) descriptively presented β coefficients with a smaller magnitude after the crisis, which indicates a decrease in sensitivity to the market factor after the crisis in the Brazilian market. SBM was relevant for around half of the portfolios, both before and after the crisis, with WML presenting statistical relevance for little more than half of the portfolios

considered in each one of the periods. HML presented statistical relevance for most of the portfolios before the crisis (11 out of 12), and after the crisis sensitivity to this factor reduced, with only six out of 12 portfolios having coefficients that were statistically significant to 5%.

Regressions for the crisis period were also carried out and the results are available upon requesting them from the authors. The main difference was in the reduction in the importance of HML in the crisis and post-crisis periods when compared with the pre-crisis period.

Table 3 Results from the time series regressions for the CAPM and 3 and 4 factor models in the pre-crisis period, using the modified method for forming portfolios

Pre-crisis	Portfolio											
	LSLos	LSWin	LBLos	LBWin	MSLos	MSWin	MBLos	MBWin	HSLos	HSWin	HBLos	HBWin
α	0.018	0.033	0.007	0.010	0.011	0.025	-0.005	-0.001	0.007	0.014	-0.016	-0.005
p	0.164	0.000	0.312	0.109	0.072	0.008	0.286	0.925	0.569	0.351	0.040	0.422
β	0.789	0.521	0.850	0.839	0.669	0.509	0.844	1.117	0.614	0.512	0.826	0.553
p	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.022	0.000	0.000
R ² a (%)	18.8	29.0	50.8	52.7	41.0	15.6	66.8	62.6	11.6	5.8	42.5	29.3
α	-0.016	0.019	0.003	0.003	0.001	0.005	0.003	-0.003	0.006	0.007	-0.005	0.000
p	0.179	0.004	0.653	0.685	0.840	0.585	0.604	0.737	0.625	0.567	0.529	0.955
β	1.094	0.668	0.783	0.852	0.812	0.851	0.797	1.089	0.956	1.061	0.788	0.592
p	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SMB	1.169	0.533	-0.089	0.141	0.458	1.037	-0.216	-0.029	0.741	1.261	-0.243	-0.007
p	0.000	0.000	0.541	0.314	0.001	0.000	0.037	0.854	0.005	0.000	0.146	0.964
HML	-0.246	-0.097	-0.070	-0.078	-0.050	-0.075	0.066	-0.035	0.144	0.178	0.103	0.084
p	0.000	0.001	0.027	0.010	0.084	0.042	0.003	0.291	0.010	0.002	0.004	0.009
R ² a (%)	44.4	44.6	54.2	55.7	48.1	43.4	70.4	62.3	32.0	43.9	47.5	34.9
α	-0.011	0.018	0.007	0.001	0.004	0.003	0.005	-0.004	0.016	-0.001	-0.001	0.001
p	0.340	0.007	0.282	0.861	0.509	0.698	0.342	0.592	0.110	0.939	0.899	0.883
β	1.132	0.660	0.811	0.840	0.833	0.842	0.812	1.077	1.027	1.001	0.819	0.597
p	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SMB	1.438	0.479	0.110	0.059	0.606	0.970	-0.113	-0.110	1.250	0.837	-0.027	0.028
p	0.000	0.001	0.436	0.687	0.000	0.000	0.277	0.511	0.000	0.001	0.867	0.860
HML	-0.256	-0.096	-0.077	-0.075	-0.056	-0.072	0.063	-0.033	0.126	0.192	0.095	0.083
p	0.000	0.001	0.008	0.012	0.045	0.049	0.003	0.329	0.004	0.000	0.004	0.010
WML	-0.740	0.147	-0.548	0.224	-0.406	0.183	-0.284	0.222	-1.398	1.168	-0.592	-0.095
p	0.002	0.267	0.000	0.115	0.003	0.294	0.006	0.168	0.000	0.000	0.000	0.532
R ² a (%)	51.0	44.8	62.5	56.7	53.8	43.5	73.2	62.8	58.8	58.6	56.1	34.4

Note. Models (1), (2), and (3) are separated in this order by the lines in the table. In bold are the coefficients that presented statistical significance to 5%.

Source: Elaborated by the authors.

Table 4 Results from the time series regressions for the CAPM and 3 and 4 factor models in the post-crisis period, using the modified method for forming portfolios

Post-crisis	Portfolio											
	LSLos	LSWin	LBLos	LBWin	MSLos	MSWin	MBLos	MBWin	HSLos	HSWin	HBLos	HBWin
α	0.022	0.014	0.019	0.010	0.011	0.000	0.007	0.003	-0.009	-0.006	-0.010	-0.016
p	0.010	0.014	0.000	0.033	0.109	0.964	0.091	0.502	0.175	0.228	0.109	0.001
β	0.625	0.575	0.695	0.448	0.648	0.553	0.954	0.659	0.600	0.358	0.999	0.746
p	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
R ² a (%)	17.3	29.1	51.7	27.9	24.3	32.8	66.1	53.0	21.6	17.4	48.3	51.4
α	-0.002	0.004	0.012	0.001	0.014	-0.001	0.006	0.000	0.007	0.007	0.007	-0.005
p	0.785	0.553	0.039	0.875	0.099	0.884	0.353	0.995	0.377	0.246	0.420	0.422
β	1.029	0.716	0.734	0.441	0.864	0.709	1.024	0.639	0.730	0.411	1.038	0.741
p	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SMB	1.605	0.544	0.087	-0.183	1.120	0.768	0.319	-0.144	0.932	0.476	0.484	0.160
p	0.000	0.010	0.601	0.285	0.000	0.000	0.061	0.354	0.000	0.004	0.039	0.349
HML	-0.314	-0.121	-0.066	-0.073	-0.036	-0.047	-0.034	-0.017	0.107	0.091	0.135	0.093
p	0.000	0.021	0.112	0.089	0.538	0.256	0.414	0.666	0.060	0.026	0.021	0.031
R ² a (%)	62.1	37.9	52.3	32.8	46.2	50.7	67.2	52.5	47.9	38.7	58.3	56.3
α	-0.004	0.005	0.009	0.003	0.008	0.001	0.001	0.002	0.004	0.009	0.003	-0.003
p	0.620	0.536	0.108	0.628	0.290	0.917	0.911	0.666	0.584	0.123	0.723	0.572
β	0.948	0.730	0.582	0.529	0.587	0.775	0.802	0.741	0.609	0.507	0.870	0.804
p	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
SMB	1.502	0.562	-0.108	-0.071	0.765	0.852	0.035	-0.013	0.778	0.599	0.268	0.240
p	0.000	0.014	0.505	0.694	0.001	0.000	0.810	0.933	0.002	0.001	0.256	0.191
HML	-0.328	-0.118	-0.092	-0.058	-0.083	-0.036	-0.072	0.001	0.086	0.108	0.107	0.104
p	0.000	0.028	0.020	0.176	0.104	0.393	0.039	0.984	0.126	0.009	0.059	0.019
WML	-0.254	0.044	-0.481	0.276	-0.877	0.208	-0.701	0.322	-0.381	0.303	-0.533	0.197
p	0.234	0.824	0.002	0.089	0.000	0.194	0.000	0.028	0.076	0.048	0.014	0.230
R ² a (%)	62.5	36.7	60.0	35.3	61.2	51.4	78.9	56.0	50.1	42.1	62.2	56.7

Note. Models (1), (2), and (3) are separated in this order by the lines in the table. In bold are the coefficients that presented statistical significance to 5%.

Source: Elaborated by the authors.

The Chow test (Wooldridge, 2014), presented in Table 5, confirms the existence of differences between the results from all of the models estimated before the crisis (Table 3) and after the crisis (Table 4), indicating the existence of a structural break in the 2008 crisis. Using a 10% level

of significance, the results from the CAPM and the three factor model present differences for 67% of the portfolios evaluated (8 out of 12 portfolios), while for the four factor model the differences appear for 58% of the portfolios (7 out of 12 portfolios).

Table 5 *p* value results from the Chow test for comparing the models estimated in the pre-crisis and post-crisis periods

Portfolio	Model		
	CAPM	3 factors	4 factors
LSLos	0.4623	0.3416	0.5265
LSWin	0.0459	0.0712	0.0431
LBLos	0.4131	0.0325	0.4739
LBWin	0.0477	0.0256	0.0475
MSLos	0.0328	0.0040	0.0251
MSWin	0.0049	0.6921	0.7361
MBLos	0.0016	0.0001	0.0001
MBWin	0.0121	0.0495	0.3953
HSLos	0.3933	0.6599	0.0102
HSWin	0.6341	0.0355	0.0084
HBLos	0.0850	0.0025	0.0426
HBWin	0.0439	0.5128	0.6328

Note. In bold are the coefficients that presented statistical significance to 10%.

Source: Elaborated by the authors.

After the time series regressions, the estimated coefficients were used in a second, cross-sectional regression, in order to obtain the predicted return for each one of the 12 portfolios in accordance with what was expected by each one of the three models analyzed. Table 6 presents two different metrics (RMSE and MAPE) for the prediction error in the cross-sectional regression, with for each statistic used, the lower the value, the better the prediction quality. Analysis of the table indicates that, generally, the four factor model presented the best prediction results, which was expected due to it containing a greater number of variables. However, in some sub-periods this superiority was marginal. If parsimony is a criterion for choosing the model, the three factor one can be taken into consideration. Other information that warrants

attention is the significant improvement in the predictiveness of the three and four factor models for the post-crisis period, which can be credited to the lack of “contamination” in the data caused by the crisis.

The fourth and fifth columns in Table 6 present the gain in prediction quality by using the multifactor models instead of the CAPM, in which we observe a significant improvement in predictability. As is to be expected, the model with the greatest number of factors presents the best prediction results. However, the gain is marginal in the pre-crisis and crisis periods. For the post-crisis period, an improvement of more than 40% is noted by using the four factor model instead of the three factor one.

Table 6 Prediction quality measures of the CAPM (capital asset pricing model) and the three and four factor models using a constant for the pre-crisis, crisis, post-crisis, and total periods, using the modified method for forming portfolios

Period	Model			Prediction quality gain in relation to the CAPM (%)		
	CAPM	Three factors	Four factors	Three factors	Four factors	
RMSE	Pre-crisis	0.0112	0.0063	0.0059	44.19*	47.93*
	Crisis	0.0157	0.0079	0.0078	49.57*	49.64*
	Post-crisis	0.0104	0.0035	0.0021	63.33**	80.03**
	Total	0.0102	0.0047	0.0046	53.84**	54.03**
MAPE	Pre-crisis	169.57	72.17	65.02	57.44	61.66
	Crisis	116.63	55.11	52.66	52.75	54.85
	Post-crisis	91.47	33.43	27.98	63.45**	69.41**
	Total	98.50	48.96	48.32	50.29*	50.94*

Note. In bold are the lowest measures between the models used.

MAPE = mean absolute percentage error; RMSE = root mean square error.

*: prediction superior to the CAPM for 5% significance; **: prediction superior to the CAPM for 1% significance.

Source: Elaborated by the authors.

6 CONCLUSION

This study analyzed the single-factor asset pricing model and the multifactor ones with three and four factors for the Brazilian equities market in the period from 2002 to 2013. Risk premiums, time series regressions, and the predictive power of the models were analyzed in the complete period and dividing the period using the 2008 crisis.

We found a market risk premium consistent with the theory, however with a lower value than that of other studies of the Brazilian market, which we considered normal due to the period evaluated in this paper considering the 2008 crisis. The premium found regarding SMB was positive, in accordance with what was expected, however different from studies for the Brazilian market, such as those from Málaga and Securato (2004) and from Mussa, Famá, and Santos (2012). The premium regarding HML presented a different sign from expected, while WML was statistically insignificant. These premiums are anomalies by nature. In the case of SMB, smaller companies tend to generate an abnormally higher return than larger companies, which was confirmed in our paper. With regards to the HML factor, we also expected a positive sign, as in the findings from Fama and French (1993), but we may not have managed to confirm previous studies due to “errors in the variables” that may be present in less liquid shares and due to not having true growth shares in Brazil, as is the case in the United States of America, primarily characterized by technology companies. In the case of WML, we did not obtain significance, possibly because of the different behaviors from those found by Fama and French (1993) in the case of losing shares, in which the momentum effect appears to have an opposite effect.

The results from the time series regressions reveal that the market risk factor is the most important for explaining portfolio returns, however it is not the only one with

statistical significance. For most of the portfolios, the three and four factor models obtain a significant improvement in the adjusted R^2 , confirming the existence of anomalies in the Brazilian equities market, as was verified in other studies (Argolo et al. 2012; Málaga & Securato, 2004; Mussa, Famá & Santos, 2012), in which the factors that represent statistically relevant anomalies for most of the portfolios were HML and WML.

Future studies are necessary in which portfolio assets are separated using different criteria, such as using the asset beta, as suggested by Lewellen, Nagel, and Shanken (2010). By analyzing the results from the various sub-periods it is observed that the variations in the estimates for the beta of the same portfolio are large, reaching up to 100%, which may indicate that the beta must vary over time, as in the study from Bollerslev, Engle, and Wooldridge (1988) and Bortoluzzo, Toloi, and Morettin (2010), for the autoregressive model of conditional duration. Our analysis indicated greater betas during the crisis, indicating that systematic risk gains importance in the crisis period.

Analysis of the predictive power of the models indicated a significant gain in predictive quality by using the multifactor model instead of the single factor one. For the total sample period, the gain was an approximately 54% reduction in RSME. Moreover, we observed that for the most recent sub-period, the inclusion of WML into the three factor model from Fama and French generated an expressive improvement in prediction quality. Finally, we observed a significant improvement in the predictability of the four factor model for the post-crisis period, contradicting common sense that the use of a longer period in the sample generates better results.

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