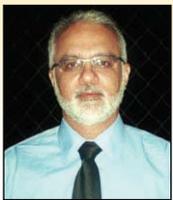


## Bending moments in beams of reinforced concrete buildings

## *Momentos fletores em vigas de edifícios de concreto armado*



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### Abstract

Residential buildings with slab systems supported by reinforced concrete beams are widely used in building industry. For design purposes, the computation of the loads on supporting beams is performed using simplified procedures, in which the slab is analyzed as an isolated element. New Brazilian code keeps the same recommendation presented in the old code allowing that, in the case of rectangular slabs with uniform load, the reactions on supporting beams may be computed assuming that those reactions correspond to the loads acting on triangles or trapezoids determined from the yield lines of the slab. In a simplified way, it is still allowed that these reactions may be considered as uniformly distributed on the slabs supports. The work shows through illustrative examples that these recommendations can some times lead to unsafe results and proposes a correction to overcome the problems.

**Keywords:** reinforced concrete beams, bending moments, reinforced concrete slabs.

### Resumo

Edifícios residenciais compostos de sistema de lajes maciças apoiadas em vigas de concreto armado são largamente utilizados na indústria da construção civil. Na prática, o cálculo das reações destas lajes sobre as vigas é feito através de processos simplificados, nos quais os painéis de laje são analisados de forma isolada. A NBR 6118 [1] mantém em seu texto o mesmo teor da antiga NB-1 [2] permitindo que, no caso de lajes maciças retangulares com carga uniforme, as reações possam ser calculadas admitindo-se que elas correspondem às cargas atuantes nos triângulos ou trapézios determinados através do método das charneiras plásticas. De maneira aproximada, a norma brasileira permite ainda que estas reações possam ser consideradas como uniformemente distribuídas sobre as vigas de apoio. O trabalho demonstra através de exemplos práticos que tal recomendação pode conduzir a resultados contrários à segurança e propõe uma correção para o cálculo das reações de apoio em lajes maciças sobre as vigas de concreto armado.

**Palavras-chave:** vigas de concreto armado, momentos fletores, lajes maciças de concreto armado.

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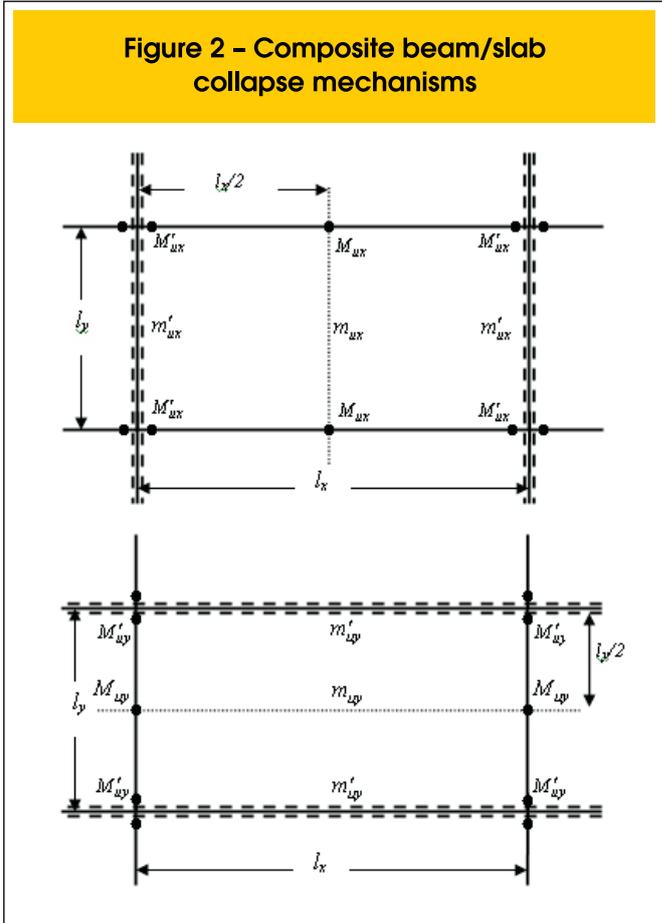
### 1. Introduction

Reinforced concrete slabs supported on beams on all sides of each panel, generally termed two-way slabs, have been widely used as structural system in reinforced concrete building in Brazil in last decades. For design purposes, it is usual to take into account approaches from new Brazilian code NBR 6118:2003 that allow considering the loads on supporting beams as uniformly distributed over its length, as it was permitted by the old one (NBR 6118, section 14.7.6.1). To calculate these reactions the code allows using several procedures such as Strip Method, Theory of Elasticity or Plastic Analysis and it is also possible to consider the slabs yield line as straight lines drawn from each corner of the panel which make 45°, 60° or 90° angles with the panel borders, when a plastic analysis is not performed. This work discusses the efficacy of this approach through the study of practical examples showing that in some cases one can obtain unsafe results since collapse mechanisms involving the formation of yield lines on slabs and plastic hinges on beams lead to underestimated ultimate design loads [3] and system equilibrium is not always satisfied.

### 2. Investigation of a typical interior slab panel

To study the design of beams supporting reinforced concrete slabs consider a typical two-way floor slab showed in Figure 1, where an interior panel with  $L_x$  and  $L_y$  spans is highlighted. In ultimate state, the highlighted panel develops collapse mechanisms that involve both the slabs and the supporting beams. Figure 2 shows such mechanisms in x and y directions, respectively, in which yield line run across the floor and plastic hinges form in the beams.

Considering the equilibrium of a slab segment by taking moments about supporting beam axis, as indicated in Figure 2, one can



show [4] that the total equilibrating moment in beams and slabs in x and y direction are, respectively:

$$M'_{ux} + M_{ux} + l_y (m'_{ux} + m_{ux}) = \frac{(q_x l_y)^2}{8} \tag{1}$$

$$M'_{uy} + M_{uy} + l_x (m'_{uy} + m_{uy}) = \frac{(q_y l_x)^2}{8} \tag{2}$$

where:  $m'_{ux}$ ,  $m_{ux}$ ,  $m'_{uy}$ ,  $m_{uy}$  are negative and positive ultimate resisting bending moments of the slab per unit width, considered uniform, in each direction and  $M'_{ux}$ ,  $M_{ux}$ ,  $M'_{uy}$ ,  $M_{uy}$  are negative and positive ultimate resisting bending moments of beams. The concept of total equilibrating moment can easily be understood verifying its computation for a simply supported beam with uniform load and moments applied to the ends, as indicated in Figure 3. From this figure one can conclude that for the equilibrium it is required that the sum of positive and negative bending moments is equal the total equilibrating moment  $-\frac{q l^2}{8}$ , in this case. This condition is found in equations (1) and (2) for the beams and for the slab of Figure 2, in both directions, and the design of beams so

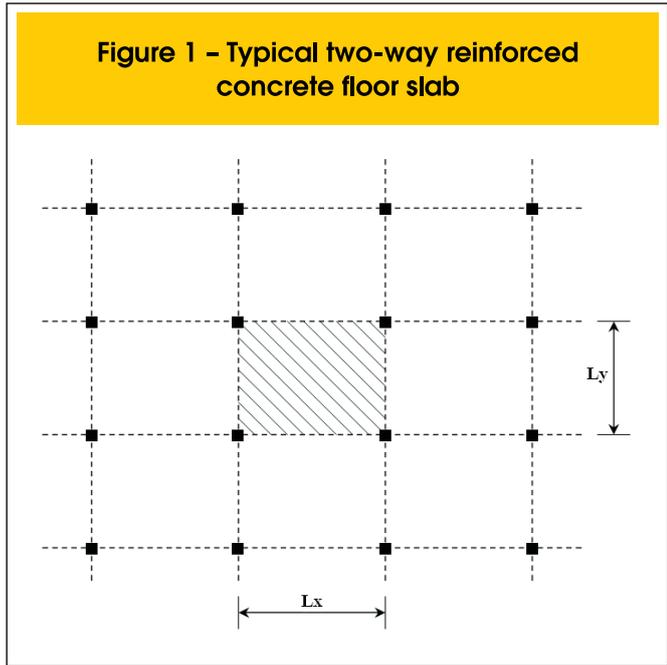
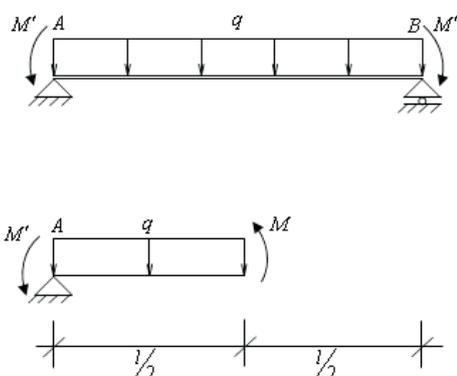


Figure 3 – Total equilibrating moment



Moment equilibrium with respect to A

$$\sum M_A = M' + M - \left(\frac{ql}{2}\right)\frac{l}{4} = 0$$

$$\therefore M' + M = \frac{ql^2}{8} = M_o = \text{equilibrating moment}$$

that these equations are satisfied will assure that the slab ultimate load will not be less than  $q_u$ .

If one desires to consider the reactions of slabs on supporting beams as uniformly distributed, as permitted in NBR 6118, it is necessary to assure the condition of total equilibrating moment in each direction of the panel. This can be achieved if in each direction the total equilibrating moment is never lower than:

$$M_o = \frac{ql_2 l_n^2}{8}$$

(3)

where:

$q$  = uniformly distributed load per unit area of slab;

$l_n$  = span in the direction where moments are being calculated;

$l_2$  = span in transverse direction.

Brazilian code recommendation does not necessarily assure the total equilibrating moment condition explained above. Also it does not satisfy the half-panel equilibrium condition nor does it assure the system load capacity.

To show this, consider the analysis of the interior panel of Figure 1, detailed in Figure 4. The design method admits that the beams are strong enough and the slabs are designed as isolated elements under stiff supports. Next, beams are designed to carry reactions from slabs.

Design bending moments to compute slab reinforcement can be obtained using one of the procedures listed below, which seek to assure the ultimate load obtained from the yield line pattern of the slab showed in Figure 5:

- a) Tables based on elastic theory of plates;
- b) Marcus' Method;
- c) Finite Element Method;
- d) Grid Analogy Method;
- e) Yield Line Theory.

Tables based on the elastic theory of plates present systematization of the solution for the governing equations for special cases of load and support conditions. Material is considered homogeneous, linear-elastic and isotropic and these tables usually

give maximum bending moments in both direction of the slab. In the paper tables developed by KALMANOK [5] and adapted by ARAÚJO [6] were used.

Marcus' method represents an adaptation of the strip method to design rectangular slabs. It is based on the approach that the applied load is carried entirely by bending. Marcus observed that strip method resulted in some large positive bending moments and proposes, therefore, a correction - through the use specific coefficients - to bring the obtained results closer to those resulting from theory of plates. He developed several tables to compute bending moments in reinforced concrete rectangular slabs for different support conditions that can be easily found in literature [7].

The Finite Element Method (FEM) is a powerful tool for numerical analysis of structures in general. In performed analysis, the slabs were modeled by thin plate elements and supporting beams by usual three-dimensional frame elements with T cross-section and tributary width of slab in accordance with Brazilian code. STRAP [8]

Figure 4 – Layout of the analyzed panel

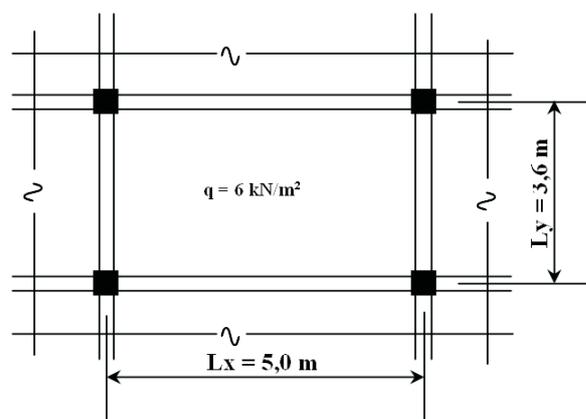
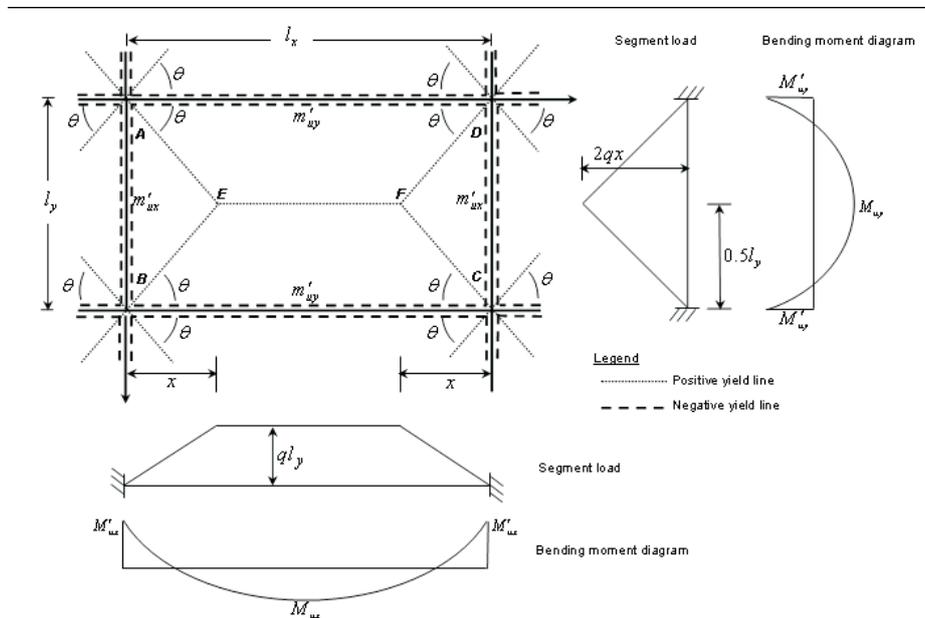


Figure 5 – Simple collapse mechanism of the slab



was the FEM package used. Since practical use of FEM to design structures still demands intensive work to analyze and interpret results, analysis using grid analogy, which is a common approach often used in commercial software to design reinforced concrete buildings [9], [10], was also carried out. The grid analogy method besides being easy to apply is based on concepts with immediate physical meaning, which make its use very simple and attractive. Yield Line Design is a method of designing slabs that uses yield line theory to investigate the collapse mechanisms at the ultimate limit state. The design of slabs using this method presents important advantages such as economy, simplicity and versatility. The economy and simplicity were demonstrated in real situations on the building of the European Concrete Building Project at Cardington [11] where, in each floor, different methods of design and detailing of slabs were constructed and compared. Decrease in

flexural reinforcement at about 14% was achieved on the floor detailed using yield line design when compared to traditional elastic methods used to design the other floors. Moreover the obtained reinforcement layouts were easier to execute and exhibited adequate behavior in service and ultimate limit state. In the examples where yield line method was used an addition of 10% to the design bending moments was considered to take into account corner fan effects, as recommended in [11], and the ratio of bending moment in x-direction to y-direction was taken as 0.4. This value is close to that obtained from a simplified optimization of the cost of reinforcement of the slab.

A 6.0 kN/m<sup>2</sup> uniformly distributed load was used to compute slab bending moments for all procedures listed above.

Table 1 shows positive and negative design bending moments to compute reinforcement of the analyzed slab obtained from each

Table 1 – Design bending moments and ultimate load – Slab with  $L_x=5$  m and  $L_y=3,6$  m

Method	Bending moments (kNm/m)				Ultimate load (kN/m <sup>2</sup> )
	$m^+_x$	$m^+_y$	$m^-_x$	$m^-_y$	
Tables	1,42	2,60	-4,40	-5,58	10,0
Marcus	1,20	2,30	-2,65	-5,11	8,30
FEM	1,43	2,65	-4,20	-5,47	9,90
Grid Analogy	1,50	2,42	-4,75	-5,28	9,90
Yield line	0,90	2,15	-1,80	-4,30	6,80

**Table 2 – Total equilibrating moments of the panel – Slab with  $L_x=5$  m and  $L_y=3,6$  m**

Method	Total equilibrating moments (kNm)			
	Short Span		Long Span	
	$m_d$	$m_d/m_{eq}$	$m_d$	$m_d/m_{eq}$
Tables/NBR	58,40	1,20	64,08	0,95
Marcus/NBR	54,50	1,12	56,81	0,84
Yield lines/NBR	49,75	1,02	52,84	0,78
FEM	48,60	1,00	67,50	1,00
Grid analogy	49,20	1,02	68,07	1,01

$m_d$  = design moment;  $m_{eq}$  = equilibrating moment

one of the methods described before as well as the respective ultimate load, computed using yield line design analysis applied to the simple collapse mechanism of Figure 5.

As can be seen from this table, the ultimate load obtained using yield line method produced a number that is closest to the target value of  $6.6 \text{ kN/m}^2$  whereas the collapse load corresponding to the use of plate design tables for slabs presented the most conservative value (about 67% higher than the load used to design the slab). This behavior occurs because the tables often give maximum values for design bending moments and this fact contributes to an uneconomic design of the slab.

Table 2 shows the total equilibrating moments for two slab directions. In this table, the slab bending moments were computed for each previously defined method and when it is indicated by NBR abbreviation, the reaction of slabs on the beams, used to compute their design bending moments, were determined taking into account the simplified procedure allowed by Brazilian code in section 14.7.6.1 (a). Total equilibrating moments in both directions obtained from equation (3) are: x-direction =  $48.60 \text{ kNm}$  and y-direction =  $67.50 \text{ kNm}$ .

One can observe from Table 2 that:

- In the long span direction, the total equilibrating moment condition is not satisfied for none of the simplified procedures used to compute isolated slab reinforcement when one assume uniformly distributed reaction of the slabs on the beams, according NBR 6118;
- The worst ratio of resisting to equilibrating moment came from yield line design/NBR which presented a reduction of 22% in ultimate load of the slab; the best ratio was obtained from plate tables with a 5% reduction;
- From safety point of view, the design of slabs using plate tables is more acceptable than the other two simplified procedures. On the other hand, if economy is desired, it is not justifiable to overdesign the slabs to ensure their stability. The best solution is to design the slabs taking into account the simple collapse mechanism and then to design the supporting beams by a limit design approach based on composite beam/slab collapse mechanisms, assuring the condition of total equilibrating moment. As the supporting beams have larger height, the amount of reinforcement will be smaller as demonstrated in [11];
- Marcus' method/NBR, which had already been traditionally

used in Brazil, presented an ultimate load that is 16% underdesigned in the long span direction. This can be covered by safety coefficients and showed not cause collapse, necessarily. However, the authors have seen cases where cracks in beams are reported, mainly in long span direction, which are often misunderstood as shrinkage effects;

- It is not acceptable that code recommendation depend on methods that overdesign the slabs to assure equilibrium for design load combination.

### 3. Reinforced concrete slabs systems with stiff beams

Design of beams supporting uniformly loaded reinforced concrete slabs which have been designed by yield line theory can be performed through the following procedures [4]:

I – method based on composite beam/slab mechanism;

II – method based on the loading transferred from slab to the beams.

The first procedure is to design slabs considering that the supporting beams are strong enough to carry the ultimate load of the slab and, once this approach is satisfied, the beams are designed using limit analysis based on composite beam/slab mechanisms in order to the ultimate load of the system has been reached.

Consideration of equilibrium of a slab segment about supporting beams axis (see Figure 2) leads to equations (1) and (2) showed before and the design of beams requiring satisfaction of these equations in each direction will assure a simultaneous collapse of the systems (beams and slab) when ultimate load of the slab is reached.

The second procedure takes into account the loading transferred to the beams from the slabs as it can be seen in Figure 5.

A question that is very important, however, is that yield line theory does not imply the way in which slab reactions are distributed on the supporting beams. A way to solve this question is to admit that this distribution follows the shape of segments of the yield line pattern, i.e: triangular loading distribution for the short span beams and trapezoidal loading distribution for the long span ones. Taking into account this approach it is possible to show that if consideration of equilibrium for the beams in every direction is verified, equations (4) and (5) are obtained.

Short span direction

$$M'_{wy} + M''_{wy} = \frac{(q l_y) l_y^2}{6} \quad (4)$$

Long span direction

$$M'_{lx} + M''_{lx} = \frac{q l_y l_x^2}{8} - \frac{q l_y l_y^2}{6} \quad (5)$$

Equilibrium of segment BEFC of the yield line pattern about BC edge (see Figure 5) result in equation (6).

$$q \frac{l_y l_x^2}{6} = \frac{q l_x l_y^2}{8} - l_x (m'_{wy} + m''_{wy}) \quad (6)$$

In a similar way, taking moments about line CD for segment CFD leads to equation (7) below.

$$q \frac{l_y l_y^2}{6} = l_y (m'_{lx} + m''_{lx}) \quad (7)$$

It is possible to show that substituting the right side of equation (4) by equation (6) and the right side from equation (5) by equation (7) one would obtain the following equations:

$$M'_{wy} + M''_{wy} = \frac{q l_x l_y^2}{8} - l_x (m'_{wy} + m''_{wy}) \quad (8)$$

$$M'_{lx} + M''_{lx} = \frac{q l_y l_x^2}{8} - l_y (m'_{lx} + m''_{lx}) \quad (9)$$

Equations (8) and (9) are identical to equations (1) and (2), aspect that allows concluding that both procedures showed before lead to the same expression for the total equilibrating moment in any direction. In other words, if the beams are designed using one of these procedures, the systems loading capacity will be assured.

In order to avoid plastic analysis of slabs to compute beams loading, codes of practice sometimes allow approximating their collapse mechanisms considering that yield lines are inclined with base angles varying according to slab support conditions. For slabs interior panels these angles are  $\theta=45^\circ$  (see Figure 5). Using Theory of Elasticity is possible to show that these simplifications are justified in the case of very stiff beams only [4].

A question that plays an important role is how to know if the

supporting beams are, in fact, rigid enough to be considered as rigid supports.

A possible solution is the use of the criterion adopted for the Canadian code [12] that states that a stiff supporting beam is defined as one in which the following relationship is verified:

$$RSP = \frac{b_w h_b^3}{l_n h_s^3} > 2,0 \quad (10)$$

where:

RSP = Relative Stiffness Parameter of the slab supporting beam;

$b_w$  = width of beam web;

$h_b$  = depth of beam;

$h_s$  = depth of slab;

$l_n$  = clear span of beam.

Once the supporting beams are deemed sufficiently stiff and if one desires to consider the reactions of slabs on beams as uniformly distributed it is also necessary to guarantee that total equilibrating moments in two directions of the panel are satisfied as it has already been showed before. A way to do this is to compute an equivalent uniformly loading ( $\bar{q}$ ) that satisfies that condition. In the long span direction, the expression for computing this loading can be obtained through the following equation:

$$\frac{\bar{q} l_x^2}{8} = \frac{q l_y l_x^2}{8} - \frac{q l_y l_y^2}{6} \therefore \bar{q} = q l_y - \frac{4 q l_y l_y^2}{3 l_x^2} \quad (11)$$

Taking into account that  $\theta=45^\circ$  ( $l_2 = \frac{l_y}{2}$ ) and that the loading used

in equation (5) comes from the contribution of two slab panels, the equivalent uniformly distributed loading on beams in long span direction can be computed by the following equation:

$$\bar{q} = \frac{q l_y (3 - m^2)}{3} \quad (12)$$

where  $m = \frac{l_y}{l_x}$

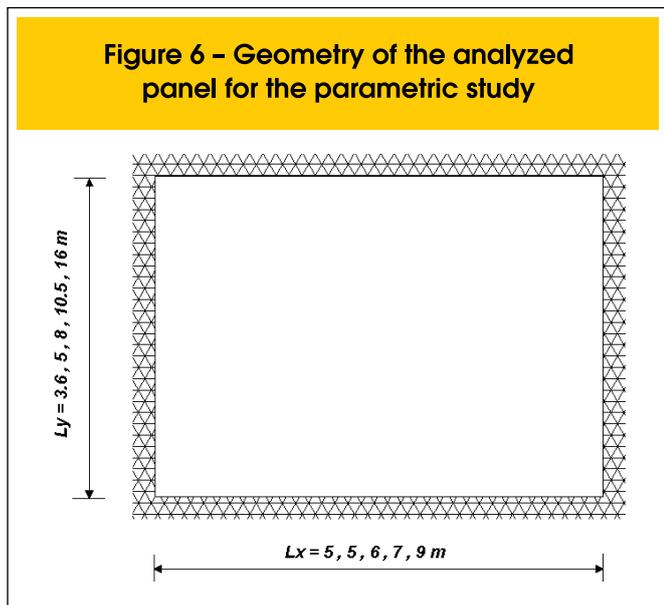
In a similar way it is possible to show that the equivalent uniformly distributed loading on beams in the short span direction can be

$$\bar{q} = \frac{q l_y}{3} \quad (13)$$

computed using equation (13).

Equations (12) and (13) allow computing the equivalent uniformly loading that assure the total equilibrating moment condition in both directions of the panel and must be used if one intends to consider the reaction from slabs on supporting beams as uniformly distributed. Canadian code [12] uses this approach to compute design bending moments of

**Figure 6 – Geometry of the analyzed panel for the parametric study**



beams supporting uniformly loaded reinforced concrete slabs. The way the Brazilian code recommends considering slab reaction on beams does not assure the total equilibrating moment condition in both directions of the panel and, therefore, correc-

tions are necessary.

#### 4. Comparative study

In order to evaluate the efficiency of the available procedures to compute reactions of slabs on supporting beams, a comparative study was performed using the following methods:

- NBR 6118 simplified approach and Marcus' Method (NBR-M);
- NBR 6118 simplified approach and Theory of Plates (NBR-P)
- Actual slab yield line pattern (SYLP-AC);
- Approximate slab yield line pattern (SYLP-AP);
- Equivalent Distributed Loading (EDL);
- Grid Analogy (GAN) and
- Finite Element Method (FEM);

Geometry of the panels investigated is shown in Figure 6, with spans lengths indicated in both directions.

For each span ratio ( $\lambda=L_y/L_x$ ) positive and negative slab bending moments were computed using Marcus' Method and Theory of Plates and the reaction of slabs on the beams were obtained through NBR 6118 simplified approach. Obtained results are shown in Table 3.

Actual yield line pattern is obtained using the procedure described in [3] and NBR 6118 approach was used to get the approximate yield line pattern. Finite element analyses were performed as well as analysis using grid analogy, which is a common approach adopted in several commercial software to design and detail reinforced concrete structures.

**Table 3 – Bending moments computed by Marcus' Method and Theory of Plates with reactions on beams according to NBR 6118**

$L_y/L_x$	Slab bending moments by Marcus (kNm/m)				Slab bending moments by Theory of Plates (kNm/m)				Reactions on beams (kN/m)	
	mx	mx'	my	my'	mx	mx'	my	my'	Short Span	Long Span
0,72	1,2	2,6	2,3	5,1	1,42	4,40	2,6	5,58	5,4	6,9
1,00	2,7	6,3	2,7	6,3	3,16	7,67	3,16	7,67	7,5	7,5
1,33	6,0	13,7	3,4	7,7	6,98	15,08	4,06	12,18	11,3	9,0
1,50	9,2	20,5	4,1	9,1	10,54	22,04	4,98	16,64	14,0	10,5
1,78	16,9	36,8	5,4	11,6	16,08	38,91	6,62	27,28	19,4	13,5

**Table 4 – Total equilibrating moments of the panel (slab/beams) – Short Span**

$L_y/L_x$	Total equilibrating moments (kNm)						
	NBR-M	NBR-P	SYLP-AC	SYLP-AP	EDL	FEM	GAN
0,72	54,50	58,40	58,59	60,33	60,33	48,60	49,20
1,00	91,87	101,03	107,50	107,50	107,50	93,75	94,38
1,33	238,60	257,48	259,61	265,60	265,60	216,00	217,26
1,50	440,48	470,72	465,48	483,35	483,35	385,88	387,60
1,78	1.132,58	1.153,22	1.155,39	1.223,70	1.223,70	972,00	969,84

**Table 5 – Total equilibrating moments of the panel (slab/beams) – Long Span**

$L_y/L_x$	Total equilibrating moments of the Panel (kNm)						
	NBR-M	NBR-P	SYLP-AC	SYLP-AP	EDL	FEM	GAN
0,72	56,81	64,08	71,19	69,52	69,52	67,50	68,07
1,00	91,87	101,03	107,50	107,50	107,50	93,75	94,38
1,33	247,40	278,24	306,43	300,60	300,60	288,00	288,98
1,50	478,28	537,21	602,40	585,46	585,46	578,80	580,13
1,78	1.394,60	1.546,70	1.760,66	1.698,75	1.698,75	1.728,00	1.729,59

Table 4 and Table 5 exhibit total equilibrating moments for slab/beam system in short and long span directions, computed using procedures listed above.

Results obtained show that in the long span direction (Table 5) and in most studied examples, NBR 6118 approach that permits to consider reaction of slabs on beams as uniformly distributed generally underestimated the total equilibrating moment condition to slab/beam system when compared with results obtained from finite element analyses which are identical to those using equation (3). Slab with ratio of side lengths of  $\lambda=1,78$  exhibited the most severe underestimating of about 20%. Table 5 results also show that, except for the NBR 6118 approach, any one of the others methods studied lead to similar results and, therefore, are possible design alternatives.

### 5. Effect of flexibility of supporting beams

The effect of flexibility of supporting beams in global behavior of the panel was analyzed taking into account a variation in their stiffnesses. To perform the study, the slab indicated in Figure 4 was analyzed using six values for the Relative Stiffness Parameter (RSP) of the supporting beams, according equation (10), as follows: RSP=0.5, RSP=1.0, RSP=1.5, RSP=2.0, RSP=2.5 e RSP=3.0. In performed analyses, reactions from slabs on the beams were computed using simplified NBR 6118 recommendations combined with Marcus' Method to compute slab bending moments, referred as ISO-SL (isolated slabs) in Table 6 and Table 7. Finite element results, grid analogy results and Canadian code recommendations

results were also showed.

Long and short span results showed in Table 6 and Table 7, respectively, indicates that changes in supporting beams stiffness leads to an increase in total bending moments of slabs and decrease in total bending moment of the beams, behavior that is not captured using Brazilian code recommendations. In fact, if one compares finite element results with isolated slabs procedure, it is possible to observe that, when the flexibility of supporting beams is high (RSP=0.5), total bending moment of slab in long span direction increase in about 127% which means an undesirable situation for design that will contribute to underdesign the slab. Other aspect that becomes clear from the obtained results is that, except for combined design using Marcus' Method and NBR 6118 recommendation, all the others methods satisfy the total equilibrating moment condition.

### 6. Conclusions

This work demonstrates through illustrative practical examples that NBR 6118 recommendations which permits to consider the reactions of slabs on supporting beams as uniformly distributed does not assure the total equilibrating moment condition for both directions and, therefore, does not satisfy panel equilibrium conditions. As a result, usage of such procedures may underdesign the beams supporting slabs. Although one does not have information about collapses in slabs exclusively due to this cause, the authors are aware of several examples of cracking in beams supporting reinforced concrete slabs in the long span direction which are often

**Table 6 – Support flexibility effect – Long Span**

RSP	Slab total moment (kNm)				Beam total moment (kNm)				Total equilibrating moment (kNm)			
	ISO-SL	CAN	FEM	GAN	NBR	CAN	FEM	GAN	NBR	CAN	FEM	GAN
0.5	13,68	-	23,29	23,15	43,13	-	44,21	44,92	56,81	-	67,50	68,07
1.0	13,68	-	18,13	18,20	43,13	-	49,37	49,87	56,81	-	67,50	68,07
2.0	13,68	12,0	15,19	15,38	43,13	55,84	52,31	52,69	56,81	67,84	67,50	68,07
2.5	13,68	12,0	14,49	14,71	43,13	55,84	53,01	53,36	56,81	67,84	67,50	68,07
3.0	13,69	12,0	13,98	14,23	43,13	55,84	53,52	53,84	56,81	67,84	67,50	68,07

Table 7 – Support flexibility effect – Short Span

RSP	Slab total moment (kNm)				Beam total moment (kNm)				Total equilibrating moment (kNm)			
	ISO-SL	CAN	FEM	GAN	NBR	CAN	FEM	GAN	NBR	CAN	FEM	GAN
0.5	37,00	-	27,09	27,02	17,50	-	21,51	22,18	54,50	-	48,60	49,20
1.0	37,00	-	25,61	25,78	17,50	-	22,99	23,42	54,50	-	48,60	49,20
2.0	37,00	32,91	24,91	25,26	17,50	23,33	23,69	23,94	54,50	56,24	48,60	49,20
2.5	37,00	32,91	24,76	25,15	17,50	23,33	23,84	24,05	54,50	56,24	48,60	49,20
3.0	37,00	32,91	24,65	25,09	17,50	23,33	23,95	24,11	54,50	56,24	48,60	49,20

attributed to shrinkage.

If it is desired to assume that the reactions of slabs on supporting beams are uniformly distributed loading one must use the equivalent loading according to Canadian code [12] or use the reaction obtained from slab yield line design. Computation of bending moments of beams using these two procedures combined with bending moments of slab also obtained using such procedures lead to total equilibrating moments in both directions of slab which satisfies the equilibrium condition of slab/beam system.

Taking into account the study of the effect of flexibility of supporting beams it is possible to conclude that:

- For RSP of supporting beams higher than 2 it is acceptable to design slabs on unyielding supports and it is suggested consideration by the Brazilian NBR 6118 code in section 14.7.2.2 of a clause to classify supports as sufficiently stiff;
- For RSP of supporting beams less than 2 it is not recommended to design slab systems as isolated panels on stiff supports;
- To compute design bending moments of beams, the approach which considers the reactions of slabs as uniformly distributed loading should be only permitted for stiff beams with adjusted loading. The RSP in equation (10) and equivalent distributed loading from equations (12) and (13) are more consistent ways than those recommended by the Brazilian code;
- If the beams supporting slabs could not be considered stiff, one should use finite element method or grid analogy to design slab/beam system.

Dunas, Rio Grande do Sul, 2003, v2, 2 ed.

- [07] ROCHA, A. M. Curso Prático de Concreto Armado, Editora Científica, Rio de Janeiro, 1971, v. 1.
- [08] STRAP 8.0. Structural Analysis Program. Version 8.0, 2000 – ATIR - Manual do Usuário.
- [09] TQS. Sistemas computacionais – Manual do Usuário
- [10] EBERICK. Sistema Para Projetos de Edificações em Concreto Armado – Manual do Usuário.
- [11] KENNEDY, G., GOODCHILD, C. Practical Yield Line Design, British Cement Association, Crowthorne, Berkshire, 2003
- [12] CSA A23.3-04, Design of Concrete Structures. Canadian Standards Association, 2004.

## 7 References

- [01] ABNT. Norma Brasileira NBR-6118: Projeto de estruturas de concreto. Rio de Janeiro, 2003
- [02] ABNT. Norma Brasileira NB-1: Projeto e execução de obras de concreto armado. Rio de Janeiro, 1978.
- [03] SILVA, F. A. N.; HOROWITZ, B., Momentos Fletores em Vigas Rígidas de Pisos em Concreto Armado. 43º Congresso Brasileiro do Concreto, Foz do Iguaçu, 2001.
- [04] PARK, R.; GAMBLE, W. L. Reinforced Concrete Slabs, John Wiley & Sons, New York, 1980.
- [05] KALMANOK, A. S. Manual Para Cálculo de Placa. Editora Interciência, Montevideo, 1961.
- [06] ARAÚJO, J. M. Curso de Concreto Armado, Editora