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# Global stability analysis of reinforced concrete buildings using the $\gamma_z$ coefficient

# *Análise da estabilidade global de edifícios de concreto armado utilizando o coeficiente $\gamma_z$*



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## Abstract

Global stability analysis is becoming increasingly important in the design of reinforced concrete buildings, especially in the slender ones, due its sensitivity to lateral displacement. The loss of stability is usually associated with the intensity of the second order effects and, in that sense, the gamma-z ($\gamma_z$) coefficient is an important evaluation parameter for this problem. This work aims to verify the $\gamma_z$ efficiency as a global stability parameter based on the buckling load factors of the structures and their respective critical buckling modes. To this purpose, a comparative analysis is performed in several idealized structures, from which an approximate equation for calculating the critical load factor based on the $\gamma_z$ coefficient is obtained. This equation was verified by numerical analysis of Finite Elements Method models of real reinforced concrete buildings. It was concluded that the proposed equation presents satisfying results within a certain range of $\gamma_z$.

***Keywords***: global stability, second order global effects, gamma-z coefficient, critical load factor, reinforced concrete buildings.

## Resumo

A análise da estabilidade global tem se tornado cada vez mais importante no projeto de edifícios de concreto armado, sobretudo nos mais esbeltos, por serem mais sensíveis aos deslocamentos laterais. A perda de estabilidade é usualmente associada à intensidade dos efeitos de 2ª ordem e, nesse sentido, o coeficiente gama-z ($\gamma_z$) torna-se um importante parâmetro de avaliação deste problema. O objetivo deste trabalho é verificar a eficiência do $\gamma_z$ como parâmetro de estabilidade global, tomando como base os fatores de carga de flambagem das estruturas e os respectivos modos críticos de instabilidade. Para esta finalidade, é realizada uma análise comparativa de diversas estruturas idealizadas, de onde obteve-se uma equação para o cálculo aproximado do fator de carga crítica em função do coeficiente $\gamma_z$. A validação dessa equação foi realizada por meio da análise numérica de modelos em Elementos Finitos de edifícios reais de concreto armado. Constatou-se que a equação proposta oferece resultados satisfatórios para um certo intervalo de $\gamma_z$.

***Palavras-chave:*** estabilidade global, efeitos globais de 2ª ordem, coeficiente gama-z, fator de carga crítica, edificios de concreto armado.

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## 1. Introduction

The global stability verification is a fundamental requisite on the design of a reinforced concrete building for it doesn't present future problems that would affect its safety and, consequently, increase its risk of collapse. Tall and slender buildings are, generally, more sensitive to lateral displacements and designers should consider the effects.

A rigorous stability analysis involves the prediction of the structures' equilibrium path, just as the determination of its critical loads and instability modes. However, in most of structural analysis, the main interest is merely to determine critical loads and respective instability modes.

Most precise global stability analysis is not a simple process, being sophisticated computational resources necessary. It evaluates the current condition of the structure regarding its stability limit through the relation of its critical load to the applied vertical load. In addition, this analysis provides the most critical deformed configuration of the structure.

Usually, during reinforced concrete buildings design, the global stability analysis is limited to considering or not the additional forces due the second order effects. Thus, one notes that there is no concern in evaluating the structure's safety regarding its global instability critical load.

A simple manner to estimate the second order effects without the need for a geometrically non-linear analysis is through the gamma-z coefficient ($\gamma_z$), a parameter obtained from a linear analysis that aims to evaluate the magnitude of the second order effects, being frequently used by designers as a reference parameter for global stability analysis.

Brazilian Standards 6118 [1] recommends that the $\gamma_z$ coefficient should be applied, within certain limits, in the evaluation of the importance of the second order global effects, as well as in the amplification of the first order effects for the estimation of final forces in the structure. However, these standards do not provide a superior limit that aims to restrain the magnitude of the second order effects in a way that the structures be free of global instability problems.

This paper aims is to establish a relation between the $\gamma_z$ coefficient and the critical global buckling load factor according to concepts presented on the literature and trough the analysis of idealized structures with simplified geometry. This relation will further be transformed into an approximate equation which allows to estimate the critical load factor from the $\gamma_z$ coefficient. Later, some examples of real buildings are analyzed in order to validate the proposed equation.

For the modeling and processing of the structures, both idealized and real buildings, the computational software SAP2000® V16.0.0, one of the most known structural analysis systems in the market, was used.

## 2. Second order effects

The second order effects appear when the equilibrium equation is taken considering the deformed configuration of the structure, which causes a geometrically non-linear behavior.

According to Wight and Macgregor [2], by a second order analysis it is possible to verify the global stability of the structure, once the instability occurs due the loss of equilibrium of the deformed structure. Kimura [3] states that the larger the second order effects are, less stable the structure is and because of that the stability of a building may be evaluated by the calculation or estimative of these effects.

As a way to simplify these analysis, the NBR 6118 [1] allows one to disregard the second order effects when they are not superior than 10% to the first order effects. This criterion is equivalent to the one adopted by the Eurocode 2 [4]. However, it is not suggested in none of these codes a superior limit that aims to prevent the collapse of the structure due loss of stability caused by excessive lateral displacements.

The ACI 318 [5] proposes that the consideration or not of the second order effects must be assessed in each floor of the building, obeying a limit of 5% relative to the first order effects, in order to be ignored. This code also specifies a superior limit of 40% for the total second order moments relative to the first order ones, ensuring the global stability of the structure when this condition is satisfied.

As the second order effects require a nonlinear analysis, there can be used reference parameters for performing a simplified verification of the importance of these effects and, consequently, of the global stability. For this purpose, the Brazilian Standards recommends the use of the alfa ($\alpha$) and gamma-z ($\gamma_z$) coefficients. Only the latter will be discussed in this paper because it is the most commonly used.

Besides the mentioned parameters, another method to evaluate the second order effects in reinforced concrete buildings uses the ratio between the total vertical load and the critical global load, named *instability index* by MacGregor and Hage (apud Fonte [6]). This parameter and the $\gamma_z$ coefficient are discussed in the following sections.

## 3. $\gamma_z$ coefficient

The $\gamma_z$ is a parameter created by Franco and Vasconcelos [7], which aims to evaluate the importance of the second order effects in frame structures of at least four stores based on a first order linear analysis, being very convenient for structural analysis.

Vasconcelos [8] explains that this method is based on the hypothesis that the successive elastic lines, induced by the applied vertical load on the deformed structure follow a geometric progression. The NBR 6118 [1] determines that, for each load combination, the $\gamma_z$ coefficient is calculated by:

$$\gamma_z = \frac{1}{1 - \frac{\Delta M_{tot,d}}{M_{1,tot,d}}} \tag{1}$$

Where:

$\Delta M_{tot,d}$ is the sum of the products of the total applied vertical forces on the structure in the considered combination by the horizontal displacements of their respective application points obtained from the first order analysis and;

$M_{1,tot,d}$ is the sum of the moments generated by all the horizontal forces of the considered combination taking the basis of the structure as reference.

Feitosa and Alves [9] explain that changes in the horizontal loads do not influence $\gamma_z$, for the second order forces would be modified proportionally to the first order ones, in this case. Thus, the factors that alter this coefficient are the vertical load and structure's stiffness.

To consider the physical non-linearity is mandatory for design and can be performed in an approximate manner by reducing the stiffness of the structural members as follows:

Slabs: $(EI)_{sec} = 0{,}3E_{ci}I_c$ **(2)**

Beams: $(EI)_{sec} = 0{,}4E_{ci}I_c$ for $A'_s \neq A_s$ and **(3)**

$(EI)_{sec} = 0{,}5E_{ci}I_c$ for $A'_s = A_s$ **(4)**

Columns: $(EI)_{sec} = 0{,}8E_{ci}I_c$ **(5)**

Where $I_c$ is the second moment of inertia of concrete, including, for T beams, the flange contribution; $E_{ci}$ is the concrete's initial elasticity modulus; $A_s$ is the area of steel in tension and $A'_s$ is the area of steel in compression.
The $\gamma_z$ coefficient also has the advantage can be used as an average amplifier of the first order effects for the approximate calculation of the final forces of the structure. The NBR 6118 [1] admits that the horizontal internal forces may be multiplied by $0{,}95\gamma_z$ so the second order effects to be considered, since $\gamma_z$ is no greater than 1,30.
The Brazilian Standards don't propose a superior limit for $\gamma_z$ to ensure the global stability of the structure. Vasconcelos and França [10] states that for values greater 1,30 the structure is excessively flexible, requiring further analysis by other methods in order to avoid problems due vibrations and resonance. As reported by Kimura [3], buildings with $\gamma_z$ above 1,30 have a high degree of instability. In addition, the author recommends 1,20 as the maximum value to be used during design.

## 4. Critical global buckling load factor (λ)

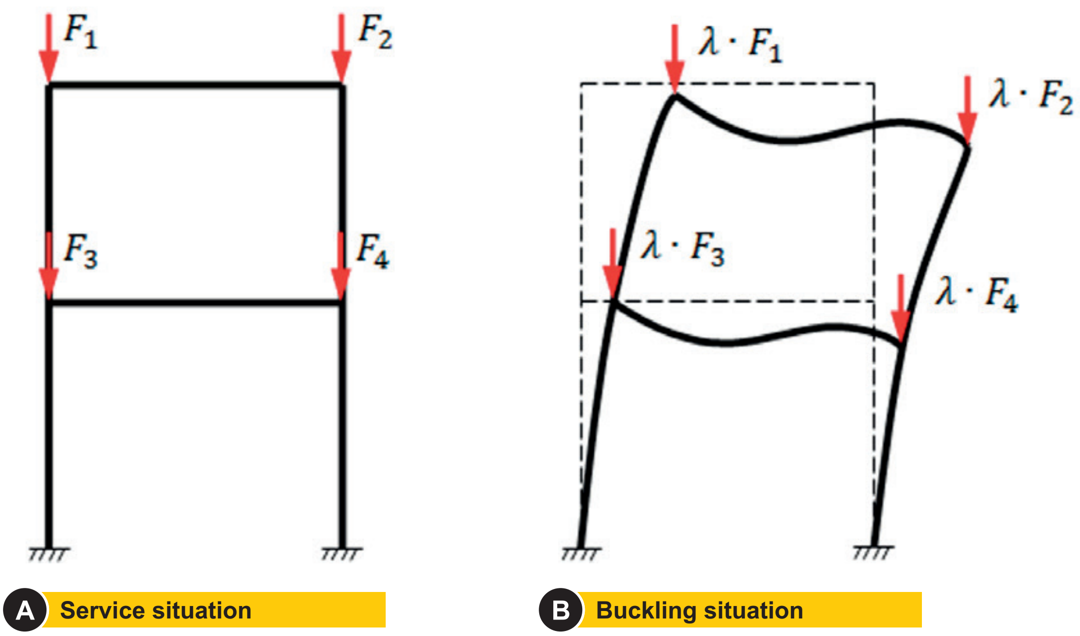
The *critical global buckling load factor* ( $\lambda$ ) of a building is also a parameter that indicates the degree of stability of the structure and it is defined as the ratio of the critical global buckling load ( $F_{cr}$ ) to the applied vertical load ( $F$ ):

$$\lambda = \frac{F_{cr}}{F} \quad (6)$$

According to Oliveira [11], $\lambda$ must multiply the vertical loads at their respective application points, resulting in the critical global load of the structure. This concept is better understood by observing Figure 1, where $\lambda$ is represented in a simple plane frame example. The sum of the applied load multiplied by $\lambda$ is the critical global buckling load of the structure.
Its value is determined through the resolution of an eigenvalues and eingenvectors problem, in which the first corresponds to the load factors and the later represents the multiple buckling modes. The equation that defines this type of problem is the following:

$$\{[K_e] - \lambda \cdot [K_g]\} \cdot \{d\} = 0 \quad (7)$$

Where, $[K_e]$ is the elastic stiffness matrix, $[K_g]$ is the geometric stiffness matrix and $\{d\}$ is the displacements vector. The eigenvalues are the values of $\lambda$ for which the vector $\{d\}$ is a nontrivial solution. The eigenvectors $\{d\}$ are the critical modes respective to each eigenvalue.
Burgos [12] explains that for the calculation of the critical global buckling load factor it is admitted the hypothesis that there will be no significant change in the distribution of forces when the




*From:* OLIVEIRA [11], adapted by the author.

**Figure 1**
Definition of the critical global load factors

vertical loads are multiplied by $\lambda$. In addition, this analysis does not include the second order effects, since it is admitted that the displacements vary linearly with the loads' increase.

The same author remarks that in practical situations it is important to know the first two critical loads in order to verify a possible interaction or proximity between the buckling modes. And draws attention for the fact that $\lambda$ must be used only as a reference parameter, since there are cases where the structure may suffer collapse due to a load considerably lower than the estimated.

MacGregor and Hage (apud Fonte [6]) denominate *instability index* ($Q$) the ratio between the total vertical load applied to the critical global buckling load. Therefore, this parameter is the inverse of the critical load factor, as described in equation (8):

$$\mathrm{Q} = \frac{F}{F_{cr}} = \frac{1}{\lambda} \tag{8}$$

The authors also suggest an amplification factor which is similar
The authors also suggest an amplification factor which is similar
to the $\gamma_z$ coefficient, which aims to evaluate the magnitude of the second order effects as a function of the instability index of the structure. This amplification factor is calculated as follows:

$$f_a(\lambda) = \frac{1}{1-Q} \tag{9}$$

In terms of the critical global buckling load, the equation (9) is rewritten as:

$$f_a(\lambda) = \frac{\lambda}{\lambda - 1} \tag{10}$$

Based on comparisons and statistical studies, these authors conclud-
Based on comparisons and statistical studies, these authors conclud-
ed that a first order analysis is sufficient for structures where the Q is equal or inferior than 0,0475, which corresponds to $\lambda$ superior than 21 and $f_a$ ($\lambda$) inferior than 1,05. When Q is bigger than 0,2, that is, $\lambda < 5$ and $f_a$ ($\lambda$) > 1,25, the collapse risk increases rapidly, thus it is not recommended that this limit is exceeded.

Comparing these limits with what is prescribed by the NBR 6118 [1] and with the values commonly adopted by structural engineers in Brazil, one gets:

$f_a(\lambda) \leq 1{,}1$ → Fixed nodes structures (the first order analysis is sufficient);

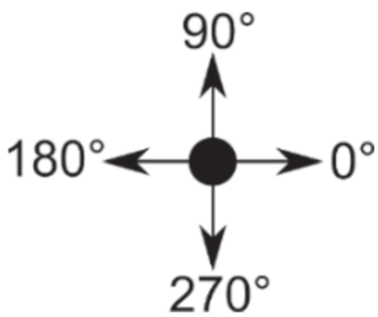



**Figure 2**
Wind directions incidence

$1{,}1 < f_a(\lambda) \leq 1{,}3$ → Free nodes structures (second order analysis is required);

$f_a(\lambda) > 1{,}3$ → Collapse probability increases.

In terms of critical global load factor:

$\lambda \geq 11$ → Fixed nodes structures (the first order analysis is sufficient);

$4{,}33 \leq \lambda < 11$ → Free nodes structures (second order analysis is required);

$\lambda < 4{,}33$ → Collapse probability increases.

It stands out that the limit of 1,25 for the amplification factor, as indicated by McGregor and Hage (apud Fonte [6]) to avoid loss of stability, was extended to 1,40 in the ACI 318 [5], for which the ë corresponds to 3,50.

The NBR 6118/1980 [13] used to fix a inferior limit for the critical load. These standards admitted that the safety of the structure was guaranteed when the buckling load was less than three times the characteristic load. Therefore, the structure should be considered unstable when $\lambda$ is lower than 3, what corresponds to an amplification factor $f_a(\lambda)$ equals 1,50. A limit for the second order effects magnitude related to global instability is not given by the Brazilian Standards.

## 5. Methodology

This paper aims to presents a comparative study between the $\gamma_z$ coefficient and the critical global buckling load in order to formulate an equation that properly relates these parameters. For this purpose, multiple idealized structures based on the same plan are analyzed, having its column section and number of stores varying, resulting in spatial frames with varying global stiffness.

Aiming to verify the applicability of the $\gamma_z$ coefficient in the calculation of the global critical load, three real reinforced concrete buildings, built or under construction in the city of Belem/Pará, were analyzed. These projects were provided by the A. C. Athayde Neto Projetos Estruturais company. Some of the criteria adopted in this paper may differ from the original design. Therefore, the results obtained can't be, in any way, be compared to the original ones.

For this numerical modeling and analysis, the SAP2000® system version 16.0.0 was used. This commercial software for structural analysis has vastly application on the market and was chosen because it performs the stability analysis in an automatic fashion, determining the global critical loads and the instability modes of the structure.

The analysis of the structures, constituted by columns, beams and slabs, were made through the Finite Elements Method (FEM). The columns and beams were represented by frame elements with the addition of rigid beam-column connection, as specified by the NBR 6118 [1], and the slabs wee modelled by plate elements, considering the rigid diaphragm in the distribution of lateral forces.

The physical non-linearity was considered in an approximate manner through the reduction of the stiffness of the structural members, as recommended by NBR 6118 [1] for the global analysis of frame structures with a minimum of 4 stores, as indicated in equations (2), (3) e (5).

The wind loads were calculated following NBR 6123 [14] were applied on the models as horizontal point loads concentrated on each pavement according to the directions shown in Figure 2.

The $\gamma_z$ coefficient was calculated using the horizontal displacements obtained from the linear analysis. To determine it, it was

**Table 1**
Load combination

| | |
|---|---|
| **Comb. 1** | 1,4 · dead load + live load |
| **Comb. 2** | 1,4 · dead load + 1,4 · (live load. + 0,6 · wind 90°) |
| **Comb. 3** | 1,4 · dead load+ 1,4 · (live load + 0,6 · wind 270°) |
| **Comb. 4** | 1,4 · dead load + 1,4 · (live load + 0,6 · wind 0°) |
| **Comb. 5** | 1,4 · dead load + 1,4 · (live load + 0,6 · wind 180°) |
| **Comb. 6** | 1,4 · dead load + 1,4 · (wind 90° + 0,5 · live load |
| **Comb. 7** | 1,4 · dead load + 1,4 · (wind 270° + 0,5 · live load) |
| **Comb. 8** | 1,4 · dead load + 1,4 · (wind 0° + 0,5 · live load) |
| **Comb. 9** | 1,4 · dead load + 1,4 · (wind 180° + 0,5 · live load) |

considered the safety formulation recommended by the NBR 6118 [1], by which the calculated second order effects are amplified by $\gamma_f / \gamma_{f3}$ and later by $\gamma_{f3}$, where $\gamma_f$ and $\gamma_{f3}$ equal 1,40 and 1,10, respectively. The ultimate normal combinations adopted for its calculation are shown in Table 1.
In obtaining the critical load factor and the instability modes only the service values of the vertical loads were considered. The physical nonlinearity in this case was also considered in an approximate manner.

## 6. Idealized structures

### 6.1 Structures' description

The Typical Floor (or just Typical) drawn is used as an outline for all the modelled idealized structures is presented in Figure 3. It

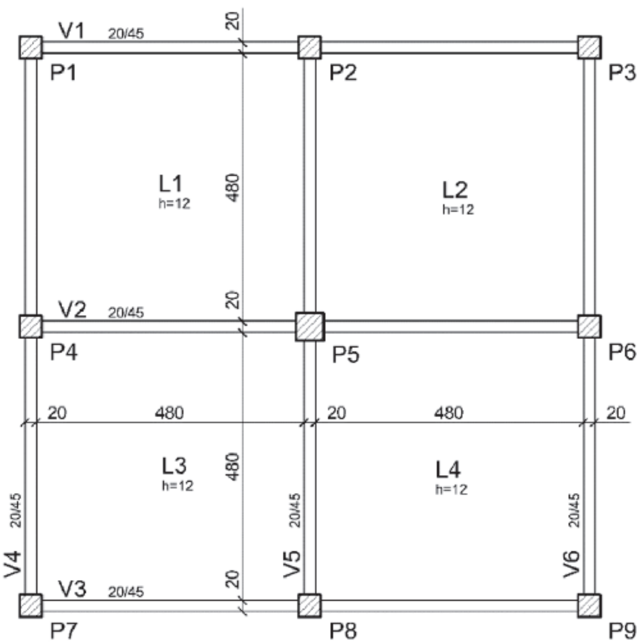



**Figure 3**
Outline of the typical floor of the idealized buildings

is composed by 6 beams of rectangular cross-section of 20cm x 45cm and 4 solid 12 cm thick slabs. The sections columns were not intentionally indicated because they vary in the several cases studied
5 groups of structures were created and named A, B, C, D and E, where each group is composed by models of equal number of stores. The quantity of Typical floors varies linearly from group to group: group A has 5 Typical; group B has 10 Typical and this way successively until group E, constituted by 25 Typical floors. Each of these groups has 5 subgroups, numbered 1 to 5, that differ in the cross-section of the columns. This was made to generate a variety of $\gamma_z$ values within a group.
Therefore, it were analyzed 25 different structures, all having the same blueprint as outline. It is important to warn that in this procedure there was no concern if the design of the columns regarding their ultimate limit states or minimum dimensions recommended by the Brazilian standards. The only aim in their design was to obtain a variety of $\gamma_z$ and $\lambda$ parameters.
In addition, it was admitted that distance between each floor and the depth of the foundations don't vary and are equal to 3,00 m and 1,50 m, respectively. In Table 2 are presented the main dimensions of these models.

### 6.2 Material properties

For all the structures, the compressive strength of the concrete ($f_{ck}$) was 30 MPa, resulting in an initial elasticity modulus of 31 GPa, according to the NBR 6118 [1] and admitting granite as coarse aggregate.

### 6.3 Applied loads

The applied vertical loads are summarized in Table 3. For the definitions of these loads, the buildings were considered to have residential purpose and that all the beams support brick walls. Regarding the horizontal loads, it was admitted that the wind loads are the only acting. The criteria adopted are shown in Table 4.

### 6.4 Structural analysis using SAP2000®

The idealized models were analyzed using SAP2000®, from

**Table 2**
Characteristics of the idealized structures

| Group | Number of floors | Total height (m) | Case | Pillar dimensions (cm) | |
|---|---|---|---|---|---|
| | | | | P1 a P4, P6 a P9 | P5 |
| A | 5 | 13,5 | 1 | 22 / 22 | 25 / 25 |
| | | | 2 | 16 / 16 | 19 / 19 |
| | | | 3 | 14 / 14 | 19 / 19 |
| | | | 4 | 13 / 13 | 18 / 18 |
| | | | 5 | 12 / 12 | 17 / 17 |
| B | 10 | 28,5 | 1 | 32 / 32 | 45 / 45 |
| | | | 2 | 20 / 20 | 28 / 28 |
| | | | 3 | 17 / 17 | 26 / 26 |
| | | | 4 | 16 / 16 | 24 / 24 |
| | | | 5 | 15 / 15 | 21 / 21 |
| C | 15 | 43,5 | 1 | 50 / 50 | 55 / 55 |
| | | | 2 | 25 / 25 | 27 / 27 |
| | | | 3 | 20 / 20 | 31 / 31 |
| | | | 4 | 20 / 20 | 21 / 21 |
| | | | 5 | 18 / 18 | 20 /20 |
| D | 20 | 58,5 | 1 | 73 / 73 | 80 / 80 |
| | | | 2 | 31 / 31 | 40 / 40 |
| | | | 3 | 25 / 25 | 33 / 33 |
| | | | 4 | 22 / 22 | 30 / 30 |
| | | | 5 | 19 / 19 | 27 / 27 |
| E | 25 | 73,5 | 1 | 95 / 95 | 100 / 100 |
| | | | 2 | 38 / 38 | 45 / 45 |
| | | | 3 | 30 / 30 | 30 / 30 |
| | | | 4 | 25 / 25 | 36 / 36 |
| | | | 5 | 21 / 21 | 30 / 30 |

**Table 3**
Vertical loads applied on the idealized structures

| Dead loads | | Live load |
|---|---|---|
| Brick walls | Slab cover | Overload |
| 5,40 kN/m | 1,00 kN/m² | 1,50 kN/m² |

**Table 4**
Criteria for the definition of the wind loads

| Criteria | | Value |
|---|---|---|
| Basic Wind velocity ($V_0$) | | 30 m/s |
| Topography factor ($S_1$) | | 1,00 |
| Roughness category ($S_2$) | | IV |
| Statistic factor ($S_3$) | | 1,00 |
| Building classes | Structure A | A |
| | Structure B | B |
| | Structure C | B |
| | Structure D | C |
| | Structure E | C |

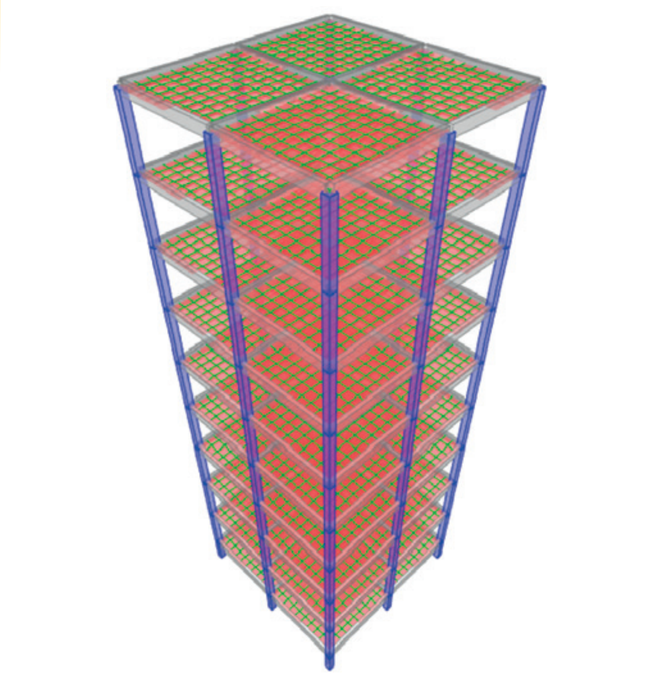


**Figure 4**
Spatial perspective of one of the idealized buildings in SAP2000®

which it was obtained the data needed for the calculation of the $\gamma_z$ coefficient and the results regarding the elastic instability analysis (i.e. instability modes and critical global factor), which are necessary for the later calculation of the amplification coefficient $f_a(\lambda)$.
Figure 4 shows one the models analyzed through SAP2000®. All the models studied which were symmetric in direction X and Y, the results are equal for both directions. Therefore, it was calculated a single pair of values of $\gamma_z$ and $\lambda$ for these structures, which are valid for both principal directions.

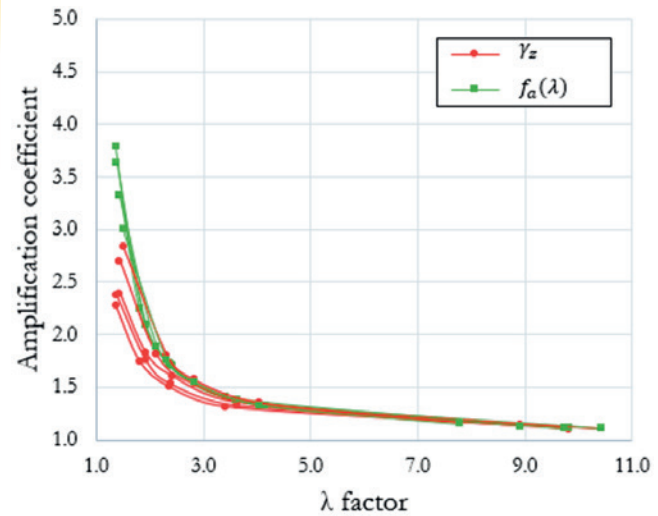



**Figure 5**
Relation of $\gamma_z$ and $f_a$ (λ) coefficients and λ factor for the idealized structures

### 6.5 Results and discussion

The calculated values for the $\gamma_z$ coefficient, critical global load factor $\lambda$ and amplification coefficient $f_a(\lambda)$ are presented in Table 5. In these analysis, there was no concern with ultimate or serviceability limit states, for the objective was only to establish a relation between the global stability parameters calculated, without considering the design of the structural members.
For purposes of comparison, the calculated values are show graphically in Figure 5, in which the horizontal axis corresponds to the $\lambda$ factor and the vertical axis is the amplification factor of the first order forces.
It is observed that, until a value close to 1,50, there is good agreement between $\gamma_z$ and $f_a(\lambda)$, which permits to establish the following approximated relation:

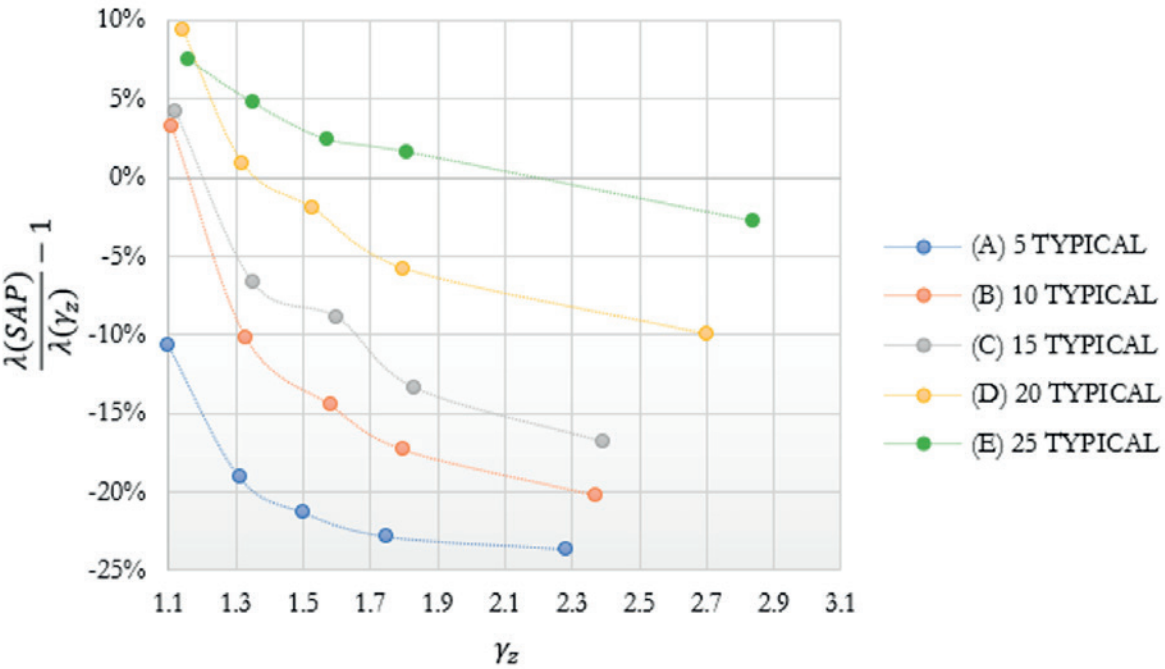



**Figure 6**
Percentage variation of λ obtained using SAP2000 e through $\gamma_z$

**Table 5**
Calculated values of the coefficients $\gamma_z$, $\lambda$ and $f_a$ ($\lambda$) for the idealized structures

| Structure | Case | $\gamma_z$ | $\lambda$ ($\gamma_z$) | $\lambda$ (SAP) | $f_a$ ($\lambda$) | Coarse |
|---|---|---|---|---|---|---|
| A (5 Typical) | 1 | 1,10 | 11,00 | 9,82 | 1,11 | Translational |
| | 2 | 1,31 | 4,23 | 3,42 | 1,41 | |
| | 3 | 1,50 | 3,00 | 2,36 | 1,74 | |
| | 4 | 1,75 | 2,33 | 1,80 | 2,25 | |
| | 5 | 2,28 | 1,78 | 1,36 | 3,78 | |
| B (10 Typical) | 1 | 1,11 | 10,09 | 10,42 | 1,11 | Translational |
| | 2 | 1,33 | 4,03 | 3,62 | 1,38 | |
| | 3 | 1,58 | 2,72 | 2,33 | 1,75 | |
| | 4 | 1,80 | 2,25 | 1,86 | 2,16 | |
| | 5 | 2,37 | 1,73 | 1,38 | 3,63 | |
| C (15 Typical) | 1 | 1,12 | 9,33 | 9,73 | 1,11 | Translational |
| | 2 | 1,35 | 3,86 | 3,60 | 1,38 | |
| | 3 | 1,60 | 2,67 | 2,43 | 1,70 | |
| | 4 | 1,83 | 2,20 | 1,91 | 2,10 | |
| | 5 | 2,39 | 1,72 | 1,43 | 3,33 | |
| D (20 Typical) | 1 | 1,14 | 8,14 | 8,91 | 1,13 | Translational |
| | 2 | 1,32 | 4,13 | 4,16 | 1,32 | |
| | 3 | 1,53 | 2,89 | 2,83 | 1,55 | |
| | 4 | 1,80 | 2,25 | 2,12 | 1,89 | |
| | 5 | 2,70 | 1,59 | 1,43 | 3,33 | |
| E (25 Typical)v | 1 | 1,16 | 7,25 | 7,79 | 1,15 | Translational |
| | 2 | 1,35 | 3,86 | 4,04 | 1,33 | |
| | 3 | 1,57 | 2,75 | 2,82 | 1,55 | |
| | 4 | 1,81 | 2,25 | 2,27 | 1,79 | |
| | 5 | 2,84 | 1,54 | 1,50 | 3,00 | |

$$\gamma_z \cong f_a(\lambda) \tag{11}$$

Using (10) and (11) it is possible to relate the $\gamma_z$ coefficient and the critical global load $\lambda$ as follows:

$$\gamma_z = \frac{\lambda}{\lambda - 1} \tag{12}$$

Hence, in order to find $\lambda$ as a function of $\gamma_z$ it is sufficient to isolate it in equation (12):

$$\lambda = \frac{\gamma_z}{\gamma_z - 1} \tag{13}$$

This relation is valid for $\gamma_z$ within 1,00 and 1,50. The inferior limit is due the impossibility of the division. The superior limit was fixed based on the great discrepancy observed in higher values, as one can see in Figure 6. With exception of the five-story buildings, the error found for $\lambda$ when calculated as a function of $\gamma_z$ are lower than 15% for the cases against safety and 10% for the in favor of security. It is concluded that under a limit of 1,50, the approximated equation provides satisfactory results. It is important to notice that even for $\gamma_z$ higher than 2,00, the error found was inferior than 25%.

The established relation holds only for the cases where the critical instability modes coincide with the principal directions for which $\gamma_z$ is calculated, that is, when the instability mode is translational in the X and Y directions.
Having equations (12) and (13), it is now possible to determine the recommended limits of the main normative codes in function of the parameters $\gamma_z$ and $\lambda$.
The criterion for ignoring the second order global effects varies in the different standards. In Eurocode 2 [4] and NBR 6118 [1] consider that when the second order effects are equal or lower than 10% the first order ones ($\gamma_z = 1{,}10$), corresponding to $\lambda = 11$, they can be neglected. However, ACI 318 [5], where the verification is performed floor by floor, the limit is taken as 5% and $\lambda$ = 20. With regard to the global stability verification, NBR 6118/1980 [13] suggested a safety buckling coefficient equals 3 ($\lambda = 3{,}00$), which, using equation (13), corresponds to $\gamma_z = 1{,}50$. Considering the ACI 318 [5], that recommends a superior limit of 1,40 for the ratio between the final global effects and the first order ones ($\gamma_z = 1{,}40$), the value of $\lambda$ equals 3,50.
For the $\gamma_z$ limited to 1,30 by the actual NBR 6118 [1] when used for the simplified calculation of the global second order forces, the relative $\lambda$ equals 4,33. All these limits are presented in Table 6.

**Table 6**
Limits for $\gamma_z$ and $\lambda$ calculated using different codes

| Code | Second order effects may be neglected | | Second order effects may be approximately calculated | | Global instability verification | |
|---|---|---|---|---|---|---|
| | $\gamma_z$ | $\lambda$ | $\gamma_z$ | $\lambda$ | $\gamma_z$ | $\lambda$ |
| NBR 6118/2014 [1] | 1,10 | 11,0 | 1,30 | 4,33 | - | - |
| EUROCODE 2 [4] | 1,10 | 11,0 | - | - | - | - |
| ACI 318 [5] | 1,05 | 20,0 | - | - | 1,40 | 3,50 |
| NBR 6118/1980 [13] | - | - | - | - | 1,50 | 3,00 |

## 7. Real buildings

### 7.1 Building 1

This residential building (Figure 7) is composed by 12 floors and it is 32,75 m high. The floors are: ground floor, 8 typical, water box and its cover. The height of the floors are presented in Table 7. The concrete's compressive strength ( $f_{ck}$ ) adopted is 25 MPa and its tangent elastic modulus is 28 GPa. The floors measure 15,93 x 47,58 m. The structure is constituted by solid slabs, pillars and beams of rectangular cross-section (Figure 8).

### 7.2 Building 2

This residential building (Figure 9) is 110,38 m high. The heights of the floors are presented in Table 8. The concrete's compressive strength ( $f_{ck}$ ) adopted is 30 MPa and its tangent elastic modulus is 31 GPa. The structure is constituted by solid slabs, pillars and beams of rectangular cross-section (Figure 10). In addition, a L-shaped column of great stiffness is placed around the elevator shaft.

### 7.3 Building 3

This commercial building (Figure 9) is 108,2 m high. The heights

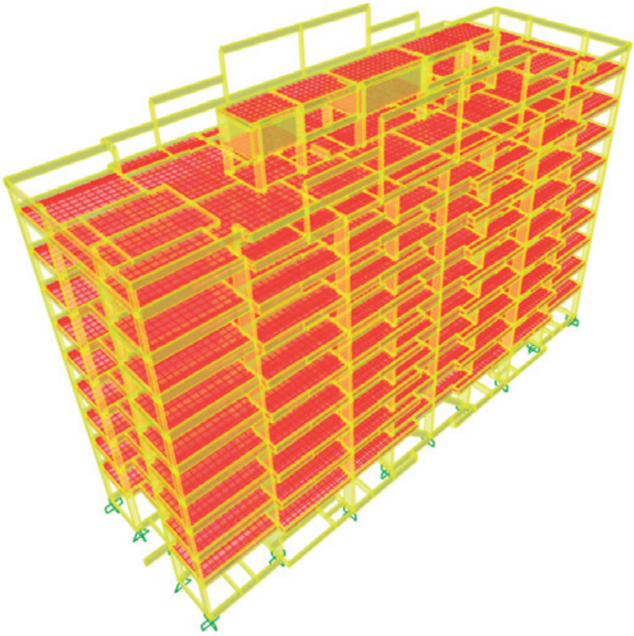


**Figure 7**
Building 1

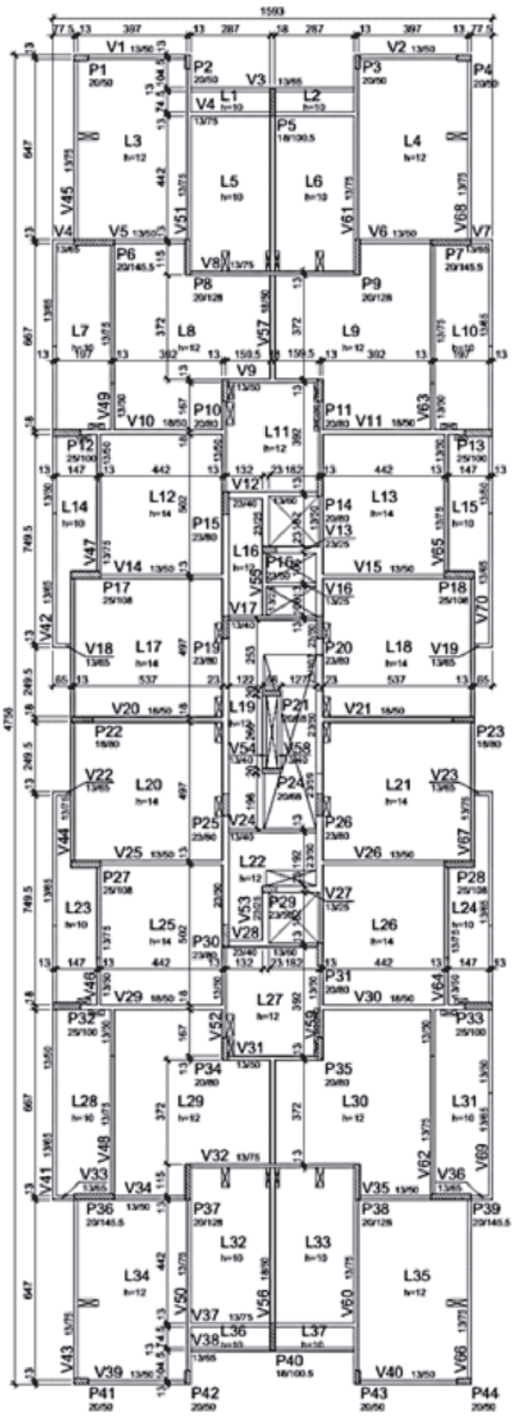


**Figure 8**
Outline of the typical floor of Building 1

**Table 7**
Floor heights for Building 1

| Floor | Floor heights (m) | Level (m) |
|---|---|---|
| Water reservoir cover | 2,15 | 32,75 |
| Water reservoir | 2,40 | 30,60 |
| Barrilete | 3,00 | 28,20 |
| 2° to 8° typical (7x) | 3,00 | 25,20 |
| First typical | 4,20 | 4,20 |
| Ground floor | 0,00 | 0,00 |

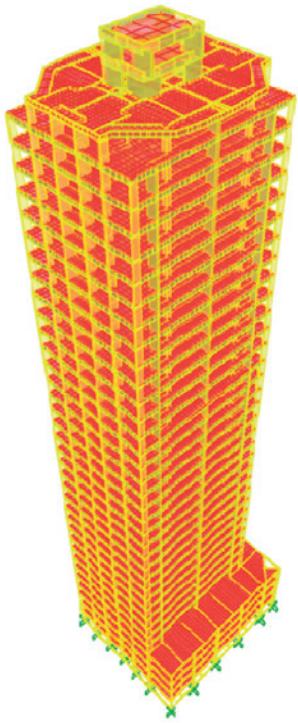


**Figure 9**
Building 2

**Table 8**
Floor heights for Building 2

| Floor | Floor heights (m) | Level (m) |
|---|---|---|
| Water reservoir cover | 2,50 | 104,26 |
| Water reservoir | 2,50 | 101,76 |
| Ceiling | 2,90 | 99,26 |
| Roof | 2,90 | 96,36 |
| 2° to 30° typical (29x) | 2,90 | 93,46 |
| 1° Typical | 3,24 | 9,36 |
| Mezzanine | 3,06 | 6,12 |
| Pilotis | 3,06 | 3,06 |
| Ground floor | 3,06 | 0,00 |
| Basement 1 | 3,06 | -3,06 |
| Basement 2 | 0,00 | -6,12 |

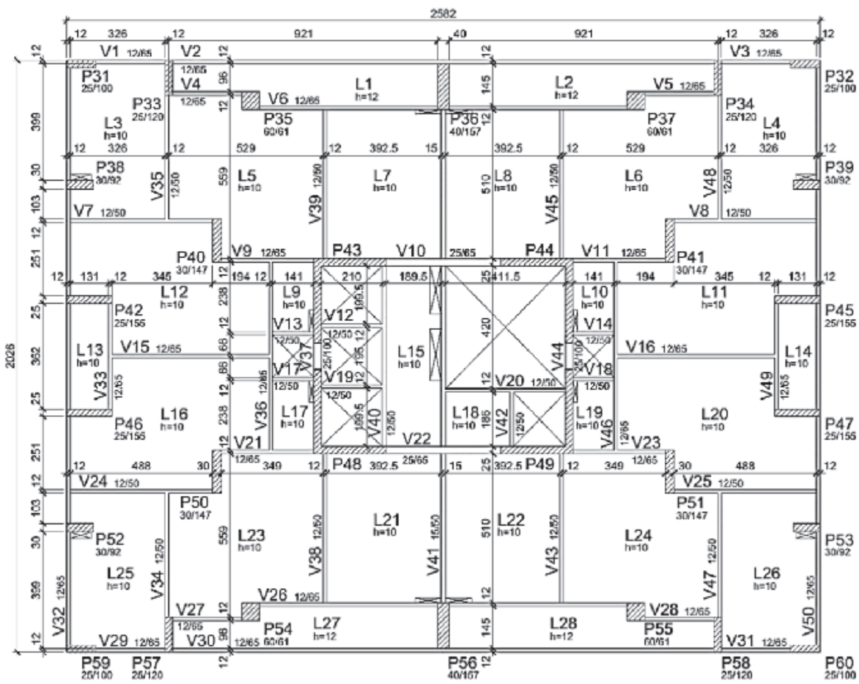



**Figure 10**
Outline of the typical floor of Building 2

**Table 9**
Floor heights for Building 3

| Floor | Floor heights (m) | Level (m) |
|---|---|---|
| Water box cover | 2,42 | 100,42 |
| Barrel | 1,65 | 98,00 |
| Penthouse 2 | 1,35 | 96,35 |
| Penthouse 1 | 3,00 | 95,00 |
| 9° to 28° typical (20x) | 3,00 | 92,00 |
| 8° typical | 5,70 | 32,00 |
| 7° typical | 4,20 | 26,30 |
| 2° to 6° typical (5x) | 3,00 | 22,10 |
| 1° typical | 3,10 | 7,10 |
| Mezzanine | 4,00 | 4,00 |
| Ground floor | 2,60 | 0,00 |
| 1° to 2° garage (2x) | 2,60 | -5,20 |
| Garage 3 | 0,00 | -7,80 |

of the floors are presented in Table 9. The concrete's compressive strength ( $f_{ck}$ ) adopted is 40 MPa and its tangent elastic modulus is 35 GPa. The structure is constituted by solid slabs, pillars and beams of rectangular and L-shaped cross-sections (Figure 12). Some of the beams were prestressed, but these effects weren't considered in the present analysis.

### 7.4 Applied loads

The vertical applied loads were the same used during the buildings' design process, which are based on the recommendations of the NBR 6120 [15]. The dead load of the concrete members was calculated using a specific weight of 25 kN/m³. For the walls, it was considered a load of 1,80 kN/m². The floor covering weight was 1,30 kN/m². Beyond the dead load, a live load of 1,50 kN/m² for the residential buildings and 2,00 kN/m² for the commercial buildings were considered. The remaining load followed NBR 6120 [15] recommendations.
The horizontal loads were considered due only the wind action and

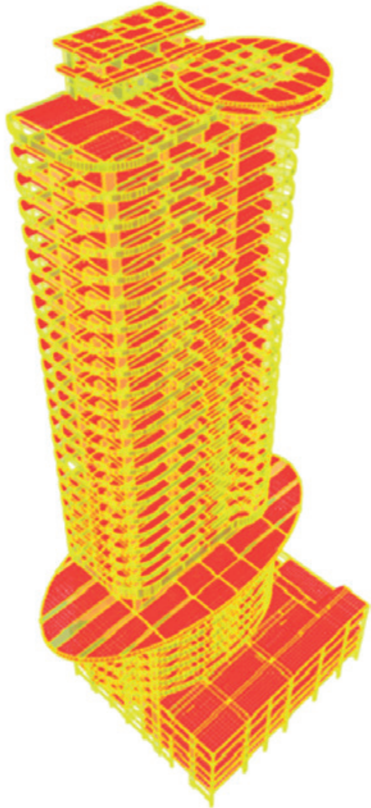


**Figure 11**
Building 3

were applied in the 4 directions indicated in Figure 2. The parameters for the determination of these loads were defined accordingly to the suggestions of NBR 6123 [14] and are exposed in Table 10. The Drag coefficients calculated for each building are presented in Table 11.

### 7.5 Results and discussion

Table 12 shows the values obtained for the $\gamma_z$ coefficient considering different wind directions. Table 13 presents the description of the first three buckling modes with their respective critical load

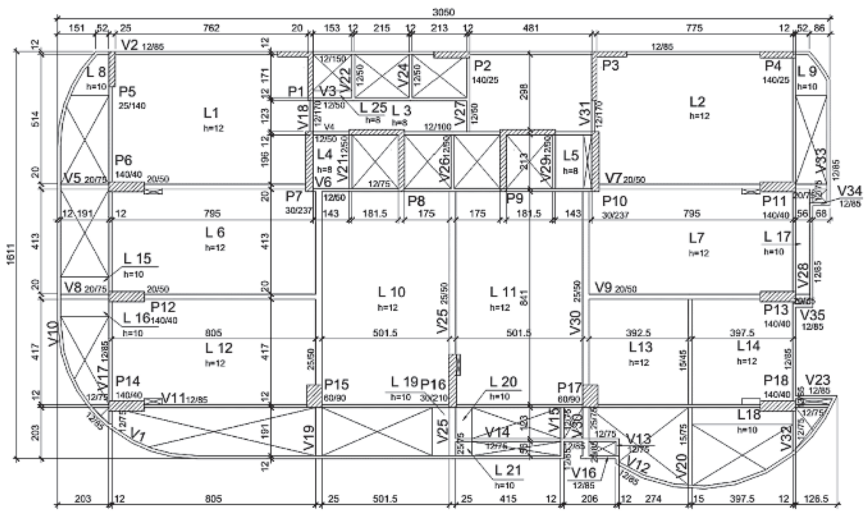


**Figure 12**
Outline of the typical floor of Building 3

factors $\lambda$, obtained from the elastic stability analysis performed in SAP2000®, and the amplification factors $f_a(\lambda)$ for the cases in which the buckling occurs due translation.

The results displayed in Table 12 are graphically presented in Figure 15, as well as its relation to the $\gamma_z$ and corresponding $\lambda$ limits. The first band represents the interval for which the second order global effects can be neglected. The second band is the range in which the second order nonlinear analysis can be performed approximately using $\gamma_z$. The next band contains the cases with high second order effect, but, admitting a limit of $\lambda(\gamma_z)$ equals 3,00, there is still reasonable safety regarding the global stability for X and Y directions. Lastly, there are the cases that present a high risk of collapse due loss of stability.

The graph shows that the buildings are very stiff, having low $\gamma_z$. However, for the where its value is higher than 1,10, a second order global effects analysis is obligatory.

A comparison between $\gamma_z$ and $f_a(\lambda)$ is shown in Figure 13. Due the stiffness of the buildings studied, the difference of these parameters is negligible.

In Table 14, the values of $\lambda$ calculated by the simplified method equation (13) and by SAP2000® are presented. Its percentage variation is graphically displayed in Figure 14. All the variations are positive, indicating that the critical global factor $\lambda$ obtained by equation (13) are inferior than the calculated by SAP2000®. Therefore, although in some cases the differences are bigger, the approximated equation provided reasonable and higher values

Nevertheless, it is important to remember that this estimative of $\lambda$ (using the $\gamma_z$ coefficient) is not a sufficient condition in the assessment of the global stability of the structure, since the calculation of $\gamma_z$ presupposes the most critical instability would occur in the X and Y directions, and not always the translational modes are the most critical, existing cases where the most critical mode

**Table 10**
Adopted parameters for the calculation of wind loads

| Criteria | Values |
|---|---|
| Basic wind load ($V_0$) | 30 m/s |
| Topographic terrain factor ($S_1$) | 1,00 |
| Roughness category ($S_2$) | IV |
| Statistic factor ($S_3$) | 1,00 |
| Building class | C |

**Table 11**
Calculated drag coefficient for each building

| Direction | Drag coefficient | | |
|---|---|---|---|
| | Building 1 | Building 2 | Building 3 |
| 90° | 0,78 | 1,41 | 1,08 |
| 270° | 0,78 | 1,41 | 1,08 |
| 0° | 1,24 | 1,32 | 1,15 |
| 180° | 1,24 | 1,32 | 1,15 |

**Table 12**
Results for the $\gamma_z$ coefficient

| Buildings | Wind direction | |
|---|---|---|
| | 0° and 180° | 90° and 270° |
| Building 1 | 1,13 | 1,08 |
| Building 2 | 1,12 | 1,12 |
| Building 3 | 1,08 | 1,17 |

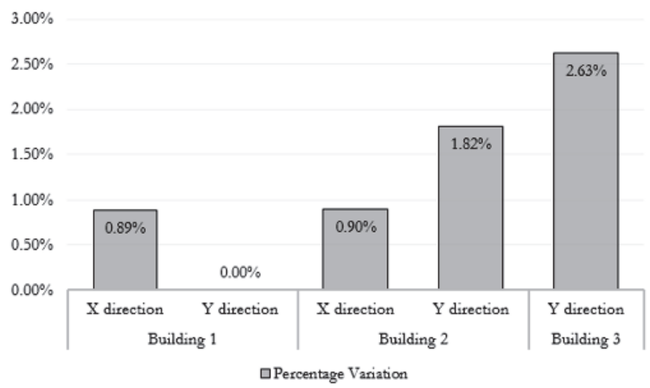



**Figure 13**
Percentage variation among the $\gamma_z$ and $f_a(\lambda)$ coefficients of the real buildings

**Table 13**
Results of the elastic stability analysis

| Buildings | Buckling modes | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1° | $\lambda_1$ | $f_a(\lambda_1)$ | 2° | $\lambda_2$ | $f_a(\lambda_2)$ | 3° | $\lambda_3$ | $f_a(\lambda_3)$ |
| Building 1 | Translation X | 9,49 | 1,12 | Torsion | 10,73 | - | Translation Y | 14,03 | 1,08 |
| Building 2 | Torsion | 4,98 | - | Translation X | 10,05 | 1,11 | Translation Y | 10,75 | 1,10 |
| Building 3 | Translation Y | 8,33 | 1,14 | Torsion | 11,21 | - | Bending Y | - | 964.59 |

**Table 14**
λ factor calculated as a function of $\gamma_z$ and by SAP2000®

| Buildings | Wind direction | $\gamma_z$ | λ ($\gamma_z$) | λ (SAP) |
|---|---|---|---|---|
| Building 1 | X | 1,13 | 8,69 | 9,49 |
| | Y | 1,08 | 13,50 | 14,03 |
| Building 2 | X | 1,12 | 9,33 | 10,05 |
| | Y | 1,12 | 9,33 | 10,75 |
| Building 3 | Y | 1,17 | 6,88 | 8,33 |

is torsional. This situation is evident in Building 2, having first a torsional instability mode and other two translational modes in the directions X and Y.

To clarify this situation, an extra analysis of Building 2 was performed in SAP2000®, imposing a $\gamma_z$ coefficient equals 1,30. This state was reached by reducing the elasticity modulus of the concrete to a value close to 13 GPa. For this situation, the calculated critical load factor $\lambda$ was 2,47 and the most critical buckling load remained torsional. The structure would not attend NBR 6118/1980 [13] $(\lambda = 3,00)$, although the corresponding $\gamma_z$ is equal 1,30, value generally accepted.

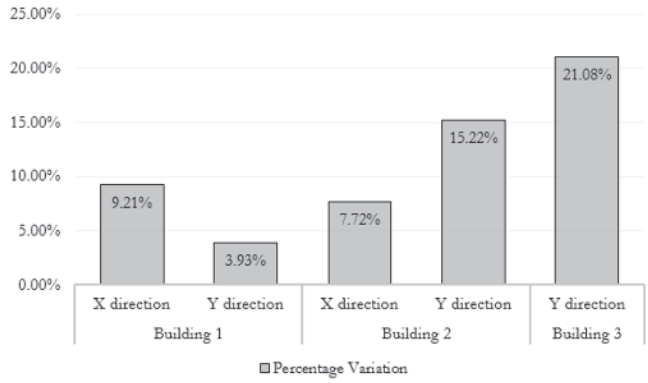



**Figure 14**
Percentage variance of the calculated λ as a function of $\gamma_z$ and by SAP2000®


**Figure 15**
State of the real buildings in relation to $\gamma_z$ and λ coefficients

## 8. Conclusions

A comparison between the $\gamma_z$ coefficient and $f_a(\lambda)$ was performed using idealized structures and it was proved the results variation is negligible under 1,50.

Assuming the equality of these parameters up until a specific limit, an approximate relation of them was established equation (13). This equation, therefore, permits to estimate the state of the structure with respect to its critical instability point in function only of the $\gamma_z$ coefficient. In the idealized structures, the calculated errors in favor of and against safety were lower than 10% and 15%, respectively.

The results for the real buildings provided by the proposed equation were satisfactory, providing values of $\lambda$ lower than the obtained from SAP2000®.

However, the global stability analysis by means of the $\gamma_z$ coefficient was proved not to be sufficient for the cases where the most critical instability mode corresponds to a torsional configuration of the structure.

In a further analysis of Building 2 with $\gamma_z$ imposed and equals 1,30, the global stability criterion suggested by NBR 6118/1980 [13] was not satisfied. This case shows a clear inefficiency of $\gamma_z$ when the most critical mode is torsional.

## 9. Acknowledgment

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