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Keyhole: Equal signs as bridges between the phenomenological and theoretical dimensions

Buraco de Fechadura: sinais de igual como pontes entre as dimensões fenomenológica e teórica

Fabiana B Kneubil*[1], Ricardo Karam[2]

[1]Department of Physics, Federal University of Santa Catarina, Florianópolis, SC, Brazil
[2]Department of Science Education, University of Copenhagen, Copenhagen, Denmark

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Physics is a peculiar way to reason about the world that often makes the invisible visible. If one aims at understanding what physics is about, recognise how experimental measurements and mathematical reasoning are intertwined is essential. In this work we exemplify such entanglement by analysing three case studies. In the first one, the explanation of the Hall effect underlines how we indirectly penetrate into the microscopic structure of a wire and manage to evaluate the number of charged particles per unit volume inside it from a set of macroscopic measurements. The second case shows how our reasoning allows us to determine the radius of a hydrogen atom from the experimental measure of the atom's binding energy. The third example comes from kinetic gas theory and illustrates how it is possible to estimate the number of gas particles per unit volume from the experimental values of pressure and temperature. These three case studies show that the equal sign of certain equations can be seen as a bridge (keyhole) connecting the empirical and theoretical dimensions. We argue that epistemological reflections should be an essential part of science education if we aim at delivering an authentic picture of the nature of physics.
Keywords: Epistemology of Physics, Theory-Experiment Relationship, Keyhole, Understanding Physics Equations, Physics Education

A física é uma maneira peculiar de raciocinar sobre o mundo que muitas vezes torna visível o invisível. Se é objetivo compreender o que a física é, reconhecer como as medições experimentais e o raciocínio matemático estão interligados é fundamental. Neste trabalho exemplificamos tal entrelaçamento por meio da análise de três estudos de caso. No primeiro, a explicação do efeito de Hall é enfatizada sobre o como se pode penetrar indiretamente na estrutura microscópica de um fio e avaliar o número de partículas carregadas por unidade de volume no seu interior, a partir de um conjunto de medidas macroscópicas. O segundo caso mostra como o nosso raciocínio nos permite determinar o raio do átomo de hidrogênio, através da medida experimental da energia de ligação do átomo. O terceiro exemplo vem da teoria cinética dos gases e ilustra como é possível calcular o número de partículas por unidade de volume de gás a partir dos valores experimentais de pressão e temperatura. Estes três estudos de caso mostram que o sinal de igual de certas equações pode ser visto como uma ponte (buraco de fechadura), que liga as dimensões empírica e teórica. Argumentamos que estas reflexões epistemológicas devem ser uma parte essencial da ensino da física, se pretendemos entregar uma imagem autêntica da natureza da física.
Palavras-chave: Epistemologia da Física, Relação Teoria-Experimento, Buraco de Fechadura, Equações Físicas, Ensino de Física

*Endereço de correspondência: fkneubil@gmail.com.

1. Introduction

> If matter escapes us, as that of air and light, by its extreme tenuity, if bodies are placed far from us in the immensity of space, [...] if the actions of gravity and of heat are exerted in the interior of the earth at depths which will be always inaccessible, mathematical analysis can yet lay hold of the laws of these phenomena. It makes them present and measurable, and seems to be a faculty of the human mind destined to supplement the shortness of life and the imperfection of the senses ([1], 1822, p.8).

The explanatory power of physics lies in its particular combination of theoretical and empirical investigations. Even though these two dimensions are commonly treated as dichotomies (e.g. empirical-theoretical, physical-mathematical, sensorial-mental) a much more genuine picture of physics is delivered if we think of them as dualities. A good metaphor for this relationship is to consider each dimension as one leg in the act of walking[1]. Both legs are essential and equally important.

Physics has a peculiar way to establish a dialogue between these two worlds (empirical and theoretical), namely via idealised/simplified models of reality (see, for instance, e.g. [2]; [3] and [4]). In this work, we analyse three case studies that exemplify this complex entanglement and show how the equal signs in some physics equations can be seen as bridges between the phenomena - accessible through experiments - and theory - only accessible through our minds.

[1]We thank Prof. Robilotta for this metaphor.

Although modelling has been thoroughly investigated in the science education literature (e.g. [5]; [6] and [7]) few of these studies have taken a careful look on how this process is expressed in physics equations. One possible reason is that many authors conceive a distinction between the physical and mathematical models. As our three examples will show, this distinction is often misleading and many physical magnitudes are essentially theoretical as they cannot be assessed through measurements. Considering the educational value of our approach, we will argue that by interpreting equal signs of equations as bridges between two worlds and explicitly highlighting differences between magnitudes accessible by experiments and by reasoning, the usual view of equations as mere calculation tools can be substituted by a rich epistemological discussion about the nature of physics.

2. Hall effect

The Hall effect was discovered by Edwin Herbert Hall in 1879 and is an important phenomenon related to a microscopic behavior of metals. At a time when the electron was not known, this effect showed itself to be relevant for the understanding of electric conduction. It consists of the appearance of a voltage in the surface of an electrical conductor, when a magnetic field is applied.

Considering a wire connected to a battery, the free electrons inside the metal move anticlockwise, as illustrated in figure 1(a), and an electric current appears in the opposite direction (1-b). Applying a magnetic field orthogonally to the wire, a potential difference ΔV is verified and measured between points A and B, as shown in figure 1(c).

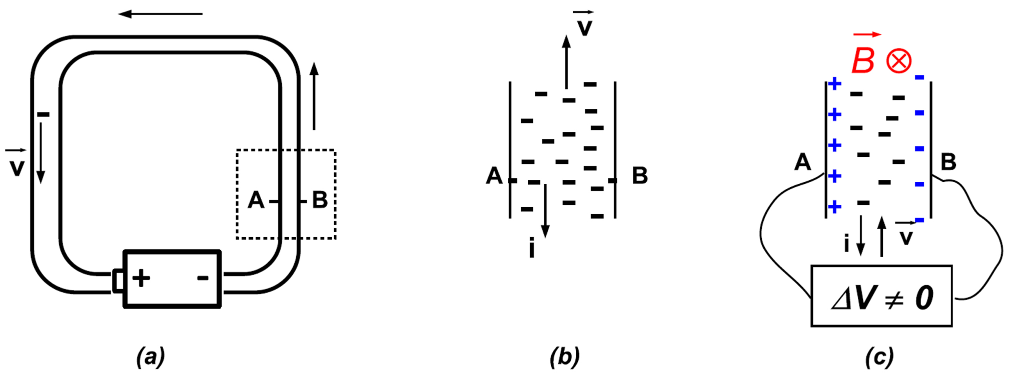

Figure 1: Microscopic view of Hall effect

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The magnetic force $F_M = qvB$ acts on the free electrons and makes them move orthogonally to both velocity $\vec{v}$ and $\vec{B}$. The potential difference ΔV is due to the accumulated charge on the inner surface of wire. According to the Lorentz force, if the moving charges were positive the magnetic force would be directed to the left, whereas if they were negative the force would be pointing to the right (see figure 2). With the voltage measured between A and B, it is possible to know the sign of the charge carriers, which was not known at that time.

These accumulated charges give rise to the electric field $\vec{E}$, which in turn acts over other electrons by means of the electric force $F_E = qE$. As shown in the figure 3, the electric force is opposite to the magnetic and when the equilibrium is established, both forces have the same magnitude and, therefore, $qvB = qE$. Thus, in this situation, the relationship between the electric and magnetic fields is $E = vB$.

The potential difference ΔV is proportional to both electric field and wire's thickness $\Delta V \;=\; E\, d$, which allows one to write

$$\Delta V \;=\; v\, B\, d \quad . \qquad (1)$$

The velocity v of the charge carriers inside a conductor depends on the current I, the area of the wire's cross section S and the number of charge

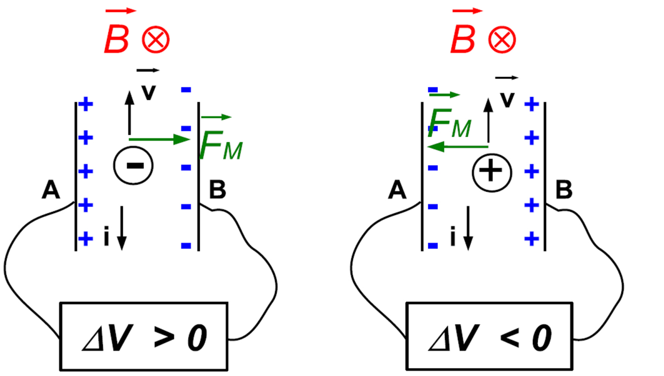

Figure 2: Magnetic force over charge carriers

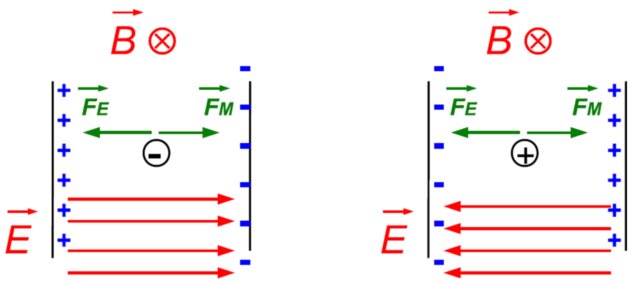

Figure 3: Magnetic and electric forces over charge carriers

carriers per unit volume n. The current is defined by $I = \Delta Q/\Delta t$, where ΔQ is the amount of charge passing through the area S in a time interval Δt. All charges within a distance $l = v\Delta t$ from a given section will cross it after a time interval Δt, i.e. all charges inside the volume $vol = l.\, x.\, d$ (see figure 4). Since the charge is discrete, it depends on the number of moving electrons given by $\Delta Q = Ne$, where e is the elementary charge. The number N is related to n by $n = N/vol$. Thus, the velocity v of charge carriers yields

$$v \;=\; \frac{I}{e\, n\, S} \quad , \qquad (2)$$

and, combining eqs.(1) and (2), we have the expression for the number of charge carriers per unit volume

$$n \;=\; \frac{I\, B}{e\, x\, \Delta V} \quad . \qquad (3)$$

This expression was obtained by several steps involving both theoretical and phenomenological elements. Between the application of the magnetic field and the verification of an electric potential there is a lack of understanding of what happens inside the conductor wire. In order to model the microscopic behaviour of free electrons it is necessary to conceive a sequence of events, which together construct a model to explain that potential difference. These steps are shown in the figure 5.

In eq.(3), all entities on the right-hand side are either measured by instruments or known constants. On the other side, the number n is a microscopic entity and cannot be measured directly, it can only be assessed via reasoning. In this case, the knowledge

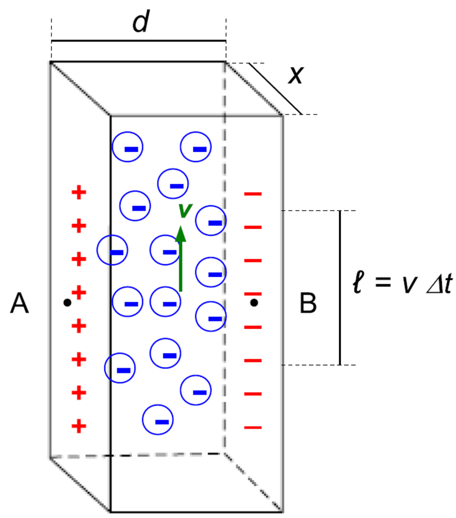

Figure 4: Piece of rectangular wire

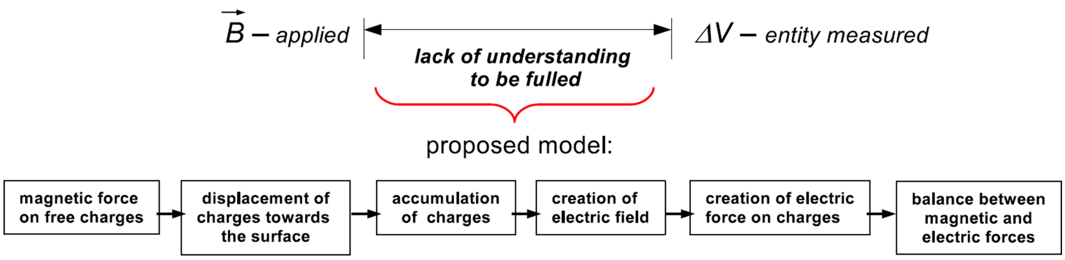

Figure 5: Model to explain the Hall effect

of n is not made by instruments, but rather through a chain of reasonings, supported by theory. The act of knowing n takes place with the interplay between theoretical-mathematical deductions and experimental measurements. In this equation, the equal sign works like an interface between two worlds, a *keyhole* which allows one to 'see' how the microscopic world is. A pictorial image of the keyhole is shown in the figure 6.

It can be interesting to consider a numerical example, for instance, to calculate n for a silver plate. Replacing feasible values for the experimental measurements ([8], 2008, p.906) in the eq.(3), such as $x = 1\ mm$, $I = 2,5\ A$, $B = 1,25\ T$, $e = 1,6 \times 10^{-19}\ C$ and $\Delta V = 0,334\ \mu V$, one obtains

$$\begin{aligned} n &= \frac{2,5\ \times\ 1,25}{1,6 \times 10^{-19}\ \times\ 1 \times 10^{-3}\ \times\ 0,334 \times 10^{-6}} \\ n &= 5,85 \times 10^{28}\ \ \text{electrons/m}^3 \quad , \end{aligned}$$

which is an impressively great number of particles to fit into a cubic meter. With this result, we can also calculate the velocity v of the charge carriers by replacing in eq.(2). Considering d equal to x, we have

$$\begin{aligned} v &= \frac{I}{enS} \\ &= \frac{2,5}{1,6 \times 10^{-19} \times 5,85 \times 10^{28} \times 1 \times 10^{-3} \times 1 \times 10^{-3}} \\ &= 0,2671 \times 10^{-3}\ \ \text{m/s} = 0,2671\text{mm/s} \quad , \end{aligned}$$

showing that the electrons' drift velocity is actually quite small (1/4 of a mm per second!), which probably contradicts our first intuition.

With the development of science, the physical knowledge is no longer merely constructed from phenomenological observations, but rather there is an intentionality which guides the experimental measurements. As we can see from this example, with the help of a voltmeter and imbued with an intentionality, the scientist has access to the inner structure of the wire. This kind of procedure illustrates how physical knowledge is constructed and exemplifies an imbricated interplay between theory, model and phenomenology which is characteristic of physics.

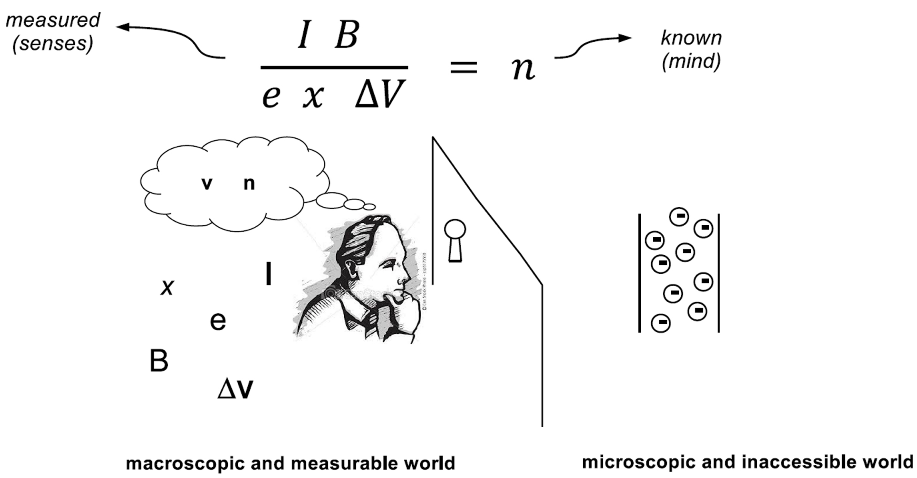

Figure 6: Keyhole for the Hall effect

3. Hydrogen Atom

In the Hydrogen atom both electron and proton are moving and the total energy of the system is

$$E_T = K_e + K_p + U \ , \tag{4}$$

where K_e and K_p are the kinetic energies of the electron and the proton respectively and U the potential energy of the system. Since the proton is about two thousand times heavier than the electron, its kinetic energy is too low. Hence, it is a good approximation to consider the proton at rest in the center of the system and the electron turning around it, as shown in figure 7. Considering this assumption, the total energy of the Hydrogen atom is given by

$$E_T = K_e + U \ . \tag{5}$$

The lowest energy state of the atom is called *ground state* and can be understood using a classical approach, where the kinetic and potential energies are calculated considering that the electron performs a circular orbit of radius R. One calculates the velocity v of the electron by using the Coulomb force and Newton's second law:

$$\begin{aligned} F &= \frac{m\,v^2}{R} = \frac{e^2}{4\,\pi\,\varepsilon_0}\,\frac{1}{R^2} \\ v &= \sqrt{\frac{e^2}{4\,\pi\,\varepsilon_0\,m\,R}} \ . \end{aligned} \tag{6}$$

The kinetic energy of the electron is

$$K = \frac{m\,v^2}{2} = \frac{1}{2}\,\frac{e^2}{4\,\pi\,\varepsilon_0}\,\frac{1}{R} \ . \tag{7}$$

The electrostatic potential energy of the system electron-proton corresponds to the necessary work to construct the system, from the situation in which the charges are infinitely distant. In the case of the Hydrogen atom, this energy depends on the radius R and is negative, given by

$$U = -\frac{e^2}{4\pi\varepsilon_0}\,\frac{1}{R} \ . \tag{8}$$

Replacing the eqs.(7) and (8) in (5), we have the total energy:

$$\begin{aligned} E_T &= K + U \\ &= \frac{1}{2}\,\frac{e^2}{4\pi\varepsilon_0 R} - \frac{e^2}{4\pi\varepsilon_0}\,\frac{1}{R} \\ E_T &= -\frac{1}{2}\,\frac{e^2}{4\,\pi\,\varepsilon_0}\,\frac{1}{R} \ . \end{aligned} \tag{9}$$

This result represents the total energy of Hydrogen and has two important features, namely the energy is proportional to 1/R and is always *negative*, independently of the radius. The orbit radius R can be interpreted as how attached to the proton the electron is, i.e., the smaller the radius the more connected the electron is to the proton. On the other hand, if R is bigger, the electron is far away from the proton and when $R \to \infty$, the electron is no longer attached to the proton and the atom is dismantled and looses its meaning. The dependency between E_T and R can be represented by means of an energy diagram. In figure 8 one shows the potential and the total energies of the Hydrogen atom. The former (U) is the Coulomb well, represented by a blue line, and the latter contains two different examples of the total energy of the atom, E_1 and E_2.

With the help of this diagram the relationship between E_T and R is clearly understood. When the atom is at the state of energy E_1, its orbit radius is R_1. For a state with higher energy, such as E_2, the radius of electron is also higher and corresponds

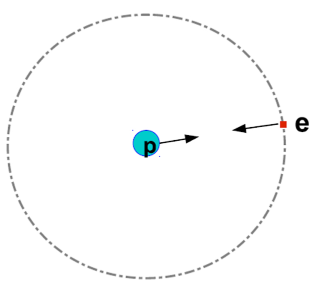

Figure 7: Hydrogen atom

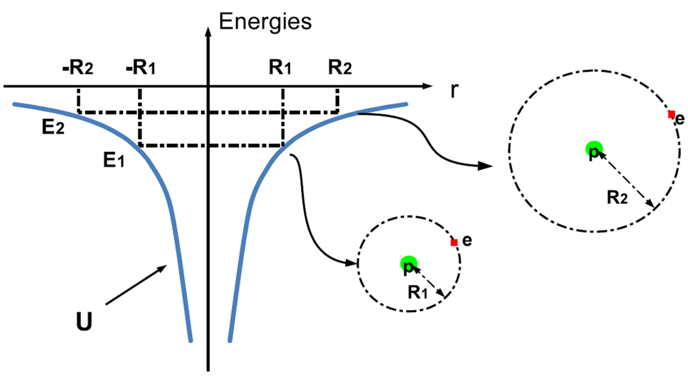

Figure 8: Energy levels of atom

to R_2. When one increases the energy of the atom, its size is also increased. If one provides energy continuously to the system, the electron goes away until it disconnects itself from the proton. In this process, the energy becomes closer and closer to the horizontal axis, until it reaches $E_T = 0$, in which the radius tends to infinity. This can be verified mathematically in eq.(9), where $E \to 0$ when $R \to \infty$.

The lowest energy of the atom is the one which the radius of the electron has its smallest possible value and, therefore, it is more strongly connected to the proton. This energy is the necessary amount given to the atom in order to break it and is called *binding energy*, represented by B. The understanding of the experimental process for breaking the Hydrogen atom, by providing energy, is represented in the following, where in figure 9(a) one shows an excited state and in 9(b), the breaking of the atom.

Figure 9(a) represents the excitation of the atom by providing an external energy (W), which takes it to a higher excited state, shown in 9a(iii). In the transition, the input energy W provokes the increase of the radius and the conservation of energy is written as

$$E_{exc} = \; W \; + \; E_1 \quad . \tag{10}$$

Therefore, W is a positive number given by

$$W \;\; = \;\; - \; \frac{1}{2} \, \frac{e^2}{4\,\pi\,\varepsilon_0} \, \frac{1}{R_{exc}} - (-\frac{1}{2} \, \frac{e^2}{4\,\pi\,\varepsilon_0} \, \frac{1}{R_1})$$

$$W \;\; = \;\; - \; \frac{1}{2} \, \frac{e^2}{4\,\pi\,\varepsilon_0} \; (\frac{1}{R_{exc}} - \frac{1}{R_1}).$$

This situation refers to an atomic excitation, whereas in the figure 9(b), the input energy W makes the electron go away from the nucleus, which corresponds to $R \to \infty$. The total energy of the system becomes zero and the provided energy W is *numerically equal* to the binding energy ($W = B$). Using the eq.(10) for the energy conservation, one has

$$\begin{aligned} E_{exc} &= B + E_1 \\ 0 &= B + E_1 \\ B &= -E_1 \end{aligned} \tag{11}$$

This result, although apparently simple, is full of subtleties. The energy E_1 is the total energy of Hydrogen in the ground state, shown previously in eq.(9), which means that one can combine it with eq.(11) and obtain the expression which relates the binding energy B to radius R:

$$B \;\; = \;\; \frac{1}{2} \, \frac{e^2}{4\,\pi\,\varepsilon_0} \, \frac{1}{R_1} \quad . \tag{12}$$

This expression came from several reasoning steps that are schematized in figure 10. We departed from the hypothesis that the Hydrogen atom at the ground state could be considered as classical and, by using theoretical tools, such as Newton's

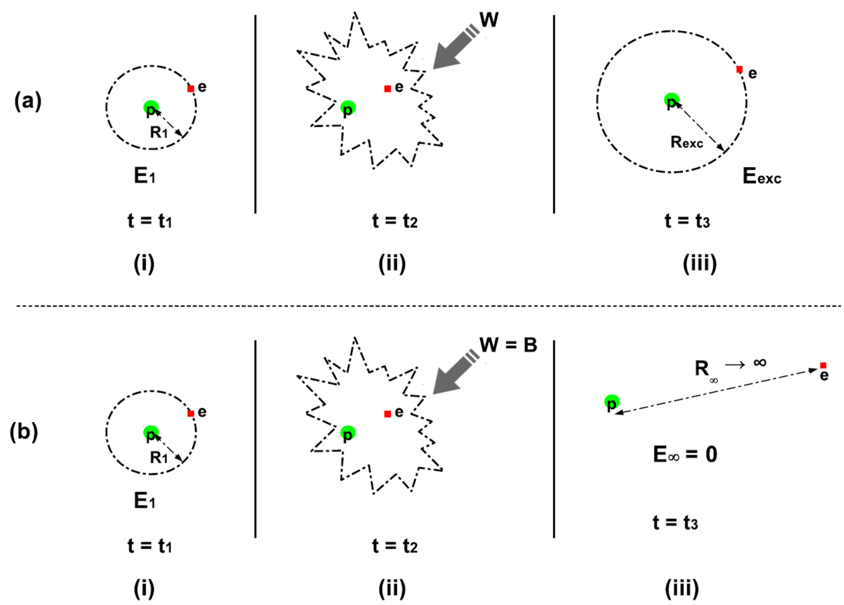

Figure 9: Breaking the atom

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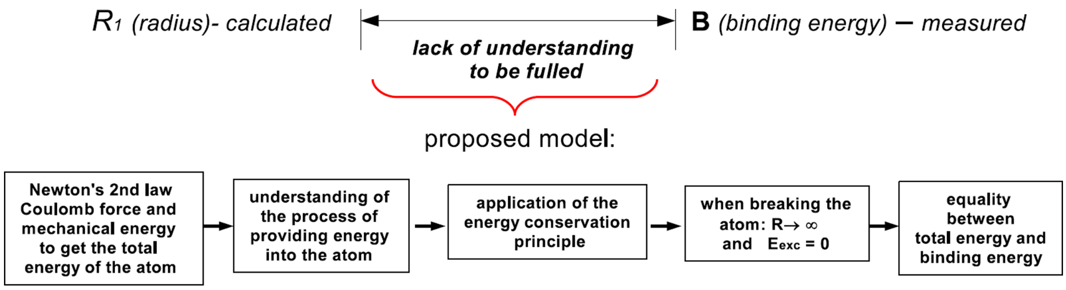

Figure 10: Model to explain the equality $B = -E_1$

second law and equation for the Coulomb potential, we obtained the expression for the total energy of the system, represented by eq.(9). The energy E_1 depends on the radius R_1 and both cannot be measured directly. Hence, eq.(9) can be considered as belonging to a theoretical dimension of knowledge, since it was derived from theoretical assumptions. Afterwards, one uses the images of excitation and breaking of the atom (figure 9) to interpret the meaning of the binding energy in terms of the ground state energy ($B = -E_1$).

Thus, one could write B = - eq.(9), which is a conclusion absolutely nontrivial, since it carries a series of constructions and a chain of reasonings to be obtained. On the other hand, the binding energy of Hydrogen at the ground state B is measured experimentally by means of spectral lines and is $13,6\ eV$. This allows one to write $E_1 = -13,6\ eV$, which corresponds to the height of the Coulomb well when the Hydrogen is at the ground state. This understanding of the ground state energy is added in the diagram of the Coulomb well in the figure 11.

The result (12) mixes together two different worlds, which involve physical entities that can be measured and others that cannot. On the left side there is an experimental measure and on the right, a composition of constants and the radius R, which is *inaccessible* by instruments. With a single measurement - the binding energy - it is possible to calculate the radius of atom R and, consequently, the velocity of electron v. By using eq.(12) and the Electron-Volt to Joule conversion $1\ eV = 1,6 \times 10^{-19}\ J$, one has $E_1 = -13,6\ eV = 21,76 \times 10^{-19}\ J$ and

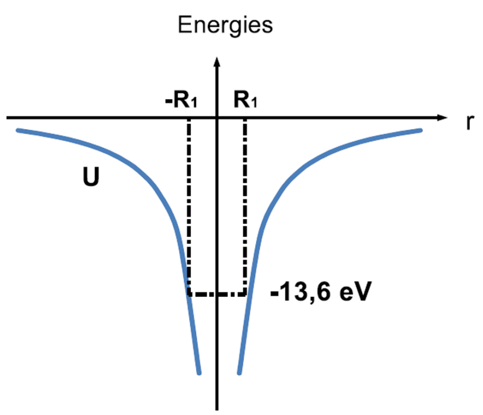

Figure 11: Ground state of Hydrogen

$$
\begin{aligned}
R_1 &= -\frac{1}{2}\,\frac{e^2}{4\,\pi\,\varepsilon_0}\,\frac{1}{E_1} \\
&= -\frac{1}{2}\,\frac{(1,6\times 10^{-19})^2 \;\;\times\;\; 9\times 10^9}{21,76\times 10^{-19}} \\
R_1 &= 0,5292\times 10^{-10} m \\
v &= \sqrt{\frac{e^2}{4\,\pi\,\varepsilon_0\,m\,R_1}} \\
&= \sqrt{\frac{(1,6\times 10^{-19})^2\times 9\times 10^9}{9,11\times 10^{-31}\;\;\times\;\;0,5292\times 10^{-10}}} \\
v &= 2,186\times 10^6\ m/s \quad .
\end{aligned}
$$

A rather fast electron moving around the proton in quite a small circle.

Similarly to the previous case, the equal sign in eq.(12) works as an interface between the microscopic and macroscopic worlds, as a *keyhole*, represented in figure 12, whereby the scientist has access to a physical entity. The physical quantities R, v, K and U, which cannot be measured directly, are known from a theoretical structure and the use of the atom model. In this sense, it is possible to *know* and understand the microscopic world, no longer through measuring instruments, but rather by means of theoretical reasoning. The theories and models allow one to reach an intangible domain, where no instrument could possibly reach, regardless of how accurate it is.

measured (senses)

$$\frac{1}{B}\frac{e^2}{4\,\pi\,\varepsilon_0}\frac{1}{2} = R$$

known (mind)

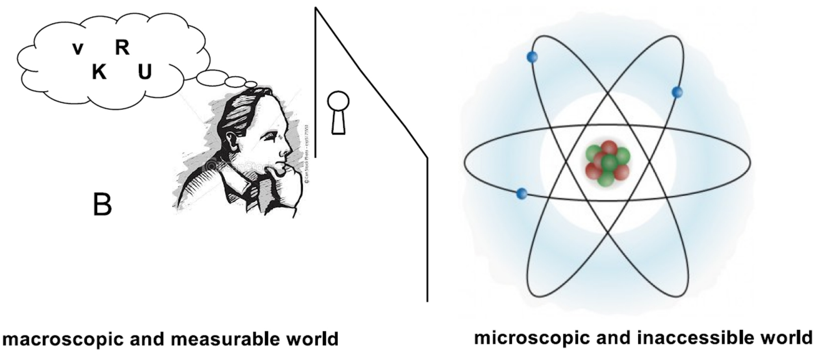

Figure 12: Keyhole of the Hydrogen atom

The act of knowing in physics encompasses nuances and intentionality degrees, where the scientist's conscience is a fundamental factor. This knowing is made with the mind more than with the senses. In this example, to *know* the orbit radius, to *know* the velocity of electron, to *know* its kinetic and potential energy, means having access to quantities through a complex network of information, which is a blend of theoretical and experimental elements.

4. Kinetic gas theory

Our last case study comes from the kinetic gas theory, whose basic idea is to consider a gas as constituting of a very large number of particles (atoms and molecules) in random motion. It involves an elaborate combination of mechanics - laws governing the motion of particles - with statistics - due to the impossibility of describing the states of single particles. Thus, this theory is a great source of examples of the imbricate relationship between experiment and theory that we are trying to put forward with the *keyhole* metaphor. In this section, we illustrate how the equal sign can be interpreted as a bridge between the micro (theoretical) and macro (experimental) in one important equation of this theory, namely the pressure of a monoatomic gas.

Kinetic gas theory assumes that pressure is due to collisions of particles with the walls of a container. The following derivation is inspired on Krönig [9], a work that influenced Clausius' 1857 seminal paper *The Nature of the Motion which we Call Heat* [10], which marks the establishment of kinetic gas theory as a paradigm, of course not uncontroversial, for scientific investigation ([11]).

Consider a monoatomic gas being made of N particles moving inside a cubic recipient of side L. For simplicity reasons and without loss of generality, since there is no direction preferred, we assume that the particles' velocities are equally distributed in six directions $(+x, -x, +y, -y, +z, -z)$. Thus, we can say that 1/6 of the particles are moving in one direction $(+x)$ towards one of the faces (see figure 13).

The particles are moving with a (mean) speed $\overline{v}$ and the ones located within a distance $s = \overline{v}\,\Delta t$ from that face will collide against it during a time interval Δt $(s = L)$. Thus, the number of particles

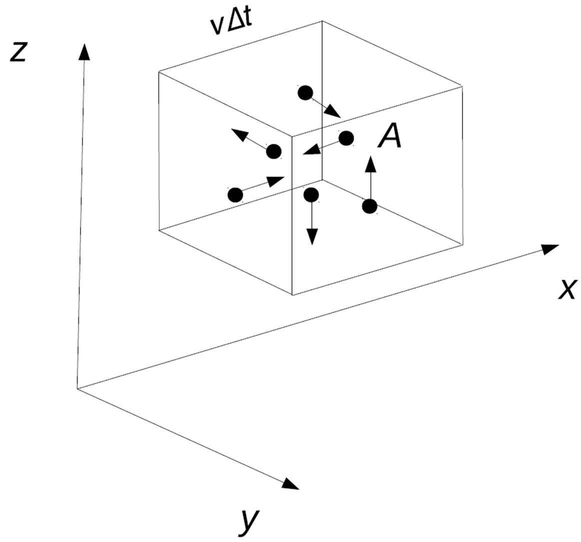

Figure 13: Particle motion inside a gas

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inside the cube moving in the $+x$ direction is given by

$$\frac{1}{6}\,\frac{N}{V}\,A\,\overline{v}\,\Delta t \quad . \tag{13}$$

Moreover, we postulate that the collisions are elastic, which means that each particle colliding with a velocity $\overline{v}$, perpendicular to the wall, will return with the same velocity $\overline{v}$ in the opposite direction. The momentum transferred to the wall after each collision is, therefore, $2\,m\,\overline{v}$. Multiplying the momentum transferred by one collision by the number of particles that will hit this wall in a time interval Δt (eq.13) we obtain $\frac{1}{6}\,\frac{N}{V}\,A\,\overline{v}\,\Delta t\,2\,m\,\overline{v}$, which is the total momentum transferred to the wall.

To obtain the force, we divide the last expression by Δt using $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$. Since pressure is force divided by the area, we arrive at an equation for the pressure (P) of the gas which can be rearranged as

$$PV = \frac{1}{3}Nm\overline{v}^2 \quad . \tag{14}$$

Similarly to the examples from the previous sections, eq.(14) is put in the *keyhole* form, in which quantities experimentally accessible (left) are separated from the theoretical ones (right). However, now the situation is more complicated because the right-hand side contains not only one, but three quantities inaccessible by experiment, namely the number of particles, their individual masses and mean velocities. By measuring the pressure and volume of a gas, we cannot obtain either of those quantities separately with this equation.

A plausible attempt to reduce the number of inaccessible quantities in the right-hand side of eq.(14) is to substitute $m\,\overline{v}^2$ by twice the mean kinetic energy of the individual particles ($\overline{E}_{kin}$). Then we can write $PV = \frac{2}{3}N\overline{E}_{kin}$, even though the problem is not solved yet, since we are still left with two quantities (number of particles and average kinetic energy) that we do not have empirical access to.

This is where the rather 'mysterious' concept of temperature[2] comes into play. In fact, before the kinetic theory was proposed, a number of empirical laws relating pressure, volume and temperature were known[3]. If considered all together, these empirical laws state that the product of pressure by volume, divided by temperature, is constant for a specific amount of rarefied and stable (ideal) gases ($\frac{PV}{T} = a$). This "constant" a would, however, change for samples with different amounts (masses) of the same gas ($\frac{PV}{T} \propto m$) and for different gases, i.e., with different molecular weights M ($\frac{PV}{T} \propto \frac{1}{M}$). By bringing both the mass of the gas and its molecular weight into consideration it is possible to obtain a *universal* gas constant R and write the ideal gas law in its well known form

$$PV = nRT \quad , \tag{15}$$

where n is the ratio between the mass of a sample and its molecular weight, which is the amount of substance (in moles). Now let us get back to equation $PV = \frac{2}{3}\,N\overline{E}_{kin}$ and compare it with $PV = nRT$. There is no unique way to connect the quantities in the right-hand side, but kinetic gas theory chose to assume that (absolute) temperature is proportional to the mean kinetic energy of the individual particles by the following relation [4]

$$\overline{E}_{kin} = \frac{3}{2}\,kT \quad , \tag{16}$$

where k is called Boltzmann constant ($k = 1,381 \times 10^{-23}JK^{-1}$). Now one can write the ideal gas law in another form

$$PV = kNT \quad . \tag{17}$$

By comparing eq.(15) with eq.(17), one can say that the first is more 'chemical', since it refers to chemical properties of the gas (e.g. molecular weight), whereas the second is more 'physical', since it relates directly to the number of particles. After isolating N in eq.(17) we finally arrive at an equation that has the 'pure' keyhole structure we were looking for

$$N = \frac{PV}{kT} \quad . \tag{18}$$

Once again, we do not have empirical access to the number of particles in a gas[5]. In fact, if we

[2] The concept of temperature has a fascinating history that is brilliantly told in [12].

[3] Among them are Boyle-Mariotte's, Gay-Lussac's, Charles' and Amonton's laws.

[4] This is valid for monoatomic gases, where the particles can move freely in three directions (3 degrees of freedom). For the general case, each degree of freedom is assigned to $\overline{E}_{kin} = \frac{1}{2}kT$ and the total kinetic energy will be a sum of the energies of each degree of freedom, which can involve both translational and rotational motion.

[5] But we can *calculate* it using this eq.(18). Considering a monoatomic gas (e.g. helium) and substituting standard values, such as $P = 1\ atm = 101.325\ N/m^2$, $V = 1\ l = 10^{-3}\ m^3$

think deeply about it, the existence (or not) of these particles is actually a matter of metaphysical belief. However, we have developed a clever way to look (with our minds) through the keyhole and actually determine how many particles are inside a gas (figure 14).

By exploring this mechanistic model quantitatively, we managed to explain some regularities observed in nature, for instance, why pressure, volume and temperature relate to each other the way they do (ideal gas law). A reasoning chain connecting the empirical laws relating pressure, volume and temperature with the assumption that gases are made of a large number of particles in random motion is sketched in figure 15.

Kinetic gas theory goes much deeper into the particles framework than this example may suggest. It analyses gases with different structures (monoatomic, diatomic, etc.) and supposes not only translational, but also rotational motion of the molecules. It goes even further to estimate the molecules' diameters and a rather curious quantity called *mean free path*, which is the mean distance travelled by a particle between two collisions. After working for a while with this theory and seeing its explanatory power, it is very hard not to believe in the (ontological) existence of these cute little particles. However, most scientists take a pragmatic position and leave such ontological questions out of the scientific discussions.

5. Educational implications

We do not have direct access to the inner structures of wires, atoms or gases. Physics makes these invisible phenomena visible [13] through a peculiar combination of theoretical models and experimental data. In sum, this science is neither a quantitative description of empirical regularities nor an abstraction game totally disconnected from the material world - both views often depicted in educational settings. Instead, as shown by the analysis of these case studies, physics considers theory and experiment as equally important dimensions and combines them in a unique way. Understanding this relationship

and $T = 0°C = 273K$, one gets $N = 2,7 \times 10^{22}$, which is an incredibly large amount of particles to fit in one liter.

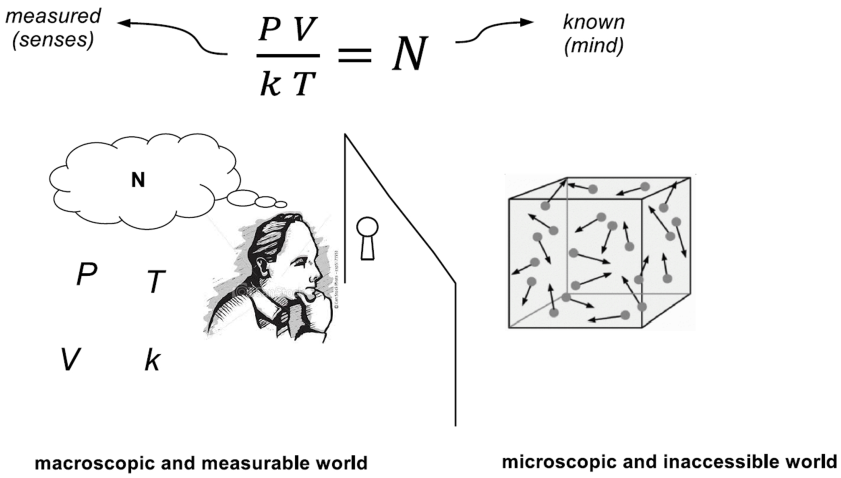

Figure 14: Keyhole for the kinetic gas theory

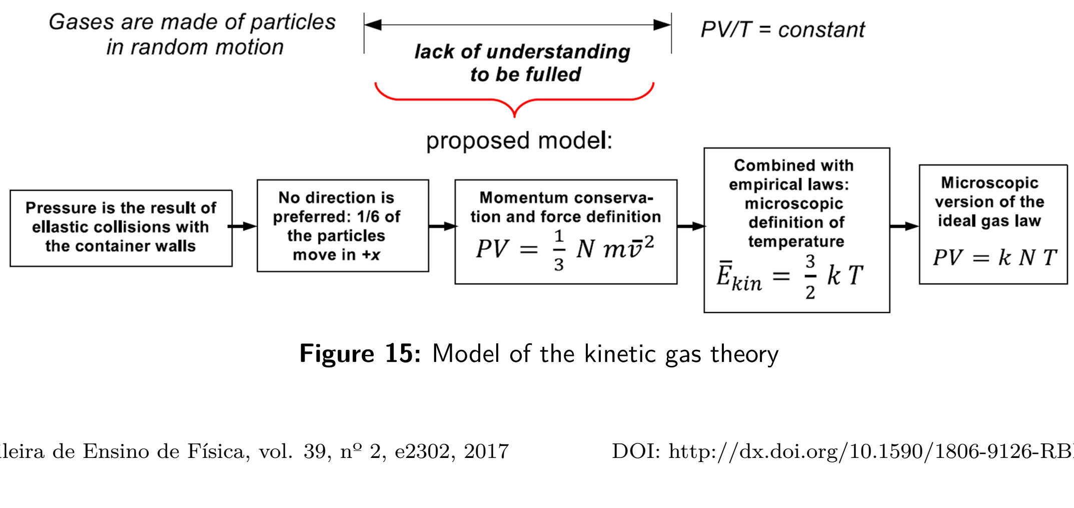

Figure 15: Model of the kinetic gas theory

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more deeply is crucial for understanding the *nature of physics*, which is a major educational goal.

Furthermore, our analysis invites one to think critically about the meaning of physics equations. The discussion aimed at highlighting a different meaning of the equal sign of certain[6] physics equations when compared with the usual mathematical representation of equality. Differently from the mathematical identity expressed in $5 = 2 + 3$, where the three elements represent numbers, the components of physics equations often have different natures. In the equations presented, the equal sign appears to function as a *keyhole* that allows the connection between macroscopic/experimental entities on one side with microscopic/theoretical ones (only assessed through reasoning) on the other. Therefore, it has a completely different meaning from a traditional (tautological) equality.

This has important educational consequences. As educational research has shown, students often treat equations as mere calculation tools to solve problems. This is due, in part, both to a lack of conceptual discussions about equations and a specific focus on their formal aspects - arguably because the latter are more easily assessable through exams. When physical quantities are treated irrespectively as mere variables, physics becomes a meaningless game of using equations to find the unknown quantity from a set of given ones. The alternative approach presented here illustrates how epistemological discourses about equations can be conducted in physics lessons.

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[6]Obviously, we by no means imply that the *keyhole* metaphor is valid for *all* physics equations. $\Delta s = v\ \Delta t$, $\vec{F} = m\ \vec{a}$ and $v = \lambda\ f$ are simple examples where it does not hold.