Volume 7, Number 1 (February 2014) p. 68-99 • ISSN 1983-4195

IBRACON

REVISTA IBRACON DE ESTRUTURAS E MATERIAIS
IBRACON STRUCTURES AND MATERIALS JOURNAL

Evaluation of deflection in reinforced concrete structures using damage mechanics

Avaliação de deflexões em estruturas de concreto armado utilizando mecânica do dano

J. J. C. PITUBA [a]
julio.pituba@pq.cnpq.br

R. G. DELALIBERA [a]
dellacivil@gmail.com

Abstract

This work intends to present a contribution about a proposed approach for the estimative of displacements in reinforced concrete structures submitted to service loads. This work is restrictive to C20 up to C35 classes of strength. The approach used in this work consists in the decreasing of cracking element elastic moduli by damage model. That constitutive model takes into account induced anisotropy, plastic deformations and bimodular elastic response and a simplified version is used in order to simulate the concrete behavior, while an elastoplastic behavior is admitted for the reinforcement. Initially, a set of beams are analyzed and some parameters related to the problem are modified, such as: compression strength, span length, cross section, reinforcement rates and support conditions. The numerical responses are compared with the ones obtained by NBR 6118:2007 Procedure. Statistical analyses are carried on in order to identify the major variables in the problem. Finally, some possible proposals to obtain cracking moment and displacement values in RC structures are discussed based on numerical and statistical analyses performed in this work in order to contribute in the improvement to Brazilian Technical Code procedure.

Keywords: reinforced concrete, technical code, damage mechanics.

Resumo

Este trabalho visa apresentar uma contribuição sobre uma proposta de abordagem para o cálculo de deslocamentos em estruturas de concreto armado em regime de serviço, sendo restrito o estudo ao caso de concretos C20 a C35. A abordagem utilizada leva em conta a penalização dos módulos elásticos dos elementos fissurados através de um modelo de dano. O modelo em questão leva em conta a anisotropia, deformações plásticas e resposta bimodular induzidas pelo processo de danificação, sendo uma versão mais simplificada usada para simular o comportamento do concreto fissurado, enquanto que um comportamento elastoplástico é admitido para a armadura. Inicialmente, são analisadas séries de vigas com variação de diversos parâmetros relacionados ao problema, tais como: resistência à compressão, arranjo das barras de aço da armadura, dimensões da seção transversal, vão e condições de apoio. As respostas numéricas são confrontadas com aquelas obtidas com o emprego do procedimento sugerido pela NBR 6118:2007. As análises numéricas são complementadas por análises estatísticas dos resultados empregando-se a metodologia ANOVA. Por fim, baseadas nas análises realizadas discutem-se algumas proposições possíveis para o cálculo do momento de fissuração e de deslocamentos em estruturas de concreto armado, como forma a contribuir no aperfeiçoamento do procedimento sugerido pela Norma Brasileira.

Palavras-chave: concreto armado, norma técnica, mecânica do dano.

[a] *Department of Civil Engineering, Federal University of Goiás, Catalão, GO, Brasil.*

Received: 21 Nov 2012 • Accepted: 25 Jan 2014 • Available Online: 13 Feb 2014

© 2014 IBRACON

1. Introduction

This paper is a sequence of what is being accomplished aiming at proposing an alternative methodology to the Procedure recommended by NBR 6118:2007 [1] for the evaluation of displacements in reinforced concrete structures, [2]. The problem to be solved for the estimative of displacements in reinforced concrete structures using PTV (Principle of Virtual Work), for instance, is not trivial. This is due to the fact that the elements of the reinforced concrete are heterogeneous, composed by concrete and steel, with distinct elasticity modules leading to different stiffness to bending. Furthermore, there is the strong possibility of the occurrence of different behaviors in a same structure subjected to loadings of low intensity (service regime), namely, regions where the tensioned concrete presents cracking process (Stage II) and regions where the concrete is intact (Stage I), not presenting noticeable cracks. Then, the inertia reduction is due to cracking process that contributes for the loss of resistance to the bending movement, where only the reinforcement resists to tension stresses.
In order to propose an alternative methodology, numerical results, obtained from the employment of a damage model [3], associated to comparisons with experimental ones of reinforced concrete structures are used. Therefore, this procedure adopted on this work is an alternative way to the experimental tests which are expensive to be performed. Besides, the use of the numerical and experimental analyses is complimented by statistical analyses based on ANOVA Methodology (Variance Analysis) that it is used to verify the main variables involved in the problem taking into account the numerical and analytical analyses with the use of NBR 6118:2007 Procedure [1].
In [2] has been presented results of the analyses performed in beams with three different spans, transversal sections and reinforcement configurations, however, in that work only the case of concrete with f_{ck}=30 MPa and two boundary conditions (simply supported and bi-fixed beams) have been studied. Moreover, numerical analyses in conjunction with statistical ones have led to the determination of expressions for the estimative of the cracking moment (M_{cr}) depending on the f_{ck} used, however such expressions need an investigation with more parameters involved in the problem.
In the present work, such parameters are taken into account in order to obtain expressions for M_{cr} which deal with called conventional concretes of classes C20 to C35. Furthermore, a discussion about the possible propositions of formulas for the evaluation of displacements in reinforced concrete structures is presented in the end of the paper.
The damage model developed by [3] is used in the analyses of reinforced concrete beams submitted to permanent and accidental variable loadings with the changes in the support conditions, span length, compression strength of the concrete, transversal section and reinforcement arrangement. The validation of the numerical responses obtained by the damage model as well as the parametric identification, can be found in [2], [3] and [4]. The modeling used describes the process of rigidity loss that leads to larger displacements, through decreasing of the elasticity module of the material at a certain point of the structure and not in the decreasing of the inertia moment of the studied section and, also, in the representation of this loss by an equivalent inertia in the whole beam, as if the whole beam was homogeneously cracked as it is considered by the NBR 6118:2007 [1]. These issues in conjunction with the reliability of the numerical responses presented by the damage model so far, associated to the low cost of the numerical analyses against the high cost of the experimental ones, have motivated the discussion of the problem presented in this work.
In the item 2 of this work is briefly presented the models used, such as: damage model, ANOVA and NBR 6118:2007 Procedure. In item 3, the prototypes numerically tested are presented, as well as information about the concretes used in the production of the prototypes. In item 4, the numerical and analytical results are presented and discussions considering ANOVA methodology are developed. Besides, the proposals for the estimative of the cracking moment and a discussion about possible approaches for the evaluation of displacements in reinforced concrete structures are presented. Finally, in item 5, the work presents some conclusions.

2. Modeling used

2.1 Damage model

The concrete is assumed as an initially isotropic material that starts to present transverse isotropy and bimodular responses induced by the damage. Moreover, the model tries to respect the principle of energy equivalence between damaged real medium and equivalent continuous medium established in the Continuum Damage Mechanics (CDM), [3].
Here in after, the damage model is briefly described. So, for the tension dominant states, the following damage tensor is adopted:

$$\mathbf{D}_T = f_1(D_1, D_4, D_5)\,(\mathbf{A} \otimes \mathbf{A}) + 2 f_2(D_4, D_5)\left[(\mathbf{A} \underline{\overline{\otimes}} \mathbf{I} + \mathbf{I} \underline{\overline{\otimes}} \mathbf{A}) - (\mathbf{A} \otimes \mathbf{A})\right] \tag{1}$$

where $f_1(D_1, D_4, D_5) = D_1 - 2\, f_2(D_4, D_5)$ and $f_2(D_4, D_5) = 1 - (1-D_4)(1-D_5)$.
The variable D_1 represents the damage in the orthogonal direction to the transverse isotropy local plane of the material, while D_4 is representative of the damage generated by the sliding movement between the crack faces. The third damage variable, D_5, is only activated if a previous compression state accompanied by damage has occurred.
In the Eq. (1), the tensor $\mathbf{I}$ is the second-order identity tensor and the tensor $\mathbf{A}$, by definition, is formed by the dyadic product of the unit vector perpendicular to the transverse isotropy plane for itself. The tensor product operations between the tensors of second order $\mathbf{I}$ and $\mathbf{A}$ that arise in Eq. (1) and which will be used during all the formulation are described in [3].
For the compression dominant states, the following damage tensor is adopted:

$$\mathbf{D}_C = f_1(D_2, D_4, D_5)\,(\mathbf{A} \otimes \mathbf{A}) + f_2(D_3)\left[(\mathbf{I} \overline{\otimes} \mathbf{I}) - (\mathbf{A} \otimes \mathbf{A})\right] + 2 f_3(D_4, D_5)\left[(\mathbf{A} \underline{\overline{\otimes}} \mathbf{I} + \mathbf{I} \underline{\overline{\otimes}} \mathbf{A}) - (\mathbf{A} \otimes \mathbf{A})\right] \tag{2}$$

where $f_1(D_2, D_4, D_5) = D_2 - 2\, f_3(D_4, D_5)$,$f_2(D_3) = D_3$ and $f_3(D_4, D_5) = 1 - (1-D_4)(1-D_5)$.
Note that in the compression damage tensor expression two additional scalar variables are introduced: D_2 and D_3. The variable

D_2 (damage perpendicular to the transverse isotropy local plane of the material) reduces the Young's modulus in that direction. On the other hand, the variable D_2 together with D_3 (that represents the damage in the transverse isotropy plane) degrades the Poisson's ratio on the perpendicular planes to the one of transverse isotropy. Finally, the resultant constitutive tensors E_T and E_C may be described as follow:

$$\begin{aligned} E_T &= \lambda_{11}[I\otimes I] + 2\mu_1[I\overline{\otimes}I] - \lambda_{22}^{+}(D_1,D_4,D_5)[A\otimes A] \\ &-\lambda_{12}^{+}(D_1)[A\otimes I + I\otimes A] - \mu_2(D_4,D_5)[A\overline{\otimes}I + I\overline{\otimes}A] \end{aligned} \quad (3)$$

$$\begin{aligned} E_C &= \lambda_{11}[I\otimes I] + 2\mu_1[I\overline{\otimes}I] - \lambda_{22}^{-}(D_2,D_3,D_4,D_5)[A\otimes A] - \lambda_{12}^{-}(D_2,D_3)[A\otimes I + I\otimes A] \\ &-\lambda_{11}^{-}(D_3)[I\otimes I] - \frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^{-}(D_3)[I\overline{\otimes}I] - \mu_2(D_4,D_5)[A\overline{\otimes}I + I\overline{\otimes}A] \end{aligned} \quad (4)$$

where $\lambda_{II} = \lambda_0$ and $\mu_I = \mu_0$. The remaining parameters will only exist for no-null damage, evidencing in that way the anisotropy and bimodularity induced by damage. Those parameters are given by:

$$\begin{aligned} &\lambda_{22}^{+}(D_1,D_4,D_5) = (\lambda_0 + 2\mu_0)(2D_1 - D_1^2) - 2\lambda_{12}^{+}(D_1) - 2\mu_2(D_4,D_5) \\ &\lambda_{12}^{+}(D_1) = \lambda_0 D_1 \,;\ \mu_2(D_4,D_5) = 2\mu_0[1-(1-D_4)^2(1-D_5)^2] \\ &+\frac{(\nu_0-1)}{\nu_0}\lambda_{11}^{-}(D_3) - 2\mu_2(D_4,D_5) \\ &\lambda_{12}^{-}(D_2,D_3) = \lambda_0[(1-D_3)^2 - (1-D_2)(1-D_3)] \\ &\lambda_{11}^{-}(D_3) = \lambda_0(2D_3 - D_3^2)\,;\ \mu_2(D_4,D_5) = 2\mu_0[1-(1-D_4)^2(1-D_5)^2] \end{aligned} \quad (5)$$

In [3], a hypersurface is defined either in the stress or strain space in order to identify the bimodular constitutive response to be used. A particular form is adopted for the hypersurface in the strain space: a hyperplane g(ε) defined by the unit normal $\mathbf{N}$ ($\|\mathbf{N}\| = 1$) and characterized by its dependence of the strain and damage states. Therefore, the following relation is proposed:

$$g(\varepsilon, \boldsymbol{D_T}, \boldsymbol{D_C}) = \boldsymbol{N}(\boldsymbol{D_T}, \boldsymbol{D_C}) \,.\, \varepsilon_e = \gamma_1(D_1,D_2)\,\varepsilon_V^e + \gamma_2(D_1,D_2)\,\varepsilon_{II}^e \quad (6)$$

where $\gamma_1(D_1,D_2)=\{1+H(D_2)[H(D_1)-1]\}\eta(D_1)+\{1+H(D_1)[H(D_2)-1]\}\eta(D_2)$ and $\gamma_2(D_1,D_2) = D_1+D_2$.

The Heaviside functions employed above are given by:

$$H(D_i) = 1 \text{ for } D_i > 0;\ H(D_i) = 0 \text{ for } D_i = 0 \quad (i = 1,\ 2) \quad (7)$$

The $\eta(D_1)$ e $\eta(D_2)$ functions are defined, respectively, for the tension and compression cases, assuming for the first one that there was no previous damage in compression affecting the present tension damage variable D_1. Analogously, for the second one it is assumed that has not had previous damage in tension affecting variable D_2.

$$\eta(D_1) = \frac{-D_1 + \sqrt{3-2D_1^2}}{3}\,;\ \eta(D_2) = \frac{-D_2 + \sqrt{3-2D_2^2}}{3} \quad (8)$$

Regarding the damage criterion, it is convenient to separate it into two criteria: the first one is used only to indicate damage incipience when the material is no longer isotropic and the second one is used for loading and unloading when the material is already considered as transverse isotropic.

The criterion for initial activation of the damage processes in tension or compression is given by:

$$f_{T,C}(\sigma) = W_e^* - Y_{0T,0C} < 0 \quad (9)$$

where W_e^* is the complementary elastic strain energy of an isotropic and virgin medium whereas $Y_{0T} = \frac{\sigma_{0T}^2}{2E_0}$ or $Y_{0C} = \frac{\sigma_{0C}^2}{2E_0}$ is a reference value obtained in uniaxial tension or compression tests, respectively. The σ_{0T} e σ_{0C} parameters are limit elastic stresses.

Therefore, $D_T = 0$ (i.e, $D_1 = D_4 = 0$) for tension dominant states or $D_C = 0$ (i.e., $D_2 = D_3 = D_5 = 0$) for compression dominant states, where the response regime of the material is linear elastic and isotropic.

For the case of $g(\varepsilon, \mathbf{D_T}, \mathbf{D_C}) > 0$, the complementary elastic energy of the damaged medium is given by the relation:

$$\begin{aligned} W_{e+}^* &= \frac{\sigma_{11}^2}{2E_0(1-D_1)^2} + \frac{(\sigma_{22}^2+\sigma_{33}^2)}{2E_0} - \frac{\nu_0(\sigma_{11}\sigma_{22}+\sigma_{11}\sigma_{33})}{E_0(1-D_1)} - \frac{\nu_0\sigma_{22}\sigma_{33}}{E_0} \\ &+\frac{(1+\nu_0)}{E_0(1-D_4)^2(1-D_5)^2}(\sigma_{12}^2+\sigma_{13}^2) + \frac{(1+\nu_0)}{E_0}\sigma_{23}^2 \end{aligned} \quad (10)$$

On the other hand, for compression dominant states ($g(\varepsilon, \mathbf{D_T}, \mathbf{D_C}) < 0$), the complementary elastic energy is expressed by:

$$\begin{aligned} W_{e-}^* &= \frac{\sigma_{11}^2}{2E_0(1-D_2)^2} + \frac{(\sigma_{22}^2+\sigma_{33}^2)}{2E_0(1-D_3)^2} - \frac{\nu_0(\sigma_{11}\sigma_{22}+\sigma_{11}\sigma_{33})}{E_0(1-D_2)(1-D_3)} - \frac{\nu_0\sigma_{22}\sigma_{33}}{E_0(1-D_3)^2} \\ &+\frac{(1+\nu_0)}{E_0(1-D_4)^2(1-D_5)^2}(\sigma_{12}^2+\sigma_{13}^2) + \frac{(1+\nu_0)}{E_0}\sigma_{23}^2 \end{aligned} \quad (11)$$

Considering a general situation of the damaged medium in tension dominant regime, the criterion for the identification of damage evolution is represented by the following relation:

$$f_T(\sigma) = W^*_{e+} - Y^*_{0T} \leq 0 \tag{12}$$

where the reference value Y^*_{0T} is defined by the maximum complementary elastic energy determined during the damage process until the actual state. For the damaged medium in compression dominant regime, analogue relations are valid to the case of tension.
In the loading case, i.e., when or , one needs to update the values of the scalar damage variables that appear in the $\mathbf{D}_T$ and $\mathbf{D}_C$ tensors, considering their evolution laws.
In the numerical applications presented in this work, the monotonic loading is considered. The evolution laws for the scalar damage variables have been proposed according to the experimental results. Thus, the general form proposed is

$$D_i = 1 - \frac{1 + A_i}{A_i + \exp\left[B_i(Y_i - Y_{0i})\right]} \quad com\ i = 1,\ 2 \tag{13}$$

where Ai, Bi and Y0i are parameters that must be identified. The parameters Y0i are understood as initial limits for the damage activation, Eq. (9).
When the damage process is activated, the formulation starts to involve the tensor $\mathbf{A}$ that depends on the knowledge of the normal to the transverse isotropy plane. Therefore, it is necessary to establish some rules to identify its location for an actual strain state. Therefore, the following assert is assumed as valid: " In the principal strain space, if two of the three strain rates are extension, shortening or null, the plane defined by them will be the transverse isotropy local plane of the material."
For this work is interesting observe that the uniaxial tension is an example of the case above where the transverse isotropy plane is perpendicular to the tension stress direction. The same observation is valid for uniaxial compression case.
The one-dimensional version of the damage model has been implemented in a program for bars structures analysis with finite layered elements. The damage mode previously described is assumed to govern the concrete layers behavior and for the longitudinal reinforcement bars, an elastoplastic behavior is admitted. In the transversal section, a certain layer can contain steel and concrete. It is defined, for each layer, an elastic modulus and an inelastic strain equivalent, by using homogenization rule.
On the other hand, adopting direction 1 as longitudinal bar direction, the relations of the models in its one-dimensional version are summarized as follows:

$$E := \begin{cases} E_C & se \quad g(\varepsilon, D_T, D_C) < 0, \\ E_T & se \quad g(\varepsilon, D_T, D_C) > 0, \end{cases} \tag{14}$$

$$E_T = E_0(1 - D_1)^2(1 - D_2)^2 \tag{15}$$

$$E_C = E_0(1 - D_2)^2 \tag{16}$$

$$W^*_{e+} = \frac{\sigma^2_{11}}{2E_0(1-D_1)^2(1-D_2)^2}\ ; \quad W^*_{e-} = \frac{\sigma^2_{11}}{2E_0(1-D_2)^2} \tag{17}$$

$$Y_T = \frac{\partial W^*_{e+}}{\partial D_1} = Y_1; \quad Y_C = \frac{\partial W^*_{e-}}{\partial D_2} = Y_2 \tag{18}$$

$$Y_1 = \frac{\sigma^2_{11}}{E_0(1-D_1)^3(1-D_2)^2}\ ; \quad Y_2 = \frac{\sigma^2_{11}}{E_0(1-D_2)^3} \tag{19}$$

2.2 NBR6118:2007 Procedure

The evaluation models of displacements in reinforced concrete beams consider the behavior of the structural elements subjected to bending moment in the Stage I (intact section without crack, considering the tension stress in the concrete) and Stage II (section with cracks, the contribution of the concrete submitted to tension stress is not considered for the equilibrium of the transversal section).
The NBR 6118:2007 [1] presents a criterion for the estimative of the excessive displacement in concrete beams subject a bending moment, based in weight procedure of the inertia moments of Stages I (I1) and II (I2), resulting in equivalent inertia moment, I_{eq}. This equivalent inertia moment is calculated by Eq. (20). Such procedure is valid since the acting moment in the critical section, M_a, is higher than the bending moment that initiates the cracking process, M_r.

$$I_{eq} = \left(\frac{M_r}{M_a}\right)^3 \cdot I_c + \left[1 - \left(\frac{M_r}{M_a}\right)^3\right] \cdot I_2 \leq I_c \tag{20}$$

In Eq. (20), I_c, is the inertia moment of the intact section, without consideration of the reinforcement bars in the transversal section (section homogenization).
The cracking moment, M_r , is calculated by the Eq. (21). It can be ob-

served in Eq. (21) that the Brazilian Code do not consider the favorable effect of the reinforcement bars, decreasing, therefore, the value of M_r.

$$M_r = \frac{\alpha \cdot f_{ct} \cdot I_c}{y_t} \quad (21)$$

The value of α used in Eq. (21) is equal to 1.2 for transversal sections T or double T and it is equal to 1.5 for rectangular transversal section. The tension strength of the concrete (f_{ct}) is calculated by Eq. (22), and y_t, is the distance from the gravity center of the transversal section to the most tensioned fiber of the transversal section.

$$f_{ct} = 0{,}21 \cdot f_{ck}^{2/3} \quad (22)$$

where f_{ck} is the compression strength of the concrete.
However, the bending moment on the critical section, M_a, is determined by an quasi-permanent combination of loads. This combination reduces the intensity of the live loads, through a statistical coefficient Ψ_2, which value can be equal to 0,3, 0,4 and 0,6, depending for what purpose is designed the use of the structure. The almost-permanent condition is calculated by Eq. (23).

$$F_{d,ser} = \sum_{i=1}^{n} F_{gi,k} + \sum_{i=1}^{m} \psi_{2j} \cdot F_{qj,k} \quad (23)$$

In Eq. (23), F_g represents the values of the intensities of the dead load and F_q represents the values of the intensities of the variables live loads.
Having considered the beam equivalent stiffness that represents an average behavior of the whole beam, it can proceed to the estimative of the immediate deflection δ by means of the equations of Materials Strength which are valid for constant sections along the structural element, i.e.:

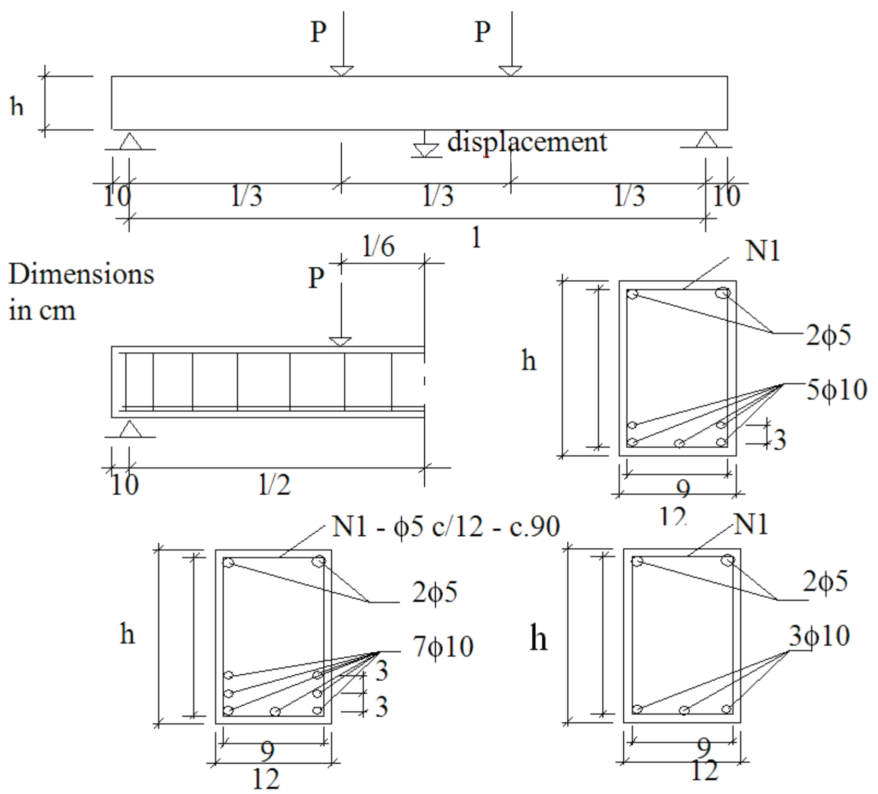
$$\delta = \frac{\alpha_c p l^4}{(EI)_{eq}} \quad (24)$$

where:
- αc is a coefficient which depends on the boundary conditions of the beam and on the kind of acting loads;
- p is the load applied;
- l is the span length of the beam.
- (EI)eq is the equivalent stiffness of the cracked beam given by the Elasticity Modulus of the concrete and the inertia moment in the Stage II (eq. (20)).

2.3 ANOVA Methodology

The Variance Analysis (ANOVA) is a statistic test used by analysts, and seeks fundamentally to verify if there is a meaningful difference between the averages and if the factors carry influence in some dependent variable, [5].
The factors proposed can be of qualitative or quantitative origin, but the dependent variable necessarily must be continuous. The

Figure 1 – Geometry properties of the test prototypes



main application of ANOVA is the comparison between averages coming from different groups, also called treatments.
There are two types of problems to be solved by ANOVA: fixed factors or random factors. The randomness determines the question of the problem. In most cases, it deals with fixed factors; after all, the second type of problem (random) will only arise when there is a study involving a random choice of factors.
In the variance analysis developed in this work, fixed factors have been used and it has been chosen five study variables: compressive strength of the concrete; boundary conditions; transversal section; the effective span length; the quantity of steel bars of the longitudinal reinforcement. The chosen variables reached one hundred eight cases of combinations, they are: span length, steel area, inertia moment and correlations between span length and steel area, span length and inertia moment and, finally, steel area and inertia moment. Further information of the methodology developed can be seen in [2].

3. Test models

The one-dimensional version of the damage model has been implemented in a program for bars structures analysis with finite layered elements. For calculus purposes, the weight of the beams has been taken into account in the finite element models as permanent loading.
Now, the finite element models of the beams used in order to verify the influence of some parameters in the estimative of displacements are described. In this work, these models are called "test prototypes", and they have been used in the numerical analyses in order to compare with analytical responses given by the NBR Procedure [1].
In this work, some parameters involved in the problem have been changed, such as: effective span length, height of the cross section, reinforcement distribution, compression strength of the concrete and the boundary conditions. Therefore, the testing models set add up a total of 324 cases, where there are three types of concretes, three types of reinforcement distribution, three different span lengths, three different heights of the transversal section and four types of boundary conditions, beyond the possible combinations of these cases.. The finite element models are named according to the properties contained in the Table 1 and their geometries are described in the Fig. 1. It is important to note that the Fig. 1 can represent the boundary conditions as simply supported, fixed and simply supported, bi-fixed, fixed and free ends (cantilever beam).
In order to check the vertical displacement obtained by numerical analyses presented in this work, it has been calculated analytically the vertical displacements of the RC beams submitted to the action of bending moment, using the criteria suggested by NBR Procedure [1], where it has been considered as permanent loads the weight of the beams and as accidental variable loads, the force

Table 1 – Properties of the test prototypes for simply supported beam

Beam	fck (MPa)	Span (m)	As (cm^2)	Beam	fck (MPa)	Span (m)	As (cm^2)	Beam	fck (MPa)	Span (m)	As (cm^2)
V31 - 12x30	30.8	2.4	2.36	V32 - 12x40	30.8	2.4	2.36	V33 - 12x50	30.8	2.4	2.36
V51- 12x30	30.8	2.4	3.93	V52 - 12x40	30.8	2.4	3.93	V53 - 12x50	30.8	2.4	3.93
V71 - 12x30	30.8	2.4	5.5	V72 - 12x40	30.8	2.4	5.5	V73 - 12x50	30.8	2.4	5.5
V34 - 12x30	30.8	3	2.36	V35 - 12x40	30.8	3	2.36	V36 - 12x50	30.8	3	2.36
V54 - 12x30	30.8	3	3.93	V55 - 12x40	30.8	3	3.93	V56 - 12x50	30.8	3	3.93
V74 - 12x30	30.8	3	5.5	V75 - 12x40	30.8	3	5.5	V76 - 12x50	30.8	3	5.5
V37 - 12x30	30.8	2	2.36	V38 - 12x40	30.8	2	2.36	V39 - 12x50	30.8	2	2.36
V57 - 12x30	30.8	2	3.93	V58 - 12x40	30.8	2	3.93	V59 - 12x50	30.8	2	3.93
V77 - 12x30	30.8	2	5.5	V78 - 12x40	30.8	2	5.5	V79 - 12x50	30.8	2	5.5
V311 - 12x30	30	2.4	2.36	V322 - 12x40	30	2.4	2.36	V333 - 12x50	30	2.4	2.36
V511- 12x30	30	2.4	3.93	V522 - 12x40	30	2.4	3.93	V533 - 12x50	30	2.4	3.93
V711 - 12x30	30	2.4	5.5	V722 - 12x40	30	2.4	5.5	V733 - 12x50	30	2.4	5.5
V344 - 12x30	30	3	2.36	V355 - 12x40	30	3	2.36	V366 - 12x50	30	3	2.36
V544 - 12x30	30	3	3.93	V555 - 12x40	30	3	3.93	V566 - 12x50	30	3	3.93
V744 - 12x30	30	3	5.5	V755 - 12x40	30	3	5.5	V766 - 12x50	30	3	5.5
V377 - 12x30	30	2	2.36	V388 - 12x40	30	2	2.36	V399 - 12x50	30	2	2.36
V577 - 12x30	30	2	3.93	V588 - 12x40	30	2	3.93	V599 - 12x50	30	2	3.93
V777 - 12x30	30	2	5.5	V788 - 12x40	30	2	5.5	V799 - 12x50	30	2	5.5
V3111 - 12x30	25	2.4	2.36	V3222 - 12x40	25	2.4	2.36	V3333 - 12x50	25	2.4	2.36
V5111- 12x30	25	2.4	3.93	V5222 - 12x40	25	2.4	3.93	V5333 - 12x50	25	2.4	3.93
V7111 - 12x30	25	2.4	5.5	V7222 - 12x40	25	2.4	5.5	V7333 - 12x50	25	2.4	5.5
V3444 - 12x30	25	3	2.36	V3555 - 12x40	25	3	2.36	V3666 - 12x50	25	3	2.36
V5444 - 12x30	25	3	3.93	V5555 - 12x40	25	3	3.93	V5666 - 12x50	25	3	3.93
V7444 - 12x30	25	3	5.5	V7555 - 12x40	25	3	5.5	V7666 - 12x50	25	3	5.5
V3777 - 12x30	25	2	2.36	V3888 - 12x40	25	2	2.36	V3999 - 12x50	25	2	2.36
V5777 - 12x30	25	2	3.93	V5888 - 12x40	25	2	3.93	V5999 - 12x50	25	2	3.93
V7777 - 12x30	25	2	5.5	V7888 - 12x40	25	2	5.5	V7999 - 12x50	25	2	5.5

Note: In case of bi-fixed beams are added the letters be at the end of the name, as well as ea to fixed-supported and e to cantilever beam.

Table 2 – Parameters of the damage model for the concretes used in this work

	Concrete fck = 25.0 MPa		Concrete fck = 30.8 MPa		Concrete fck = 30.0 MPa	
	Tension	Compression	Tension	Compression	Tension	Compression
Y_{01} / Y_{02} (MPa)	1.137×10^{-4}	Y_{02} =0.5×10^{-5}	Y_{01} = 0.72×10^{-4}	Y_{02} =0.5×10^{-3}	Y_{01} = 0.72×10^{-4}	Y_{02} =1.7×10^{-3}
A_1 / A_2	A_1 = 5.33	A_2 = -0.0086	A_1 = 50	A_2 = -0.9	A_1 = 50	A_2 = -0.8
B_1/B_2 (MPa^{-1})	B_1 = 5660	B_2 = 5.71	B_1 = 6700	B_2 = 0.4	B_1 = 6700	B_2 = 1.1

values of the F_r and $3F_r$, applied to the l/3 distances and 2l/3 from the support of the left of the beam (see Fig. 1). The force Fr has been obtained by Eq. (25) and its value depends on the cracking moment value (Eq. 21).

$$F_r = \left(M_r - \frac{g \cdot \ell^2}{8} \right) \cdot \frac{2}{\ell} \tag{25}$$

In Eq. (25), F_r is the value of the force intensity that composes the cracking process, g represents the weight of the reinforced concrete beam and l is the span length of the beam.

Note that, the parametric identification of the damage model for the concretes with compression strength of 25MPa, 30 MPa and 30,8 MPa used in this work is presented in [2] and [4], as well as the employment of the damage model in the numerical analyses of RC beams and frames is presented in [6], [7] and [8]. Those results are compared with experimental ones in order to validate the employment of the damage model. The parameters are presented in Table 2.

Table 3 – Displacement values obtained by the NBR 6118:2007 Procedure and numerical tests for Fr. (cantilever beam, 30.8 MPa)

Beam	P = Fr				
	Fr NBR (KN)	Fr Num (KN)	Disp. NBR (cm)	Disp. Num (cm)	Difference (%)
V31e- 12x30	2.24	1.84	0.20	0.105	47.50
V51e- 12x30	2.24	1.88	0.20	0.105	47.50
V71e - 12x30	2.24	1.90	0.20	0.105	47.50
V34e - 12x30	1.30	1.51	0.29	0.170	41.38
V54e - 12x30	1.30	1.50	0.29	0.162	44.14
V74e - 12x30	1.30	1.51	0.29	0.162	44.14
V37e - 12x30	3.08	2.22	0.14	0.080	42.86
V57e - 12x30	3.08	2.28	0.14	0.080	42.86
V77e - 12x30	3.08	2.30	0.14	0.080	42.86
V32e - 12x40	4.46	3.27	0.15	0.083	44.67
V52e - 12x40	4.46	3.31	0.15	0.082	45.33
V72e- 12x40	4.46	3.36	0.15	0.082	45.33
V35e - 12x40	2.92	2.60	0.23	0.127	44.78
V55e - 12x40	2.92	2.60	0.23	0.124	46.09
V75e - 12x40	2.92	2.52	0.23	0.119	48.26
V38e - 12x40	5.87	3.87	0.11	0.060	45.45
V58e - 12x40	5.87	3.82	0.11	0.060	45.45
V78e - 12x40	5.87	3.87	0.11	0.060	45.45
V33e - 12x50	7.41	5.02	0.12	0.067	44.17
V53e - 12x50	7.41	5.12	0.12	0.069	42.50
V73e - 12x50	7.41	5.21	0.12	0.069	42.50
V36e - 12x50	5.12	4.01	0.19	0.110	42.11
V56e - 12x50	5.12	3.94	0.19	0.101	46.84
V76e - 12x50	5.12	4.00	0.19	0.101	46.84
V39e - 12x50	9.55	5.76	0.09	0.050	44.44
V59e - 12x50	9.55	5.90	0.09	0.050	44.44
V79e - 12x50	9.55	6.00	0.09	0.050	44.44

According with experimental data reported in [6], the first concrete has tension strength of 2.3 MPa and elasticity modulus of 32,300 MPa. The second concrete has tension strength of 2.25 MPa and elasticity modulus of 29,200 MPa, [8]. The third one, according with [7], has 30,400 MPa for the elasticity modulus. The steel used in the reinforcement has Es = 196,000 MPa and yielding stress of 500 MPa. It is important to note that the finite element models have been tested in order to obtain the objectivity of the meshes used here, [2] and [4]. Therefore, in the numerical analyses the geometry symmetries has been taken into account and only half beam has been analyzed. The longitudinal discretization has been composed by 16 finite elements whereas for the cross section, 15 layers representing concrete and/or steel have been employed.

4. Numerical, analytical and statistical results

Due to the high number of results, in the following tables are described some of them. The results consist in vertical displacements in the middle of the span of each test prototype obtained by the employment of the NBR 6118:2007 Procedure [1], as well as those obtained in the numerical analyses. The values P=Fr and P=3Fr have been considered in order to analyze the behavior of the NBR 6118:2007 [1] Procedure related to the evolution of the damage process on the beams.

The percentage values of the difference between the results have been calculated adopting the ones recommended by NBR 6118:2007 [1]as reference values.

It can be observed on the tables above the conservatism of the calculation procedure of NBR 6118:2007 [1], being reflected, in most cases, in differences about 30% to 50%. In the case of bi-fixed beams, the difference becomes more relevant (Table (4)). In general, it can even be observed that the differences between the displacement values decrease with the increase of the applied load Fr. The model adopted by NBR6118:2007 [1] approaches the beam stiffness using just only one value to whole beam leading to high displacement values. Otherwise, the damage model degrades selectively the longitudinal elasticity modulus of each concrete layer in each finite element along beam giving a more realistic simulation of the damage process on the beam, which usually results in smaller displacements than those obtained with NBR6118:2007 [1] analytical model. It can be observed that the tensioned concrete

Table 4 – Displacement values obtained by the NBR 6118:2007 Procedure and numerical tests for Fr. (bi-fixed beam, 30.0 MPa)

Beam	P = Fr				
	Fr NBR (KN)	Fr Num (KN)	Disp. NBR (cm)	Disp. Num (cm)	Difference (%)
V311be- 12x30	13.85	12.06	0.10	0.018	82.00
V511be- 12x30	13.85	12.33	0.10	0.018	82.00
V711be - 12x30	13.85	12.44	0.10	0.018	82.00
V344be - 12x30	10.72	11.47	0.15	0.032	78.67
V544be - 12x30	10.72	11.71	0.15	0.032	78.67
V744be - 12x30	10.72	11.80	0.15	0.032	78.67
V377be - 12x30	16.92	19.02	0.07	0.017	75.71
V577be - 12x30	16.92	19.18	0.07	0.017	75.71
V777be - 12x30	16.92	15.53	0.07	0.014	80.00
V322be - 12x40	24.99	28.04	0.07	0.020	71.43
V522be - 12x40	24.99	28.50	0.07	0.020	71.43
V722be- 12x40	24.99	28.63	0.07	0.020	71.43
V355be - 12x40	19.50	15.98	0.11	0.034	69.18
V555be - 12x40	19.50	16.36	0.11	0.022	80.00
V755be - 12x40	19.50	16.58	0.11	0.022	80.00
V388be - 12x40	30.38	27.37	0.05	0.017	66.00
V588be - 12x40	30.38	27.56	0.05	0.011	78.00
V788be - 12x40	30.38	27.41	0.05	0.017	66.00
V333be - 12x50	39.38	36.54	0.06	0.022	63.33
V533be - 12x50	39.38	36.66	0.06	0.022	63.33
V733be - 12x50	39.38	36.63	0.06	0.022	63.33
V366be - 12x50	30.9	32.07	0.09	0.025	72.22
V566be - 12x50	30.9	32.73	0.09	0.025	72.22
V766be - 12x50	30.9	33.18	0.09	0.025	72.22
V399be - 12x50	47.75	47.14	0.04	0.017	57.50
V599be - 12x50	47.75	47.57	0.04	0.017	57.50
V799be - 12x50	47.75	47.73	0.04	0.017	57.50

Table 5 – Displacement values obtained by the NBR 6118:2007 Procedure and numerical tests for 3Fr. (simply supported beam, 30.8 MPa)

Beam	P = 3.Fr				
	Fr NBR (KN)	Fr Num (KN)	Disp. NBR (cm)	Disp. Num (cm)	Difference (%)
V31- 12x30	27.42	18.84	0.57	0.35	38.60
V32 - 12x40	49.83	31.17	0.5	0.31	38.00
V33 - 12x50	78.87	61.05	0.45	0.38	15.56
V34 - 12x30	20.85	14.67	0.85	0.53	37.65
V35 - 12x40	38.40	23.55	0.75	0.43	42.67
V36 - 12x50	61.26	46.17	0.69	0.55	20.29
V37 - 12x30	33.78	22.11	0.40	0.24	40.00
V38 - 12x40	60.96	50.76	0.35	0.31	11.43
V39 - 12x50	96.12	53.79	0.32	0.18	43.75
V51- 12x30	27.42	19.32	0.44	0.26	40.91
V52 - 12x40	49.83	31.98	0.37	0.21	43.24
V53 - 12x50	78.87	62.67	0.33	0.26	21.21
V54 - 12x30	20.85	14.73	0.66	0.38	42.42
V55 - 12x40	38.40	24.15	0.56	0.31	44.64
V56 - 12x50	61.26	47.37	0.51	0.38	25.49
V57 - 12x30	33.78	22.68	0.31	0.18	41.94
V58 - 12x40	60.96	75.42	0.26	0.33	-26.92
V59 - 12x50	96.12	55.11	0.23	0.13	43.48
V71 - 12x30	27.42	19.53	0.40	0.23	42.50
V72 - 12x40	49.83	32.46	0.32	0.18	43.75
V73 - 12x50	78.87	63.78	0.28	0.22	21.43
V74 - 12x30	20.85	14.91	0.60	0.33	45.00
V75 - 12x40	38.40	24.51	0.49	0.26	46.94
V76 - 12x50	61.26	48.21	0.43	0.31	27.91
V77 - 12x30	33.78	22.92	0.28	0.16	42.86
V78 - 12x40	60.96	53.07	0.23	0.18	21.74
V79 - 12x50	96.12	56.01	0.2	0.1	50.00

between cracks is taken into account in the resistance to the bending moment according to the damage model, such fact does not happen in the formulation of the model used by NBR6118:2007 [1]. It can also be observed that NBR6118:2007 [1] provides one only value of M_r regardless the reinforcement arrangement disposed in the beam. However, the numerical analyses show a variation in M_r value, which would be more natural because the mechanical behavior of the beam will obviously be influenced by the reinforcement arrangement from the beginning of the cracking process to its collapse, among other factors, [2].

In [2] are presented the statistical analyses performed with ANOVA methodology for the case of the simply supported and bi-fixed beams with the variation of parameters already mentioned in the introduction of this work. Then, in [2] has been observed that the transversal section and span length are the most important variables in the problem when the beam is subjected to moderate val-

Table 6 – Simply supported beam, fck=25 MPa, analytical values, F = Fr

Simply supported beam		C25	$F=F_R$	Analytical	
Ultimate force					
Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical, 0.5}$ N=26
ℓ	8.6×10^{-3}	2	4.3×10^{-3}	4.3×10^{-3}	3.37
A_s	0	2	0	0	3.37
A_c	4.2×10^{-3}	2	2.1×10^{-3}	2.1×10^{-3}	3.37
ℓ X A_s	0	4	0	0	2.74
ℓ X A_c	4×10^{-4}	4	10×10^{-5}	10×10^{-5}	2.74
A_s X A_c	0	4	0	0	2.74
Error	0	8	0	0	–
Total	0.0013	26	–	–	–

ues of the service loads. However, when the loading value increases, the transversal section keeps the most important variable, but the reinforcement distribution becomes a more important variable than the span length. This change is due to the very intense damage process which occurs in the beam in this loading stage.
In this work, some additional parameters are introduced, such as: one more concrete with compression strength of 25 MPa and two new boundary conditions (cantilever beam and fixed and simply supported beam). Moreover, in order to overcome a gap left in the work [2], the change of span length from 4 m to 2 m has been made. The, now, the cantilever beam cases have analytical and numerical possible results, in order to contribute for the statistical analyses.
Once again, it is necessary the presentation of some tables with statistical results referring to concrete C25 in simply supported beams, as example. In fact, the analyses lead to the making of 48 tables.
Closing, it is related here that the results of the statistical analyses do not show any evident change of the behavior of the problem

Table 7 – Simply supported beam, fck=25 MPa, numerical values, F = Fr

Simply supported beam		C25	F=F_R	Analytical	
Ultimate force					
Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical, 0.5}$ N=26
ℓ	1.064×10^{-3}	2	5.32×10^{-4}	8.02	3.37
A_s	3.5×10^{-5}	2	1.733×10^{-5}	0.261	3.37
A_c	1.026×10^{-3}	2	5.13×10^{-4}	7.734	3.37
ℓ X A_s	3.653×10^{-4}	4	9.133×10^{-5}	1.377	2.74
ℓ X A_c	7.06×10^{-4}	4	1.765×10^{-4}	1.015	2.74
A_s X A_c	2.693×10^{-4}	4	6.733×10^{-5}	2.661	2.74
Error	5.307×10^{-4}	8	6.633×10^{-5}	–	–
Total	3.996×10^{-3}	26	–	–	–

Table 8 – Simply supported beam, fck=25 MPa, analytical values, F = 3 Fr

Simply supported beam		C25	F=3F_R	Analytical	
Ultimate force					
Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical, 0.5}$ N=26
ℓ	0.391	2	0.195	1.407×10^{4}	3.37
A_s	0.123267	2	0.062	4.438×10^{3}	3.37
A_c	0.051	2	0.026	1.854×10^{3}	3.37
ℓ X A_s	0.011	4	2.694×10^{-3}	194	2.74
ℓ X A_c	4.489×10^{-3}	4	1.122×10^{-3}	3.2	2.74
A_s X A_c	1.778×10^{-4}	4	4.444×10^{-5}	80.8	2.74
Error	1.111×10^{-4}	8	1.389×10^{-5}	–	–
Total	0.581	26	–	–	–

Table 9 – Simply supported beam, fck=25 MPa, numerical values, F = 3Fr

Simply supported beam		C25	F=3F_R	Numerical	
Ultimate force					
Factors	Squares sum	Freedom degrees	Squares average	F_0	$F_{critical, 0.5}$ N=26
ℓ	0.122	2	0.061	96.974	3.37
A_s	0.072634	2	0.036	57.826	3.37
A_c	0.032	2	0.016	25.656	3.37
ℓ X A_s	0.01	4	2.558×10^{-3}	4.073	2.74
ℓ X A_c	0.029	4	7.248×10^{-3}	4.392	2.74
A_s X A_c	0.011	4	2.758×10^{-3}	11.541	2.74
Error	0.005024	8	6.28×10^{-4}	–	–
Total	0.282	26	–	–	–

Figure 2 – Cracking moment for the concrete C25

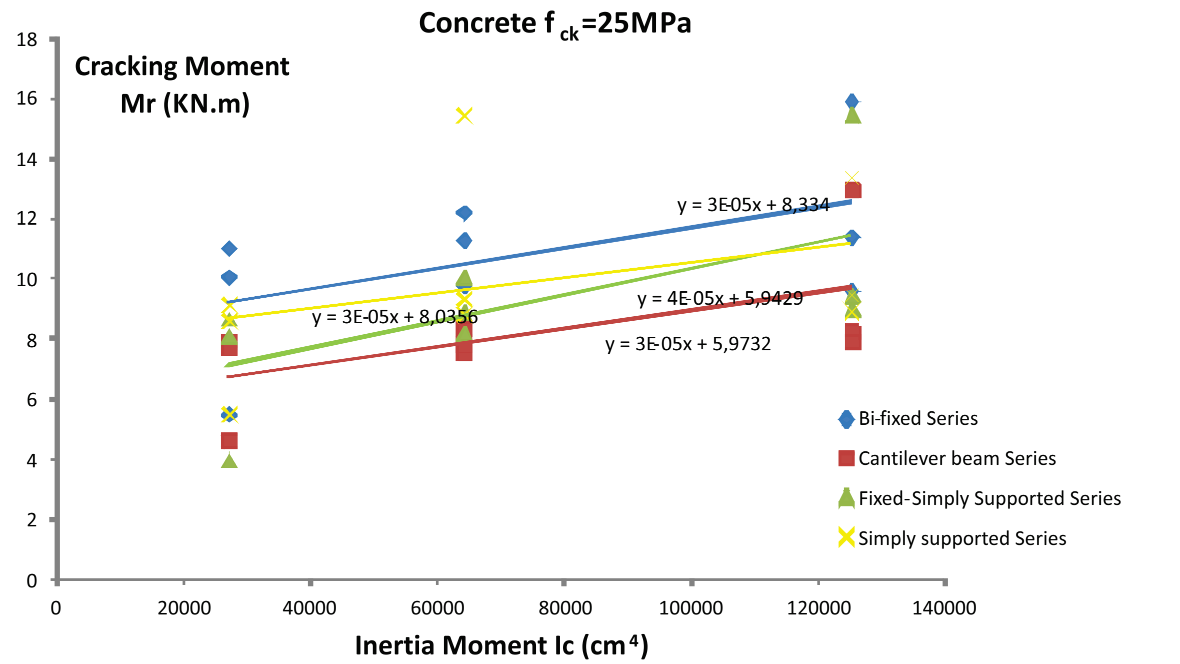


Figure 3 – Cracking moment for the concrete C30

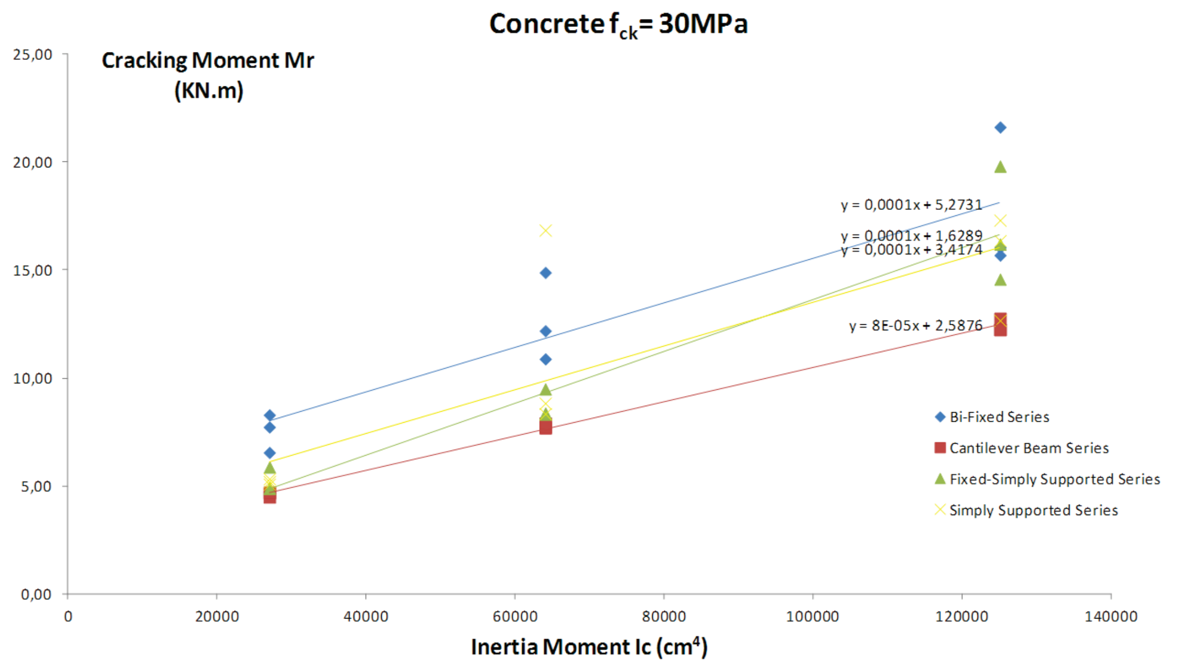
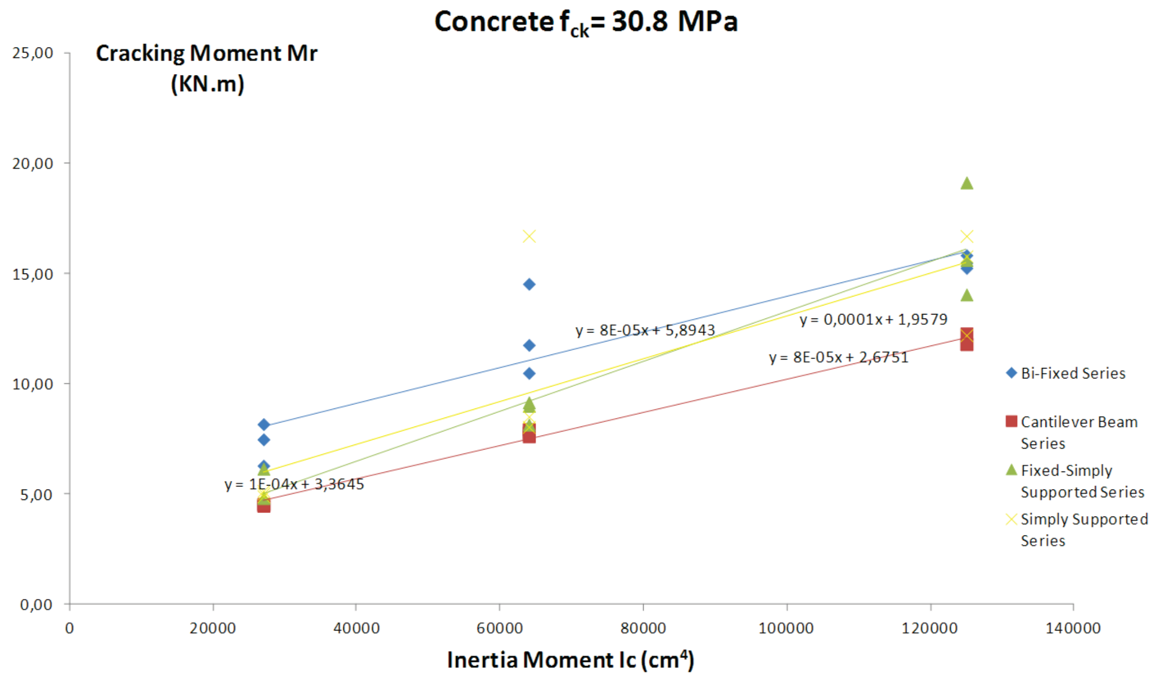


Figure 4 – Cracking moment for the concrete C30,8



variables when the new parameters have been introduced in this work. Therefore, in the beginning of the cracking process in $F = F_r$, the transversal section of the beam is the most influent variable in the problem, followed by the span length. When the cracking process is more evident, the steel reinforcement area starts to gain importance because the concrete does not resist efficiently to the efforts, mainly in the tensioned area of the beam.

4.1 Discussion about the proposals to the evaluation of the deflection in reinforced concrete structures

Based on the results obtained so far, both in this work or in previous one [2], in Fig. (2) it is illustrated a graphic containing the transversal section inertia versus the cracking moment of the numerical analyses for the case of concrete C25 and working with beams in domain 2 (5ø10.0mm). For each boundary condition case, it has been adopted a regression in a manner as simple as possible (linear) to obtain an expression for the estimation of the cracking moment related to the initial inertia of the transversal section. Note that, in this work, formulations as simplest as possible are adopted always thinking in the practical applicability of the study. The same procedure has been performed to the concrete C30 (Figure 3) and concrete with compression strength of 30.8 MPa (Figure 4).

Therefore, the equations below are proposals to be used for the called "conventional concretes", i. e., concretes that belongs to the classes C20 until C 35. Such statement is justified by the compression strength used in this work, where it is possible to extrapolate the results obtained for concretes in classes in the neighborhood of the concretes addressed here.

$$M_r = 0.000041 I_c + \beta_1 \text{ p/ concretos C20 e C25} \quad (26)$$

$$M_r = 0.000151 I_c + \beta_2 \text{ p/ concretos C30 e C35} \quad (27)$$

where, in the equations above, the values are expressed in kN.m for M_r and cm^4 for I_C. The values of β_1 and β_2 are given in Table 10. The proposed equations have been used in the analyses of this work and compared with the values recommended by NBR6118:2007 [1]

Table 10 – Values of the coefficients related to support conditions

Boundary conditions	β_1	β_2
Bi-fixed/simply supported	8	4.5
Cantilever beam/ fixed-supported	6	2

Table 11 – Values of the cracking moment given by proposed method and NBR6118:2007 procedure (KN.m)

I_c (cm^4)	Support conditions	Proposal C20/C25	NBR C20	NBR C25	dif. for C25 (%)	dif. for C20 (%)	Proposal C30/C35	NBR C30	NBR C35	dif. for C30 (%)	dif. for C35 (%)
27x10^3	Bi-fixed	9.08	5.97	6.93	-31.11	-52.14	8.55	7.82	8.67	-9.33	1.35
64x10^3	Bi-fixed	10.56	10.61	12.31	14.23	0.47	14.1	13.90	15.41	-1.42	8.49
125x10^3	Bi-fixed	13	16.58	19.24	32.42	21.58	23.25	21.72	24.07	-7.03	3.43
27x10^3	Cantilever beam	7.08	5.97	6.93	-2.233	-18.63	6.05	7.82	8.67	22.64	30.19
64x10^3	Cantilever beam	8.56	10.61	12.31	0.474	19.32	11.6	13.90	15.41	16.57	24.71
125x10^3	Cantilever beam	11	16.58	19.24	2.82	33.65	20.75	21.72	24.07	4.48	13.81
27x10^3	Fixed-supported	7.08	5.97	6.93	-2.233	-18.63	6.05	7.82	8.67	22.64	30.19
64x10^3	Fixed- supported	8.56	10.61	12.31	0.474	19.32	11.6	13.90	15.41	16.57	24.71
125x10^3	Fixed- supported	11	16.58	19.24	2.82	33.65	20.75	21.72	24.07	4.48	13.81
27x10^3	Simply supported	9.08	5.97	6.93	-31.11	-52.14	8.55	7.82	8.67	-9.33	1.35
64x10^3	Simply supported	10.56	10.61	12.31	14.23	0.47	14.1	13.90	15.41	-1.42	8.49
125x10^3	Simply supported	13	16.58	19.24	32.42	21.58	23.25	21.72	24.07	-7.03	3.43

(see Table 11). In general, the values obtained by the proposal are smaller, however, it is emphasized here that the numerical analyses have been performed with the use of a damage model which considers the cracking distributed in the structural element, then it is natural the contribution of the boundary conditions in this cracking panorama. It is possible to note on the table above, in general, the proposed model for M_r presents lightly superior values than NBR's for the cases of small inertias of the transversal section. Moreover, the proposed model presents results more closer to the NBR Procedure for the cases of medium inertias and presents smaller values than NBR's in the case of bigger inertias. It can be noted that the proposed model for C30/C35 has presented a better behavior than the proposal for C20/C25 when compared with the values presented by NBR.

On the other hand, in the case of the evaluation of the displacements, according to Materials Strength Theory, such calculation in structures can be a given function, in a general way, by:

$$\delta = \frac{\alpha_{ap} \cdot p \cdot l^3}{EI_c} \qquad (28)$$

where α_{ap} is a constant dependent on the boundary condition, p is the acting loading, l is the effective span length, I_c is the inertia moment of the transversal section and E is the concrete elasticity modulus. It can be observed that the main parameters involved in the problem, according to the results of ANOVA methodology, are contemplated in Eq. (28).

Nowadays, NBR6118:2007 [1] uses a procedure where the inertia moment of the transversal section is decreased when the cracking process takes place. This penalization procedure is homogeneous leading to only one value for the inertia moment to the whole beam. In this work, the stiffness degradation is focused on the decreasing of the Elasticity Modulus according to the approach given by Continuum Damage Mechanics.

It can be observed that when there is a cracking processes in progress, the Elasticity Modulus is function of a variable that defines the concrete cracking stage. This variable can be understood as damage (D). However, the own damage is dependent on the deformation of the structural system and related stresses. Such stress and strain states depend on the loading level applied in the structure, i.e., there is a non-linear relation in this whole process.

It can be observed that a relation that selectively degrades the stiffness of the structure by means of the Elasticity Modulus of the cracked concrete in different phases until its collapse, it is desirable. Even more if the parameters involved in this relation are of current use in the Structural Engineering. Therefore, it is proposed that the Elasticity Modulus be used in Eq. (28) as:

$$E = (1 - D)E_0 \qquad (29)$$

where E_0 is the Elasticity Modulus of the virgin concrete obtained by NBR 6118:2007 [1]. On the other hand, the damage process is dependent on the variables involved in the problem, such as: cracking moment, compression and tension strength of the concrete. It is also proposed that the damage variation is given by a non-linear relation illustrated in Fig. (5) and that it is dependent on the class of the concrete.

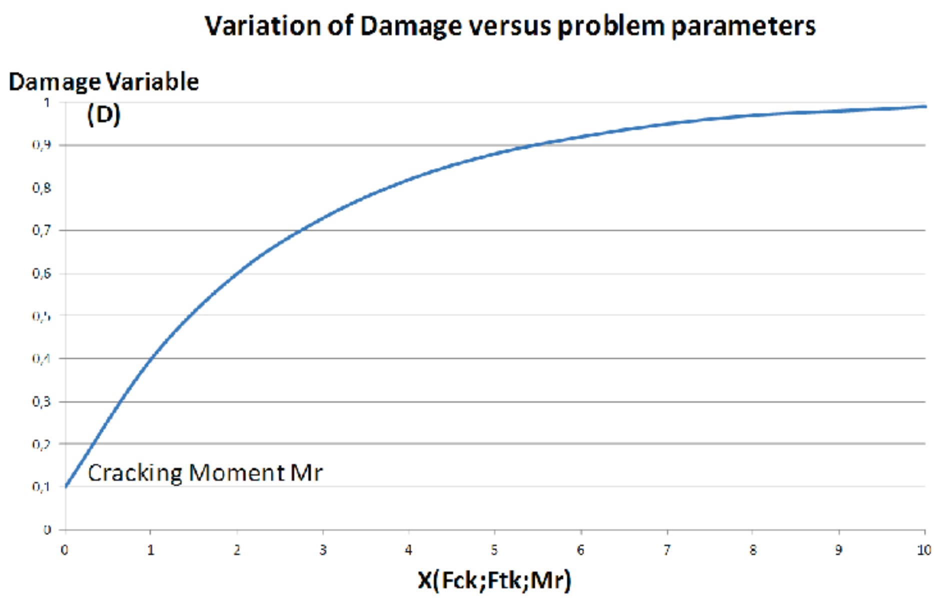
However, there are two ways to follow: it can be proposed an equation for variable D with the important parameters obtained by ANOVA or, it can be proposed an equation for D based in a regression, taking as a basis the numerical results obtained so far.

It is adopted the first option because it works with parameters with a more tangible physical meaning for the engineers, always remembering that this is the philosophy used here. Moreover, the second option demands the complexity of working in several dimensions with a enormous range of results. This can be studied in a future work.

Therefore, following the chosen option, in a given state of the efforts x displacements, it can be calculated the stiffness of the structural element keeping unchanged the inertia moment of the transversal section and using the Elasticity Modulus updated by Eq. (29) for a given acting bending moment on the most loaded section, since the acting bending moment be superior to the cracking moment of the structural element calculated by Eqs. (26) or (27). After some studies and, having as a basis a simple but efficient damage model, it is proposed the following expression for estimation of the variable D:

$$D = 1 - \frac{M_r(1-A)}{M_a} - \frac{A}{e^{\left[\frac{f_{ctm}}{1000}(M_a - M_r)\right]}} \qquad (30)$$

Figure 5 – General proposal for the damage variable related to the parameters involved in the problem



where M_a and M_r , are given in KN.m, A parameter is a value dependent on the concrete class and f_{ctm} is the medium direct tension strength or characteristic of the concrete given by Eq. (31) in MPa.

$$f_{ctm} = 0{,}3\, f_{ck}^{2/3} \qquad (31)$$

Fig. (6) presents a D x M_a graphic for a class C25, M_{cr} = 9,35 KN.m and A = 0,9. The use of Eq. (30) leads to a non pronounced stiffness degradation what generates a more realistic structural behavior.

Finally, the proposed model is used in the case of the beam tested in reference [6]. Such beam has been chosen because there is detailed information about the experimental test, as well as about the obtained values, giving a reliability for the comparison of results.

In Table 12, the values experimentally obtained for $M_{a,exp}$ (acting bending moment), $M_{r,exp}$ (cracking moment), δ_{exp} (displacement of the middle span), are compared with the values analytically

Figure 6 – Study about the variation of D related to bending moment acting on the critical section

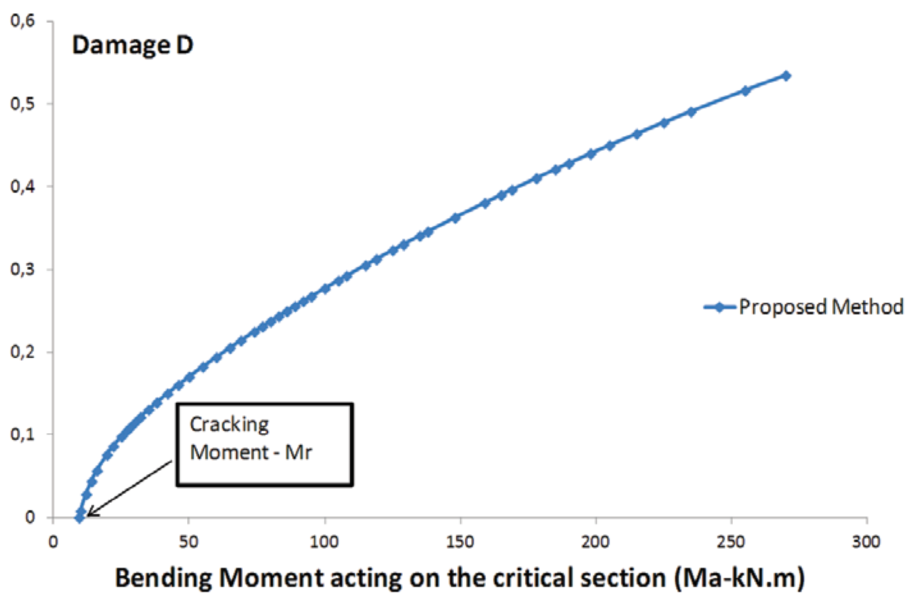


Table 12 – Values of the acting and cracking moments (KN.m) and displacement (mm) given by the experimental responses, proposed method and NBR6118:2007 procedure

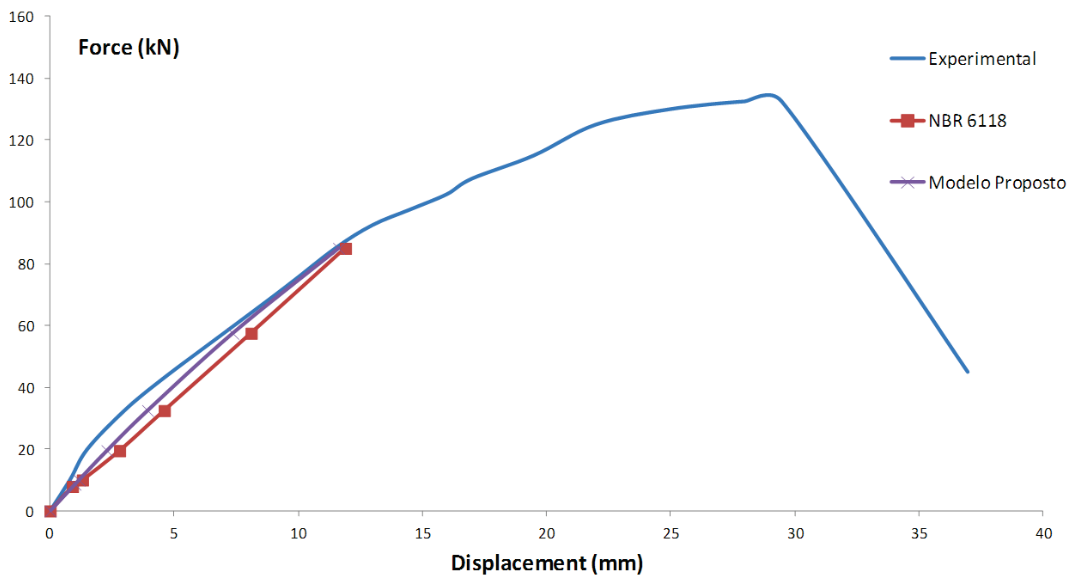
$M_{a,exp}$	$M_{r,exp}$	$M_{r,NBR}$	$M_{r,proposal}$	δ_{exp}	δ_{NBR}	$\delta_{proposal}$
–	19.69	8.66	9.35	–	–	–
10.64	–	–	–	0.8	1.3	1.16
19.69	–	–	–	2.3	2.8	2.29
32.02	–	–	–	3.0	4.6	3.93
55.77	–	–	–	7.0	8.1	7.36
81.89	–	–	–	11.5	11.9	11.62

obtained by NBR 6118:2007 ($M_{r,NBR}$, δ_{NBR}) and by the proposed methodology ($M_{r,proposto}$, $\delta_{proposto}$).
It can be observed results more realistic obtained by the use of the proposed model related to the experimental results than the ones obtained with the use of the procedure suggested by NBR. However, it is necessary to think about the existence of some safety reservation for the evaluation of displacement. Fig. (7) shows the comparison of the experimental results with those obtained by NBR and by the methodology proposed in this work.
Note that the proposed model to the evaluation of the deflections in the reinforced concrete structures depends on a deeper study on the results obtained so far for the conventional concretes (C20 to C35). In this sense, it is necessary to obtain more reliable experimental results for the validation of the proposal of this work, as well as for studies about its limitation and verification related to safety use. These features will be studied in a future work.

5. Conclusions

In this work a damage model for the concrete proposed by [3] has been used in the evaluation of deflections in reinforced concrete structures.
The parameters involved in the problem and its combinations have been found and, a total of 324 prototypes have been numerically analyzed using the damage model and, analytically analyzed using NBR6118:2007 Procedure. The application of ANOVA methodology confirms the conclusions obtained in [2], even with the inclusion of new parameters in the problem. In other words, it can be observed that the cracking moment does not take into account the reinforcement distribution. Moreover, it can be observed that the displacements obtained from the analytical analyses are greater than those ones obtained through numerical and experimental analyses. It is due to the fact that NBR6118:2007 [1] estimates an average value for stiffness of the whole beam leading to high dis-

Figure 7 – Comparison between experimental response and results obtained by the proposed method and NBR procedure



placement values. On the other hand, the damage model degrades the stiffness in a selective way, therefore it is possible to consider the contribution of tensioned concrete between cracks. However, the existence of a safety reservation always must be necessary.
In a general way, the ANOVA methodology shows the variables that must be contained in an eventual alternative formulation to the NBR6118:2007 Procedure [1]. Such proposal has been presented at the end of this work, where the focus about stiffness penalization becomes the Elasticity Modulus, following the basis given by Continuum Damage Mechanics. Initial tests have been performed in this work and the results shown the potentialities of the proposed methodology employment, but its effective validation and use limitation study will be objects of future studies. Besides, it is necessary to verify the use safety of the proposal in practical applications of the Structural Engineering. In sum, the results presented in this work encourage the authors to proceed in the development of this proposed methodology.

7. Acknowledgement

The financial support from CNPq (National Council for Scientific and Technological Development) is gratefully acknowledged.

8. References

[01] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. NBR 6118:2007, Projeto de estruturas de concreto – Procedimento. Rio de Janeiro, 2004.
[02] PITUBA, J. J. C., DELALIBERA, R. G., and RODRIGUES, F. S.. Numerical and statistical analysis about displacements in reinforced concrete beams using damage mechanics. Computers and Concrete, an International Journal, Vol. 10(3), 307-330, 2012.
[03] PITUBA, J. J. C.. and FERNANDES, G. R.. An anisotropic damage for the concrete. Journal of Engineering Mechanics - ASCE, Vol. 137(9), 610-624, 2011.
[04] PITUBA, J. J. C.. and LACERDA, M. M. S.. Simplified damage models applied in the numerical analysis of reinforced concrete structures. IBRACON Structures and Materials Journal, Vol. 5(1), 26-37, 2012.
[05] MONTGOMERY, D. C.. Design and analysis of experiments, Arizona State University, 4th Edition, John Wiley & Sons, 1996.
[06] DELALIBERA, R. G.. .Análise teórica e experimental de vigas de concreto armado com armadura de confinamento. Dissertação de Mestrado, Escola de Engenharia de São Carlos, Universidade de São Paulo, 2002.
[07] VECCHIO, F. J. and Emara, M. B.. Shear deformations in reinforced concrete frames. ACI Structural Journal, Vol. 89, n. 1, p. 46-56, 1992.
[08] ÁLVARES, M. S.. Estudo de um modelo de dano para o concreto: formulação, identificação paramétrica e aplicação e emprego do método dos elementos finitos. Dissertação de Mestrado, Escola de Engenharia de São Carlos, Universidade de São Paulo, 1993.